2017

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Assisted Self-Persuasion:
Advertising with Consumer Adjustment to Choice

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Working Paper 14

Ph.D. Program in Economics
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New York, NY 10016
March 2017

I am grateful to Ben Ho, Fahad Khalil, Jacques Lawarrée, and seminar participants at the University of Toronto, University of Washington, Vassar College, and ISMS Marketing Science conference for helpful comments. Shay Culpepper provided excellent research assistance.

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ABSTRACT

I develop a new theory of persuasive advertising in which consumers rationally adjust to (i.e., improve their attitude toward) the products they choose and advertising facilitates adjustment. Advertising’s price effects depend on whether marginal or inframarginal consumers are most heavily targeted, consistent with the literature. But they also depend on advertising’s role as an overall adjustment intensifier, whence variation in the cost of adjustment with the strength of the consumer’s initial product preference determines the equilibrium price level. Whether too much or too little advertising is provided in equilibrium depends on the sign and size of advertising’s price effect, the relative density of marginal consumers, and the relative extent to which advertising’s adjustment cost reductions benefit marginal consumers.

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I. INTRODUCTION

Advertising is like aspirin: while it is known to work, its functional mechanism is still not entirely clear. The two leading economic theories as to why consumers respond to advertising both have significant limitations.

One theory holds that advertising is persuasive, altering consumers’ tastes for a firm’s product and spuriously creating product differentiation.\(^1\) Persuasive advertising is typically modeled as increasing demand, making demand more inelastic, or both. But such notions are at odds with the prevailing economic view that consumers’ tastes are fixed; and, more broadly speaking, the persuasion theory offers no explanation as to why advertising should elicit a response at all from a rational consumer.

The other leading theory holds that advertising is informative. According to its conception, advertising influences consumers to the extent that it provides useful information of some kind on the product - its features, price, availability, and so forth. Even advertising messages that appear non-substantive (e.g., image-oriented) convey the information that the product is advertised and must be of sufficient quality to have elicited a costly advertising expenditure (Nelson 1974). But the information theory cannot explain the efforts devoted to the crafting of message and image in ads otherwise devoid of informational content. If the purpose is just to show that money is being spent and the ad is not in some measure intended to be persuasive, why should such details matter?

This paper presents a new theory of advertising as assisting self-persuasion. The theory is based on the conception of a consumer who rationally adjusts to the choices she makes. That is, it assumes a consumer who does not simply make utility-maximizing choices

\(^1\) For a more extensive discussion of the theory of advertising as well as a literature survey, see Bagwell (2007).
but additionally, and simultaneously, invests effort to increase the utility obtained conditional on her choice. This proposition is built on extensive recent psychological research showing that consumers routinely undergo a sort of mental re-positioning relative to choices they have made, changing both their stated preferences and the physiological manifestations of their hedonic responses. Indeed, it is common sense that a person should work to get comfortable with the inevitable, and this is something that, introspection suggests, we all do to the extent we find we are able. Advertising, in this context, provides the persuasive ammunition that consumers need to get themselves “psyched up” about the product. Such a conception of advertising as a tool for the consumer is consistent with the Elaboration Likelihood Model, which considers the conditions under which consumers approach advertising with active thinking as a part of their decision-making process (Petty and Cacioppo 1986). Separately, studies of cognitive dissonance reveal that consumers actively seek persuasive advertising messages that have the potential to reduce doubts about product choices (Ehrlich et al. 1957, Mills 1965).

I posit a Hotelling model of differentiated product competition in which consumers differ as to their initial tastes for two competing products. A consumer can, at a cost, adjust to the product he intends to choose – in essence, “moving closer” to it, and thereby avoiding some of the transportation cost associated with imperfect taste matching. Advertising for a particular product is modeled generally as reducing the cost of such adjustment. By

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2 Studies offering evidence of preference change based solely on subject ratings of chosen alternatives include Lieberman et al. (2001), Kitayama et al. (2004), Sharot et al. (2010), and Wakslak (2012). Studies that additionally measured changes using functional magnetic resource imaging (fMRI) of subjects’ brains include Sharot et al. (2009), van Veen et al. (2009), Izuma et al. (2010), Jarcho et al. (2011), Qin et al. (2011), Kitayama et al. (2013), and Izuma and Adolphs (2013).

3 Advertising is also conceived as a consumer tool in the uses and gratifications literature: see O’Donohoe (1994), Ko et al. (2005), Aitken et al. (2008), and Phillips and McQuarrie (2010). Advertising in the model moreover may be viewed as a stand-in for other forms of costly marketing communication, particularly salesperson communications to customers, which have been discussed as facilitating post-purchase dissonance reduction. See Hunt (1970), Milliman and Decker (1990), and Grewal and Sharma (1991).
incorporating adjustment as a step in a model of rational choice, the theory allows the induced outcomes to be subsumed into “final” preferences such that the conventional techniques for analyzing choices (including the axioms of revealed preference) may be applied to them. This approach avoids many of the complications associated with previous efforts to model taste change.

Though adjustment implies intensification of preference, prices do not rise unambiguously with advertising. Advertising’s effects accrue to its roles as a *shaper* of relative adjustment facility levels across consumers (i.e., it has the potential to reduce the relative marginal adjustment costs experienced by consumers with stronger or weaker initial product preferences, thereby skewing price levels) and overall *intensifier* of adjustment (i.e., in reducing adjustment costs, advertising causes more adjustment by consumers across the board and so intensifies adjustment’s own effects on prices). The sign of the latter effect in turn depends on how consumers’ facility with adjustment varies based on the strength of their initial product preference.

While firms’ motivations to advertise accrue to their interest in increasing sales at the margin and increasing profit on inframarginal consumers through increased prices, consumers’ benefits from advertising accrue to the reduction in their adjustment costs. In general, these goals are not aligned, whence it is found that advertising may be over- or under-provided in equilibrium. Factors affecting the balance include advertising’s price effect, the density of consumers at the margin between the firms, and the relative extent to which adjustment cost reductions benefit marginal consumers relative to the overall mass of consumers.
The rest of this paper is structured as follows. Section II introduces the model. Section III derives the equilibrium. Section IV relates the level of advertising that results in equilibrium to the social welfare-optimizing level. Section V concludes and discusses opportunities for future research. The Appendix contains proofs and derivations of all lemmas and propositions.

II. THE MODEL

Consider two products, indexed 0 and 1, each produced by an independent firm correspondingly named. The firms are located at opposite ends of a segment of length 1 representing the product space. Following Hotelling’s model (1929), each consumer is characterized by a location \( x \in [0,1] \), identifying his relative taste for the two products. Consumers are assumed distributed on this segment according to an arbitrary distribution function \( F \) with full support and continuous density function \( f \). They buy at most one unit of a single product. I assume the baseline utility of a consumer at \( x \) buying product \( j \) to be given by

\[
U_x = V - p_j - t|x - j|
\]

where \( V \) is the common reservation price for the product, \( p_j \) is the price of product \( j \), and \( t \) parameterizes the utility loss due to \( j \)’s not being the consumer’s ideal choice – the standard “transportation” cost, linear in the consumer’s distance from \( j \).

Suppose that the consumer faces the possibility of adjusting to a product, defined as relocating on the segment to be closer to it, thereby paying less transportation cost. The process is quite naturally viewed as an incremental one, involving incremental investment of costly or aversive effort that pays off with an incremental improvement in attitude toward the
product. In this context, let us conceive of advertising for a given product as making such adjustment easier with respect to that product, that is, improving the consumer’s product-specific technology of adjustment so that the same amount of attitude improvement toward the product may be achieved at lower cost. These concepts are represented by the following adjustment marginal cost function associated with product $j$,

$$g^j(i, x, A_j) = \phi(A_j)g^{j, 1}(i, x) + \left(1 - \phi(A_j)\right)g^{j, 0}(i, x)$$

where $i$ is the distance from $x$ and closer to $j$’s position, $g^{j, k}(i, x) > 0$ for $j, k = 0, 1$, and $A_j$ represents firm $j$’s advertising expenditure. Here we let the $g^{j, k}(\cdot)$ be continuous on their support with $g^{j, k}(i, \bar{x})$ increasing and strictly convex in $i$ (i.e., $g_i^{j, k} > 0$ and $g_{ii}^{j, k} > 0$) and

$$\lim_{i \to x} g^{0, k}(i, x) = \infty \quad \text{and} \quad \lim_{i \to 1-x} g^{1, k}(i, x) = \infty.$$  

We make the following assumptions about advertising’s effects on the marginal cost of adjustment:

ASSUMPTION 1: $\phi(\cdot)$ is continuous on its support.

ASSUMPTION 2: $\phi(\cdot)$ is increasing and strictly concave.

ASSUMPTION 3: $\lim_{A_j \to 0} \phi(A_j) = 1$.

ASSUMPTION 4 (Advertising-driven dominance): $g^{j, 1}(i, x) < g^{j, 0}(i, x)$ for $j = 0, 1$.

Notice the generality of the formulation. Advertising is conceived as reducing product-specific adjustment costs asymptotically from a zero-advertising level given by $g^{j, 0}(i, x)$ toward a limit (lower bound) given by $g^{j, 1}(i, x)$. Other than Assumption 4, there is
no restriction on the shape of \( g^{j_1}(i,x) \) relative to \( g^{j_0}(i,x) \). Advertising generally does not affect all consumers equally; some may experience greater relative effects, seeing their marginal adjustment costs reduced more than others if, say, the advertising “targets” them.

One may view the function \( g^j(\cdot) \) as representing a set of adjustment curves

\[ G^j := \{ g^j(i) = g^j(i,x) : x \in [0,1] \} \]

characterized by differing values of \( x \), whereby each curve represents the cost, at each state of attitude improvement \( i \), of incremental “movement toward” \( j \) for the consumer located initially at \( x \). Let us refer to \( G^j \) as an adjustment map for product \( j \). Figure 1 illustrates an adjustment map for product 0. Figure 2 displays the adjustment map \( G^0 \) along with pre-advertising and limiting adjustment maps for product 0.

Whereas Assumption 4 sets forth the intuitive notion that advertising for a product makes consumers no worse at adjusting to it and perhaps better, the next assumption reflects the same idea with respect to relative preference for products:

ASSUMPTION 5 (Preference dominance): For all \( x \in [0,1] \), \( -\frac{\partial g^0}{\partial x} < \frac{\partial g^0}{\partial x} \) (and \( -\frac{\partial g^1}{\partial x} < \frac{\partial g^1}{\partial x} \)).

That is, the more preferred a product is initially, the lower the marginal cost of adjustment at any particular location achieved through accumulated adjustment. This “dominance” condition implies that a person who initially prefers a product more than another person finds it less costly to achieve a given attitude toward that product through adjustment than the other person. This gives rise to adjustment maps of non-crossing nested contours, similar to
well-behaved indifference maps. It follows that adjustment results in a better “net” attitude the greater the individual’s initial proximity to a product.

For consumers for whom adjusting to a given product is preferred over leaving one’s attitude unchanged, one can speak of adjustment productivity: how much attitude improvement with respect to the product the consumer will attain, given his preferences, his particular capabilities at adjusting to it, and the transportation cost (i.e., his opportunity cost of adjusting). Define the set \( X_j(t) = \{ x : g_j^0(0,x) < t \} \); since \( g_j^0(0,x) \), while continuous, is not required to be monotonic in \( x \), \( X_j(t) \) may contain (compact) gaps. One may then define the implicit function \( i_x^j(t) \) on \( X_j(t) \times \{ t > 0 \} \rightarrow \mathbb{R}^+ \) such that \( g_j^0(i_x^j(t),x) = t \) as consumer \( x \)’s “adjustment productivity given \( t \).” One may similarly define \( i_{x,A_j}(t) \) and \( i_{x,A_j}(x,A_j,r) \) based on \( g_{j,1}(i_{x,A_j}(t),x,t,A_j,r) = t \) and \( g_j(i_{x,A_j}(x,A_j,r),x,A_j,r) = t \), respectively. Note that, for \( x \notin X_j(t) \), \( i_x^j(t) = 0 \). Thus the adjustment model nests non-adjustment as a sub-case (i.e., \( X_j(t) = \emptyset \)).

The following lemma advances some useful results that follow from the definition of adjustment productivity:

**LEMMA 1:** (i) \( i_x^{0,0} < 1 \) (and \( i_x^{0,1} > 1 \)); (ii) \( i_{A_j}^{0,0} \geq 0 \) (and \( i_{A_j}^{0,1} \geq 0 \)); (iii) \( \lim_{A_j \rightarrow \infty} i_{A_j}^{0,0} = 0 \) (and \( \lim_{A_j \rightarrow \infty} i_{A_j}^{0,1} = 0 \)); (iv) \( i_t^{0,0} = 1/g_{t,0}^{0,0} > 0 \) (and \( i_t^{0,1} = 1/g_{t,1}^{0,1} > 0 \)).

Accounting for adjustment, the utility of a consumer at \( x \) buying product 0 is given by
One can see that utility losses accruing to choosing a non-ideal product equal the sum of adjustment cost and transportation cost components and are a function of the consumer’s adjustment productivity. Figure 3 displays these losses graphically as areas under the adjustment and transportation cost curves.  

Following Bloch and Manceau (1999), but extended to the adjustment case, I impose what is in effect a restriction on the size of $V$ relative to $t$ and to the rate of change of $g^{0,0}$ with respect to $x$:

ASSUMPTION 6: \[
\left\{ V - t \left[ x - i^{0,0}(x) \right] - \int_0^{i^{0,0}(x)} g^{0,0}(i,x,A_0) \, di \right\} F(x) \text{ is increasing for all } x \in [0,1].
\]

The assumption is a sufficient condition for the market to be covered under adjustment, as stated by the following lemma:

LEMMA 2: Given Assumption 6, the market is covered in equilibrium.

\[^4\text{Note that the setup in (3) and (4) is isomorphic to a traditional Hotelling model with nonlinear transportation costs. The adjustment map construct adds value relative to such a model by making the costs accruing to adjustment (as opposed to “transportation”) structurally and visually explicit so that adjustment’s distinct effects are clearly visible.}\]
To simplify the analysis and avoid corner cases, the analysis of advertising will be performed in the context of the following minimally-restrictive assumption about consumers’ adjustment productivity:

**ASSUMPTION 7 (Strong adjustment feasibility):** Let $x^*$ be the location of the indifferent consumer when there is no adjustment. Then $[x^*, 1] \subset X_0(t)$ and, correspondingly, $[0, x^*] \subset X_1(t)$.

This assumption provides that all consumers have the incentive to adjust to at least one product, and moreover that all consumers that a firm might target with its advertising (i.e., marginal, or else inframarginal to its rival) have positive adjustment productivity with respect to its product. Given this assumption, advertising does not need to be expended to bring consumers to the point where adjustment becomes feasible; rather its purpose is to improve the adjustment productivity of consumers who are already at least minimally productive at adjustment.

The location $x_E^*$ of the indifferent consumer under adjustment can be derived by setting $U_0 = U_1$. Thus it is defined implicitly by

$$
\Theta\left(x_E^*, t, p_0, p_1, A_0, A_1, r\right) \equiv p_1 - p_0 + t - 2tx_E^* - t\left[\int^{i^1\left(x_E^*, t, A_1, r\right)}_{i^0\left(x_E^*, A_0, r\right)} g^1\left(i, x_E^*, A_1, r\right) di - \int^{i^0\left(x_E^*, A_0, r\right)} g^0\left(i, x_E^*, A_0, r\right) di\right] = 0
$$

---

5 The existence of an $x^*$ is guaranteed by Lemma 2.
Based on this, one may define market shares for the two products as $D_0 = F(x_F^*)$ and $D_1 = 1 - F(x_F^*)$.

There are two periods. At $t = 1$, firms choose advertising expenditures, taking each other’s advertising expenditure choices as given. At $t = 2$, they choose prices, taking each other’s prices and their previous advertising choices as given. Firms recognize that their prices and their rivals’ prices will depend on their prior advertising choices and so treat these strategically with respect to their advertising decisions in $t = 1$. At the end of $t = 2$, consumers choose products and adjust to the product they choose; they receive utility, and the firms earn profits. I seek subgame perfect Nash equilibria to this game.

Given demand, profits of the firms are given by

$$
\Pi_0 = p_0(A_0, A_1)F(x_F^*) - aA_0
$$

$$
\Pi_1 = p_1(A_0, A_1)\{1 - F(x_F^*)\} - aA_1
$$

where I specify $a$ as the unit cost of advertising.

As a final assumption, I employ a variant on a distributional restriction by Caplin and Nalebuff (1991), which they showed constitutes a sufficient condition for the existence of a unique equilibrium in a broad class of games. Bloch and Manceau (1999) demonstrated the use of the Caplin-Nalebuff assumption in a model of persuasive advertising. The present variant generalizes that assumption to the model involving adjustment by imposing a set of complementary restrictions on the consumer distribution $f$ and the adjustment functions $g^{j,k}$. In the Appendix, it is demonstrated that the assumption applies to a rather general set of $f$ and $g$ functional form combinations.
ASSUMPTION 8: \( F(\cdot) \) is log concave in \( p_0 \) (and \( 1 - F(\cdot) \) is log concave in \( p_1 \)).

IV. EQUILIBRIUM

I begin by establishing a standard equilibrium existence result for the advertising and pricing game played by the firms and then derive the effect of advertising on prices as a comparative static result that characterizes the equilibrium. Equilibrium in the two-stage game is solved by backward induction: one first must determine the Nash equilibrium price-setting strategies of the firms in \( t = 2 \), then the \( t = 1 \) advertising strategies that take account of the \( t = 2 \) decisions. I obtain the following: 6

PROPOSITION 1: Assume the following hold: (i) the initial marginal productivity of advertising \( \phi_{A_j} \big|_{A_j=0} \) is sufficiently large \((j = 0, 1)\), (ii) \( \phi \) is sufficiently concave, (iii)

\[
\frac{\left[1 - F(x_E^*)\right]}{f(x_E^*)} \left[ \frac{dg_{A_0}}{dx_e} i_{A_0}^0 + \int_0^{i_{A_0}} \frac{\partial^2 g_{A_0}}{\partial x_e \partial A_0} di \right] > \int_0^{i_{A_0}} g_{A_0}^0 di \text{, and (iv) } \\
\frac{-F(x_E^*)}{f(x_E^*)} \left[ \frac{dg_{A_1}}{dx_e} i_{A_1}^1 + \int_0^{i_{A_1}} \frac{\partial^2 g_{A_1}}{\partial x_e \partial A_1} di \right] > \int_0^{i_{A_1}} g_{A_1}^1 di .
\]

Then the two-period game in which firms choose advertising levels at \( t = 1 \) and price levels contingent on these at \( t = 2 \) has a unique subgame perfect equilibrium \((A_0^*, A_1^*, p_0^*, p_1^*)\) at which \( A_0^*, A_1^* > 0 \), \( p_0^* = -F(x_E^*)/f(x_E^*) \frac{\partial x_E^*}{\partial p_0} \), and \( p_1^* = \left[1 - F(x_E^*)\right]/f(x_E^*) \frac{\partial x_E^*}{\partial p_1} .\)

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6 All proofs of propositions are in the Appendix.
The intuition behind pre-conditions (i) and (ii) on the function $\phi$ is straightforward: (i) is necessary if at least some advertising is to be profitable in equilibrium, given advertising’s positive incremental cost; and (ii) ensures the uniqueness of each firm’s optimizing advertising level. Pre-condition (iii) is essentially a requirement that the marginal revenue product of advertising for firm 0 be positive. (Pre-condition (iv) states the same for firm 1.) It represents the balance of the two relevant effects of advertising on firm 0’s revenues – effects that are related to two competing influences that advertising has with respect to consumer adjustment. The left-hand side comprises an expression, weighted by 

$$\left[1 - F\left(x_0^*\right)\right]/f\left(x_0^*\right),$$

that reflects how advertising, via the structure of the adjustment map, influences the price sensitivity of demand. I will interpret the components of this expression in detail later in discussing Proposition 2. Advertising induces the firm’s rival to raise price when the left-hand side is positive and to cut price when it is negative. The right-hand side of (iii) comprises the effect of advertising on the accumulated adjustment costs of the marginal consumer. In effect, it represents advertising’s direct effect on firm 0’s sales under the Nash conjecture that firm 1’s advertising expenditure will be held constant. The sign of the effect is always negative; correspondingly, the direct effect of advertising on sales is always positive. For (iii) to be satisfied, then, it is necessary that the effect of firm 0’s advertising in inducing firm 1 to reduce price must not be large relative to the effect of the advertising in marginally increasing firm 0’s sales directly. Otherwise, advertising in any amount will not be profitable to the firm.

Now let us turn to advertising’s price effect in equilibrium. It is sufficient to consider the effect of varying the exogenous unit cost of adjustment-facilitating advertising, $a$. I eliminate sectoral effects by confining the analysis to a context in which both demand and
adjustment are symmetric with respect to products. Taking the price function

\[ p_j(A_0(a), A_1(a)) \]

the price effects of \( a \) through advertising are given by (for \( j = 0 \)):\(^7\)

\[
\frac{\partial p_0}{\partial a} = \frac{\partial p_0}{\partial A_0} \frac{\partial A_0}{\partial a} + \frac{\partial p_0}{\partial A_1} \frac{\partial A_1}{\partial a} + \frac{\partial p_1}{\partial A_0} \frac{\partial A_0}{\partial a} + \frac{\partial p_1}{\partial A_1} \frac{\partial A_1}{\partial a}
\]

Signing the expression, one obtains the following result:

**PROPOSITION 2**: Assume (1) \( f(x) = f(1 - x) \) and (2)

\[ g^{0,k}(i, x) = g^{1,k}(i, 1 - x) \forall i \geq 0, x \in [0,1] \]. Then, advertising’s effect on prices is the sum of:

(i) a *shaping effect*, arising from altering how consumers’ adjustment facility varies with the strength of initial product preference; and

(ii) an *intensification effect*, arising from advertising’s amplification of adjustment’s effect on price based on the existing adjustment map.

Specifically, advertising increases prices if

\[
i^{(a)}(i^*_{A_0}) \int_0^i \frac{\partial^2 g^1}{\partial x \partial A_0} di \cdot \frac{dA_0}{dx} i^{(a)} > 0 \quad \text{(whence, by symmetry)}, \int_0^i \frac{\partial^2 g^0}{\partial x \partial A_0} di \cdot \frac{dA_0}{dx} i^{(a)} < 0
\]

and decreases them if the inequalities are reversed.

Proposition 2 shows that advertising affects prices by influencing consumers’ technology of self-persuasion in two ways. First, advertising *changes the shape of the adjustment map*, that is, it alters how marginal adjustment costs vary across consumers. I refer to this effect as the “shaping effect.” The solid red and solid blue adjustment maps in Figure 4 illustrate an example of the shaping effect: movement from the former to the latter

\(^7\)It is straightforward to show that \( a \) affects \( p_j \) only through advertising (i.e., there is no direct effect).
represents advertising that reduces marginal adjustment costs relatively more for consumers with less intense initial product preferences (i.e., marginal consumers). Such advertising results in lower prices. In contrast, advertising that reduces marginal adjustment costs more in relative terms for inframarginal consumers would result in higher prices.

The theory’s predictions with regard to the shaping effect are consistent with recent analysis of the price effects of targeted advertising. Erdem et al. (2008) interpret advertising in the traditional sense as a tool for increasing consumers’ willingness to pay (WTP). Under that conception, advertising that targets marginal consumers increases their WTP, making the demand curve flatter and causing the market price to fall. Meanwhile, advertising that targets inframarginal consumers and their WTP makes the demand curve steeper and less elastic, resulting in a higher price. Their empirical findings across an array of consumer brands reveal the sort of variation in advertising price effects predicted based on their conceptual model given the targeting strategies presumably preferred by their managers.

Second, according to the proposition, advertising affects prices by moving the adjustment map downward, holding constant relative marginal costs of adjustment across consumers. I refer to this effect as the “intensification effect,” in that it intensifies the influence that the adjustment process itself has on prices; it does this by, in effect, making all consumers better at – and therefore more intensely involved in – adjustment. This effect has no counterpart in Erdem et al.’s conceptual analysis of traditional persuasive advertising. It is an effect that depends uniquely on the peculiar nature of adjustment and one therefore that becomes apparent only when one conceives of advertising as facilitating self-persuasion.
The sign of the intensification effect depends on the post-advertising shape of the adjustment map. Consider the two panels of Figure 5. The first panel shows a symmetric adjustment map in which the curves grow flatter, at first, as one moves from $x^*_E$ toward positions of stronger initial preference. The second panel shows a symmetric map in which the curves grow steeper. In the first case, a price increase for one of the products would move to the margin previously-inframarginal consumers who find adjustment more productive at improving their attitude than the consumer at $x^*_E$. These consumers, if they switched products, would forgo lower total costs (i.e., transportation plus adjustment costs) from the product they left than would the consumer at $x^*_E$. Given symmetry, they would also incur greater total costs from their new product relative to the consumer at $x^*_E$. Thus demand is less price-sensitive, all else equal, when the adjustment map has this particular shape. Since advertising intensifies adjustment, it would intensify adjustment’s reduction of price sensitivity in this case, causing prices to rise, all else equal.

< INSERT FIGURE 5 APPROXIMATELY HERE >

In the second case, a price increase moves to the margin previously-inframarginal consumers who find adjustment less productive than the consumer at $x^*_E$. These consumers, if they switched products, would forgo greater total costs from the product they left than would the consumer at $x^*_E$. An adjustment map with this particular shape sets up an increased incentive for switching, whence demand becomes more price-sensitive. Since advertising intensifies adjustment, it would intensify adjustment’s increase in price sensitivity in this
case, causing prices to fall, all else equal. The example of the intensification effect shown in Figure 4 is of this variety: advertising-induced movement from the solid blue to dashed blue adjustment map represents an intensification of price-sensitivity-increasing adjustment and would cause prices to fall.

The patterns of marginal adjustment cost represented by the maps shown in the two panels of Figure 5 signify contrasting views as to what psychological processes dominate adjustment. The top panel, displaying adjustment maps that flatten toward positions of extreme preference, suggests that adjustment proceeds based on the halo effect – a well-established tendency by which individuals in certain situations tend to infer unknown objective, or undecided subjective, qualities of an object from their overall impressions. According to the halo effect, a positive impression about an object causes the individual to fill in the unformed aspects of his attitude toward it more positively; a negative impression has the opposite effect. Consumers whose judgments about products are characterized by the halo effect would thus find adjustment easier the stronger their initial preference for a product; that is, a strong positive initial impression would facilitate the building of yet a stronger positive attitude.

The bottom panel, displaying adjustment maps that steepen toward positions of extreme preference, proposes that adjustment proceeds according to regression to the mean (RTTM). Rather than proposing adjustment as being skewed deterministically by existing attitude, this conception casts adjustment as part of a learning process whereby random exposures to information provide the raw material for attitude change. If individuals are

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8 Note that what is critical to our result in both cases is what happens near $x^*_E$; in both cases, the curves must eventually become increasingly steep as one approaches $x = 0$ or $x = 1$, as this follows from our limiting assumptions on the $g^{1/2}$. 

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given access to the same information, then those with an initially weak preference for an object possess the greatest unexploited opportunities to adjust. These individuals should find adjustment easier. Individuals with the most intense initial preferences, in contrast, have exhausted the most persuasive information and arguments and should find further adjustment relatively more costly. Thus final preferences should tend to converge toward a “mean.”

Note that the effects of adjustment-facilitating advertising on price sensitivity and price levels that we have just derived based on the model’s assumptions do not depend on the distribution of consumers. They depend only (1) how the progression of marginal adjustment costs of consumers over the adjustment process varies depending upon strength of their initial preferences, as represented by the shape of the adjustment map; and (2) which consumers the advertising targets or influences most heavily.

V. WELFARE

Let us consider now how the level of adjustment-facilitating advertising chosen by the firms in equilibrium varies relative to the welfare optimum. The benchmark level is obtained when \((A_0,A_1)\) are chosen to maximize

\[
W = W_{CS} + \Pi_0 + \Pi_1 = W_{CS} + \Pi_T
\]

where consumer surplus is measured as the aggregation of utility across the mass of consumers, based on (3) and (4),
Comparing these to the privately-chosen levels of advertising yields the following result:

**PROPOSITION 3 (Welfare – General):** Assume (1) $f(x) = f(1-x)$ and (2) $g^0(i, x) = g^1(i, 1-x) \forall i \geq 0, x \in [0,1]$. Then if

$$\frac{1}{2f^{'}(\frac{1}{2})} \left[ \int_0^{\frac{t}{2}} g_{A_0}^0 \left( \frac{dE^*}{dx} \right) dx + \int_0^{\frac{t}{2}} g_{A_0}^0 \left( \frac{dx}{dx} \right) dx \right] - \int_0^{\frac{t}{2}} g_{A_0}^0 \left( \frac{dx}{dx} \right) dx + 2 \int_0^{\frac{t}{2}} g_{A_0}^0 \left( \frac{dx}{dx} \right) dx$$

is positive, the firms advertise too much; if it is negative, they advertise too little.

Proposition 3 indicates generally that the wedge between private incentives and the social net benefits of advertising accrues to the conflict inherent in the firm’s focus on winning the marginal consumer versus social welfare’s dependence on the sum of benefits across all consumers. Consistent with this, the first term shows that the density of consumers at the margin (i.e., midpoint in the symmetric case) mediates the role of advertising’s price effect in the divergence between private and social net benefits. The firm’s desire to advertise more when doing so increases prices drives the greatest wedge relative to social net benefits when the density of consumers at the midpoint is lowest. In this situation, the firm weighs the
low cost of losing sales to sparse marginal consumers against the high revenue benefit of raising prices on inframarginal consumers. When instead the density of consumers at the midpoint is greater, the firm’s incentives not to lose sales to marginal consumers align more closely with the social welfare objective of keeping prices low across all consumers. Along similar lines, the second and third terms show the private-social gap widening when advertising’s “efficiency” effect of reducing accumulated adjustment costs accrues more to the benefit of marginal consumers relative to the mass of consumers. Advertising benefits enjoyed by consumers at the margin are readily recaptured by the firm through incremental sales and increased prices. Benefits accruing to inframarginal consumers are irrelevant to sales or to the possibility of incrementing price.

To obtain a more intuitive understanding of the proposition, let us now consider how private and social incentives for advertising diverge in a benchmark case.

PROPOSITION 4 (Welfare – Benchmark Case): Consider the case in which the benefit accruing to the marginal consumer from advertising is twice the average consumer benefit. Then firms advertise too much if the price effect of advertising is positive and too little if it is negative.

It can be seen easily that the special case proposed in Proposition 4 is one in which the last two terms in the lemma are equal and drop out and, consequently, the welfare conclusions are simplified. What does this signify? The benchmark represents the particular relative level of benefit accruing to the marginal consumer at which the value firms are able to recoup through marginal sales gain from advertising exactly balances the social benefit.
that accrues to advertising across the mass of consumers. This leaves firms’ net benefit from advertising’s effect on prices as the sole private source of motivation for advertising that has no counterpart in social benefit. Thus when advertising increases prices firms are motivated to advertise too much, and when it decreases prices they are motivated to advertise too little. The case arises if the benefit from advertising declines linearly from that accruing to the marginal consumer at \( x = \frac{1}{2} \) to the consumer with extreme preference for either product 0 or 1, who reaps no benefit from advertising because his adjustment productivity is fixed at zero.

One can easily construct from the benchmark an alternative case in which advertising has no price effect but is under-provided in equilibrium. Consider a RTTM pattern adjustment map like the one in the bottom of Figure 5. From Assumption 5 it follows that the consumer at \( x = \frac{1}{2} \) is adjusting less than double the amount the consumer at \( x = \frac{1}{4} \) is adjusting; she in turn is adjusting less than double the amount of the consumer at \( x = \frac{1}{8} \), and so forth. Thus the benefit accruing to the \( x = \frac{1}{2} \) consumer from advertising that simply moves the adjustment map downward without affecting its shape is less than double that of the average consumer. This suggests firms would advertise too little in such a case even when the price effect of advertising is zero. As we know, however, pure intensification advertising reduces price under the RTTM pattern. Instead suppose we consider advertising that targets inframarginal consumers such that it shifts the adjustment map from a RTTM pattern to a neutral pattern. Based on Proposition 2, such advertising increases price. Therefore there exists advertising that moderately targets inframarginal consumers that would be price neutral, as illustrated in Figure 6. Because the benefits of the advertising are distributed more toward the inframarginal consumers relative to pure intensification advertising, the firms
incentives to under-provide are increased. Thus such advertising is under-provided in equilibrium.

V. CONCLUSION

The paper has offered a theory of persuasive advertising as a phenomenon distinct from informative advertising and with its own efficient purpose. The updated conception of advertising’s role arises from a new model of consumer behavior that recognizes the phenomenon of adjustment to choice. When rational consumers desire to adjust to their choices, it becomes possible to conceive of how persuasion might influence them. The influence is, indeed, consensual.

Traditional advertising theory has predicted conflicting price effects for advertising, accruing to the increased market power of brands on the one hand (the effect of advertising as persuasion) and to increased consumer access to price and product information on the other (the effect of advertising as information). Accordingly predictions of global effects based on advertising are often misleading. Recent empirical efforts, such as Erdem et al. (2008), have therefore sought a more complex understanding by focusing on how advertising differentially influences the willingness of pay of different groups of consumers.

The recent methodological advances in the empirical analysis of advertising’s effects have created an opportunity for rich theories to advance complex hypotheses of effects that can more accurately describe reality than past conceptions. Here the present model has offered two notable advances, which we consider in the context of Erdem et al.’s (2008)
important findings. First, it provides a framework, based on the conception of advertising as assisting self-persuasion, for predicting an array of differential price effects attributable to consumer heterogeneity. These effects can accrue not only to differential targeting of different groups of consumers, as posited by Erdem et al. (2008), but also to different response tendencies to the same advertising by consumers with different levels of initial preference for a brand. The theoretical bases for variations in consumers’ adjustment facility by preference intensity level suggested in section III – the halo effect and preference regression to the mean – provide some examples of how the model may be applied. Such conceptions may then be taken to data. With new approaches, including neural data, it may be possible to observe sources of heterogeneity in consumers’ responses and so test the theory’s predictions in ways not previously possible.

Second, the model offers a general structure for looking at how advertising that assists adjustment can shape the heterogeneous responses of different consumers differentially. In doing so, it can advance testable hypotheses for the price effects of complex advertising strategies that target different messages at different audiences. Thus whereas Erdem et al. (2008) consider the special implications of Heinz’ “horizontal” strategy, future empirical studies might use the present model as a basis for examining the impacts of multifarious campaigns.

This paper has only initiated the process of understanding the implications of consumer adjustment, and there are numerous opportunities for further research. I focus here on just a few interesting avenues relating to advertising’s role. First, it would be helpful not just to understand the general welfare implications of adjustment-facilitating advertising, but also the determinants of which consumers benefit. Second, while I have focused on the
symmetric case with respect to price effects, it would be useful to study how advertising affects pricing and competition in the case where the market is biased in favor of one of the firms. How does a dominant firm use adjustment-facilitating advertising versus how an upstart firm might use it? What are the implications for equilibrium pricing and welfare when the market is lopsided?

Third, it would be informative to extend the model to consider firms having a choice among advertising strategies that shape the adjustment map in different ways. This conception is realistic and could be revealing with respect to a number of welfare-relevant phenomena. Consider, for example, a case in which advertising reduces adjustment costs and flattens adjustment curves. One might characterize three strategic options for the firm: (i) target all consumers with the advertising, (ii) target only inframarginal consumers, or (iii) target only consumers at or near the margin. Figure 7 illustrates the three cases and their effects on the adjustment map: the dashed red curves show the pre-advertising map, while the solid blue curves show the post-advertising map. Which strategy would the firms choose? Would the choice depend upon certain parameter values? What is interesting in this scenario is the possibility that the firms might prefer (ii) to (i). We know from Proposition 2 that the adjustment map resulting from (ii) entails higher prices. On this basis, it would not be surprising if the firms favor it. If they do, a “shrouding equilibrium” could result in which the firms and perhaps some consumers benefit substantially from advertising, while those consumers that are indifferent between the products are left “in the dark” and experience far less benefit. (If the price increase due to advertising is large enough, all consumers may be worse off.) Note that, if (ii) is preferred by firms in Nash equilibrium, it is a dominant strategy, meaning neither firm would have an independent incentive to enlighten the median
consumers. Thus, as in other scenarios involving shrouding equilibria (e.g., Gabaix and Laibson 2006), a suboptimal outcome results despite competition in the market.

<INSERT FIGURE 7 APPROXIMATELY HERE>

APPENDIX

A. Applicability of log concavity of $F(.)$ in $p_j$

In this section, I will show that log concavity of $F(.)$ in $p_0$ – a critical condition for the existence of an interior equilibrium in prices – may be met (1) for the general class of symmetric adjustment map pairs for any log concave distribution $f$, and (2) for an example of a non-symmetric adjustment map pair when $f$ is Beta distributed with shape parameters $(\alpha, \beta) = (3,3)$. The main issue in the case of non-symmetric map pairs is that, approaching the extreme locations $x = 0$ and $x = 1$, consumers’ marginal adjustment costs approach infinity for the nearby product. Thus, unless marginal adjustment costs for the distant product similarly grow without limit, sensitivity of demand to price rises precipitously at the extremes, making it potentially profitable for firms to attempt to drop price from any candidate interior maximum to a low enough level to take the whole market. This situation is avoided if the density of consumers at the extremes is sufficiently low, as with some log-concave distributions such as the Beta. So, to summarize, an interior price equilibrium will result whenever the incentive to de-stabilize such an equilibrium is mitigated by adjustment symmetry; or when there are not enough consumers with extreme tastes for firms to want to de-stabilize an interior price equilibrium despite non-symmetry.
We may define the log concavity of $F(.)$ in $p_0$ as \( f\left(x^*_E\right) \frac{\partial x^*_E}{\partial p_0} / F\left(x^*_E\right) \) being decreasing in $p_0$ or, equivalently, $-F\left(x^*_E\right) / f\left(x^*_E\right) \frac{\partial x^*_E}{\partial p_0}$ decreasing in $p_0$. Suppose first that $F(.)$ is log concave in its own argument $x$; then $\partial^2 x^*_E / \partial p_0^2 < 0$ is then a sufficient condition for log concavity of $F(.)$ in $p_0$. Now, from (A7) we derive

\[
\frac{\partial^2 x^*_E}{\partial p_0^2} = -\left[ -2t + \int_0^{i^{1.0}(x^*_E)} \frac{dg^{1.0}}{dx^*_E} di - \int_0^{i^{0.0}(x^*_E)} \frac{dg^{0.0}}{dx^*_E} di \right]^2.
\]

\[
\begin{bmatrix}
\int_0^{i^{1.0}(x^*_E)} \frac{d^2 g^{1.0}}{dx^*_E^2} \frac{\partial x^*_E}{\partial p_0} di - \int_0^{i^{0.0}(x^*_E)} \frac{d^2 g^{0.0}}{dx^*_E^2} \frac{\partial x^*_E}{\partial p_0} di + \frac{dg^{1.0}}{dx^*_E} i^{1.0} \frac{\partial x^*_E}{\partial p_0} - \frac{dg^{0.0}}{dx^*_E} i^{0.0} \frac{\partial x^*_E}{\partial p_0} \\
\int_0^{i^{1.0}(x^*_E)} \frac{d^2 g^{0.0}}{dx^*_E^2} \frac{\partial x^*_E}{\partial p_0} di - \int_0^{i^{0.0}(x^*_E)} \frac{d^2 g^{1.0}}{dx^*_E^2} \frac{\partial x^*_E}{\partial p_0} di + \frac{dg^{1.0}}{dx^*_E} i^{1.0} \frac{\partial x^*_E}{\partial p_0} - \frac{dg^{0.0}}{dx^*_E} i^{0.0} \frac{\partial x^*_E}{\partial p_0}
\end{bmatrix}
\]

\[
= \left( \frac{\partial x^*_E}{\partial p_0} \right)^2 \left[ \int_0^{i^{1.0}(x^*_E)} \frac{d^2 g^{1.0}}{dx^*_E^2} \frac{\partial x^*_E}{\partial p_0} di - \int_0^{i^{0.0}(x^*_E)} \frac{d^2 g^{0.0}}{dx^*_E^2} \frac{\partial x^*_E}{\partial p_0} di + \frac{dg^{1.0}}{dx^*_E} i^{1.0} \frac{\partial x^*_E}{\partial p_0} - \frac{dg^{0.0}}{dx^*_E} i^{0.0} \frac{\partial x^*_E}{\partial p_0} \right]
\]

We note in (A7) that $\frac{\partial x^*_E}{\partial p_0}$ is not a function of $i$, so it may be pulled outside the integral, allowing us to write the sufficient condition as

\[(A1) \quad -\frac{\partial x^*_E}{\partial p_0} \cdot \left[ \int_0^{i^{1.0}(x^*_E)} \frac{d^2 g^{1.0}}{dx^*_E^2} \frac{\partial x^*_E}{\partial p_0} di - \int_0^{i^{0.0}(x^*_E)} \frac{d^2 g^{0.0}}{dx^*_E^2} \frac{\partial x^*_E}{\partial p_0} di + \frac{dg^{1.0}}{dx^*_E} i^{1.0} \frac{\partial x^*_E}{\partial p_0} - \frac{dg^{0.0}}{dx^*_E} i^{0.0} \frac{\partial x^*_E}{\partial p_0} \right] \leq 0\]

Consider now the set of pairs of symmetric adjustment maps, \( \{\mathcal{S}^{0.0}, \mathcal{S}^{1.0}\} \). For these pairs, for each adjustment curve in $\mathcal{S}^{0.0}$ corresponding to a given location $x \in (0,1)$, the corresponding curve in $\mathcal{S}^{1.0}$ would be its mirror image about $x$. An example of a (sub)set of such a map pairs, for $j = 0,1$ and $\rho \in [-1,1]$, is given by

\[(A2) \quad \mathcal{S}^{j.0} = \begin{cases} \frac{x}{2(x-\rho)} & \text{for } x \in [0,\frac{1}{2}] \\ \frac{\rho(1-x)}{2(1-\rho)} + \frac{1-\rho}{2} & \text{for } x \in [\frac{1}{2},\frac{1}{2}] \\ \frac{\rho(1-x)}{2(1-\rho)} + \frac{1-\rho}{2} & \text{for } x \in [\frac{1}{2},\frac{3}{4}] \\ \frac{1}{2(1-\rho)} & \text{for } x \in [\frac{3}{4},1] \end{cases}\]
Two example map pairs from this set are displayed in Figure 8, corresponding to the values \( \rho = 1 \) and \( \rho = -1 \). In the first example (with \( \rho = 1 \)), adjustment curves in both maps flatten monotonely as one moves toward the midline \( x = \frac{1}{2} \). These maps conceive of consumers quite intuitively as being more facile with adjustment to both options if they are initially more indifferent between their options; consumers who feel strongly about an option initially are less able to find good opportunities to adjust to either their preferred option, because they are already almost perfectly satisfied with it; or its alternative, because they simply find it hard to conceive of how they might get comfortable with that distant option. One may verify that for these \( \frac{\partial g^{1/0}}{\partial x} < 0 \) for \( x \in [0, \frac{1}{2}] \), while \( \frac{\partial g^{1/0}}{\partial x} > 0 \) for \( x \in [\frac{1}{2}, 1] \). In the second example (with \( \rho = -1 \)), adjustment curves flatten at first toward the midline, but then (beyond \( x = \frac{1}{4} \) or \( x = \frac{3}{4} \)) steepen to reach a local maximum steepness at \( x = \frac{1}{2} \). These maps conceive of the most indifferent consumers as being more rigid than those just a bit closer to one option or another; perhaps consumers the most initially dissatisfied with their options become pig-headed or embittered and therefore are inflexible. One may verify that for these \( \frac{\partial g^{1/0}}{\partial x} < 0 \) for \( x \in [0, \frac{1}{4}] \) and \( x \in [\frac{3}{4}, 1] \), while \( \frac{\partial g^{1/0}}{\partial x} > 0 \) for \( x \in [\frac{1}{4}, \frac{1}{2}] \) and \( x \in [\frac{1}{2}, 1] \).

<INSERT FIGURE 8 APPROXIMATELY HERE>

With symmetric adjustment map pairs, the following conditions clearly hold: (i) \( \frac{d s^{0/0}}{d x} = \frac{d s^{1/0}}{d c} \), (ii) \( \frac{d^2 s^{00}}{d c^2} = \frac{d^2 s^{10}}{d c^2} \), (iii) \( i_{x}^{0,0} = i_{x}^{1,0} \), and (iv) \( i_{x}^{0,0} = i_{x}^{1,0} \). It may be verified, based on these, that the sufficient condition (A1) above for log concavity is met. That is, we have log concavity of \( F(x(p_0)) \) for any log concave distribution \( f \).

More generally, if \( f \) is not log concave, it must be the case that
for log concavity of \( F(.) \) in \( p_0 \). Consider now an example of a non-symmetric adjustment map pair, given by \( g^{0,0}(i,x) \equiv x/2(x-i) \) and \( g^{1,0}(i,x) \equiv (1-x)/2(1-x-i) \) for \( x \in [0,1] \).

These functions have the property that \( g^{0,0}(0,x) = g^{1,0}(0,x) = 1/2 \). Observe further that \( i^{0,0}(x,t) = \frac{2t-1}{2t}x \) is defined for \( t \geq \frac{1}{2} \), whence \( i^* < x \); similarly \( i^{1,0}(x,t) = \frac{2t-1}{2t}(1-x) \), whence \( i^* < 1-x \). We also have \( i_{x}^{0,0} = \frac{2t-1}{2t} \) and \( i_{x}^{1,0} = -\frac{2t-1}{2t} \). Let us now take the first and second derivatives with respect to \( x \):

\[
\frac{\partial g^{0,0}}{\partial x} = \frac{-i}{2(x-i)^2} \leq 0; \quad \frac{\partial^2 g^{0,0}}{\partial x^2} = \frac{i}{(x-i)^3} \geq 0
\]

and

\[
\frac{\partial g^{1,0}}{\partial x} = \frac{i}{2(1-x-i)^2} \geq 0; \quad \frac{\partial^2 g^{1,0}}{\partial x^2} = \frac{i}{(1-x-i)^3} \geq 0
\]

Now we evaluate (A1) at \( i = 0 \) (i.e., the position at which the indifferent consumer evaluates his decision between product options), for all possible \( x \) (i.e., \( x \in [0,1] \)). Using integration by parts:

\[
\begin{align*}
&\int_{0}^{x} \frac{d^2 g^{1,0}}{dx^2} \, dx - \int_{0}^{x} \frac{d^2 g^{0,0}}{dx^2} \, dx + \frac{d g^{1,0}}{dx} \bigg|_{x}^{x} - \frac{d g^{0,0}}{dx} \bigg|_{x}^{x} \\
&= \int_{0}^{x} \frac{i}{(1-x-i)^3} \, dx - \int_{0}^{x} \frac{i}{(x-i)^3} \, dx + \frac{i}{2(1-x-i)^2} \left( -\frac{2t-1}{2t} \right) - \frac{-i}{2(x-i)^2} \left( \frac{2t-1}{2t} \right) \\
&= \frac{i}{2(1-x-i)} - \frac{1}{2(1-x-i)} \left[ \frac{2t-1}{2t} \right]_{0}^{2t-1} - \frac{i}{2(x-i)^2} \left[ \frac{2t-1}{2t} \right]_{0}^{2t-1} \\
&= \frac{(4t^2 - 4t + 1)(2x-1)}{2x(1-x)}
\end{align*}
\]
Substituting into (A7) for our example functions we obtain \( \frac{\partial x_F}{\partial p_0} = -1/(1 - \ln \frac{1}{2t}) \).

Now assume \( f \) is distributed Beta with shape parameters \((\alpha, \beta) = (3, 3)\). We have:

\[
f(x) = \frac{[x(1-x)]^2}{\int_0^1 [u(1-u)]^2 \, du}; \quad F(x) = \int_0^x [u(1-u)]^2 \, du
\]

\[
\Rightarrow f'(x) = \frac{2x(1-x)(1-2x)}{\int_0^1 [u(1-u)]^2 \, du}
\]

Thus,

\[
\frac{f'(x^*_E)}{f(x^*_E)} = \frac{f(x^*_E)}{F(x^*_E)} = \frac{2x(1-x)(1-2x)}{[x(1-x)]^2} - \frac{[x(1-x)]^2}{\int_0^x [u(1-u)]^2 \, du}
\]

(A5)

\[
= \frac{2(1-2x)}{x(1-x)} - \frac{30[1-x]^2}{x[6x^2 - 15x + 10]}
\]

One may verify, using (A4), (A5), and \( \frac{\partial x_F}{\partial p_0} = -1/(1 - \ln \frac{1}{2t}) \), that (A3) holds for all \( x \in (0,1) \), and for any \( t > \frac{1}{2} \).

**B. Proofs and Derivations of Lemmas, Propositions, and Remarks**

B1. DERIVATION OF LEMMA 1.

Begin with the expression \( g^j(i^{*j}(x,t,\theta,A_j,r_j),\theta,x,A_j,r_j) = t \) which implicitly defines \( i^{*j} \) and expand (here, shown for \( j = 0 \), with the arguments of \( i \) suppressed and \( g \) suppressed but for \( i \) and \( x \)):

\[
\phi(A_0)r\left[ g^{0,1}(i^{*0},x) - g^{0,0}(i^{*0},x) \right] + g^{0,0}(i^{*0},x) = t
\]

Totally differentiating yields
\[
\begin{align*}
g_x^0 dt^{i_0^0} &= -g_A^0 dA_0 - g_r^0 dr - g_x^0 dx - g_\theta^0 d\theta + dt \quad \iff \quad g_r^0 dt^{i_0^0} = -\left\{ \phi_A r \left( g^{0.1} - g^{0.0} \right) \right\} dA_0 - \left\{ \phi(A_0) \left( g^{0.1} - g^{0.0} \right) \right\} dr - g_x^0 dx - d\theta + dt
\end{align*}
\]

Using Cramer’s rule, it follows from Assumption 7 that
\[
i^{i_0^0}(x,A_0) = \frac{-g_x^0}{g_r^0} < \frac{g_x^0}{g_r^0} = 1
\]

Also using Cramer’s rule one obtains
\[
i^{i_0^0}(x,A_0, r, t) = -\frac{g_A^0}{g_r^0} = \frac{\phi_A r \left( g^{0.0} - g^{0.1} \right)}{g_r^0} \geq 0
\]

whence it follows from Assumption 5 that \( \lim_{A_0 \to \infty} i^{i_0^0} = 0 \). Additionally,
\[
i^{i_0^0}(x,A_0, r, t) = \frac{-g_r^0}{g_r^0} = \frac{\phi(A_0) \left( g^{0.0} - g^{0.1} \right)}{g_r^0} \geq 0
\]

\[
i^{i_0^0}(x,A_0, r, t) = 1/g_r^0 > 0 \text{ and } i^{i_0^0} = -\frac{g_\theta^0}{g_r^0} = -\frac{1}{g_r^0} < 0
\]

Corresponding results can be derived along the same lines for \( j = 1 \).

**B2. PROOF OF LEMMA 2.**

The proof is an extension of the proof of Bloch and Manceau’s (1999) Lemma 1.

Assume no advertising, and suppose that the market is not covered, that is, at equilibrium prices \( \left( p_0^*, p_1^* \right) \) there exists a consumer \( x \) for whom

\[
\begin{align*}
V - p_0^* - t \left[ x - i^{i_0^0}(x) \right] - \int_0^{i^{i_0^0}(x)} g^{0.0}(i,x) di < 0 \quad \text{and} \\
V - p_1^* - t \left[ 1 - x - i^{i_1^0}(x) \right] - \int_0^{i^{i_1^0}(x)} g^{1.0}(i,x) di < 0
\end{align*}
\]
One can show these prices do not constitute a Nash equilibrium, in that firm 0 can increase its profit by lowering its price \( p_0 \) without altering the profit, hence strategy, of firm 1. Begin by noting that, under \( (p_0^*, p_1^*) \), because there is a consumer for whom neither good provides nonnegative utility somewhere between the firms, the profit of firm 0 can be written

\[
\Pi_0 = p_0^*(x_0) F(x_0) \equiv \left\{ V - t \left[ x_0 - i^{*0,0}(x_0) \right] - \int_0^{\tilde{i}^{0}(x_0)} g^{0,0}(i, x_0) di \right\} F(x_0)
\]

where \( x_0 \) is the position of the consumer who, at prices \( (p_0^*, p_1^*) \), is just indifferent between buying product 0 and buying nothing. By assumption, \( \partial \Pi_0 / \partial x_0 > 0 \). Now note that

\[
\frac{\partial p_0}{\partial x_0} = -t + \tilde{i}^{0,0}_x - g^{0,0}(i^{0,0}, x_0) i^{0,0}_x - \int_0^{\tilde{i}^{0}(x_0)} \frac{dg^{0,0}}{dx_0} di
\]

\[
= -t - \int_0^{\tilde{i}^{0}(x_0)} \frac{dg^{0,0}}{dx_0} di < -t + \int_0^{\tilde{i}^{0}(x_0)} \frac{dg^{0,0}}{di} di
\]

\[
= -t + g(i^{*0,0}(x_0), x_0) - g(0, x_0) = -g(0, x_0) < 0
\]

which follows from Assumption 7. Since \( \partial \Pi_0 / \partial x_0 = (\partial \Pi_0 / \partial p_0)(\partial p_0 / \partial x_0) \), it follows that \( \partial \Pi_0 / \partial p_0 < 0 \). Therefore a small downward deviation in the price \( p_0 \) from \( p_0^* \) increases firm 0’s profits while not affecting firm 1’s profits. This contradicts the assertion that \( (p_0^*, p_1^*) \) constitute an equilibrium.

Since the market is covered when there is no advertising, it follows that it is also covered when there is advertising.

B3. PROOF OF PROPOSITION 1.

Let us begin by considering the firms’ Nash price-setting strategies at \( t = 2 \).

Differentiating firm 0’s profit equation in (6) with respect to price yields
\[ \frac{\partial \Pi_0}{\partial p_0} = F(x_E^*) + p_0 f(x_E^*) \frac{\partial x_E^*}{\partial p_0} \]

where \( \frac{\partial x_E^*}{\partial p_0} \) is derived by applying Cramer’s rule to (5),

\[ \frac{\partial x_E^*}{\partial p_0} = - \frac{\partial x_E^*}{\partial p_1} \cdot \begin{bmatrix} 1 \end{bmatrix} \]

\[ = - \begin{bmatrix} 1 \end{bmatrix} \left[ \begin{array}{c} -2t + \int_0^1 \frac{dg}{dx_{E^*}} \, di - \int_0^1 \frac{dg_0}{dx_{E^*}} \, di \end{array} \right] < 0 \]

Using (A6), one obtains \( \left( \frac{\partial \Pi_0}{\partial p_0} \right)_{p_0=0} = F(x_E^*) \bigg|_{p_0=0} > 0 \) : non-zero demand for product 0 is guaranteed at \( p_0 = 0 \) by \( t > 0 \) and \( g^j(i,x) > 0 \). Moreover,

\[ \left( \frac{\partial \Pi_0}{\partial p_0} \right)_{p_0=0} = p_0 f(0) \left( \frac{\partial x_E^*}{\partial p_0} \right) < 0 , \text{ where } p_0 \bigg|_{x_E^*} > 0 \). Because, following from Assumption 8, \(- F(x_E^*)/f(x_E^*) \frac{\partial x_E^*}{\partial p_0} \) must be decreasing in \( p_0 \), it follows that there exists a unique solution to the first-order condition \( \frac{\partial \Pi_0}{\partial p_0} = 0 \) and that it is \( p_0^* = - \frac{F(x_E^*)}{f(x_E^*)} \frac{\partial x_E^*}{\partial p_0} \). A corresponding analysis of firm 1’s problem yields the unique solution to the first-order condition \( \frac{\partial \Pi_1}{\partial p_1} = 0 \) at \( p_1^* = \left[ 1 - F(x_E^*)/f(x_E^*) \right] \frac{\partial x_E^*}{\partial p_1} \).

Now consider the firms’ advertising choices at \( t = 1 \). We focus on analyzing firm 0’s problem. Differentiating firm 0’s profit equation in (6) with respect to \( A_0 \) yields

\[ \frac{\partial \Pi_0}{\partial A_0} = p_0 f(x_E^*) \left[ \frac{\partial x_E^*}{\partial A_0} + \frac{\partial x_E^*}{\partial p_0} \frac{\partial p_0}{\partial A_0} + \frac{\partial x_E^*}{\partial p_1} \frac{\partial p_1}{\partial A_0} \right] + \frac{\partial p_0}{\partial A_0} F(x_E^*) - a \]

Firm 0 knows it will choose price optimally at \( t = 2 \) taking its advertising choice as given; accordingly the first-order condition for firm 0’s advertising decision involves substituting the first-order condition for price and the expression \( p_0^* = - \frac{F(x_E^*)}{f(x_E^*)} \frac{\partial x_E^*}{\partial p_0} \) into (A8) and setting the resulting expression equal to zero. Thus (A8) reduces to
\[
\frac{\partial \Pi_0}{\partial A_0} = F(x^*_E) \left( -\frac{\partial s_E}{\partial x^*_E} \frac{\partial x^*_E}{\partial p_1} + \frac{\partial p_1}{\partial A_0} \right) - a
\]

which is the sum of advertising’s effect on revenue through the prices of both firms. (The direct effect of advertising on 0’s sales drops out due to the envelope theorem.)

It can be shown easily that the marginal revenue product of firm 0’s advertising – the first term in (A9) – is a linear function of \( \phi_{A_0} \). Thus it is necessary only that the first unit of advertising be sufficiently productive (i.e., \( \phi_{A_0} \bigg|_{A_0=0} \) sufficiently large) and the marginal revenue product of advertising be positive for \( \frac{\partial \Pi_0}{\partial A_0} > 0 \) at \( A_0 = 0 \). [By manipulating (A9); establishing that there exists \( \bar{A}_0 \) such that, for \( A_0 > \bar{A}_0 \), \( \frac{\partial \Pi_0}{\partial A_0} < 0 \); and establishing the conditions under which under which \( \frac{\partial^2 \Pi_0}{\partial A_0^2} < 0 \), we may show.] Our first task, then, is to establish the equivalence of pre-condition (iii) of the proposition to a positive marginal revenue product of advertising for firm 0 (and, by extension, the equivalence of (iv) to a positive marginal revenue product of advertising for firm 1).

We may derive an expression for \( \frac{\partial p_1}{\partial A_j} \) by totally differentiating the first-order condition in price for firm 1 and applying Cramer’s rule:

\[
\frac{\partial p_1}{\partial A_j} = -\frac{f(x^*_E) \frac{\partial s_E}{\partial x^*_E} + p_1 f(x^*_E) \frac{\partial x^*_E}{\partial p_1} + p_1 f(x^*_E) \frac{\partial x^*_E}{\partial A_0}}{\frac{\partial^2 s_E}{\partial A_0} \left[ 2 f(x^*_E) + p_1 f(x^*_E) \frac{\partial x^*_E}{\partial p_1} \right] + p_1 f(x^*_E) \frac{\partial^2 x^*_E}{\partial p_1^2}}
\]

Using this, and \( \frac{\partial s_E}{\partial p_1} = -\frac{\partial s_E}{\partial p_0} \), it can be shown that

\[
-\frac{\partial s_E}{\partial x^*_E} \frac{\partial x^*_E}{\partial p_0} + \frac{\partial p_1}{\partial A_0} = -f(x^*_E) \left\{ -\frac{\partial s_E}{\partial A_0} + p_1 \frac{\partial x^*_E}{\partial A_0} \frac{\partial x^*_E}{\partial x^*_E} p_1 \left( \frac{\partial x^*_E}{\partial p_1} \right)^2 \right\}
\]
Because the denominator is known to be positive by the second-order conditions for a maximum in price, signing this expression positive is equivalent to signing the curly-bracketed expression negative.

Recalling from Proposition 1 that \( p_i^* = \left[ 1 - F(x_E^*) \right] / f(x_E^*) \frac{\partial^2 x_E^*}{\partial x_i \partial \lambda_x} \), the curly bracketed expression in (A11) may be written

\[
\frac{\partial^2 x_E^*}{\partial \lambda_x \partial p_i} + \frac{\left[ 1 - F(x_E^*) \right] \frac{\partial^2 x_E^*}{\partial x_i \partial \lambda_x}}{f(x_E^*) \left( \frac{\partial^2 x_E^*}{\partial \lambda_x \partial p_i} \right)^2} = \frac{\left( 1 - F(x_E^*) \right) \frac{\partial^2 x_E^*}{\partial x_i \partial \lambda_x}}{f(x_E^*) \left( \frac{\partial^2 x_E^*}{\partial \lambda_x \partial p_i} \right)^2}.
\]

Using (A7), we derive

\[
\frac{\partial^2 x_E^*}{\partial x_i \partial \lambda_x} = \left[ -2t + \int_0 \frac{dg_0^1}{dx_E^*} di - \int_0 \frac{dg_0^0}{dx_E^*} di \right]^2.
\]

(A13)

\[
\frac{\partial^2 x_E^*}{\partial x_i \partial p_1} = \left[ -2t + \int_0 \frac{dg_0^1}{dx_E^*} di - \int_0 \frac{dg_0^0}{dx_E^*} di \right]^2.
\]

(A14)

These allow us to relate \( \frac{\partial^2 x_E^*}{\partial x_i \partial \lambda_x} \) to \( \frac{\partial^2 x_E^*}{\partial x_i \partial \lambda_x} \) by the following expression:
\[ \frac{\partial^2 s_y}{\partial p_i \partial A_0} = \frac{\partial x_y}{\partial p_i} \frac{\partial s_y}{\partial A_0} - \left[ \frac{dg_0^0}{dx_E} \frac{i_0}{A_0} + \int_0 \frac{\partial^2 g_0^0}{\partial x_0 \partial A_0} di \right] \left( \frac{\partial s_y}{\partial p_i} \right)^2 \]

Substituting into (A12) we obtain

\[ \left[ 1 - F(x_E^*) \right] \left[ \frac{dg_0^0}{dx_E} \frac{i_0}{A_0} + \int_0 \frac{\partial^2 g_0^0}{\partial x_0 \partial A_0} di \right] \left( \frac{\partial s_y}{\partial p_i} \right)^2 \]

Applying Cramer’s rule to (5) and using (A7),

\[ \frac{\partial s_y}{\partial A_0} = \frac{\int_0 \frac{g_0^0 - g_0^0}{dx_E} di}{\int_0 w_1(x_E^*) dx_E} = \frac{\int_0 g_0^0 di}{w_1(x_E^*)} \]

Using this expression, along with (A15) and \( \frac{\partial s_y}{\partial A_0} = -\frac{\partial s_y}{\partial x_0} \), we obtain as a necessary condition for an interior equilibrium in advertising

\[ \left[ 1 - F(x_E^*) \right] \left[ \frac{dg_0^0}{dx_E} \frac{i_0}{A_0} + \int_0 \frac{\partial^2 g_0^0}{\partial x_0 \partial A_0} di \right] > \int_0 g_0^0 di \]

Analogous analytics show that the marginal revenue product of advertising for firm 1 is positive if and only if

\[ \frac{-F(x_E^*)}{f(x_E^*)} \left[ \frac{dg_1^1}{dx_E} \frac{i_1}{A_1} + \int_0 \frac{\partial^2 g_1^1}{\partial x_1 \partial A_1} di \right] > \int_0 g_1^1 di \]

It remains to show the proposition follows from pre-condition (ii). We have already established that a positive marginal revenue product of advertising and \( \phi_{A_0} \big|_{A_0=0} \) large enough imply \( \partial \Pi_0 / \partial A_0 > 0 \) at \( A_0 = 0 \). Meanwhile, by Assumption 3 and Lemma 1, respectively, as
\( A_0 \) grows large, \( \phi_{A_i} \) and \( i_{A_0}^{\phi} \) approach zero. Inspection of (A16) and (A10) reveals that the components of the marginal revenue product of advertising that multiply \( \phi_{A_i} \) are bounded above with respect to increases in \( A_0 > 0 \), implying that there exists \( \overline{A}_0 \) such that, for \( A_0 > \overline{A}_0 \), \( \partial \Pi_0 / \partial A_0 < 0 \). Because the marginal revenue product of advertising is a linear function of \( \phi_{A_i} \), this term will decline monotonely with \( A_0 > 0 \), implying that there exists \( \overline{A}_0 \) such that, for \( A_0 > \overline{A}_0 \), \( \partial \Pi_0 / \partial A_0 < 0 \) is guaranteed, whence, given parallel results for firm 1, the existence of a unique subgame perfect equilibrium

\[ (A_0^*, A_1^*, p_0^*, p_1^*) \text{ with } A_0^*, A_1^* > 0. \]

B4. PROOF OF PROPOSITION 2

To sign the effect of \( a \) on price, begin by observing that in the symmetric case \( \partial A_0 / \partial a = \partial A_1 / \partial a < 0 \). The other components of (7) are obtained by totally differentiating the first-order condition in price for firm 0 (which is obtained by setting (A6) equal to zero) and using Cramer’s rule:

\[
\frac{\partial p_0}{\partial p_1} = -f(x_E^*) \frac{\partial x_E^*}{\partial p_1} + p_0 f'(x_E^*) \frac{\partial x_E^*}{\partial p_0} \frac{\partial x_E^*}{\partial p_1} + p_0 f(x_E^*) \frac{\partial^2 x_E^*}{\partial p_0 \partial p_1}
\]

and, for \( j = 0,1 \),

\[
\frac{\partial p_0}{\partial A_j} = -f(x_E^*) \frac{\partial x_E^*}{\partial A_j} + p_0 f'(x_E^*) \frac{\partial x_E^*}{\partial p_0} \frac{\partial x_E^*}{\partial A_j} + p_0 f(x_E^*) \frac{\partial^2 x_E^*}{\partial p_0 \partial A_j}
\]

(A17)

\[(A10)\text{ provides an analogous expression for firm 1’s pricing decision with respect to changes in advertising.}\]
When incorporating these expressions in the evaluation of (7), certain other facts will be useful. From (A13) and (A14), we observe \( \frac{\partial^2 x_E^{*}}{\partial p_i \partial A_i} = \frac{\partial^2 x_E^{*}}{\partial p_i} = - \frac{\partial^2 y^{*}}{\partial p_0 \partial p_i} \) and \( \frac{\partial^2 x_E^{*}}{\partial p_0 \partial A_i} = - \frac{\partial^2 y^{*}}{\partial p_0 \partial A_i} \), respectively. Moreover, consistent with the derivation of (A14), we obtain

\[
\frac{\partial^2 x_E^{*}}{\partial p_0 \partial A_i} = - \left[ -2t + \int_{0}^{\xi_E^{*}} \frac{dg_i^{2}}{dx_i} di - \int_{0}^{\xi_E^{*}} \frac{dg_i^{3}}{dx_i} di \right]^2.
\]

(A18)

\[
\begin{bmatrix}
\int_{0}^{\xi_E^{*}} \left[ \frac{\partial^2 x_E^{*}}{\partial x_i \partial A_i} + \frac{\partial^2 x_E^{*}}{\partial x_i \partial A_i} \right] di - \int_{0}^{\xi_E^{*}} \frac{\partial^2 x_E^{*}}{\partial x_i \partial A_i} di + \frac{\partial^2 x_E^{*}}{\partial x_i \partial A_i} \left( i_1 + \frac{i^1}{x_i} \frac{\partial p_0}{\partial A_i} \right) - \frac{\partial^2 x_E^{*}}{\partial x_i \partial A_i}
\end{bmatrix}
\]

Additionally, under the symmetric case, \( x_E^{*} = \frac{1}{2} \), whence \( f'(x_E^{*}) = 0 \), \( F(x_E^{*}) = 1 - F(x_E^{*}) = \frac{1}{2} \), and \( \partial x_E^{*}/\partial A_0 = - \partial x_E^{*}/\partial A_i \) (using (A16)). Using these facts, \( \partial p_0/\partial p_i = \frac{1}{2} \).

It may be observed from (A18) that when \( \frac{dg_i^{0}}{dx_i} i_0^0 + \int_{0}^{\xi_E} \frac{dg_i^{0}}{dx_i \partial A_0} di > 0 \) (and, by

\[
\frac{\partial^2 x_E^{*}}{\partial x_i \partial A_i} \left( i_1 + \frac{i^1}{x_i} \frac{\partial p_0}{\partial A_i} \right) - \frac{\partial^2 x_E^{*}}{\partial x_i \partial A_i}
\]

symmetry, \( \frac{dg_i^{1}}{dx_i} i_1^1 + \int_{0}^{\xi_E} \frac{dg_i^{1}}{dx_i \partial A_0} di < 0 \), the numerators of \( \frac{\partial^2 x_E^{*}}{\partial p_0 \partial A_0} \) and \( \frac{\partial^2 x_E^{*}}{\partial p_0 \partial A_i} \) are both increased by the same positive quantity, whereby it follows that \( \partial p_0/\partial A_0 > - \partial p_i/\partial A_i \). By symmetry, \( \partial p_i/\partial A_i > - \partial p_i/\partial A_0 \). Thus the second and third pair of terms in (7) are both positive in this case, thus prices increase with advertising if \( \frac{dg_i^{0}}{dx_i} i_0^0 + \int_{0}^{\xi_E} \frac{dg_i^{0}}{dx_i \partial A_0} di > 0 \) (and

\[
\frac{\partial^2 x_E^{*}}{\partial x_i \partial A_i} \left( i_1 + \frac{i^1}{x_i} \frac{\partial p_0}{\partial A_i} \right) - \frac{\partial^2 x_E^{*}}{\partial x_i \partial A_i}
\]

\( \frac{dg_i^{1}}{dx_i} i_1^1 + \int_{0}^{\xi_E} \frac{dg_i^{1}}{dx_i \partial A_0} di < 0 \).
Suppose instead $\frac{dg^0}{dx^* i^0} + \int_0 i^0 \frac{d^2 g^0}{dx^0 \partial A_0} \, di < 0$ (and by symmetry

$\frac{dg^1}{dx^* i^0} + \int_0 i^0 \frac{d^2 g^0}{dx^0 \partial A_0} \, di > 0$). Then the numerators of $\frac{d^2 g^0}{\partial p_0 \partial A_0}$ and $\frac{d^2 g^0}{\partial p_0 \partial A_1}$ are both decreased by the same positive quantity, whereby $\partial p_0 / \partial A_0 < -\partial p_0 / \partial A_1$ and, by symmetry,

$\partial p_1 / \partial A_1 < -\partial p_1 / \partial A_0$. It follows that prices decrease with advertising if

$\frac{dg^0}{dx^* i^0} + \int_0 i^0 \frac{d^2 g^0}{dx^0 \partial A_0} \, di < 0$ (and $\frac{dg^1}{dx^* i^1} + \int_0 i^1 \frac{d^2 g^0}{dx^0 \partial A_1} \, di > 0$).

B5. PROOF OF PROPOSITION 3.

The welfare-maximizing social planner sets $A_0, A_1$ simultaneously to maximize $W$ as given in (8). Based on (9), the derivatives of total profits $(j = 0, 1)$ and consumer surplus with respect to advertising are, respectively,

$$\frac{\partial \Pi_T}{\partial A_j} = \frac{\partial p_j}{\partial A_j} F(x_E^*) + p_j f(x_E^*) \left( \frac{\partial^2 x^*}{\partial A_j} + \frac{\partial x^*}{\partial A_j} \frac{\partial p_j}{\partial A_j} \right)$$

$$+ \frac{\partial p_j}{\partial A_j} \left[ 1 - F(x_E^*) \right] - p_j f(x_E^*) \left( \frac{\partial^2 x^*}{\partial A_j} + \frac{\partial x^*}{\partial A_j} \frac{\partial p_j}{\partial A_j} \right) - a$$

$$= \left( \frac{\partial p_j}{\partial A_j} - \frac{\partial p_j}{\partial A_j} \right) F(x_E^*) + (p_0 - p_1) f(x_E^*) \left( \frac{\partial^2 x^*}{\partial A_j} + \frac{\partial x^*}{\partial A_j} \frac{\partial p_j}{\partial A_j} \right) + \frac{\partial p_j}{\partial A_j} - a$$
\[
\frac{\partial W_{CS}}{\partial A_0} = \left( p_1 - p_0 + t - 2tx^*_E + ti^*q \left( x^*_E \right) - ti^*l \left( x^*_E \right) \right) \frac{i^*}{i^*} \left( x^*_E \right) + \int_0^{x^*_E} g^0 \left( x^*_E \right) di + \int_0^{x^*_E} g^1 \left( x^*_E \right) di
\]

\[
\frac{\partial W_{CS}}{\partial A_1} = \left( p_1 - p_0 + t - 2tx^*_E + ti^*q \left( x^*_E \right) - ti^*l \left( x^*_E \right) \right) \frac{i^*}{i^*} \left( x^*_E \right) + \int_0^{x^*_E} g^0 \left( x^*_E \right) di + \int_0^{x^*_E} g^1 \left( x^*_E \right) di
\]

Thus the first-order condition with respect to the planner’s choice of firm 0’s advertising level simplifies to

\[
\begin{align*}
&\left( t \left( 1 - 2x^*_E + ti^*q \left( x^*_E \right) - ti^*l \left( x^*_E \right) \right) - \int_0^{x^*_E} g^0 \left( x^*_E \right) di \right) + \int_0^{x^*_E} g^1 \left( x^*_E \right) di + \left( p_1 - p_0 \right) \left[ 1 - f \left( x^*_E \right) \right] \\
&+ \left( \frac{\partial p_0}{\partial A_0} - \frac{\partial p_0}{\partial A_0} \right) \left[ x^*_E - F \left( x^*_E \right) \right] + \int_0^{x^*_E} \left( t - g^0 \left( i^*q \right) \right) i^* q \left( x^*_E \right) dx - \int_0^{x^*_E} g^0 \left( x^*_E \right) dx = a
\end{align*}
\]

Under symmetry, all but the last term of (A19) drops out. An analogous expression corresponds to firm 1’s advertising.

Meanwhile, recall that firm 0 chooses to advertise to the point represented by the first-order conditions of its profit maximization, to wit,

\[
p_0 f \left( x^*_E \right) \left( \frac{\partial x^*_E}{\partial A_0} \frac{\partial x^*_E}{\partial A_0} \frac{\partial p_0}{\partial A_0} \frac{\partial p_0}{\partial A_0} \frac{\partial x^*_E}{\partial A_0} \frac{\partial x^*_E}{\partial A_0} \right) \frac{\partial x^*_E}{\partial A_0} \frac{\partial p_0}{\partial A_0} F \left( x^*_E \right) = a
\]

So social desirability of the level of advertising chosen by firm 0 depends on the sign of

\[
\frac{\partial x^*_E}{\partial A_0} \frac{\partial x^*_E}{\partial A_0} \frac{\partial p_0}{\partial A_0} \frac{\partial p_0}{\partial A_0} \frac{\partial x^*_E}{\partial A_0} \frac{\partial x^*_E}{\partial A_0} \frac{\partial x^*_E}{\partial A_0} \frac{\partial p_0}{\partial A_0} F \left( x^*_E \right) + \int_0^{x^*_E} g^0 \left( x^*_E \right) dx = a
\]

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If positive, then the firm advertises too much; if negative, too little.

We may simplify (A20) by noting that, similar to the expressions (A18), but corresponding to firm 1’s price,

\[
\frac{\partial^2 x_E}{\partial p_0 \partial A_0} = \left[ -2t + \int_0^t \frac{\partial^2 y}{\partial x} \frac{\partial y}{\partial x} \, di - \int_0^t \frac{\partial^2 y}{\partial x} \frac{\partial y}{\partial x} \, di \right]^{-2}.
\]

Simplifying (A10) and (A17) in view of symmetry and other known facts yields

\[
(A21) \quad \frac{\partial p_0}{\partial A_0} = -\frac{p_0}{2} \frac{\partial^2 x_E}{\partial p_0 \partial A_0}, \quad \frac{\partial p_1}{\partial A_0} = -\frac{p_0}{2} \frac{\partial^2 x_E}{\partial p_0 \partial A_0}.
\]

Substituting these expressions into (A20) yields \( \frac{\partial p_0}{\partial A_0} F(x_E^*) + \int_0^t g_0^* \, didx \). Finally, we can use (A16), (A21), \( F(x_E^*) = \frac{1}{x_E^*} \), the Corollary to Proposition 6, and, based on Proposition 1,

\[
p_0^* = -1\left[ 2f(x_E^*) \frac{\partial^2 x_E}{\partial p_0} \right],
\]

to rewrite as follows:

\[
\frac{\partial p_0}{\partial A_0} F(x_E^*) + \int_0^t g_0^* \, didx = \left[ -\frac{p_0}{2} \frac{\partial^2 x_E}{\partial p_0 \partial A_0} \right] F(x_E^*) + \int_0^t g_0^* \, didx
\]

\[
= -\frac{p_0}{4} \frac{\partial^2 x_E}{\partial p_0} \left[ \int_0^t \frac{\partial^2 y}{\partial x} \frac{\partial y}{\partial x} \, di + \int_0^t \frac{\partial^2 y}{\partial x} \frac{\partial y}{\partial x} \, di \right] - \frac{1}{4} \int_0^t g_0^* \, didx + \int_0^t g_0^* \, didx
\]

\[
= \frac{1}{8} f(x_E^*) \frac{\partial^2 x_E}{\partial p_0} \left[ \int_0^t \frac{\partial^2 y}{\partial x} \frac{\partial y}{\partial x} \, di + \int_0^t \frac{\partial^2 y}{\partial x} \frac{\partial y}{\partial x} \, di \right] - \frac{1}{4} \int_0^t g_0^* \, didx + \int_0^t g_0^* \, didx
\]

\[
= \frac{1}{8} f\left(\frac{1}{2}\right) \left[ \int_0^t \frac{\partial^2 y}{\partial x} \frac{\partial y}{\partial x} \, di + \int_0^t \frac{\partial^2 y}{\partial x} \frac{\partial y}{\partial x} \, di \right] - \frac{1}{4} \int_0^t g_0^* \, didx + \int_0^t g_0^* \, didx
\]
which takes the same sign as the expression given in the statement of the proposition.

B6. PROOF OF PROPOSITION 4.

Follows trivially from the proof of Proposition 3, per argument in the text.

REFERENCES


Figure 1
An Adjustment Map

$g^0(x^*, i)$
Figure 2
Pre- and Post-Advertising Adjustment Maps

$g^{0,0}(x^*, i)$

$g^{0}(x^*, i; A_0 < \infty)$

$g^{0,1}(x^*, i)$

in the limit

pre-advertising

post-advertising

$A_0 \to \infty$
Figure 3
Components of Utility Loss from Selecting Non-ideal Product 0
Figure 4
Shaping and intensification effects
Figure 5
Flattening vs. steepening adjustment maps
Figure 6
Scenario involving under-provision of advertising
i. Target all consumers

ii. Target only inframarginal consumers

iii. Target only consumers at or near margin

Figure 7
Choice of Advertising Strategies
Figure 8
Symmetric Adjustment Map Pairs