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Simulation as a Predictor in Probability

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Abstract
In this study, we simulate bivariate normal data. We gain intuition about the bivariate normal distribution by comparing the generated data to the associated bivariate normal density surface. We also get results about covariance and correlation. We will use tools from linear algebra to discuss transformations of random normal vectors, and the use of contours.

Introduction
This project is done using the MATLAB software. In this project we simulate bivariate normal data and gain intuition about the bivariate normal distribution by comparing those data to associated bivariate normal density surface. Our work also illustrate results about covariance, correlation and allow conjectures about transformation of normal random vectors. We draw conclusions about the relationship between the density and the simulated bivariate data, make use of contour.

Simulate bivariate normal data
To begin with, we simulated 10000 pairs of values $w_1$ and $w_2$ from independent, identically distributed normal values, where the mean is 0 and the variance is $\frac{1}{100}$. We can symbolically represent it as

$$\begin{bmatrix} w_1 \\
W_2 \end{bmatrix} \sim N \left( \begin{bmatrix} \text{mean of } w_1 \\
\text{mean of } w_2 \end{bmatrix}, \begin{bmatrix} \text{covariance of } w_1 & \text{covariance of } w_2 \\
\text{covariance of } w_2 & \text{covariance of } w_2 \end{bmatrix} \right)$$

where the mean is $\mu$, the covariance matrix $\Sigma$, and the graph of the points form ellipses, which indicates a normal distribution. We verified this conjecture later in the project.

The properties of $X$
As stated before, we know that $\frac{X}{\lambda} = A \cdot \frac{W}{\lambda}$. The linear combination $X = A \cdot W$ have the following properties:

$$\mu_x = A \mu_w = A \cdot \begin{bmatrix} 0 \\
0 \end{bmatrix}$$

$$\sum_{X} = \text{Cov}(X) = A \sum_{W} A^T = A \cdot \begin{bmatrix} 1 & 0 \\
0 & 1 \end{bmatrix} \cdot A^T$$

Density function of the joint distribution
The probability density function for bivariate data is given below:

$$f(x_1, x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1 - \rho^2}} e^{-\frac{1}{2}(x_1^2\sigma_1^2 + x_2^2\sigma_2^2 - 2\rho x_1x_2\sigma_1\sigma_2)}$$

where $\rho$ is the correlation, $\sigma_1$ and $\sigma_2$ are the standard deviations of $w_1$ and $w_2$, respectively.

Contour plot and 3-D plot
As you can see on the contour plot, the points do form ellipses, and the shape of the 3-D plot also suggests that $\frac{X_1}{\lambda_1}$ are normally distributed points. To confirm this prediction, we simulated another two sets of $\frac{X}{\lambda}$.

Conclusion
In this project, we applied linear transformation to bivariate normal data. The points on the scatter plot of the resulting data form ellipses, which is an indication of normal distribution. Based on the graphs, we predicted that when a linear transformation is applied to normally distributed data, the resulting data is still normally distributed. To prove this, we found the probability function of each bivariate data set and used it to graph the contour plot and 3-D plot. The contour plots and 3-D plots confirmed our prediction.

Selected preferences