Estimation of Outer-Middle Ear Transmission using DPOAEs and Fractional-Order Modeling of Human Middle Ear

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ESTIMATION OF OUTER-MIDDLE EAR TRANSMISSION USING DPOAES AND FRACTIONAL-ORDER MODELING OF HUMAN MIDDLE EAR

by

MARYAM NAGHIBOLHOSSEINI

A dissertation submitted to the Graduate Faculty in Speech-Language-Hearing Sciences in partial fulfilment of the requirements for the degree of Doctor of Philosophy,
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THE CITY UNIVERSITY OF NEW YORK
Abstract

ESTIMATION OF OUTER-MIDDLE EAR TRANSMISSION USING DPOAES AND FRACTIONAL-ORDER MODELING OF HUMAN MIDDLE EAR

by

Maryam Naghibolhosseini

Advisor: Glenis R. Long

Our ability to hear depends primarily on sound waves traveling through the outer and middle ear toward the inner ear. Hence, the characteristics of the outer and middle ear affect sound transmission to/from the inner ear. The role of the middle and outer ear in sound transmission is particularly important for otoacoustic emissions (OAEs), which are sound signals generated in a healthy cochlea, and recorded by a sensitive microphone placed in the ear canal. OAEs are used to evaluate the health and function of the cochlea; however, they are also affected by outer and middle ear characteristics. To better assess cochlear health using OAEs, it is critical to quantify the impact of the outer and middle ear on sound transmission. The reported research introduces a noninvasive approach to estimate outer-middle ear transmission using distortion product otoacoustic emissions (DPOAEs). In addition, the role of the outer and middle ear on sound transmission was investigated by developing a physical/mathematical model, which employed fractional-order lumped elements to include the viscoelastic characteristics of biological tissues. Impedance estimations from wideband reflectance measurements were used for parameter fitting of the model. The model was validated comparing its estimates of the outer-middle ear sound transmission with those given by DPOAEs. The
outer-middle ear transmission by the model was defined as the sum of forward and reverse outer-middle ear transmissions. To estimate the reverse transmission by the model, the probe-microphone impedance was calculated through estimating the Thevenin-equivalent circuit of the probe-microphone. The Thevenin-equivalent circuit was calculated using measurements in a number of test cavities. Such modeling enhances our understanding of the roles of different parts of the outer and middle ear and how they work together to determine their function. In addition, the model would be potentially helpful in diagnosing pathologies of cochlear or middle ear origin.
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I would like to express my gratitude to my precious parents, lovely brothers, and wonderful friends for their constant love, support, and friendship. At last but not least, I would like to thank Mohsen, my love, my best friend, my husband. He was the source of inspiration and encouragement during my PhD research. I thank him with all my heart, for the great love, patience, and support.
For my wonderful husband

Mohsen Zayernouri
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<td>OAE</td>
<td>otoacoustic emission</td>
</tr>
<tr>
<td>DPOAE</td>
<td>distortion product otoacoustic emission</td>
</tr>
<tr>
<td>TM</td>
<td>tympanic membrane</td>
</tr>
<tr>
<td>I/O</td>
<td>input/output</td>
</tr>
<tr>
<td>DP</td>
<td>distortion product</td>
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<td>finite element method</td>
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List of Symbols

\( \frac{d(\cdot)}{dt} \) \hspace{1cm} First derivative
\( \alpha \) \hspace{1cm} Fractional-order of the derivative
\( \alpha_{ALCT} \) \hspace{1cm} Fractional-order derivative of the annular ligament and cochlea
\( \alpha_{MIT} \) \hspace{1cm} Fractional-order derivative of the malleus-incus complex
\( \alpha_{OJLT} \) \hspace{1cm} Fractional-order derivative of the ossicular joints and ligaments
\( \Delta x \) \hspace{1cm} Length change
\( \frac{d^\alpha}{dv^\alpha} \) \hspace{1cm} \( \alpha \)th derivative
\( \hat{P}_{Cavity} \) \hspace{1cm} Estimated pressure at the entrance of the cavity
\( \omega \) \hspace{1cm} Angular velocity
\( \hat{P}_{EC} \) \hspace{1cm} Pressure at the entrance of the ear canal in reverse transmission
\( \hat{P}_{OEC} \) \hspace{1cm} Pressure at the termination of the ear canal
\( \hat{P}_{OTM} \) \hspace{1cm} The pressure at the tympanic membrane termination in reverse transmission
\( \hat{P}_{TM} \) \hspace{1cm} Pressure at the middle ear cavity in reverse transmission
\( \hat{P}_{C} \) \hspace{1cm} Pressure at the cochlea in forward transmission
\( \hat{P}_{EC} \) \hspace{1cm} Pressure at the ear canal in forward transmission
\( A_{fp} \) \hspace{1cm} Stapes-footplate area
\( A_{TM} \) \hspace{1cm} Tympanic membrane area
\( C \) \hspace{1cm} Capacitor
\( c \) \hspace{1cm} Speed of sound in the air
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$DPOAE_{2T}$</td>
<td>$2f_1 - f_2$ DPOAE generated in the two-tone condition</td>
</tr>
<tr>
<td>$DPOAE_{3T}$</td>
<td>$2f_1 - f_2$ DPOAE generated in the three-tone condition</td>
</tr>
<tr>
<td>$e$</td>
<td>Voltage</td>
</tr>
<tr>
<td>$E(s)$</td>
<td>Laplace transform of voltage</td>
</tr>
<tr>
<td>$e_t$</td>
<td>First temporal derivative of voltage</td>
</tr>
<tr>
<td>$e_x$</td>
<td>First spatial derivative of voltage</td>
</tr>
<tr>
<td>$f_r$</td>
<td>Half-wavelength resonance frequency of the cavity</td>
</tr>
<tr>
<td>$FT$</td>
<td>Forward transmission</td>
</tr>
<tr>
<td>$G'_{EC}$</td>
<td>Ear canal gain in reverse transmission</td>
</tr>
<tr>
<td>$G'_{TM}$</td>
<td>Tympanic membrane gain in reverse transmission</td>
</tr>
<tr>
<td>$G_{EC}$</td>
<td>Ear canal gain</td>
</tr>
<tr>
<td>$G_{TM}$</td>
<td>Tympanic membrane gain</td>
</tr>
<tr>
<td>$i$</td>
<td>Current</td>
</tr>
<tr>
<td>$i_t$</td>
<td>First temporal derivative of current</td>
</tr>
<tr>
<td>$i_x$</td>
<td>First spatial derivative of current</td>
</tr>
<tr>
<td>$K$</td>
<td>Fractional capacitors</td>
</tr>
<tr>
<td>$k$</td>
<td>Wave number</td>
</tr>
<tr>
<td>$K_{ALCT}$</td>
<td>Annular ligaments plus cochlea fractional stiffness</td>
</tr>
<tr>
<td>$K_{ALCT}$</td>
<td>Stiffness of the anular ligament and cochlea</td>
</tr>
<tr>
<td>$K_{MEC}$</td>
<td>Stiffness of the middle ear cavity</td>
</tr>
<tr>
<td>$K_{MIT}$</td>
<td>Stiffness of the malleus-incus complex</td>
</tr>
<tr>
<td>$K_{OJLT}$</td>
<td>Stiffness of the ossicular joints and ligaments</td>
</tr>
<tr>
<td>$l_I$</td>
<td>Incus length</td>
</tr>
<tr>
<td>$l_M$</td>
<td>Malleus length</td>
</tr>
<tr>
<td>$l_{Cavity}$</td>
<td>Cavity length</td>
</tr>
</tbody>
</table>
$l_{EC}$  Ear canal length  
$M_{MEC}$  Mass of the middle ear cavity  
$M_{MIT}$  Mass of the malleus and incus  
$M_{ST}$  Mass of the stapes  
$MET$  Ear canal and middle ear gain  
$P_C$  Cochlear pressure  
$P_S$  Probe-microphone pressure  
$P_{Cavity}$  Pressure at the entrance of the cavity  
$P_{EC}$  Pressure at the entrance of the ear canal  
$P_{OEC}$  Pressure at the termination of the ear canal  
$P_{OTM}$  The pressure at the tympanic membrane termination  
$P_{TM}$  Pressure at the tympanic membrane  
$R_{ALCT}$  Annular ligaments plus cochlea resistance  
$R_{ALCT}$  Resistance of the annular ligament and cochlea  
$R_{EC}$  Reflection coefficient at the termination of the ear canal  
$R_{MEC}$  Resistance of the middle ear cavity  
$R_{MIT}$  Resistance of the malleus-incus complex  
$R_{OJLT}$  Resistance of the ossicular joints and ligaments  
$R_{TM}$  Reflection coefficient of the tympanic membrane at the umbo  
$RT$  Reverse transmission  
$T_{TM}$  Tympanic membrane delay  
$U$  Volume velocity  
$Z$  Impedance  
$Z_S$  Probe-microphone impedance  
$Z_T$  Output impedance at the cochlea during reverse transmission
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$Z_{0TM}$</td>
<td>Characteristic impedance of the tympanic membrane</td>
</tr>
<tr>
<td>$Z_0$</td>
<td>Characteristic impedance of the air</td>
</tr>
<tr>
<td>$Z_{ALT}$</td>
<td>Transformed impedance of the annular ligaments</td>
</tr>
<tr>
<td>$Z_{Cavity}$</td>
<td>Impedance of the cavity</td>
</tr>
<tr>
<td>$Z_{CT}$</td>
<td>Transformed impedance of the cochlea</td>
</tr>
<tr>
<td>$Z_{IEC}$</td>
<td>Ear canal input impedance during reverse transmission</td>
</tr>
<tr>
<td>$Z_{IMEC}$</td>
<td>Impedance at the middle ear cavity during reverse transmission</td>
</tr>
<tr>
<td>$Z_{ITM}$</td>
<td>The input impedance at the TM</td>
</tr>
<tr>
<td>$Z_{MEC}$</td>
<td>Impedance of the middle ear cavity</td>
</tr>
<tr>
<td>$Z_{MIT}$</td>
<td>Transformed impedance of the incus-malleus complex</td>
</tr>
<tr>
<td>$Z_{OEC}$</td>
<td>Ear canal terminating impedance</td>
</tr>
<tr>
<td>$Z_{OJLT}$</td>
<td>Transformed impedance of the ossicular joints and ligaments</td>
</tr>
<tr>
<td>$Z_{OTM}$</td>
<td>Terminating impedance at the umbo</td>
</tr>
<tr>
<td>$Z_{ST}$</td>
<td>Transformed impedance of the stapes</td>
</tr>
<tr>
<td>$Z_{TMR}$</td>
<td>Tympanic membrane input impedance during reverse transmission</td>
</tr>
<tr>
<td>CA</td>
<td>Cochlear amplification</td>
</tr>
<tr>
<td>d</td>
<td>Distance</td>
</tr>
<tr>
<td>L</td>
<td>Inductor</td>
</tr>
<tr>
<td>M</td>
<td>Mass</td>
</tr>
<tr>
<td>P</td>
<td>Pressure</td>
</tr>
<tr>
<td>R</td>
<td>Resistor</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

1.1 Middle Ear Transmission

The sounds we hear travel mainly through the outer and middle ear toward the inner ear, where they are further amplified due to a frequency-selective wave-amplification mechanism, then transduced into electrical signals. The outer ear collects the sound energy and sends it toward the tympanic membrane (TM). When a sound signal reaches the TM, part of its energy is transferred to the middle ear and the rest gets reflected back. The amount of reflection and transmission depends on the difference between the impedances of the outer and middle ear (reviewed in Rosowski, 1996; Merchant and Rosowski, 2003). These impedances are the measures of opposition that the outer and middle ear impose to a pressure wave that travels through them.

The middle ear plays an important role coupling the vibrations of the low-impedance air to the high-impedance fluid in the cochlea to maximize the flow of energy towards the inner ear (Merchant and Rosowski, 2003). The middle ear is composed of the TM, ossicles (malleus, incus, and stapes), joints between the ossicles (incudomalleolar and incudostapedial), muscles, and ligaments. The pressure change at the TM results in movements of the umbo, the tip of the malleus process, which is embedded in the TM
in humans. Movements of the umbo result in motion of the ossicles. The malleus and incus transfer the motion of the umbo to the stapes footplate, which is in contact with the cochlear fluid at the oval window. Therefore, vibrations of the stapes footplate will cause the cochlear fluid to vibrate. The rotational and translational motion of the ossicles creates complex bending and compressive movements (reviewed in Rosowski, 1996). The viscoelasticity of the middle ear ligaments, muscles, and joints affects the dynamics of the ossicles (Zhang and Gan, 2011; Bohnke et al., 2013).

The middle ear acts linearly unless it receives feedback from the central nervous system. In humans, the stapedius muscle contracts in response to moderate sound levels of approximately 65 dB SPL (Feeney et al., 2004); this phenomenon is called the *middle ear muscle reflex*. The contraction of the stapedius muscle pulls the neck of the stapes, altering its vibration. The middle ear muscle reflex provides attenuation of the sound energy to protect the inner ear from noise damage; however, there are some limitations associated with this function (reviewed in Pilz et al., 1997; Geisler, 1998). There is a latency in the middle ear muscle activation; therefore, the middle ear reflex may not protect the inner ear effectively in presence of impulse sounds. In addition, the middle ear reflex adapts to long-duration sounds and may not be able to protect the ear when it is exposed to long periods of intense sounds (Moller, 2000).

The impedance difference between the air in the ear canal and the fluid in the cochlea potentially causes reflection of part of the sound energy back to the ear canal. The middle ear partially compensates for this impedance mismatch by providing a pressure gain in the forward direction; namely, the middle ear increases the pressure of the sound that travels from the ear canal toward the cochlea. The ratio of the pressure at the stapes footplate to the pressure at the TM is defined as the *middle ear forward transmission*. The middle ear provides pressure attenuation for sounds traveling in the reverse direction from the inner ear toward the ear canal. The ratio of the pressure in the ear canal at the
TM and the pressure at the stapes footplate, when the middle ear is driven in reverse
direction, is called the middle ear reverse transmission.

1.2 Middle Ear Transmission Estimations/Measurements

Helmholtz (1868) proposed a simple model (ideal transformer model) to estimate middle
ear transmission. Helmholtz modeled the malleus and the incus as two levers and the
TM and the stapes footplate as two pistons. He proposed that the amount of pressure
gain provided by the middle ear can be estimated by the area and lever ratios (reviewed
in Dallos, 1973; Rosowski, 1996). The area ratio is referred to as the area of the TM
divided by the stapes footplate area. The TM has a larger area than the stapes footplate,
resulting in a pressure gain equal to the area ratio. The lever ratio refers to the length of
the malleus divided by the length of the incus; the malleus and the incus together work
as a lever that provides reinforcement of force, which translates into pressure increase
(reviewed in Rosowski, 1996). The pressure gain, estimated by the classical transformer
model of Helmholtz is flat across frequency. The estimated pressure gain in human
cadavers is not flat across frequency (Puria, 2003). Based on the Helmholtz model, the
amount of pressure attenuation and gain in forward and reverse directions should be
equal. In contrast to the Helmholtz model, the pressure gain and attenuation estimated
by in-vivo measurements and also in human cadavers differ from each other (Shera and
Miller, 2002; Puria, 2003; Dong and Olson, 2006). Forward pressure gain is sensitive to
the ear canal load and reverse pressure attenuation is sensitive to the cochlear load (Puria
and Rosowski, 1996; Magnan et al., 1997; Dong and Olson, 2006).

1.2.1 Invasive Experiments

Efforts to measure the middle ear transfer function invasively were made on human
cadavers (Puria and Rosowski, 1996; Puria et al., 1997; Voss et al., 2000; Aibara et al.,
2001; Puria, 2003; Nakajima et al., 2009), gerbils (Dong and Olson, 2006; Ravicz et al., 2008; Dong et al., 2012), cats (Voss and Shera, 2004), chinchillas (Songer and Rosowski, 2007; Ravicz et al., 2010; Ahn et al., 2013), and guinea pigs (Nuttall, 1974; Magnan et al., 1997). To estimate the middle ear forward transmission invasively, a pressure sensor or transducer is usually placed within the vestibule near the stapes footplate and another in the ear canal. The pressure within the vestibule near the stapes footplate divided by the pressure in the ear canal yields the forward pressure transfer function (Magnan et al., 1997; Dong and Olson, 2006). In other investigations, the ratio of the stapes footplate volume velocity and the ear canal pressure was considered as the forward velocity transfer function (Voss and Shera, 2004; Songer and Rosowski, 2007; Ahn et al., 2013). To estimate the volume velocity of the stapes, invasive techniques such as laser-Doppler vibrometry is used (Voss et al., 2000; Songer and Rosowski, 2006). To obtain measurements in the reverse direction, an intracochlear sound source is needed to drive the middle ear in reverse. The reverse middle ear transfer function has been estimated as the pressure in the ear canal divided by the pressure in the scala vestibuli near the stapes footplate (Magnan et al., 1997; Puria, 2003; Dong and Olson, 2006) or divided by the volume velocity of the stapes footplate (Voss and Shera, 2004). Insertion of the stimulus transducer in the cochlea as an intracochlear sound source causes damage to the cochlea; therefore, an alternative approach using distortion product otoacoustic emissions (DPOAEs) has been utilized (Magnan et al., 1997; Voss and Shera, 2004; Dong et al., 2012).

Otoacoustic emissions (OAEs) are signals that are generated inside the cochlea as a result of cochlear active mechanism (Kemp, 1978). When OAEs are generated by presenting two tones in the ear canal, the emissions recorded in the ear canal are called DPOAEs. Presenting two primary tones with frequencies \( f_1 \) and \( f_2 \) in the ear canal results in generation of distortion products (DP) at several frequencies inside the cochlea. The distortion product at \( 2f_1 - f_2 \) can be used as an intracochlear sound source to drive the
middle ear in reverse (Magnan et al., 1997; Voss and Shera, 2004; Dong and Olson, 2006). Although in-vivo/invasive measurements are very valuable, they are not possible in living humans. The cochlea is located in the temporal bone in humans, which makes noninvasive direct measurements of the cochlea impossible. In-vivo/invasive measurements require a long preparation time and are hard to implement. There are potential errors depending on the probe placement, drying out effects, and bleeding of the ear during the experiments (Magnan et al., 1997; Puria, 2003; Voss and Shera, 2004). In laser vibrometry, the measurement angle of the stapes velocity is important and reported to impact the result by 4 dB (Voss et al., 2000). Furthermore, invasive measurements of the outer-middle ear transmission can remain stable for only a couple of hours; this drying out effect can impact the stapes velocity by 10 dB SPL. (Voss et al., 2000). In addition, the mass of the reflective tape used for laser vibrometry may affect the measurement (Voss et al., 2000). Therefore, proposing a noninvasive technique to estimate the middle ear transfer function is needed.

1.2.2 Non-Invasive Techniques

There are not many noninvasive methods (Zwicker and Harris, 1990; Keefe, 2002; Shera and Miller, 2002) to estimate the middle ear transfer function. Zwicker and Harris used a cancellation tone to cancel the DPOAE at $2f_1 - f_2$ in two conditions. In the first condition, the level and phase of the cancellation tone was adjusted by the investigator by means of visual inspection of the DPOAE spectrum. In the second (psychoacoustic) condition, the participant adjusted the phase and level of the cancellation tone until he/she could not hear the distortion product at $2f_1 - f_2$. The levels needed for acoustic cancellation of DPOAE by the investigator were lower than the equivalent levels for the psychoacoustic cancellation. The level difference between the two cancellation tones in the two conditions was used as an estimate of the reverse middle ear transmission (Zwicker and Harris, 1990).
The estimation of the middle ear transmission by Zwicker and Harris was only done at a few discrete frequencies (Zwicker and Harris, 1990). Therefore, the resulting reverse middle ear transfer function had poor frequency resolution. Furthermore, the addition of the cancellation tone may have suppressed the OAE leading to contamination of the results.

Emissions generated in the cochlea were also used as a noninvasive intracochlear sound source to investigate the middle ear transmission (Keefe, 2001, 2002; Shera and Miller, 2002). Horizontal and vertical translations of the input/output (I/O) functions of DPOAEs across frequency were used as an estimate of the middle ear forward and reverse transfer function spectrums, respectively. The I/O function was defined as the DPOAE level as a function of $L_2$ (Keefe, 2001, 2002; Shera and Miller, 2002). The idea was based on cochlear scaling symmetry, which implies that irrespective of the place of maximum vibration on the basilar membrane, different tones travel with the same number of cycles in the cochlea (Shera and Guinan, 2008). Therefore, the shapes of the I/O functions of DPOAE are assumed to stay the same across frequency, and the translation of the I/O functions is a result of middle ear effects (Keefe, 2002). One of the drawbacks to this approach is that the cochlear scaling symmetry assumption might not be valid at all frequencies. Another issue with this technique, and also with Zwicker and Harris’ method, is that they assumed the DPOAE came from a single source in a region of the cochlea (Zwicker and Harris, 1990; Keefe, 2001, 2002). However, DPOAEs are generated in two different regions of the cochlea (Talmadge et al., 1998; Mauermann et al., 1999a), based on two different mechanisms: nonlinear distortion and linear reflection (Shera and Guinan, 1999). The two stimulus tones (called primaries) presented in the ear canal travel to the cochlea to their best frequency places on the basilar membrane. If the frequencies of the two tones are close enough, they will overlap on the basilar membrane and intermodulation distortion will occur in the overlap region.
(Kummer et al., 1995; Brown et al., 1996; Gaskill and Brown, 1996; Talmadge et al., 1997, 1998, 1999; Mauermann et al., 1999a). The acoustic energy generated by the intermodulation distortion travels as a wave both basally and apically. The backward traveling wave is recorded in the ear canal and often called the *generator component* (Talmadge et al., 1998, 1999; Dhar et al., 2002; Shaffer et al., 2003). The generator component (also called the *overlap* or *nonlinear-region component*), is generated due to nonlinear interaction between the two tones in the maximum overlap region, has a short latency and slow phase change with DPOAE frequency. The forward traveling wave travels to its best place on the BM, where it is partially reflected back due to linear reflection from cochlear pre-existing micromechanical perturbations and is called the *reflection component* (Talmadge et al., 1998, 1999; Dhar et al., 2002; Shaffer et al., 2003). The reflection component has a long latency with rapid phase change (Talmadge et al., 1998, 1999; Kalluri and Shera, 2001).

The DPOAE, recorded in the ear canal, is mainly the sum of the two components (Kemp and Brown, 1983; Brown et al., 1996; Talmadge et al., 1997). Since the two components have different sources and mechanisms of generation, they might be affected by different cochlear dysfunctions (Talmadge et al., 1998; Mauermann et al., 1999a,b). The composite DPOAEs measured with high frequency resolution has a quasiperiodic pattern with many dips and peaks and called the *fine structure* in the literature (Kemp and Brown, 1983; Gaskill and Brown, 1990; He and Schmiedt, 1993; Talmadge et al., 1997; Mauermann et al., 1999a; Kalluri and Shera, 2001; Shaffer et al., 2003). The peaks and dips occur as a result of constructive and destructive interference of the generator and reflection components (reviewed in Shera and Guinan, 2008).

The non-invasive techniques for estimating the middle ear transfer function (Zwicker and Harris, 1990; Keefe, 2001, 2002) did not separate the two components; however, the impact of the fine structure may be reduced when data from many participants are
averaged (Keefe, 2001, 2002) but not in individual ones (Zwicker and Harris, 1990).

Measurements and estimations of the middle ear transfer function along with other
data have been used to model the middle ear (Avan et al., 2000; O’Connor and Puria,
2008). Modeling a complex system such as the middle ear helps to understand the
function of the system.

1.3 Middle Ear Modeling

1.3.1 Classical Transformer Model

The main drawback of the Helmholtz model, described in section 1.2, is that it did not
account for the stiffness, damping, or elasticity of the middle ear system. Also, this model
does not consider the impact of cochlear impedance on forward transmission and the ear
canal termination impedance effect on reverse transmission.

1.3.2 Two-Port Models

A simplified model of the middle ear is a two-port model, which considers the middle ear
as a black box. Such models determine a transmission matrix that relates the pressure
and volume velocity in the input port to the pressure and volume velocity in the output
port. For middle ear forward transmission, the input is considered at the ear canal or TM
and the output is at the stapes footplate (Shera and Zweig, 1992; Puria, 2003; Songer
and Rosowski, 2007). A two-port model does not provide details about the function of
each part of the middle ear.

1.3.3 Lumped Element Models

Middle ear function and transmission characteristics depend on how different parts of
the middle ear collaborate during sound transmission. One way to model the middle ear
structures is to employ lumped element modeling (Zwislocki, 1962; Lutman and Martin, 1979; Kringlebotn, 1988; Avan et al., 2000; O’Connor and Puria, 2008). A lumped element model simulates the middle ear as a combination of idealized mechanical elements (e.g., mass, spring, and dashpot) or corresponding equivalent electrical elements (e.g., inductor, capacitor, and resistor). The inertia of the system can be modeled by mass or inductance elements. Mass elements are used in modeling the TM, ossicles, and middle ear cavities. The stiffness of the middle ear can be modeled by spring or capacitance elements. Since the inertia of the middle ear joints, ligaments, and muscles are negligible, springs and dashpots are usually used in modeling them. The system friction may be modeled by dashpot or resistance elements.

Lumped element modeling has been done by fitting the model parameters to pressure/velocity measurements along the middle ear (Zwislocki, 1962; Lutman and Martin, 1979; Avan et al., 2000; O’Connor and Puria, 2008).

1.3.4 Distributed Transmission Line

Lumped element models assume that a mechanical system is lumped into several mechanical/electrical elements. The TM is usually simulated as a single-piston (Shera and Zweig, 1992; Kringlebotn, 1988) or two-piston lumped model (Zwislocki, 1962; Shaw and Stinson, 1983; Goode et al., 1994), which limits it to lower frequencies. At higher frequencies, the motion of the TM becomes more complicated, and different parts of the TM move out of phase relative to each other. A distributed transmission line model that posits a circuit with infinitesimally small inductors, capacitors, and resistors that are distributed continuously along the line was used to model the TM to account for this problem (Puria and Allen, 1998; Puria, 2003; Parent and Allen, 2007).
1.3.5 Finite Element Models

Finite element methods (FEMs) are alternative approaches that take into account the complex geometry and movements of the middle ear (Koike et al., 2002; Sun et al., 2002; Tuck-Lee et al., 2008; Ravicz and Rosowski, 2013). The FEM models consider each part of the middle ear as composed of finite triangular or quadrilateral elements. The idea of FEM is to numerically solve the governing equations on the behavior of a system on a finite number of geometrical sub-domains, called *elements*. This approach requires the corresponding constitutive laws or the governing equations of the deformation of matter of the medium of interest. This method also needs the correct enforcement of boundary and initial conditions. In addition, the FEM model requires material properties and comprehensive morphological data to simulate a 3-D geometrical reconstruction of complex structures such as the middle ear. Histological data of the middle ear along with computer aided 3-D geometric reconstructions were used to determine the geometrics of the model (Buytaert et al., 2011). Three-dimensional reconstruction of the human middle ear can be done using optical tomography (Buytaert et al., 2011) or micro-CT/clinical-CT to obtain anatomical structure data from the middle ear (Lee et al., 2010; Puria and Steele, 2010; Yao et al., 2013). Data collection for developing an FEM model requires considerable pre- and post-processing.

1.3.6 Fractional-Order Lumped Element Modeling

Lumped element modeling has the benefit of quantitatively specifying the function of different parts of the middle ear. The more elements are incorporated into a lumped model, the more accurately the model will predict the middle ear function. However, increasing the number of elements in the model would increase the degrees of freedom and the unknown parameters. Therefore, more measurements will be needed to determine the unknown parameters. Hence, adding more elements may not be an efficient strategy
for improving lumped element models. An alternative approach is to incorporate more realistic lumped elements, which are better adapted to the physical nature of the biological materials in the human ear. For instance, the incudostapedial and incudomalleolar joints, middle ear muscles, and ligaments are all *viscoelastic* (Zhang and Gan, 2011; Bohnke et al., 2013). The viscoelastic materials have been traditionally simplified to a system of linear springs to demonstrate the elasticity in addition to linear (Newtonian) dashpots, to take the viscous effects into account (Schiessel et al., 1995). This viscous effects are usually modeled in terms of the first-order time derivative of the quantity of interest ($c \frac{d}{dt}$).

The generalization of mathematical models involving *integer-order* derivatives to those possessing *fractional-order* elements has been examined truthfully in many other biological applications (Craiem and Armentano, 2007; Magin, 2010). In these modern models, the integer-order time-derivatives are replaced with fractional-order ones ($\frac{d^\alpha}{dt^\alpha}$, $\alpha \in (0, 1)$). One example is the dynamics of human arteries; experimental investigations have shown that they are viscoelastic. Therefore, fractional-order constitutive laws are by far more stable and more realistic models to simulate the dynamics of human arteries (Craiem and Armentano, 2007). These models are ideal for simulating the relationship between stress and strain in viscoelastic materials.

### 1.4 Statement of the Work

The main objective of the proposed research is to develop a *fractional-order* lumped element model of the human middle ear. It is proposed that the dynamics of the viscoelastic ligaments, muscles, and joints can be more effectively modeled using *fractional-order* elements. It is aimed to demonstrate that such fractional-order models seamlessly incorporate the complex effects and multi-scale properties of tissues. The acoustical characteristic of the ear canal was also included in the model. Ear canal input impedance estimates and outer-middle ear transmission (OMET) estimates were utilized to set the
model parameters and validate the performance of the model.

The parameters of the proposed model were determined using ear canal input impedance estimates from wideband reflectance measurements and histological findings of published literature. The magnitude and phase of the input impedance of the ear canal was used for parameter fitting because all the model parameters contribute to the values of this impedance across frequency. The magnitude of the impedance is the ratio of the pressure amplitude to the volume velocity amplitude. The impedance phase indicates the amount by which the volume velocity lags the pressure; in the time domain, the volume velocity is shifted by the ration of phase and angular velocity later with respect to the pressure wave.

To set the model parameters, the governing equations of the fractional-order lumped element model were derived and the variables were set to a fixed value. After the unknown parameters of the model were estimated, the model was validated using the OMET estimates. The model estimates of outer-middle ear transmission were compared to the estimates using DPOAEs.

DPOAEs were used to yield OMET estimates. In addition to generating DPOAEs using the standard two external primaries (two-tone condition), DPOAEs at the same frequency were also generated using one of these external tones and a distortion product generated in the cochlea by two other external tones (three-tone condition). In the two-tone condition, the interaction between \( f_1 \) and \( f_2 \) generated \( DPOAE_{2T} \) (see Fig. 1.1). We note that in this paradigm, \( L_2 \) is presented in the ear canal and it is impacted by the forward ear canal and middle ear transmission in addition to cochlear amplification (CA) before reaching to its characteristic place in the cochlea.

A three-tone condition was employed previously by Shera and Guinan (2007). In our three-tone condition, we kept \( f_1 \) and added the auxiliary tones \( f_a \) and \( f_b \). These extra tones result in a distortion product (\( DP \)) that induces vibration of the BM at the
distortion product place (i.e., $2f_a - f_b$) and goes toward the ear canal and is recorded as $DPOAE'_{3T}$ (see Fig. 1.1). The interaction of $f_a$ and $f_b$ generates a distortion product ($L_{ab}$) that travels to its characteristic place at $2f_a - f_b = f_2$ (i.e. $L'_2$), where the interaction of $L'_2$ with $f_1$ generates $DPOAE_{3T}$. The difference between the primary tone $L_2$ in the two-tone condition and $DPOAE'_{3T}$ in the three-tone condition was assumed as an estimate for the outer-middle ear transmission, which is explained in the following.

Figure 1.1: Schematic of the two- and three-tone conditions. (a) $DPOAE_{2T}$ is generated by the interaction between the two external tones, $f_1$ and $f_2$, in the cochlea. (b) The two external tones $f_a$ and $f_b$ generate $DPOAE'_{3T}$ at $2f_a - f_b$ in the ear canal. The distortion product, generated by $f_a$ and $f_b$ interaction inside the cochlea, interacts with the external tone $f_1$ in the cochlea and generates $DPOAE_{3T}$ at the $2f_1 - f_2$ frequency.

The estimates of reverse middle ear transmission by Zwicker and Harris (1990) gave rise to the idea of comparing an external tone with a DPOAE to estimate the OMET. Zwicker and Harris (1990) asked subjects to adjust the level of an external tone until they could no longer perceive a distortion product generated by two other tones. The
stimuli needed to cancel the perceived distortion product (interpreted as an estimate of the cochlear activity) was higher than the level of a tone needed to cancel the acoustic DPOAE in the ear canal; they interpreted this difference as an approximate estimate of the amount of reverse middle ear transmission (Zwicker and Harris, 1990). Our approach permitted us to indirectly modify a stimulus of level \( L'_2 \) in the cochlea by varying \( L_b \) and comparing the I/O function to that generated when \( L_2 \) in the two-tone condition was varied. When \( DPOAE_{3T} \) and \( DPOAE_{2T} \) were similar, \( L'_2 \) (in the three-tone condition) and \( f_2 \) (in the two-tone condition) are expected to be similar. Since \( L_2 \) is affected by forward transmission (\( FT \)) and cochlear amplification (\( CA \)), the sum of these estimates approximates the value of \( L'_2 \) (Eq. 1.1). Moreover, \( L_{ab} \) is also impacted by \( CA \) such that \( L'_2 \) approximates the sum of \( L_{ab} \) and \( CA \) (Eq.1.2).

\[
L_2 + FT + CA \approx L'_2 \tag{1.1}
\]

\[
L'_2 \approx L_{ab} + CA \tag{1.2}
\]

In the ear canal, we can measure the \( DPOAE'_{3T} \), which is approximately equal to \( L_{ab} \) plus the reverse outer-middle ear attenuation. Therefore,

\[
DPOAE'_{3T} \approx L_{ab} + RT. \tag{1.3}
\]

From Eq. (1.1), (1.2), and (1.3), we obtain

\[
L_2 + FT + CA \approx DPOAE'_{3T} - RT + CA, \tag{1.4}
\]

and therefore,

\[
DPOAE'_{3T} - L_2 \approx RT + FT. \tag{1.5}
\]

Hence, the horizontal distance between the two I/O functions can be interpreted
as the difference between forward and reverse outer-middle ear transmission. The I/O function for the two-tone condition is the level of $DPOAE_{2T}$ as a function of $L_2$ and the I/O function for the three-tone condition is the level of $DPOAE_{3T}$ as a function of $DPOAE'_{3T}$ level.

In order to test the suppressive/enhancing effects of $f_a$ on the DPOAE level at $2f_1 - f_2$ in the three-tone condition, an interaction-control condition was developed. In the interaction-control condition, external tones $f_1$, $f_2$, and $f_a$ were presented and the DPOAE generated at $2f_1 - f_2$ was compared to the $2f_1 - f_2$ DPOAE level in the two-tone condition. The level difference between the DPOAE in the two-tone condition and the DPOAE in the interaction-control condition determined the amount of suppression/enhancement.

The proposed technique to estimate outer-middle ear transmission is among the few noninvasive methods (Zwicker and Harris, 1990; Keefe, 2002; Shera and Miller, 2002) for estimating the middle ear transfer function. Furthermore, this approach does not have the drawbacks of the noninvasive methods explained in 1.2.1.

1.5 Hypotheses

- DPOAE can be used to estimate the outer-middle ear transmission characteristics. The interaction between the reflection and the generator components provides a fine-structure pattern with many peaks and dips. The fine-structure pattern is due to in-phase and out-of-phase interactions of the two components at the stapes. The generator component is more stable than the composite DPOAE and does not have a fine-structure pattern. Therefore, it is hypothesized that the outer-middle ear transmission estimates would be more stable if we remove the reflection component and use the generator component to estimate the outer-middle ear transmission.

- The fractional-order lumped element model provides an efficient simulation of the
middle ear function. In addition, the viscoelastic characteristics of the middle ear elements can be demonstrated through fractional-order constitutive laws in which first-order time derivatives are replaced by fractional-order derivatives.

- The input impedance estimates of the outer ear along with outer-middle ear transmission can be used to define and validate the fractional-order lumped element model of the ear.
Chapter 2

Methods

DPOAE data were used to estimate outer-middle ear transmission. The proposed model was fit to impedance estimates from reflectance measurements (explained in section 2.2.1); and the performance of the model was evaluated by comparing the outer-middle ear transmission estimates from the model with the ones estimated using DPOAEs. To estimate the OMET by the model, the probe-microphone impedance was calculated by measurements in several test cavities.

2.1 Participants

Twelve normal hearing adults, seven females and five males, were recruited for the study. All participants passed an initial hearing evaluation, which included otoscopy, audiometry, and tympanometry. Otoscopy was done to ensure that there was nothing blocking the ear canal. Standard audiometry at half-octave frequencies between 250 – 8000 Hz was performed to measure hearing thresholds. All participants had hearing thresholds of lower than 15 dB. Participants’ middle ear/eardrum function was evaluated using 226 Hz Tympanometry (GSI 33 Middle-ear analyzer) to ensure that TPP was less than 50 daPa.

DPOAE data were obtained from all participants; however, only seven of the participants...
(four females and three males) were included in this study because more complete data sets were obtained from them. Reflectance measurements were available for only six of these seven participants (four females and two males).

2.2 Stimuli and Procedure

2.2.1 Wideband Reflectance

Part of a sound in the ear canal gets reflected back at the TM and part of its energy is absorbed by the middle ear. Reflectance is the ratio of the reflected power to the incident sound’s power. A value of 1.0 for reflectance indicates complete reflection at the TM, and a value of 0.0 indicates complete absorption by the middle ear (reviewed in Feeney et al., 2003; Allen et al., 2005). Wideband reflectance measurements provide estimates of the input impedance of the outer ear (reviewed in Robinson et al., 2013). Impedance is defined as the opposition to movement and can be calculated as the ratio of pressure to volume velocity.

Reflectance measurements were obtained using Mimosa Acoustics Hear ID Middle-Ear Power Analyzer (MEPA3) described in Jeng et al. (2008). The Thevenin-equivalent pressure and impedance of the probe-microphone was estimated in the Mimosa system; any combination of impedances and pressure sources with two terminals can be replaced by a single impedance and a single pressure source, which are called *Thevenin-equivalent impedance and pressure*, respectively. To calculate the Thevenin-equivalent pressure and impedance parameters of the probe system, the system was calibrated by placing the Etymotic ER-10C probe tip of the machine in four different calibration cavities (see section 2.3.4). Subsequently, a 60 dB SPL chirp stimulus covering the broad frequency range between 200 – 6000 Hz was delivered to the participants’ ears using the ER-10C probe. Middle ear impedance was estimated using the estimated impedance and pressure...
measures during the calibration in addition to the pressure measures in participants’ ears.

2.2.2 DPOAE

DPOAEs were generated by either two external tones $f_1$ and $f_2$ (two-tone condition) or three external tones $f_1$, $f_a$, and $f_b$ (three-tone condition). In addition, potential suppression/enhancement by $f_a$ on $2f_1 - f_2$ DPOAE level was evaluated by adding $f_a$ to $f_1$ and $f_2$ in the two-tone condition; this condition is called *interaction-control condition*.

2.2.2.1 DPOAE Data Collection

DPOAE data were obtained from the right ear of each participant while they were sitting quietly in a recliner within a double-walled IAC sound-treated booth. Continuous logarithmically sweeping tones (going up in frequency, 1 second per octave) were used as the stimuli (see section 2.2.2.3 for stimuli levels and frequency ranges).

Stimuli were generated and the ear canal response recorded using custom Mac software (OSX) interfaced with a MOTU 828 Firewire audio interface. After passing through a Tucker Davis Technology headphone buffer (TDT HB6), the signals were delivered to the ear canal using two or three Etymotic ER-2 insert earphones (for the two-tone or the three-tone conditions) coupled to the OAE probe. The ER-2 insert earphones were designed to provide flat frequency responses at the TM. The ear canal response was recorded with an Etymotic three-port ER-10A microphone/preamplifier system connected to a battery-operated Stanford Research Systems SRS 560 low-noise preamplifier. The output of the SR560 was connected to the MOTU, which digitized the signal at a sampling rate of 44.1 kHz, before it was stored on the Mac computer for offline analysis. An appropriate size GSI eartip was placed on the ER-10A probe before inserting the probe into the participants’ ear canals.

The position of the OAE probe (probe fit) was checked using broadband noise,
presented through the three ER-2s, at the beginning, in the middle, and at the end of data collection, to ensure the stability of the OAE probe assembly throughout data collection.

2.2.2.2 Selection of $f_2/f_1$ and $f_b/f_a$

In the three-tone condition, interactions between different distortion products and the primaries may impact the DPOAE level at $2f_1 - f_2$ and should therefore be considered. Specific values of $f_2/f_1$ and $f_b/f_a$ were chosen to minimize the impact of the interactions between distortion products, as detailed out in this section.

Using a frequency ratio between 1.2 and 1.25 results in the largest distortion product at $2f_1 - f_2$ in humans (Gaskill and Brown, 1990; Mauermann et al., 1999a; Dhar et al., 2005; Johnson et al., 2006). Therefore, $f_2/f_1$ and $f_b/f_a$ were limited to this range. To determine the exact ratios, potential interactions among the distortion products and the primaries were considered.

The distortion products generated by interaction of $f_b$ and $f_2$, $f_b$ and $f_1$, $f_a$ and $f_2$, and $f_a$ and $f_1$ can have large amplitudes depending their frequency ratios. These large amplitudes could have suppressive/enhancing impacts on the DPOAE estimates at $2f_1 - f_2$. Therefore, ratios of $f_b/f_2$, $f_b/f_1$, $f_a/f_2$, and $f_a/f_1$ were calculated to check for these effects. Assuming $f_2/f_1 = 1.2-1.25$ and $f_b/f_a = 1.2-1.25$, then

$$f_b/f_2 = 1.5\text{-}1.67,$$
$$f_b/f_1 = 1.8\text{-}2.07,$$
$$f_a/f_2 = 1.25\text{-}1.33,$$
$$f_a/f_1 = 1.55\text{-}1.65.$$

Since $f_a/f_2$ is between 1.25 and 1.33, the largest potential distortion product would be generated by interaction of $f_a$ and $f_2$ at $2f_2 - f_a$. This distortion product should be far enough from $2f_1 - f_2$ and also from the $f_1$ and $f_2$ overlap region (close to the $f_2$ site).
The difference between $2f_2 - f_a$ and $2f_1 - f_2$, and also between $2f_2 - f_a$ and $f_2$ are shown in Fig. 2.1 as a function of frequency ratios $f_b/f_a$ and $f_2/f_1$. It was assumed that the frequency $f_1$ ranged between $1200 - 4800$ Hz (two octaves); all calculations were done for $f_1 = 1200$ Hz.

![Figure 2.1: Difference between $2f_2 - f_a$ and $2f_1 - f_2$ (blue circles) and between $2f_2 - f_a$ and $f_2$ (red circles) shown for frequency ratios $f_b/f_a$ and $f_2/f_1$ (depicted on the x-axis). The green line depicts 100 Hz frequency.](image)

$2f_2 - f_a$ distortion products for all the combinations of frequency ratios $f_b/f_a$ and $f_2/f_1$, depicted in Fig. 2.1, are more than 350 Hz away from the overlap region (close to $f_2$ site) of the $f_1$ and $f_2$ (see the red circles in Fig. 2.1). The frequency ratios that provide large enough frequency differences (i.e., difference of larger or equal to 100 Hz for $(2f_2 - f_a) - (2f_1 - f_2)$) were chosen and are listed in the first column in Table 2.1. The suppressive/enhancing effects of $3f_2 - 2f_a$ and $4f_2 - 3f_a$ distortion products by interaction between $f_2$ and $f_a$ are minimized for all the ratios in Table 2.1, since they are more than 150 Hz away from $2f_1 - f_2$ and more than 700 Hz away from the $f_2$ site.

The distortion product at $2f_a - f_b$ has the largest value when $f_b/f_a = 1.2-1.25$. However, there are other distortion products that are at lower levels but need to still be considered (i.e., $3f_a - 2f_b$ and $4f_a - 3f_b$). The interferences of $3f_a - 2f_b$ and $4f_a - 3f_b$
with $f_1$ were taken into account, and the ratio of $f_1$ and $3f_a - 2f_b$ as well as the ratio of $f_1$ and $4f_a - 3f_b$ were calculated and shown in the second and third columns in Table 2.1.

Table 2.1: $f_1/(3f_a - 2f_b)$ and $f_1/(4f_a - 3f_b)$ for different values of $f_b/f_a$ and $f_2/f_1$ for which $(2f_2 - f_a) - (2f_1 - f_2) \geq 100$ Hz.

<table>
<thead>
<tr>
<th>Ratios</th>
<th>$f_1/(3f_a - 2f_b)$</th>
<th>$f_1/(4f_a - 3f_b)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_b/f_a = 1.2, f_2/f_1 = 1.2$</td>
<td>1.1111</td>
<td>1.6667</td>
</tr>
<tr>
<td>$f_b/f_a = 1.2, f_2/f_1 = 1.21$</td>
<td>1.1019</td>
<td>1.6529</td>
</tr>
<tr>
<td>$f_b/f_a = 1.2, f_2/f_1 = 1.21$</td>
<td>1.1257</td>
<td>1.7646</td>
</tr>
<tr>
<td>$f_b/f_a = 1.2, f_2/f_1 = 1.22$</td>
<td>1.0929</td>
<td>1.6393</td>
</tr>
<tr>
<td>$f_b/f_a = 1.2, f_2/f_1 = 1.22$</td>
<td>1.1164</td>
<td>1.7501</td>
</tr>
<tr>
<td>$f_b/f_a = 1.22, f_2/f_1 = 1.22$</td>
<td>1.1417</td>
<td>1.8804</td>
</tr>
<tr>
<td>$f_b/f_a = 1.2, f_2/f_1 = 1.23$</td>
<td>1.0840</td>
<td>1.6260</td>
</tr>
<tr>
<td>$f_b/f_a = 1.21, f_2/f_1 = 1.23$</td>
<td>1.1074</td>
<td>1.7359</td>
</tr>
<tr>
<td>$f_b/f_a = 1.22, f_2/f_1 = 1.23$</td>
<td>1.1324</td>
<td>1.8651</td>
</tr>
<tr>
<td>$f_b/f_a = 1.23, f_2/f_1 = 1.23$</td>
<td>1.1593</td>
<td>2.0194</td>
</tr>
<tr>
<td>$f_b/f_a = 1.2, f_2/f_1 = 1.24$</td>
<td>1.0753</td>
<td>1.6129</td>
</tr>
<tr>
<td>$f_b/f_a = 1.21, f_2/f_1 = 1.24$</td>
<td>1.0984</td>
<td>1.7219</td>
</tr>
<tr>
<td>$f_b/f_a = 1.22, f_2/f_1 = 1.24$</td>
<td>1.1233</td>
<td>1.8501</td>
</tr>
<tr>
<td>$f_b/f_a = 1.23, f_2/f_1 = 1.24$</td>
<td>1.1499</td>
<td>2.0031</td>
</tr>
<tr>
<td>$f_b/f_a = 1.24, f_2/f_1 = 1.24$</td>
<td>1.1787</td>
<td>2.1889</td>
</tr>
<tr>
<td>$f_b/f_a = 1.2, f_2/f_1 = 1.25$</td>
<td>1.0667</td>
<td>1.6000</td>
</tr>
<tr>
<td>$f_b/f_a = 1.21, f_2/f_1 = 1.25$</td>
<td>1.0897</td>
<td>1.7081</td>
</tr>
<tr>
<td>$f_b/f_a = 1.22, f_2/f_1 = 1.25$</td>
<td>1.1143</td>
<td>1.8353</td>
</tr>
<tr>
<td>$f_b/f_a = 1.23, f_2/f_1 = 1.25$</td>
<td>1.1407</td>
<td>1.9871</td>
</tr>
<tr>
<td>$f_b/f_a = 1.24, f_2/f_1 = 1.25$</td>
<td>1.1692</td>
<td>2.1714</td>
</tr>
<tr>
<td>$f_b/f_a = 1.25, f_2/f_1 = 1.25$</td>
<td>1.2000</td>
<td>2.4000</td>
</tr>
</tbody>
</table>

The frequency ratio $f_1/(4f_a - 3f_b)$ is more than 1.6 for all the cases in Table 2.1. Therefore, we ignored the distortion products generated by the $f_1$ and $4f_a - 3f_b$ interference. Given the values of $f_1/(3f_a - 2f_b)$, the distortion products $3(3f_a - 2f_b) - 2f_1$ and $4(3f_a - 2f_b) - 3f_1$ should be considered at ratios $> 1.09$ and $< 1.09$, respectively; the distortion products at $3(3f_a - 2f_b) - 2f_1$ and $4(3f_a - 2f_b) - 3f_1$ should be far enough from the $f_2$ site and the $2f_1 - f_2$ place. The frequencies of the distortion products at $3(3f_a - 2f_b) - 2f_1$ and $4(3f_a - 2f_b) - 3f_1$ along with $2f_1 - f_2$ for the ratios in Table 2.1
are shown in Fig. 2.2.

Figure 2.2: $2f_1 - f_2$ (red circles) along with $3(3f_a - 2f_b) - 2f_1$ (green squares) and $4(3f_a - 2f_b) - 3f_1$ (black squares) for ratios > 1.09 and < 1.09 (see Table 2.1), respectively.

The ratios that provide the largest distance between $2f_1 - f_2$ and the distortion product at either $3(3f_a - 2f_b) - 2f_1$ or $4(3f_a - 2f_b) - 3f_1$ are $f_b/f_a = 1.25$ and $f_2/f_1 = 1.25$. For the ratios in Table 2.1, $f_2$ was between 1440 – 1500 Hz, which was far enough from $3(3f_a - 2f_b) - 2f_1$ and $4(3f_a - 2f_b) - 3f_1$ distortion products for all the ratios shown in Fig. 2.2. Therefore, frequency ratios $f_b/f_a = 1.25$ and $f_2/f_1 = 1.25$ were considered for obtaining DPOAE data for the following participants: NDP1, NDP3, NDP11, and NDP12.

The ratios $f_2/f_1 = f_b/f_a = 1.225$ were also acceptable since $3(3f_a - 2f_b) - 2f_1$ distortion product was more than 200 Hz away from the $2f_1 - f_2$ distortion product. Because these frequency ratios yield the largest distortion product, $f_2/f_1 = f_b/f_a = 1.225$ were used for obtaining DPOAE from two participants, NDP5 and NDP6. For data collection from NDP2, the ratios $f_2/f_1 = 1.2$ and $f_b/f_a = 1.25$ were used before checking the interactions of the distortion products. Since $2f_2 - f_a$ was very close to $2f_1 - f_2$ when $f_b/f_a = 1.25$ and $f_2/f_1 = 1.2$ (see Fig. 2.1) for NDP2, the DP at $2f_2 - f_a$ might have impacted the DPOAE estimates at $2f_1 - f_2$. To check whether $2f_2 - f_a$ had any suppressive/enhancing
effects on the $2f_1 - f_2$ DPOAE, the interaction-control condition was used.

### 2.2.2.3 Levels and Frequency Ranges

In the two-tone condition, DPOAEs were generated using two external tones, $L_1/f_1$ and $L_2/f_2$ ($f_1 < f_2$). Since $f_b/f_a$ was chosen to be the same as $f_2/f_1$ and equal to 1.25, for NDP1, NDP3, NDP11, and NDP12, the DPOAE generated by $f_a$ and $f_b$ was considered to be two-tone condition data ($DPOAE_{2T}$) for these participants. In the three-tone condition, DPOAEs were generated using external tone $L_1/f_1$ along with the distortion product generated by the $L_a/f_a$ and $L_b/f_b$ interaction inside the cochlea.

The levels and frequency ranges used for each of the participants are shown in Table 2.2. $L_a$ was set to 75 dB SPL (or 65 dB SPL for NDP2) to increase the signal to noise ratio (SNR); The SNR was calculated by subtracting the noise level from the DPOAE level at each frequency. If $f_1$ level was fixed and $f_2$ level varied, the DPOAE level would saturate and start to decrease at higher $f_2$ levels (called turnover) (Brown and Gaskill, 1990; Whitehead et al., 1995). Therefore, DPOAE date were not collected for $L_2$ above 55 dB SPL for most participants due to the occurrence of such turnover at high $f_2$ levels.

Table 2.2: DPOAE levels and frequency ranges for all participants. All the levels are in dB SPL and the frequencies are in Hz.

<table>
<thead>
<tr>
<th>Participant</th>
<th>$L_1$</th>
<th>$L_2$</th>
<th>$L_a$</th>
<th>$L_b$</th>
<th>$f_1$</th>
<th>$f_2$</th>
<th>$f_a$</th>
<th>$f_b$</th>
<th>$f_2/f_1$</th>
<th>$f_b/f_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDP1</td>
<td>65</td>
<td>75</td>
<td>40:5:55</td>
<td>35:5:50</td>
<td>1200-4800</td>
<td>1500-6000</td>
<td>2000-8000</td>
<td>2500-10000</td>
<td>1.25</td>
<td>1.25</td>
</tr>
<tr>
<td>NDP2</td>
<td>20:5:65</td>
<td>20:5:65</td>
<td>1250-5000</td>
<td>1500-6000</td>
<td>2000-7998</td>
<td>2449-9998</td>
<td>1.2</td>
<td>1.25</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The frequency ranges for the two-tone condition for NDP5 where $f_1 = 1250 - 5000$ and $f_2 = 1500-6000$ Hz, which are not shown in the table. Such frequency ranges resulted
in $f_2/f_1 = 1.2$, which was slightly different from $f_2/f_1 = 1.225$ in the three-tone condition for this participant. It was assumed that the DPOAE generated by $f_2/f_1 = 1.2$ and 1.225 are close enough, however, this might slightly impact the estimates of outer-middle ear transmission for NDP5.

At each level, the ear canal recording from every second sweep was assigned to one of two buffers to provide two data sets for each level, minimizing contamination by probe slippage. The number of sweeps for each buffer was 240 at $L_b = 35$ and 40 dB SPL; and 180 at $L_b = 45$ and 50 dB SPL; and 120 at $L_b = 55$ dB SPL for the first four participants in Table 2.2 (i.e., NDP1, NDP3, NDP11, and NDP12). The number of sweeps in the three-tone condition for NDP5 was higher than that for the first four participants in the table; and the number of sweeps for NDP2 was fewer than that for all the other participants in the three-tone condition. A higher number of runs was used to optimize the SNR. Consequently, a larger number of runs was used in the three-tone condition in comparison with the two-tone condition for NDP5, NDP6, and NDP3.

In the interaction-control condition (for NDP2), the levels and frequency ranges of $f_1$ and $f_2$, and also the number of sweeps were the same as those in the two-tone condition for this participant. The level and frequency ranges of $f_a$ were the same as the three-tone condition.

### 2.2.3 Cavity Measurements to Estimate the Probe-Microphone Impedance

To estimate the probe-microphone impedance, Thevenin-equivalent voltage and impedance of the probe-microphone should be determined by pressure measurements in cavities with different lengths. We used a single brass cavity with a metal rod, which was inserted into one end of the cavity and positioned to approach different lengths. The ER-10A probe with a GSI eartip, smaller than those used for the participants to provide a tight fit,
was inserted into the other end of the brass cavity. The cavity lengths of 14.558, 23.804, 33.710, 56.798, and 66.78 mm were used for the pressure measurements. The inside diameter of the brass cavity was 8 mm.

Custom MATLAB software, developed by Shawn Goodman and Rachel Scheperle based on the codes provided by Stephen Neely, along with the multi-channel MATLAB audio Playrec was used for data collection and analysis (see Scheperle et al. (2011)). A MAC computer interfaced with an UltraLite-mk3 Hybrid Motu at a sampling rate of 44.1 kHz was used to present the stimulus and record the response. The stimulus was a wideband chirp across 0.25 – 10 kHz, which was delivered to the cavity by an ER-2 insert earphone coupled to the ER-10A microphone/preamplifier system. Three sets of measurements were collected by the three different ER-2s, which were used as sound sources for obtaining DPOAE in the three-tone condition. The impedance of the probe-microphone was then estimated using the measurements by each ER-2.

2.3 Data Analysis

Prior to extracting the DPOAE levels from the ear canal recordings, visual inspection of the spectrograms of the responses along with an artefact rejection algorithm were used to exclude noisy data. The artefact rejection algorithm down-weighted noisy segments of the recorded ear canal response to reduce the impact of noise on the averages of the sweeps from each buffer.

2.3.1 DPOAE Estimation

Overlapping Hann-windowed segments of the data were analyzed using a least squares fit procedure (Long and Talmadge, 1997) for estimating the primaries, DPOAE, the noise floor, and the generator and reflection components. Using least squares fit, the difference between the predicted waveform and the ear canal recording was minimized
by adjusting the phase and amplitude of the expected components (Long and Talmadge, 1997; Talmadge et al., 1997, 1999; Long et al., 2008). The Hann-window bandwidth was fixed to 8 Hz for estimating the composite DPOAE. The center frequency of the filter changed depending on the DPOAE frequency; the DPOAE phase and level were estimated at each frequency. A narrow-band filter with 2 Hz bandwidth was used to estimate the generator and reflection components. Since there is a delay in generation of the reflection component after the generator component, the reflection component was modeled as a phase-shifted (time latency) component of the generator component (Long et al., 2008). When the narrowband filter is used, only the generator component falls within the filter since there is a delay in generation of the reflection component (Long et al., 2008). When the narrow-band filter with a frequency-dependant latency function is used, the generator component would fall out of the window permitting estimation of the reflection component (Long et al., 2009).

2.3.2 Middle Ear Muscle Activation

Since the primary tone levels $L_1$, $L_a$, and the maximum of $L_b$ were 65, 75, and 55 dB SPL respectively, they may have evoked middle ear reflex activation. Activation of the middle ear reflex changes the impedance of the middle ear, thereby, impacting the primary tones measured in the ear canal (Henin et al., 2014). The changes in primary levels and phases were evaluated for all levels to check for middle ear muscle activation during data collection.

2.3.3 Estimation of Outer-Middle Ear Transmission

The input/output functions for the two- and three-tone conditions were calculated at frequencies with SNR of larger than 6 dB and level difference of larger than 2 dB between the two buffers. Since the DPOAE generated by $f_a$ and $f_b$ was considered as the two-tone
condition for participants \textit{NDP1}, \textit{NDP3}, \textit{NDP11}, and \textit{NDP12}, the I/O functions for the two-tone condition was defined as \(2f_a - f_b\) DPOAE as a function of \(L_b\) for these participants. The I/O function for the other participants in the two-tone condition was defined as \(2f_1 - f_2\) DPOAE\(_{2T}\) as a function of \(L_2\) (see Fig. 1.1). The I/O function for the three-tone condition was defined \(2f_1 - f_2 \ (f_2 = 2f_a - f_b)\) DPOAE\(_{3T}\) as a function of DPOAE\(_{3T}'\) (see Fig. 1.1). The distance between the two I/O functions provided an estimate for the forward plus the reverse outer-middle ear transmission.

In the interaction-control condition, \(f_1\), \(f_2\), and \(f_a\) were presented. The levels of the \(2f_1 - f_2\) generator components in the interaction-control condition were compared with the levels of the \(2f_1 - f_2\) generator components in the two-tone condition. The \(2f_1 - f_2\) level differences between the two-tone and the interaction-control conditions were used to evaluate amount of suppression/enhancement by \(f_a\) on \(2f_1 - f_2\) DPOAE.

### 2.3.4 Estimation of Probe-Microphone Impedance

Based on the Thevenin theorem, the probe-microphone circuit can be replaced by a voltage source (equivalent to pressure) and an impedance, depicted as \(P_S\) and \(Z_S\) in Fig. 2.3. \(Z_{\text{Cavity}}\) and \(P_{\text{Cavity}}\) denote the cavity impedance and voltage, respectively. \(Z_{\text{Cavity}}\) was determined by modeling each cavity as a viscothermal tube, considering viscous and thermal effects (explained in Keefe (1984) and Keefe et al. (1992)).

![Figure 2.3: Thevenin source equivalent circuit.](image)

By applying Kirchhoff’s voltage law (i.e., the sum of the products of the impedances
and the currents is equal to the sum of voltage sources in a closed-loop circuit) to the circuit in Fig. 2.3, the following equation was obtained:

\[
Z_{\text{Cavity}} * P_{\text{Cavity}} = Z_{\text{Cavity}} * P_S - Z_S * P_{\text{Cavity}},
\]

(2.1)

Using Eq. 2.1, a matrix of the known parameters (i.e., measured pressures and impedances of the cavities) and the unknown parameters (i.e., \(Z_S\) and \(P_S\) of the source) was built and a least squares fit was applied to determine the unknown parameters (explained in Scheperle et al., 2011).

Once one has \(P_S\) and \(Z_S\), \(P_{\text{Cavity}}\) was determined for each cavity size using Eq. 2.1. The accuracy of the estimations was then assessed comparing the estimated \(P_{\text{Cavity}}\) and the measured \(P_{\text{Cavity}}\). The estimation error was defined as the ratio of the squared difference of the measured and estimated \(P_{\text{Cavity}}\) summed over the frequency and across the cavities and the sum of squared measured cavity pressures. The error was defined by the following formula:

\[
E = \frac{\sum \sum |P_{\text{Cavity}} - \hat{P}_{\text{Cavity}}|^2}{\sum \sum |P_{\text{Cavity}}|^2} * 10000,
\]

(2.2)

where \(\hat{P}_{\text{Cavity}}\) is the estimated \(P_{\text{Cavity}}\). The formula is scaled by 10000 to bring the estimated error to the order of one.

After estimating the error, an iterative fitting procedure was followed until an error smaller than 2 was yielded. Knowing the lengths of the cavities, the half-wavelength resonance frequencies of the cavities were calculated using the following equation:

\[
f_r = \frac{c}{2 * l_{\text{Cavity}}},
\]

(2.3)

where \(f_r\), \(c\), and \(l_{\text{Cavity}}\) denote resonance frequency, speed of sound, and cavity length, respectively. The iterative algorithm searched for a frequency with maximum pressure around the expected resonance frequency \(f_r\) for each cavity. Then the estimated lengths
of the cavities were calculated using the estimated resonance frequencies (using Eq. 2.3). Using the estimated lengths of the cavities, $Z_S$ and $P_S$ were estimated again until an error smaller than 2 was obtained. The estimation error by the third source (the ER-2 used to generate $f_b$) had the lowest error (1.48); hence, the probe-microphone impedance estimated by the third sound source was used for the model. The computed cavities impedances and estimated cavities impedances using this source are shown in Fig. 2.4 by solid and dashed lines, respectively. Different cavity sizes (shown in the legend, Fig. 2.4) are depicted by different colors.

![Cavities impedances](image)

Figure 2.4: Computed impedances of the cavities (solid lines) and the estimated impedances of the cavities (dashed lines), by the ER-2 used to generate $L_b$ in the three-tone condition.

### 2.4 Fractional-Order Model of Human Ear

A network description of the human ear by Rosowski (1996) is shown in Fig. 2.5. Each block of this network can be modeled by a combination of masses, springs, and dashpots.
The proposed model in this work has all the blocks of Fig. 2.5 except the concha horn because all measurements were done by inserting the probe-microphone into the ear canal. The proposed model can be seen in Fig. 2.6. Note that inductors, capacitors, and resistors are equivalent to masses, springs, and dashpots, respectively. The pressures are denoted by $P$ and are equivalent to voltage in electrical circuits. The impedances of different parts of the ear are denoted by $Z$. The transformers account for changes in the pressure wave through the outer-middle ear due to area changes at the TM and stapes-footplate, as well as the force reinforcement by the lever mechanism of the malleus and the incus. The number of turns of the transformers, associated with changes in TM and stapes footplate area and malleus-incus force reinforcement, are written below each transformer in Fig. 2.6. $A_{TM}$, $l_I$, $l_M$, and the $A_{fp}$ denote the TM area, the incus length, the malleus length, and the stapes-footplate area, respectively. Because the masses of the ossicular ligaments, muscles, and joints are negligible, they are lumped into a fractional viscoelastic element and a resistor.
Figure 2.6: Proposed fractional-order model of the human ear.

The transformed version of the model in Fig. 2.6 is depicted in Fig. 2.7. The resistors, masses, and the fractional capacitors are depicted by $R$, $M$, and $K$, respectively. Note that the stiffness was used instead of the capacitance. The stiffness is equivalent to the inverse of the capacitance.

Figure 2.7: The transformed circuit of the fractional-order model of the human ear.

The number of turns of the transformers determines the relationships between the impedances in the original model in Fig. 2.6 and the transformed model in Fig. 2.7. The impedances in the transformed model are related to the impedances of the model in Fig. 2.6 according to the following equations:

$$Z_{MIT} = \frac{Z_{MI}}{A_{TM}^2},$$  \hspace{1cm} (2.4)
The ear canal was considered as a rigid-walled tube and was modeled by a distributed parameter lossless transmission line. Using the wave equation 2.9 for sound propagation in a lossless tube, where \( k \) is the wave number, Eq. 2.10 and 2.11 can be derived.

\[
P_{xx} + k^2 P = 0 \quad (2.9)
\]

\[
P = P^+ e^{jkd} + P^- e^{-jkd} \quad (2.10)
\]

\[
U = U^+ e^{jkd} + U^- e^{-jkd} \quad (2.11)
\]

In Eq. 2.11, \( U \) denotes the volume velocity and \( d \) is the distance from the end of the tube. The + and – superscripts denote the forward traveling wave and the reflected wave, respectively. The impedance at any point in the ear canal tube can be calculated using the following equation:

\[
Z(d) = \frac{P(d)}{U(d)} = \frac{Z_{0EC} e^{jkd} + R_{EC} e^{-jkd}}{e^{jkd} - R_{EC} e^{-jkd}}, \quad (2.12)
\]

where \( Z_{0EC} \) denotes the characteristic impedance of the air, which is \( 9.24 \times 10^6 \text{ kg/(s.m}^4) \) at 22°C. \( R_{EC} \) is the reflection coefficient at the termination of the ear canal, which is represented by the following:

\[
R_{EC} = \frac{Z_{OEC} - Z_0}{Z_{OEC} - Z_0} \quad (2.13)
\]
in which $Z_{OEC}$ represents the impedance of the ear canal termination (see Fig. 2.7). From Eq. (2.12), the impedance of the ear canal at the place of the probe is represented by

$$Z(l_{EC}) = Z_{0EC}e^{jkl_{EC}} + R_{EC}e^{-jkl_{EC}},$$

(2.14)

where $l_{EC}$ is the length of the ear canal from the probe to the TM, which was considered as an unknown parameter in the model. The wave number $k$ is equal to $\omega/c$, where $\omega$ is the angular velocity and $c$ is the speed of sound in the air, which is equal to 344.43 m/s at 22 °C. Using the classic transmission line model for the ear canal, the pressure gain provided by the ear canal can be estimated as

$$G_{EC} = \frac{P_{OEC}}{P_{EC}} = \frac{1 + R_{EC}}{e^{jkl_{EC}} - R_{EC}e^{-jkl_{EC}}},$$

(2.15)

in which $P_{OEC}$ is the pressure at the termination of the ear canal (see Fig. 2.7). The TM was modeled by a fractional-order transmission line in which fractional capacitors were used instead of integer-order capacitors. The use of fractional-order elements impacts the wave equation in ways, which will be explained in this section.

The classic lossless transmission line is shown in Fig. 2.8. The voltages and the currents are denoted by $e$ and $i$, respectively. $\Delta x$ represents very small changes in the length of the transmission line. $L$ and $C$ denote an inductor and a capacitor, respectively.

![Figure 2.8: Classic lossless transmission line circuit.](image)

By applying the Kirchhoffs voltage law to the circuit in Fig. 2.8, the following equations
for a lossless classic transmission line can be obtained

\[ e(x - \Delta x) - e(x) = L\Delta x \frac{\partial i}{\partial t}, \]  
(2.16)

and

\[ i(x) - i(x + \Delta x) = C\Delta x \frac{\partial e}{\partial t}. \]  
(2.17)

From Eq. (2.16) and (2.17), it can be derived that

\[ e_x + Li_t = 0, \]  
(2.18)

and

\[ i_x + Ce_t = 0, \]  
(2.19)

where \( e_x \) and \( i_x \) denote the first spatial derivative of voltage and current, respectively; \( e_t \) and \( i_t \) indicate the first temporal derivative of voltage and current, respectively. Eq. (2.19) is derived if the derivatives are integer-orders. If the capacitor is considered as a fractional-order element, then Eq. (2.19) turns out to be the following:

\[ i_x + C^{\alpha} \frac{\partial^{\alpha} e}{\partial t^{\alpha}} = 0, \]  
(2.20)

in which \( \alpha \) is the fractional-order of the derivative. Taking the first derivative of Eq. (2.18) with respect to \( x \) and plugging \( i_{tx} \) into Eq. (2.20), it can be shown that

\[ e_{xx} - LC^{\alpha} \frac{\partial^{\alpha+1} e}{\partial t^{\alpha+1}} = 0. \]  
(2.21)

Taking the Laplace transform of Eq. (2.21), the following is derived:

\[ \frac{d^2 E(s)}{dx^2} - LC^{\alpha} s^{\alpha+1} E(s) = 0. \]  
(2.22)
\( E(s) \) denotes the Laplace transform of \( e \). Solving Eq. (2.22) for \( E(x, s) \) results in

\[
E(x, s) = E^+ e^{\sqrt{LC^\alpha}s^{\alpha+1}d} + E^- e^{-\sqrt{LC^\alpha}s^{\alpha+1}d}.
\] (2.23)

Therefore, the input impedance of the transmission line is

\[
Z = Z_0 e^{\sqrt{LC^\alpha}s^{\alpha+1}d} + Re^{-\sqrt{LC^\alpha}s^{\alpha+1}d} - Re^{-\sqrt{LC^\alpha}s^{\alpha+1}d} e^{\sqrt{LC^\alpha}s^{\alpha+1}d},
\] (2.24)

where, \( R \) is the reflection coefficient at the termination of the transmission line.

According to Eq. 2.24, the input impedance of the fractional-order transmission line model of the TM will be

\[
Z_{ITM} = Z_{0TM} e^{T_{TM} \sqrt{s^{\alpha+1}}} + R_{TM} e^{-T_{TM} \sqrt{s^{\alpha+1}}} - R_{TM} e^{-T_{TM} \sqrt{s^{\alpha+1}}} e^{T_{TM} \sqrt{s^{\alpha+1}}}.
\] (2.25)

Here, \( Z_{0TM} \) indicates the characteristic impedance of the TM and \( T_{TM} \) denotes the delay for sound to pass through the TM. The transmission line modeling approach suggests that there are forward and reflected traveling waves on the TM surface; the reflections occur at the umbo due to the difference between the TM characteristic impedance and the input impedance at the umbo. The constructive and destructive interference of the forward and reflected waves results in formations of standing waves on the TM surface. The reflection coefficient of the TM at the umbo (i.e., \( R_{TM} \)) can be found by the following equation:

\[
R_{TM} = \frac{Z_{OTM} - Z_{0TM}}{Z_{OTM} - Z_{0TM}}.
\] (2.26)

The termination impedance of the TM at the umbo is denoted by \( Z_{OTM} \). Using the fractional-order transmission line model for the TM, the amount of pressure gain provided
by the TM can be estimated by

\[ G_{TM} = \frac{P_{OTM}}{P_{TM}} = \frac{1 + R_{TM}}{e^{T_{TM} \sqrt{s^\alpha + 1}} + R_{TM} e^{-T_{TM} \sqrt{s^\alpha + 1}}}. \] (2.27)

\( T_{TM} \) and \( Z_{0TM} \) were considered as the unknown parameters in the model. Therefore, in addition to \( l_{EC} \), the masses, resistors, and fractional capacitors in Fig. 2.7, \( T_{TM} \) and \( Z_{0TM} \) should also be considered during the parameter fitting procedure.

### 2.4.1 Parameter Fitting of the Model

The unknown parameters of the model were found by fitting the model to the impedance magnitudes and phases estimates from power reflectance measurements. To reduce the number of unknown parameters, the resistors and fractional capacitors of the annular ligaments and the cochlea were added (i.e., Eq. 2.28 and 2.29).

\[ R_{ALCT} = R_{ALT} + R_{CT} \] (2.28)

\[ K_{ALCT} = K_{ALT} + K_{CT} \] (2.29)

The initial fitting was done on the simplified version, shown in Fig. 2.9, of the transformed model to reduce the complexity (degrees of freedom) of the problem.

![Simplified model of the ear](image)

**Figure 2.9:** Simplified model of the ear.

Parameters \( R_S, M_S, \) and \( K_S \) were set first by fitting to the impedance magnitude estimate from the reflectance measurements to minimize the sum of squared error between
the impedance estimate by the model and that from the reflectance measurements. The ear canal and the TM were included in the model next and \( l_{EC}, T_{TM}, \) and \( Z_{0TM} \) were calculated. The parameters found for the mass, the stiffness, the resistor, the ear canal, and the TM in the model shown in Fig. 2.9 were used as the initial values for setting the parameters of the model in Fig. 2.7.

The parameter fitting of the transformed model in Fig. 2.7 was done in several steps. The ear canal and the TM parts of the model were included during all the fitting steps. However, in each step, a set of elements were added to the basic model including the ear canal and the TM. In the first step, the fractional capacitors were included and were set in order to minimize the sum of squared error of the impedance magnitude at low frequencies. The impedance of the fractional capacitors is presented by the following equation:

\[
Z_K = \frac{K_{ALCT}K_{OJLT}}{K_{ALCT}S^{\alpha_{OJLT}} + K_{OJLT}S^{\alpha_{ALCT}}} + \frac{K_{MIT}}{S^{\alpha_{MIT}}}. \tag{2.30}
\]

The ear canal input impedance was found using Eq. 2.14, in which \( R_{EC} \) was derived from Eq. 2.13. The TM impedance and its reflection coefficient were determined using Eq. 2.25 and Eq. 2.26. All the impedances were calculated in the Laplace domain (i.e., \( j\omega = s \)).

In the next step, masses were included in addition to the fractional capacitors. The masses were set in order to minimize the sum of squared error between the estimates of the impedance magnitude by the model and from the reflectance measurements. The impedance of the masses and fractional capacitors were found by

\[
Z = \frac{(M_{ST}.S^{(1+\alpha_{ALCT})} + K_{ALCT}).K_{OJLT}}{M_{ST}.S^{(1+\alpha_{ALCT}+\alpha_{MIT})} + K_{ALCT}.S^{\alpha_{OJLT}} + K_{OJLT}.S^{\alpha_{ALCT}}} + \frac{K_{MIT}}{S^{\alpha_{MIT}}} + M_{MIT}.S. \tag{2.31}
\]

The resistors were added next and were set to minimize the sum of squared errors.
of both the impedance magnitudes and phases estimates. The impedance of the masses, resistors, and fractional capacitors found in this step, denoted by $Z_{OTM}$, is presented by the following:

$$Z_{OTM} = Z_{MIT} + \frac{Z_{OJLT}(Z_{ST} + Z_{ALCT})}{Z_{OJLT} + Z_{ST} + Z_{ALCT}}.$$ \hspace{1cm} (2.32)

$Z_{MIT}$, $Z_{OJLT}$, $Z_{ST}$, and $Z_{ALCT}$ in Eq. 2.32 were found using the following equations:

$$Z_{MEC} = M_{MEC}.s + K_{MEC}/s + R_{MEC},$$ \hspace{1cm} (2.33)

$$Z_{MIT} = M_{MIT}.s + K_{MIT}/s^{\alpha_{MIT}} + R_{MIT},$$ \hspace{1cm} (2.34)

$$Z_{OJLT} = K_{OJLT}/s^{\alpha_{OJLT}} + R_{OJLT},$$ \hspace{1cm} (2.35)

$$Z_{ST} = M_{ST}.s, \text{and}$$ \hspace{1cm} (2.36)

$$Z_{ALCT} = K_{ALCT}/s^{\alpha_{ALCT}} + R_{ALCT}.$$ \hspace{1cm} (2.37)

The fractional-orders of the derivatives for the capacitors were then set in order to minimize the sum of squared errors of the impedance magnitudes and phases estimates. The middle ear cavity parameters (i.e., $M_{MEC}$, $R_{MEC}$, and $K_{MEC}$) were added next. Since the ear canal and the TM parameters were set taking into account only a mass, a resistor, and a capacitor, the TM and the ear canal parameters were determined again. Therefore, in this step, the parameters of the middle ear cavity, the ear canal, and the TM were set in order to minimize the sum of squared errors of the impedance magnitudes and phases estimates. In this step $Z_{OEC}$ was calculated according to the following equation:

$$Z_{OEC} = Z_{ITM} + M_{MEC}.s + K_{MEC}/s^{\alpha_{MEC}} + R_{MEC}.$$ \hspace{1cm} (2.38)

After performing the aforementioned steps for parameter fitting, a fine-tuning was done to find a more accurate impedance and a better fit.
2.4.2 Model Evaluation

The performance of the model was evaluated by comparing the OMET (i.e., the forward plus reverse transmission) estimated by the model and the DPOAE. The forward outer-middle ear transmission in the model was calculated by

\[
FT = \frac{\overrightarrow{P_C}}{\overrightarrow{P_{EC}}} = \frac{G_{EC}.G_{TM}.Z_{ITM}.Z_{CT}.Z_{OJT}}{Z_{OTM}.Z_{OEC}.(Z_{OJT} + Z_{ST} + Z_{ALCT})}.A_{TM}.l_I/A_{fp}.l_M,
\]  

(2.39)

in which \(P_C\) is the pressure in the cochlea.

During the reverse transmission, the goal was to find \(\frac{\overrightarrow{P_{EC}}}{\overrightarrow{P_C}}\) when sound travels from the cochlea toward the ear canal. The transformed model for the reverse transmission is shown in Fig. 2.10. \(Z_S\) is the estimated impedance of the probe-microphone.

Figure 2.10: Transformed fractional-order model of the outer-middle ear during reverse sound transmission.

The outer-middle ear reverse transmission by the model was calculated by

\[
RT = \frac{\overrightarrow{P_{EC}}}{\overrightarrow{P_C}} = \frac{G'_{EC}.G'_{TM}.Z_{IEC}.Z_{TMR}.Z_{OJT}}{Z_T.Z_{IMEC}.(Z_{OJT} + Z_{MIT} + Z_{TMR})}.A_{TM}.l_I/A_{fp}.l_M,
\]  

(2.40)

where \(G'_{EC}\) and \(G'_{TM}\) denote the gain of the ear canal and the TM in the reverse direction, respectively, which are represented by

\[
G'_{EC} = \frac{\overrightarrow{P_{EC}}}{\overrightarrow{P_{OEC}}} = \frac{1 + R'_{EC}}{e^{jkl_{EC}} + R'_{EC}e^{-jkl_{EC}}},
\]  

(2.41)
and

\[ G'_{TM} = \frac{\bar{P}_{TM}}{\bar{P}_{OTM}} = \frac{1 + R'_{TM}}{e^{T_{TM} \sqrt{s^{\alpha + 1}}} + R'_{TM} e^{-T_{TM} \sqrt{s^{\alpha + 1}}}}, \tag{2.42} \]

in which \( R'_{EC} \) and \( R'_{TM} \) denote the reflection coefficients at the terminations of the ear canal and the TM, respectively, in the reverse transmission and were found from

\[ R'_{EC} = \frac{Z_S - Z_0}{Z_S + Z_0}, \tag{2.43} \]

and

\[ R'_{TM} = \frac{Z_{IMEC} - Z_0}{Z_{IMEC} + Z_0}. \tag{2.44} \]

The OMET is defined as the addition of forward and reverse sound transmission and was determined in dB units using

\[ MET = 20 \log(FT) + 20 \log(RT). \tag{2.45} \]
Chapter 3

Results

The outer-middle ear transmissions (i.e., sum of forward and reverse outer-middle ear transmissions) were estimated using the generator components and the composite DPOAEs. The probe fit was checked by comparing $L_1$ and $L_a$ levels and phases at different $L_b$s. The parameters of the proposed model were set using impedance estimates from power reflectance measurements. The model was then evaluated by comparing its estimates of outer-middle ear transmission with the ones estimated using DPOAE generator components.

3.1 Estimation of Outer-Middle Ear Transmission using DPOAEs

The $2f_1 - f_2$ composite DPOAEs, generator components, and reflection components for the three-tone condition for participant NDP12 are shown in Fig. 3.1. The solid and dashed lines depict data extracted from buffer 1 and buffer 2, respectively. The two buffers are depicted to show the stability of the measurements. As can be seen in panel (a), the estimated DPOAE levels approach the noise at low frequencies; leading to an increase in the discrepancies between the two buffers at these frequencies. The generator components (depicted in panel (b)) are well above the noise floor for most frequencies except at very low frequencies. Consequently, the SNR is smaller at low frequencies,
which is reflected in the discrepancies between the two buffers. Comparing panel (a) and (b), reveals that, as expected, the generator components are less variable across frequency and less affected by noise than the composite DPOAEs. Consequently, the generator components were used to estimate the outer-middle ear transmission.

![Graphs showing (a) 2f₁ - f₂ Composite DPOAE, (b) Generator component, and (c) Reflection component at different L_b values.](image)

Figure 3.1: [NDP12] 2f₁ - f₂ composite DPOAEs (panel a), generator components (panel b), and reflection components (panel c) at different L_b values (shown in the legend) of the three-tone condition. Buffer 1 and buffer 2 are depicted by solid and dashed lines, respectively. The noise levels are shown by red dashed lines.

As seen in panel (c), the reflection components are lower in level and closer to the noise than the generator components (panel (b)), leading to lower SNRs and more
contamination of the reflection component estimates by noise. The in-phase and out-of-phase interaction of the reflection and generator components results in the composite DPOAE, which is not stable across frequency (see panel (a)).

Detailed plots of the Composite DPOAEs and their components for other participants are presented in Appendix A.

Because $f_b/f_a = 1.25$ had the same value as $f_2/f_1$, the $2f_a - f_b$ DPOAE was used as the two-tone condition for the frequency ranges where both $2f_a - f_b$ and $2f_1 - f_2$ were available for NDP1, NDP3, NDP11, and NDP12. The $2f_a - f_b$ generator components along with the $2f_1 - f_2$ generator components for NDP12 are depicted in Fig. 3.2. As seen, the $2f_a - f_b$ generator components (two-tone condition) are higher in level than the $2f_1 - f_2$ generator components (three-tone condition). Outer-middle ear transmission for NDP12 was estimated between 1638 – 3271 Hz because $2f_1 - f_2$ and $2f_a - f_b$ generator components were both available for these frequencies.

![Figure 3.2:](image)

Figure 3.2: [NDP12] $2f_1 - f_2$ generator components (three-tone condition) and $2f_a - f_b$ generator components (two-tone condition) depicted by solid and dashed lines, respectively, at different $L_b$s (shown in the legend) for the buffer 1. The red dashed lines depict the beginning and end of the frequency range for which both $2f_1 - f_2$ and $2f_a - f_b$ generator components are available.

To estimate the OMET, the DPOAE and the generator components I/O functions of the two- and three-tone conditions were extracted in the aforementioned frequency range for NDP12. The I/O functions of the generator components of the two- and three-tone conditions for NDP12 at an arbitrary frequency $2f_1 - f_2 = 3001$ Hz are depicted by red
and blue circles in Fig. 3.3. Two inclusion criteria were used for including data at each primary level and frequency across different levels and frequencies: the SNR should be larger than 6 dB SPL and the level difference between the two buffers should not exceed 2 dB SPL. All the points, shown in Fig. 3.3, passed the inclusion criteria.

The horizontal distance between the two- and three-tone conditions I/O functions was considered as an estimate for the reverse outer-middle ear transmission plus the forward outer-middle ear transmission. To find the horizontal distance between the two I/O functions, a line was fitted to each I/O function using a least squares technique (dashed lines in Fig. 3.3). The distance between the two I/O functions was estimated as the mean of the distances $a_{\text{max}}$ and $a_{\text{min}}$ (orange arrows in Fig. 3.3). Here, $a_{\text{max}}$ denotes the distance between the $DPOAE'_{3T}$ generator components and $L_2$ both associated with the maximum value of $2f_1 - f_2$ generator component in the three-tone condition. Moreover, $a_{\text{min}}$ represents the distance between the $DPOAE'_{3T}$ generator component and $L_2$ both associated with the minimum value of $2f_1 - f_2$ generator component in the two-tone condition. The mean of $a_{\text{min}}$ and $a_{\text{max}}$ was considered as an estimate of the outer-middle ear transmission.

![Graph](image)  

Figure 3.3: [NDP12] Two- and three-tone I/O functions (red and blue circles) along with fitted lsf lines for the two- and three-tone conditions (magenta and cyan dashed lines) at $2f_1 - f_2 = 3001$ Hz. The distance between the two I/O functions was estimated as the mean of $a_{\text{max}}$ and $a_{\text{min}}$.

The distances between the two- and three-tone conditions I/O functions were determined
at frequencies where the data fitted our inclusion criteria, and is plotted as a function
of frequency for \textit{NDP12} in Fig. 3.4. In addition to the estimation of the OMET from
the generator components, shown by the green stars in Fig. 3.4, the OMET was also
estimated using the composite DPOAEs, depicted by red stars. As can be seen, the
OMET estimated by the generator components looks like a lowpass filtered version
of the OMET estimated using the composite DPOAEs. The OMET estimated using
the composite DPOAEs has many dips and peaks, while, the OMET estimated by the
generator components is more stable across frequency.

![NDP12 OMET estimates using the composite DPOAEs (red stars) and the
generator components (green stars); \(L_b = 35\) to \(55\) dB SPL in \(5\) dB steps.]

Figure 3.4: \textit{NDP12} OMET estimates using the composite DPOAEs (red stars) and the
generator components (green stars); \(L_b = 35\) to \(55\) dB SPL in \(5\) dB steps.

As seen in Fig. 3.4, the OMET estimates using the generator components are negative;
the OMET estimates were calculated and are shown next to the \(2f_1 - f_2\) and \(2f_a - f_b\)
generator components levels in Fig. 3.5. The frequencies corresponding to the maxima
and minima of the OMET are marked by the red and black dashed lines, respectively, in
Fig. 3.5 (note that the OMET is negative). As depicted, the lower frequency maximum
in the OMET happens at a local maximum in the \(2f_a - f_b\) generator components and a
local minimum in the $2f_1 - f_2$ generator components. The higher frequency maximum in the OMET occurs at the deep local minimum of the $2f_1 - f_2$ generator components. The lower frequency minimum in the OMET happens at a local minimum at the $2f_a - f_b$ levels and a local maximum in the $2f_1 - f_2$ generator components levels. The higher frequency local minimum in the OMET occurs at a frequency close to a local maximum in the $2f_1 - f_2$ generator components.

Figure 3.5: [NDP12] Left panel: OMET estimates using the generator components. Right panel: $2f_1 - f_2$ and $2f_a - f_b$ generator components levels. The frequencies of the maxima and minima of the OMET estimates are marked by the red and black dashed lines, respectively, in both graphs.

In another participant, NDP11, the DPOAE data obtained when $L_b = 35$ and 55 dB SPL were not included in OMET estimations because the probe-fit measurements changed significantly due to probe repositioning and slippage at these levels. As can be seen by comparing Fig. 3.6 (NDP11) and panel (b) of Fig. 3.1 (NDP12), the generator components for NDP11 are closer to the noise floor and the discrepancies between the two buffers are larger than those for NDP12. Consequently, OMET could be estimated for fewer frequencies (Fig. 3.7, panel (a)).
Figure 3.6: [NDP11] $2f_1 - f_2$ generator components. Solid and dashed lines depict the buffer 1 and buffer 2 data, respectively. The noise floor is depicted by red dashed lines.

As seen in Fig. 3.7, panel (a), for some frequencies (e.g., around 2 kHz), the OMET estimations are missing. Although the discrepancies between the two buffers in Fig. 3.6 are larger than 2 dB SPL for those frequencies (e.g., around 2 kHz), the signals are well above the noise floor for some of the levels. Therefore, the level difference criteria for the two buffers for OMET estimation was increased to exclude points that exceed 4 dB SPL between the two buffers. OMET estimates for NDP11 by the updated criterion are shown in Fig. 3.7, panel (b), revealing that OMET estimation using the generator components which pass the updated criterion is very stable across frequency but the OMET estimated by the composite DPOAEs is very unstable.
Figure 3.7: [NDP11] OMET estimates using the composite DPOAEs (red stars) and the generator components (green stars); $L_b = 40, 45, \text{ and } 50 \text{ dB SPL}$. The data points with a difference of higher than 2 and 4 dB SPL between the two buffers were not included in panel (a) and (b), respectively.

The positions of maxima and minima of the OMET estimates for participant NDP11 are marked in Fig. 3.8. The lowest frequency maximum in the OMET occurs at a local maximum in the $2f_a - f_b$ generator components (three-tone condition). The second maximum frequency in the OMET happens at a local minimum in the $2f_1 - f_2$ generator components (two-tone condition). The highest frequency maximum in the OMET happens very close to a deep minimum of the $2f_1 - f_2$ generator components and also a local maximum in the $2f_a - f_b$ generator components. The mid-frequency local minimum in the OMET occurs in a $2f_1 - f_2$ local maximum.
DPOAE data was available at $L_b = 40, 45, 50,$ and $55$ dB SPL for $NDP1$. The data obtained when $L_b = 55$ dB SPL were not included in OMET estimation because the I/O functions were beginning to saturate at this level. The generator component I/O functions for the two- and three-tone conditions at an arbitrary frequency $2f_1 - f_2 = 1829$ Hz are depicted in Fig. 3.9.

The generator components for $NDP1$ are depicted in Fig. 3.10. As seen, the generator components are increasing markedly at frequencies above 3 kHz, and were distorted due
to over amplification of the presented stimuli by the SR560 for NDP1. The gain on the SR560 was not set properly because the data collection software was not restarted before the data collection; this did not happen for any other participant. Consequently, the DPOAE data for frequencies above 2750 Hz were not included from the OMET estimation for NDP1 (2750 Hz is marked by black dashed line in Fig. 3.10).

![Figure 3.10](image)

**Figure 3.10:** [NDP1] $2f_1 - f_2$ generator components. Solid and dashed lines depict buffer 1 and buffer 2, respectively. The noise floor is shown by the dotted lines. The red and black dashed lines are passed through $2f_1 - f_2 = 1850$ and 2750 Hz.

The OMET estimated using the generator components at $L_b = 40, 50,$ and 55 dB SPL for NDP1 is depicted in Fig. 3.11, panel (a). As seen in Fig. 3.10, the generator components around 1850 Hz (marked by the red dashed line) are well above the noise floor at several $f_b$ levels. Frequencies around 1850 Hz were not included from the OMET estimation (Fig. 3.11, panel (a)) because the difference between the two buffers was greater than 2 dB SPL. The 2 dB SPL difference criterion was changed to a difference of 4 dB SPL for NPD1, as was used for NDP11. The OMET estimates using the updated criterion are shown in Fig. 3.11, panel (b). The OMET estimation by the composite DPOAE became more unstable in panel (b) compared to using a criterion of 2 dB SPL difference for the two buffers (i.e., panel (a)). Estimation of the OMET using the generator components by the updated criterion is very stable across frequency (Fig. 3.11, panel (b)), and is similar to the result from NDP11 (see Fig. 3.7). Since the 4 dB difference between the two buffers criterion provided OMET estimates at more
frequencies while being stable across frequency, this criterion and SNR of larger than 6 dB were used as the inclusion criteria for the other participants (i.e., NDP2, NDP3, NDP5, and NDP6).

As seen in panel (b), the last two points in the OMET estimates by the generator components increase significantly in level compared to the previous frequencies. Since the generator components (see Fig. 3.10) were within the noise floor for these two points, they have not been included in the OMET estimation.

![Figure 3.11: [NDP1] OMET estimates using the composite DPOAEs (red stars) and the generator components (green stars); $L_b = 40, 45, \text{ and } 50 \text{ dB SPL}$. The data points with a difference of higher than 2 and 4 dB SPL between the two buffers were not included for panel (a) and (b), respectively.](image)

The frequencies with maximum and minima in the OMET are very close to the frequencies with local minimum and maxima in the $2f_1 - f_2$ generator components (see the right panel in Fig. 3.12).
Figure 3.12: [NDP1] Left panel: OMET estimates using the generator components. Right panel: $2f_1 - f_2$ and $2f_a - f_b$ generator components. The maxima and minimum of the of the OMET estimates are marked by the red and black dashed lines, respectively.

NDP2 is an example of a participant with low emissions, close to the noise floor. Although DPOAE data was obtained at $L_b = 20 - 65$ dB SPL in 5 dB steps for this participant but $L_b = 55, 60,$ and $65$ dB SPL were not included in OMET estimation because of occurrence of the turnover at these levels (explained in section 2.2.2.3). DPOAE data when $L_b = 20$ dB SPL were not included as well since the data was in the noise floor at this level. The I/O functions at an arbitrary frequency $2f_1 - f_2 = 1144$ Hz are depicted in Fig. 3.13. As can be seen, turnover occurs in both two-tone and three-tone conditions at $L_b > 50$ dB SPL.
The estimated OMET along with the positions of minimum and maximum for NDP2 are shown in Fig. 3.14. As seen, the generator component levels for the three-tone condition are going down toward the noise floor at frequencies above 1550 Hz. Therefore, the estimation of the OMET is not available at most frequencies above 1550 Hz. The minimum and maximum in the OMET estimation occur at positions of the maximum and minimum of the $2f_1 - f_2$ generator components in the three-tone condition, which is similar to the findings for other participants (e.g., NDP1, NDP11, and NDP12).
Figure 3.14: [NDP2] Left panel: OMET estimates using the generator components. Right panel: $2f_1 - f_2$ generator components for the two-tone (dashed lines) and three-tone (solid lines) conditions. The maximum and minimum of the OMET estimates are marked by the red and black dashed lines, respectively.

The OMET estimates using the generator components for all the participants are depicted in Fig. 3.15. Since the OMET estimates are negative, minima in OMET are associated with more attenuation and maxima are associated with less attenuation. The minima and maxima for $NDP1$, $NDP3$ (except the first maximum), and $NDP12$ happen in close proximity to each other in frequency. However, the places for local minima for $NDP11$ occur at close proximity to frequencies of the local maxima of $NDP1$, $NDP3$, and $NDP12$. The first maximum in $NDP11$ occurs very close to the frequency and level of one of the maxima for $NDP5$. The lowest minimum and maximum places for $NDP5$ happen at a maximum and a minimum place for $NDP2$, respectively. The lower frequency minimum in $NDP6$ is at a minimum for $NDP5$; the higher frequency minimum for $NDP6$ occurs at a local minimum for $NDP2$. Although the OMET estimations from $NDP2$ are missing at higher frequencies, a maximum close in level and frequency to the highest one in $NDP3$ OMET is evident.
Figure 3.15: OMET estimates using the generator components for all the participants.

The local minima and maxima frequencies and magnitudes for the OMET for all participants are shown in Table 3.1 for comparison. As can be seen there are similarities between the patterns between participants. The OMET estimates for participants \( NDP_1, NDP_2, NDP_5, NDP_6 \) have one or two local minima between \( 1.1 – 1.7 \) kHz; the OMET estimates from other participants were not available at lower frequencies. The OMET estimates for \( NDP_3, NDP_11, NDP_12 \) have one or two minima between \( 1.9 – 3.1 \) kHz. The OMET estimates for all participants have one or two local maxima between \( 1.1 – 1.9 \) kHz. The OMET estimates for \( NDP_3, NDP_11, NDP_12 \) have one or two local maxima between \( 2 – 3.1 \) kHz.
Table 3.1: Local minima and maxima frequencies and magnitudes of the OMET estimates using DPOAEs for all the participants.

<table>
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<tr>
<th>Participant</th>
<th>Local maximum frequency (Hz)</th>
<th>Maximum magnitude (dB)</th>
<th>Local minimum frequency (Hz)</th>
<th>Minimum magnitude (dB)</th>
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<td>1559</td>
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<td></td>
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</tr>
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3.1.1 Middle Ear Muscle Activation

To check the middle ear muscle activation during data collection, levels and phases of the primaries $f_1$ and $f_a$ were estimated at the entrance of the ear canal. Because the primaries $f_1$ and $f_a$ were constant for all $L_b$ levels in the three-tone condition, their estimated levels and phases in the ear canal should be the same at different $L_b$ levels. The estimated $f_1$ and $f_a$ levels for NDP12 are shown in Fig. 3.16.
Figure 3.16: [NDP12] $L_1$ and $L_a$ primaries estimated at the entrance of the ear canal at different $L_b$s (depicted in the legend) in the three-tone condition.

As seen in Fig. 3.16, $L_1$ and $L_a$ at $L_b = 35$ and 40 dB SPL are different from $L_1$ and $L_a$ when $L_b = 45$, 50, and 55 dB SPL. The difference between the primary levels of $f_1$ and $f_a$ at $L_b = 55$ dB SPL and the primary levels at $L_b = 35$, 40, 45, and 50 dB SPL are depicted in Fig. 3.17; as seen in panel (a) and panel (b), $L_1$ and $L_a$ differences have similar patterns across frequency and they do not exceed 5 dB SPL.

Figure 3.17: [NDP12] The patterns observed when subtracting $L_1$ at $L_b$s of 35, 40, 45, and 50 dB SPL from $L_1$ when $L_b = 55$ dB SPL (panel (a)). Panel (b) depicts same differences for $L_a$.

The differences between the primary levels at different $L_b$s might be due to probe
slippage during DPOAE data collection. To check the probe fit during data collection, the ear canal responses to broadband noise generated by the ER-2s used to generate $L_1$ and $L_a$ (called ear canal calibrations) before and after obtaining recordings at each level were compared. The ear canal calibrations for $L_1$ and $L_a$ are depicted in panel (a) and (b) in Fig. 3.18, respectively. As seen in panel (a) and (b), both $L_1$ and $L_a$ ear canal calibrations before and after measuring $L_b$s of 45, 50, and 55 dB SPL are very similar to each other. The $L_1$ and $L_a$ ear canal calibrations after measuring $L_b$s of 35 and 40 dB SPL deviate from the ear canal calibrations before these levels. The deviations between the ear canal calibrations decrease as the frequency approaches 4 kHz for both $L_1$ and $L_a$ ear canal calibrations.

Figure 3.18: [NDP12] $L_1$ ear canal calibrations (panel (a)) and $L_a$ ear canal calibrations (panel (b)) before and after recordings at the levels indicated in the legend.

In order to relate the primary level changes to the changes in the ear canal calibrations, the difference between the $L_1$ ear canal calibration along with the $f_1$ level difference at $L_b=55$ and 35 dB SPL are depicted in panel (a) in Fig. 3.19; the comparison when $L_b=55$ and 40 dB SPL can be seen in panel (b). As can be seen in panel (a), the
$f_1$ level difference (at $L_b = 35$ and 55 dB SPL), shown in red, is highly correlated with the difference in the ear canal calibration at $L_b = 55$ (mean of the responses before and after the level) and after $L_b = 35$ dB SPL, shown by the dark green line. The ear canal calibration difference between $L_b = 55$ and before 35 dB SPL is spiky, shown by the light green curve, which is due to the presence of noise during the ear canal calibrations.

As can be seen in panel (b), the difference between the ear canal calibrations at $L_b = 55$ and before and after $L_b = 40$ dB SPL (shown by the light and dark green lines, respectively), and the $f_1$ level differences (shown in red) follow the same pattern and are highly correlated. The difference between the mean ear canal calibrations at $L_b = 55$ dB SPL and the mean ear canal calibrations at $L_b = 40$ dB SPL (shown by the dashed black line) is very similar to the $f_1$ level change. The largest level changes as a result of middle ear muscle activation are at frequencies below the ones examined here (below 1.5 kHz) (Henin et al., 2014).

![Figure 3.19](NDP12)

Figure 3.19: [NDP12] Differences between $L_1$ ear canal calibrations when $L_b = 55$ and $L_b = 35$ (panel (a)) or 40 dB SPL (panel (b)). The difference between ear canal calibrations mean at $L_b = 55$ and before 35 or 40 dB SPL are depicted by light green lines; the difference with after 35 or 40 dB SPL are depicted by dark green lines. The differences between $f_1$ levels for $L_b = 55$ and $L_b = 35$ or 40 dB SPL are shown in red.
The difference between the $L_a$ ear canal calibrations and $f_a$ level change between $L_b = 55$ and $35$ dB SPL, and between $L_b = 55$ and $40$ dB SPL are depicted in Fig. 3.20. As shown, the amount of ear canal calibration changes and $f_a$ level differences are correlated and consistent with the findings for $L_1$ in Fig. 3.19.

Figure 3.20: [NDP12] Differences between $L_a$ ear canal calibrations when $L_b = 55$ and $L_b = 35$ (panel (a)) or $40$ dB SPL (panel (b)). The differences between $f_a$ levels for $L_b = 55$ and $L_b = 35$ or $40$ dB SPL are shown in red.

In addition to potentially changing primary levels at the entrance of the ear canal, middle ear muscle activation can change the phases of the primaries at the entrance of the ear canal. The differences between $f_1$ phase at $L_b = 55$ dB SPL and $f_1$ phases at $L_b$s of $35$, $40$, $45$, and $50$ dB SPL are shown in Fig. 3.21, panel (a); phase differences for $f_a$ can be seen in panel (b). The phase differences for both $f_1$ and $f_a$ for all $L_b$ levels are smaller than 0.08 Rad, which are negligible and within the measurement error.
Figure 3.21: [NDP12] $f_1$ phase differences (panel (a)) and $f_a$ phase differences (panel (b)) between phases when $L_b = 55$ dB SPL and phases when $L_b = 35, 40, 45$ and $50$ dB SPL.

### 3.1.2 Interaction-Control Condition

To evaluate potential enhancement/suppression by $f_a$ on $2f_1 - f_2$ DPOAE, the $2f_1 - f_2$ levels in the two-tone condition were compared to those in the interaction-control condition. Comparison between the $2f_1 - f_2$ generator components levels in the two-tone and the interaction-control conditions are depicted in Fig. 3.22. As can be seen, the generator components levels are contaminated by noise at most frequencies when $L_2 = 25$ dB SPL. To quantify the amount of suppression/enhancement, the difference between the $2f_1 - f_2$ generator components levels in the two-tone and the interaction-control conditions were calculated and are depicted in Fig. 3.23.
The difference between the $2f_1 - f_2$ generator components levels in the two-tone and the interaction-control conditions was calculated for frequencies with SNR of greater than 6 dB SPL and when the difference between the two buffers was smaller than 4 dB SPL. As can be seen in Fig. 3.23, few points are depicted at $L_b = 25$ dB SPL because the data was at the noise floor (see Fig. 3.22). As observed in Fig. 3.23, the impact of $f_a$ on $2f_1 - f_2$ is suppressive at $L_b$s of 30, 35, 40, 45, and 50 dB SPL. The suppression value was smaller than 11 dB SPL at all levels and across all frequencies except at several frequencies around 3 kHz at $L_b = 30$ dB SPL, where the suppression approached 13.5 dB SPL because the generator components were closer to the noise floor and could be contaminated by noise; these frequencies (at this level) were not included in the OMET estimates.
3.2 Parameter Fitting

Step by step parameter fitting procedures for participant NDP3 are explained below. The simplified version of the model (shown in Fig. 2.9) was fit to initialize the model parameters. $R_s$ was initially set to $4.84 \times 10^7 \text{kg/(sm}^4\text{)}$, which is the average cochlea input impedance estimated by O’Connor and Puria (2008) from the Aibara et al. (2001) measurements. $R_s$ and $K_s$ were considered first. The imaginary part of the model impedance would be

\[
\text{Imag}(Z) = \frac{K_s}{s^\alpha}.
\]  

All of the fractional-orders (e.g., $\alpha$’s) were set to 0.9 initially. Since including $K_s$ mainly affects the low frequencies, $K_s$ was fixed so that the imaginary part of the model impedance fit the impedance from the reflectance measurements for frequencies lower than 1078 Hz. Solving for $K_s$ to determine the least squares error, results in $K_s = 3.9 \times 10^{10} \text{kg/s}^2\text{m}^4$. The imaginary parts of the impedance estimated from the
reflectance measurements as well as by the model are shown in Fig. 3.24 with green and dashed black lines, respectively.

Because the value of $R_s$ impacts the impedance magnitude,

$$Z = R_s + \frac{K_s}{s^\alpha},$$

(3.2)

$R_s$ was set to $10^6 \text{ kg/sm}^4$ to approach the minimum least squares fit of the estimated impedance magnitudes by the model and from the reflectance measurements. The estimated impedance magnitudes from the reflectance measurements and by the model using the updated $R_s$ are plotted with green black dashed lines, respectively, in Fig. 3.25.
Next a mass of 190 $kg/m^4$ was added to the simplified model to satisfy the minimum least squares fit at frequencies higher than $> 3700$ Hz. The result of the fit after fixing the mass value is depicted in Fig. 3.26.

Subsequently, the ear canal length, the TM delay, and the characteristic impedance of the TM were estimated. The possible values for these three parameters are defined
Length of the ear canal was measured in 15 human cadavers by making rubber ear molds of the ear canals (Stinson and Lawton, 1989); the measured lengths ranged between 27 − 37 mm. Because the probe-microphone was inserted into the ear canal in our data collection, the effective length of the ear canal was reduced. Therefore, in parameter fitting of the proposed model the ear canal length was assumed to range between 15 − 30 mm.

The mean TM delay was estimated by (O’Connor and Puria, 2008) to be 17.6 and 75.5 µs for two sets of data from temporal measurements by Aibara et al. (2001) and (O’Connor and Puria, 2006), respectively. Hence, the TM delay was permitted to range between 1−100 µs in the proposed model.

The characteristic impedance of the TM can be found using the following equation

\[ Z_0 = \sqrt{E \cdot \rho}, \]  

(3.3)

where \( E \) denotes the Young’s modulus of the TM and \( \rho \) denotes the density of the TM. Young’s modulus value was found to be approximately 0.023 GPa (Decraemer et al., 1980) and the TM density was estimated to be \( 1.2 \times 10^3 \text{kg/m}^3 \) by Kirikae, 1960 (see Wada et al. (1992) for a review). Using these values, the characteristic impedance of the TM approximates \( 1.66 \times 10^5 \text{kg/sm}^2 \). Therefore, \( Z_{0TM} \) was assumed to fall in between \( 10^5 − 10^8 \text{kg/sm}^4 \) in the proposed model.

Considering the aforementioned ranges for \( l_{EC}, T_{TM}, \) and \( Z_{0TM} \), as well as the initial values for \( m_s, K_s, \) and \( R_s \), the parameters of the simplified model were set to minimize the sum of squared error of the impedance phase and magnitude between the impedance estimates from reflectance measurements and by the model. The impedance magnitudes and phases obtained after setting the parameters are depicted in Fig. 3.27 using a logarithmic scale for a clear visual comparison between the impedance magnitude estimates.
by the model and from the reflectance measurements. As can be seen, the estimated impedance magnitude and phase by the model have the same pattern as those estimated from the reflectance measurements; however, the values of the estimations by the model and the reflectance measurements differ across frequency. Furthermore, the frequencies and magnitudes of the minima and maxima do not match. These differences were expected because the fit was obtained using the simplified version of the model. Such differences were expected to be minimized after setting the parameters of the transformed model from Fig. 2.7.

Figure 3.27: [NDP3] Impedance magnitude and phase estimates from the simplified model (dashed black lines) and from the reflectance measurements (green lines).

The values of the parameters of the simplified model (i.e., $M_s$, $K_s$, $R_s$, $l_{EC}$, $T_{TM}$, and $Z_{0TM}$) were used as initial values for finding the parameters of the transformed model in Fig. 2.7. To estimate the parameters of the transformed model in Fig. 2.7, first, the fractional capacitors were considered in addition to the ear canal and the TM. The fractional capacitors were set to fit to the impedance magnitude estimations from the reflectance measurements at frequencies lower than 1078 Hz. The estimated impedance magnitude by the model after setting the fractional capacitors is plotted along with the estimated impedance magnitude from the reflectance measurements in Fig. 3.28.
Figure 3.28: [NDP3] Impedance magnitude estimates from the model (black dashed lines) and from reflectance measurements (green lines), after fixing the values of the fractional capacitors.

Cochlear and ossicles’ masses and resistors, which were initialized in parameter fitting of the simplified model, were added to the model next. The fitting procedure was repeated to match the impedance phases and magnitudes of the model and estimations from reflectance measurements. The result can be seen in Fig. 3.29.

Figure 3.29: [NDP3] Impedance magnitude and phase estimates from the model (black dashed lines) and from the reflectance measurements (green lines), after setting the cochlear and ossicles’ masses and resistors.
The fractional-orders of the derivatives belonging to the fractional capacitors and the middle ear cavity parameters were set next. The magnitudes and phases of the impedance estimates by the model and from the reflectance measurements after setting these parameters are shown in Fig. 3.30

![Impedance magnitude and phase estimates](image)

Figure 3.30: [NDP3] Impedance magnitude and phase estimates from the model (black dashed lines) and from reflectance measurements (green lines) after setting the fractional-orders of the fractional capacitors and the middle ear cavity parameters.

As can be seen in Fig. 3.30, both magnitude and phase of the impedance estimate by the model match the estimated impedance magnitude and phase from reflectance measurements; however, there are small discrepancies around the maxima and minima in both magnitudes and phases. Therefore, after setting all the parameters, the parameters were fine-tuned to minimize the discrepancies. The final impedance magnitude and phase fits from the transformed model for NDP3 are depicted in Fig. 3.31. As seen, the estimates from the model and from reflectance measurements are highly correlated. The normalized root mean squared error for estimation of the impedance magnitude and phase were 5.51% and 6.29%, respectively.
Figure 3.31: \([NDP3]\) Impedance magnitude and phase estimates from the transformed model (black dashed lines) and from reflectance measurements (green lines).

The same fitting procedure was employed for \(NDP1\), \(NDP2\), \(NDP5\), \(NDP11\), \(NDP12\); the step by step parameter fitting for \(NDP1\) can be seen in Appendix C.1. The final impedance magnitudes and phases from the model for each of these participants are depicted in Fig. 3.32. As can be seen the patterns of the impedance magnitudes and phases from reflectance measurements are captured by the model for all participants. However, there are some discrepancies between the model estimates and reflectance measurements estimates in both impedance magnitudes and phases, which are more noticeable for \(NDP2\) and \(NDP12\). As can be observed, maximum discrepancies in impedance magnitude estimates for \(NDP2\) (panel (c)) and \(NDP12\) (panel (k)) occur at frequencies higher than 1.4 kHz. The discrepancies between the magnitudes estimates are largest around the minimum for both of these participants. The discrepancies between the phases estimates from the model and from reflectance measurements for \(NDP2\) (panel (d)) and \(NDP12\) (panel (m)) occur at lower frequencies than the magnitudes discrepancies. The maximum discrepancies in phase estimates for \(NDP2\) and \(NDP12\) are between 1 – 2 kHz and 1 – 4 kHz, respectively.
Figure 3.32: Impedance magnitudes and phases estimates by the transformed model (black dashed lines) and from reflectance measurements (green lines). The impedance magnitudes are depicted in left panels; the right panels show the impedance phases. The participants’ identifiers are depicted in the low left corners of the impedance magnitudes graphs.
To quantify the discrepancies between impedance estimates from the model and from reflectance measurements, normalized root mean squared errors between the estimates of the impedance magnitudes and phases for all participants were calculated and are shown in Table 3.2. Since the impedance magnitudes are large (i.e., range between $5 \times 10^6 - 10^8 \text{kg/sm}^4$), the normalized root mean squared log errors were also calculated for the impedance magnitudes estimates (third column of Table 3.2). The normalized rms errors of impedance magnitudes and phases are smaller than 20% and 27%, respectively. The normalized rms log errors of impedance magnitudes do not exceed 3%. The lowest errors of magnitude and phase estimates belong to NDP1 and NDP3 with values of 4.03 and 6.29%, respectively. As can be seen in the second or third column, larger magnitude errors belong to ND2 and NDP12; larger phase errors also belong to these participants (see the fourth column of the table).

Table 3.2: Normalized RMS errors/log-errors for impedance magnitudes and phases estimates from the model and from power reflectance measurements.

<table>
<thead>
<tr>
<th>Participant</th>
<th>Normalized RMS error Impedance magnitude</th>
<th>Normalized RMS log error Impedance magnitude</th>
<th>Normalized RMS error Impedance phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>NDP1</td>
<td>4.03%</td>
<td>0.89%</td>
<td>9.55%</td>
</tr>
<tr>
<td>NDP2</td>
<td>19.11%</td>
<td>2.98%</td>
<td>15.79%</td>
</tr>
<tr>
<td>NDP3</td>
<td>5.51%</td>
<td>0.36%</td>
<td>6.29%</td>
</tr>
<tr>
<td>NDP5</td>
<td>9.41%</td>
<td>1.36%</td>
<td>9.52%</td>
</tr>
<tr>
<td>NDP11</td>
<td>9.96%</td>
<td>1.63%</td>
<td>11.55%</td>
</tr>
<tr>
<td>NDP12</td>
<td>15.9%</td>
<td>2.83%</td>
<td>26.26%</td>
</tr>
</tbody>
</table>

The parameters of the proposed model for each participant (NDP1, NDP2, NDP3, NDP5, NDP11, and NDP12) are in Table 3.3.
Table 3.3: Parameters of the model for NDP1, NDP2, NDP3, NDP5, NDP11, and NDP12.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>NDP1</th>
<th>NDP2</th>
<th>NDP3</th>
<th>NDP5</th>
<th>NDP11</th>
<th>NDP12</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{EC}$ (mm)</td>
<td>22.1</td>
<td>23.43</td>
<td>21.5</td>
<td>20</td>
<td>17.8</td>
<td>15</td>
</tr>
<tr>
<td>$T_{TM}$ (µs)</td>
<td>26</td>
<td>14.83</td>
<td>12.9</td>
<td>43</td>
<td>11</td>
<td>10</td>
</tr>
<tr>
<td>$Z_{BTM}$ (kg/sm²)</td>
<td>$61 \times 10^9$</td>
<td>$124.94 \times 10^9$</td>
<td>$121.3 \times 10^9$</td>
<td>$33 \times 10^6$</td>
<td>33</td>
<td>33</td>
</tr>
<tr>
<td>$\alpha_{TM}$</td>
<td>0.9</td>
<td>0.6</td>
<td>0.91</td>
<td>0.96</td>
<td>0.98</td>
<td>0.995</td>
</tr>
<tr>
<td>$M_{MEC}$ (kg/m⁴)</td>
<td>200</td>
<td>318.5</td>
<td>277</td>
<td>500</td>
<td>1070</td>
<td>184</td>
</tr>
<tr>
<td>$K_{MEC}$ (kg/s²m⁴)</td>
<td>$10^{10}$</td>
<td>$6.8 \times 10^9$</td>
<td>$8 \times 10^9$</td>
<td>$7.7 \times 10^{10}$</td>
<td>$1.6 \times 10^{10}$</td>
<td>$9.96 \times 10^{10}$</td>
</tr>
<tr>
<td>$R_{MEC}$ (kg/sm⁴)</td>
<td>$10^6$</td>
<td>$8.05 \times 10^6$</td>
<td>$7 \times 10^6$</td>
<td>$19 \times 10^6$</td>
<td>$6.7 \times 10^6$</td>
<td>$9.71 \times 10^6$</td>
</tr>
<tr>
<td>$M_{MIT}$ (kg/sm⁴)</td>
<td>200</td>
<td>738.3</td>
<td>642</td>
<td>720</td>
<td>495</td>
<td>207</td>
</tr>
<tr>
<td>$K_{MIT}$ (kg/s²m⁴)</td>
<td>$11 \times 10^{10}$</td>
<td>$8.49 \times 10^{10}$</td>
<td>$9.99 \times 10^{10}$</td>
<td>$7.1 \times 10^{10}$</td>
<td>$3.1 \times 10^{10}$</td>
<td>$2.7 \times 10^{10}$</td>
</tr>
<tr>
<td>$R_{MIT}$ (kg/sm⁴)</td>
<td>$13 \times 10^6$</td>
<td>$5.62 \times 10^6$</td>
<td>$6.61 \times 10^6$</td>
<td>$16 \times 10^6$</td>
<td>$12.6 \times 10^6$</td>
<td>$7.1 \times 10^6$</td>
</tr>
<tr>
<td>$\alpha_{MIT}$</td>
<td>0.91</td>
<td>0.99</td>
<td>0.89</td>
<td>0.96</td>
<td>0.82</td>
<td>0.8</td>
</tr>
<tr>
<td>$R_{OJT}$ (kg/sm⁴)</td>
<td>$9 \times 10^6$</td>
<td>$8.5 \times 10^6$</td>
<td>$4.6 \times 10^6$</td>
<td>$6 \times 10^6$</td>
<td>$21 \times 10^6$</td>
<td>$10^4$</td>
</tr>
<tr>
<td>$K_{OJT}$ (kg/s²m⁴)</td>
<td>$19 \times 10^{10}$</td>
<td>$13.6 \times 10^{10}$</td>
<td>$16 \times 10^{10}$</td>
<td>$30 \times 10^{10}$</td>
<td>$24 \times 10^{10}$</td>
<td>$49 \times 10^{10}$</td>
</tr>
<tr>
<td>$\alpha_{OJT}$</td>
<td>0.89</td>
<td>0.6</td>
<td>0.895</td>
<td>0.95</td>
<td>0.94</td>
<td>0.95</td>
</tr>
<tr>
<td>$M_{ST}$ (kg/sm⁴)</td>
<td>2400</td>
<td>1700</td>
<td>2000</td>
<td>1290</td>
<td>1980</td>
<td>3619</td>
</tr>
<tr>
<td>$K_{ALCT}$ (kg/s²m⁴)</td>
<td>$10^{10}$</td>
<td>$4.25 \times 10^{10}$</td>
<td>$5 \times 10^{10}$</td>
<td>$2 \times 10^9$</td>
<td>$1.1 \times 10^{10}$</td>
<td>$1.1 \times 10^{10}$</td>
</tr>
<tr>
<td>$R_{ALCT}$ (kg/sm⁴)</td>
<td>$7 \times 10^6$</td>
<td>$12.75 \times 10^6$</td>
<td>$15 \times 10^6$</td>
<td>$20 \times 10^6$</td>
<td>$4 \times 10^6$</td>
<td>$13.46 \times 10^6$</td>
</tr>
<tr>
<td>$\alpha_{ALCT}$</td>
<td>0.99</td>
<td>0.79</td>
<td>0.985</td>
<td>0.91</td>
<td>0.91</td>
<td>0.8</td>
</tr>
</tbody>
</table>

### 3.3 Model Evaluation

To evaluate the performance of the model, the sum of the middle ear forward and reverse transmissions estimated from the model for each participant was compared to the outer-middle ear transmission estimated using DPOAE generator components from that participant.

To estimate outer-middle ear reverse transmission using the model, the probe-microphone impedance was estimated using Thevenin-equivalent estimations from cavity measurements with an estimation error of 1.48 (see section 2.3.4). The estimated probe-microphone impedance magnitude and phase are shown in Fig. 3.33.
Figure 3.33: Magnitude and phase of the estimated impedance of the probe-microphone.

To calculate forward transmission from the model, using Eq. 2.39, cochlear input impedance was needed. The sum of annular ligament and cochlear parameters (i.e., \( K_{ALC} \) and \( R_{ALC} \)) were fixed when estimating the parameters in Table 3.3 (as described in section 2.4.1). In order to minimize the error between the estimates of outer-middle ear transmission from the model and DPOAE, the fractional capacitance and resistance of the annular ligament were varied.

The forward transfer function (\( FTF \)) and the absolute value of the reverse transfer function (\( |RTF| \)) from the model for the six participants are depicted in Fig. 3.34, left panels. Since the reverse outer-middle ear transmission is negative, the absolute value of the reverse transmission (i.e., reverse attenuation) was calculated for comparison with the forward transmission, which was positive at most frequencies. The sum of the estimated outer-middle ear forward and reverse transmissions from the model and DPOAE generator components can be seen in the right panels in Fig. 3.34. Since the outer-middle ear transmission (OMET) and the reverse transfer function are both negative, minima in these functions are associated with more attenuation and maxima mean less attenuation. As can be seen in the right panels of Fig. 3.34, the OMET estimates using DPOAE generator components for all six participants are in the range of the OMET estimates.
from the model; however, the maxima and minima in OMET estimates using DPOAE generator components do not exist in the OMET estimates from the model except for \(NDP_3\). For \(NDP_3\), the lower and higher frequency maxima in the OMET estimates using DPOAE occur close to the maxima (around 1.4 and 3 kHz) in the OMET estimates from the model. Although the OMET estimates from the model do not capture all minima and maxima in the OMET estimates using DPOAE, they follow the same pattern of increase or decrease in OMET estimates using DPOAE (see panel (d) and (h)). As can be seen for \(NDP_2\) (panel (d)), the OMET estimates from the model and DPOAE both increase in level towards 4 kHz; for \(NDP_{11}\) (panel (h)), both OMET estimates decrease toward 3 kHz. The RMS errors between the outer-middle ear transmission estimates from the model and using DPOAE generator components are shown in Table 3.4. As can be seen the normalized error is lower than 16% for the six participants.

Table 3.4: Normalized RMS errors between OMET estimates from the model and using DPOAE generator components for the participants.

| \(NDP_1\) | \(NDP_2\) | \(NDP_3\) | \(NDP_5\) | \(NDP_{11}\) | \(NDP_{12}\) |
| 11.4% | 7.1% | 9.8% | 15.1% | 10.9% | 7.4% |

As can be seen in the left panels in Fig. 3.34, the values of reverse attenuations are larger than the forward gains across all frequencies for all six participants, which results in negative outer-middle ear transmission as can be seen in the right panels (green curves). As depicted in the right panels, the outer-middle ear transmission estimates from the model range from \(-48\) to \(-17\) dB between \(0.2 - 6\) kHz and have one or two maxima (minima attenuations) that occur between \(0.8 - 1.4\) and/or \(3.2 - 5.1\) kHz for all six participants.

For participant \(NDP_1\) (see panel (a)), the forward transfer function increases toward a peak at frequencies around 1.6 kHz (with magnitude 12 dB); the higher frequency resonance around 3.4 kHz with a higher magnitude (18 dB) seems to reduce this low-frequency peak, leaving a bump near 1.6 kHz. The minimum attenuation in the reverse
outer-middle ear transfer function occurs at 2.1 kHz, which is higher in frequency than
the low-frequency bump in the forward transfer function.

For $NDP2$, the forward outer-middle ear transfer function has a sharp maximum
around 3.9 kHz with magnitude of 24 dB and a minimum at 650 Hz (see panel (c)). The
maximum and minimum attenuation in the reverse transfer function occurs at 4.7 and
2.4 kHz, respectively, which are higher than the maximum and minimum frequencies in
the forward transfer function, respectively.

For another participant, $NDP3$ (see panel (i)), the forward transfer function had
two maxima around 1.3 kHz and 3.2 kHz with values of 12 and 18 dB, respectively; a
minimum occurred between the maxima positions at 1.9 kHz; this minimum was close
to the minimum attenuation frequency. The maximum attenuation (45 dB) occurred
higher in frequency (around 3.7 kHz) than the higher-frequency maximum in the forward
transfer function.

The reverse attenuation for $NDP5$ (panel (e)) increases toward a maximum, which
occurs at frequencies higher by 1 kHz than the maximum frequency in the forward transfer
function. For another participant, $NDP11$ (panel (g)), the forward transfer function has
two maxima around 1 and 5 kHz; the absolute value of the reverse transfer function
increases toward a maximum around 5 kHz as well.

For $NDP11$, the minimum attenuation (at 2 kHz) occurs 1 kHz below the minimum
frequency in the forward transmission. The forward transmission has two main resonance
frequencies; the absolute value of reverse attenuation has a resonance frequency higher
than the high-frequency maximum.

Although the forward transfer function for $NDP12$ (panel (m)) has two main resonances
around 1.2 and 4.5 kHz, the reverse transfer function is almost flat between 2 – 6 kHz.
Figure 3.34: Left panel: Estimated forward transfer function (\(FTF\)), depicted in blue, and the absolute value of the reverse transfer function (\(|RTF|\)), depicted in magenta, of the outer-middle ear from the model. Right panel: sum of the estimated forward and reverse outer-middle ear transmissions (OMET) from the model (green lines) and DPOAE generator components (black stars).
Although the model fit was based on impedance estimates between 0.2 – 6 kHz from reflectance measurements, estimates of forward and reverse transmission were calculated between 0.2 – 10 kHz for each of the six participants (panel (a) and (b), Fig. 3.35). As can be seen in panel (a), the forward outer-middle ear transfer functions range between −19 and 24 dB and has at least one resonance for all six participants; the main resonance frequency occurs between 3 – 5.1 kHz. A low frequency resonance is observed in forward transfer functions only for NDP3, NDP11, and NDP12 between 1 – 1.3 kHz; and a bump is observed between 0.9 – 2 kHz for NDP1, NDP2, and NDP5. As can be seen, a mid-frequency anti-resonance occurs at 1984 Hz for NDP3, at 3014 Hz for NDP11, and at 2.8 kHz for NDP12; and a high-frequency anti-resonance occurs for only NDP2 and NDP3 around 8 kHz.

The reverse outer-middle ear attenuation ranges from 31 to 60 dB between 0.2 – 10 kHz for the six participants (see Fig. 3.35, panel (b)). As can be observed, the patterns of the absolute value of reverse outer-middle ear transfer functions for all six participants are very similar to each other at frequencies below 1 kHz, however, the levels are different. The absolute value of the reverse transfer functions for all six participants have a low frequency maximum and minimum around 400 Hz. The main resonance frequency for all participants occurs between 3 – 6 kHz. A mid-frequency anti-resonance between 2 – 2.5 kHz can be seen for all participants except for NDP5. A high-frequency anti-resonance can be observed between 6 – 9 kHz for all participants except for NDP12. The reverse attenuation for NDP12 varies in a smaller range, 37 to 40 dB between 0.2 – 6 kHz, compared to the reverse attenuations for other participants.
Figure 3.35: Forward outer-middle ear transfer functions (panel (a)) and absolute value of reverse outer-middle ear transfer functions (panel (b)) between 0.2 – 10 kHz estimated from the model for the participants indicated in the legend.

The model was successfully fit to the impedance magnitudes and phases from reflectance measurements from all participants. The fitting error differed between participants for impedance magnitudes and phases. The result of the evaluation suggests that the model can predict outer-middle ear transmission comparable to those estimated by DPOAE generator components. However, more minima and maxima were present in the OMET estimates by DPOAEs, which might be due in part to the impact of cochlear function on DPOAE.
Chapter 4

Discussion

A noninvasive approach was used to estimate the sum of forward and reverse outer-middle ear transmission using DPOAE obtained in two conditions; in the first condition (i.e., two-tone condition) the DPOAE was generated by the interaction of two external tones \( f_1 \) and \( f_2 \). In the second condition (i.e., three-tone condition), the DPOAE was generated by the interaction of one of these external tones \( f_1 \) along with a distortion product generated by the interaction of two other external tones \( f_a \) and \( f_b \).

When DPOAE in these two conditions were similar, \( f_2 \) level in the cochlea (in the two-tone condition) and the distortion product at \( 2f_a - f_b \) (in the three-tone condition) were expected to be similar. Since \( L_2 \) was mainly affected by forward transmission and DPOAE generated by \( f_a \) and \( f_b \) was impacted by reverse transmission, the horizontal distance between the I/O functions of the two conditions (\( 2f_1 - f_2 \) DPOAE as a function of \( L_2 \) in the two-tone condition or as a function of DPOAE generated by \( f_a \) and \( f_b \) interaction in the three-tone condition) was used as an estimate of the sum of forward and reverse outer-middle ear transmission. To evaluate the suppressive/enhancing impact of \( f_a \) on \( 2f_1 - f_2 \) DPOAE in the three-tone condition, an interaction-control condition was introduced in which \( f_a \) was added to \( f_1 \) and \( f_2 \) of the two-tone condition. Comparison between \( 2f_1 - f_2 \) levels in the two-tone and interaction-control conditions provided an
estimate of the amount of suppression/enhancement of $2f_1 - f_2$ DPOAE by $f_a$. Estimating outer-middle ear transmission using DPOAEs is limited to DPOAE levels with SNR of larger than 6 dB; therefore, estimates can not be obtained from individuals with severe sensorineural or conductive hearing loss for whom DPOAE is in the noise floor.

The OMET estimates based on DPOAE generator components was used to evaluate the proposed fractional-order lumped element model. Fractional capacitors were employed to model the viscoelastic characteristics of the middle ear joints, ligaments, muscles, and the cochlea. The masses were used to model the inertia of the middle ear cavity and ossicles. The resistors modeled the damping of sound energy by the ossicles, ossicular joints (and ossicular ligaments and muscles), the annular ligament, the cochlea, and the middle ear cavity. Furthermore, the ear canal was modeled by a lossless transmission line and the TM was modeled by a fractional-order transmission line to incorporate the viscoelasticity of the TM.

The proposed model was fit to the ear canal input impedance magnitude and phase estimates from wideband reflectance measurements for each individual. The reverse outer-middle ear transfer function from the model used the probe-microphone impedance estimated from Thevenin-equivalent parameters of the probe-microphone (estimated from measurements in several test cavities). The performance of the model was then evaluated by comparing the sum of forward and reverse outer-middle ear transmission from the model to the estimates using DPOAEs for each individual.

In principle, the fractional-order modeling approach leads to a better accuracy in estimation of forward and reverse transmission compared to the existing (integer-order) lumped element modeling approaches. This is true because the main sources of inaccuracy in the lumped element modeling are: I) lumping all the higher-dimensional effects, and II) employing simplified constitutive laws. Our approach relaxes the second source of error in the model through employing more sophisticated physical laws, which translates
into more robust and more accurate predictions.

### 4.1 Outer-Middle Ear Transmission Estimates

The forward plus reverse outer-middle ear transmission estimated using DPOAEs ranged between $-33.15 \pm 4.46$ and $-21.01 \pm 4.30$ dB with the minimum of approximately $-39$ dB and an approximate maximum of $-18$ dB for seven participants. An interaction-control condition was performed on one of the participants (NDP2) in which the $2f_2 - f_a$ distortion product occurred very close to the $2f_1 - f_2$ distortion product. The impact of $f_a$ on $2f_1 - f_2$ distortion product was suppressive when $L_b = 30, 35, 40, 45, \text{ and } 50$ dB SPL at frequencies with SNR of larger than 6 dB and when the difference between the two buffers was greater than 4 dB SPL. The amount of suppression did not exceed 13.5 dB SPL across levels and frequencies. Therefore, $f_a$ suppressive effects on $2f_1 - f_2$ distortion product might have impacted the outer-middle ear transmission estimates using DPOAE in NDP2.

Since the outer-middle-ear transmission is negative, maximum OMET estimates are associated with less attenuation and minimum OMET means more attenuation. The minima OMET estimates (i.e., more attenuation) occurred mainly at $2f_1 - f_2$ generator-component maxima in the two-tone condition and $2f_1 - f_2$ generator-component minima in the three-tone condition. Less attenuation was observed near $2f_1 - f_2$ generator-component minima in the two-tone condition and $2f_1 - f_2$ generator-component maxima in the three-tone condition. Due to these relations between OMET estimates and patterns of maxima and minima in the generator components, the cochlear function might have impacted the OMET estimates.

The sum of forward and reverse middle ear transmission estimated from one cadaver was negative with attenuations ranging from 6 to 28 dB, had one maximum and two minima between $1 - 3.3$ kHz (Puria and Rosowski, 1996). The OMET estimates for our
participants all had one to three local minima, and one to three local maxima in the frequency range between $1 - 3.3$ kHz. The number of maxima and minima might be underestimated, because OMET estimates were not available at all frequencies between $1.3 - 3.3$ kHz due to low SNRs or discrepancies between the two buffers. The minimum middle ear attenuation from Puria and Rosowski (1996) measurements (i.e., 6 dB) was less than found in all our seven participants. The maximum attenuation that they found (i.e., 28 dB) was in the range of our estimations. When Puria (2003) estimated middle ear attenuation in five human temporal bones from four cadavers, the mean middle ear attenuation ranged from approximately 1 to 35 dB between $1 - 3.3$ kHz. He only reported the mean and standard deviation of the middle ear attenuation (not individual estimates of middle ear attenuation). The mean middle ear attenuation had only one minimum around 1.2 kHz, which was smaller than the minimum of the outer-middle ear attenuation in our estimates.

Both Puria and Rosowski (1996) and Puria (2003) estimated middle ear attenuation but we estimated outer-middle ear attenuation; therefore, discrepancies between the estimations are expected. Both Puria and Rosowski (1996) and Puria (2003) obtained invasive measurements in the vestibule and at the TM, which might be another reason for the differences between our estimates and their estimates. They also used different transducers. While Puria and Rosowski (1996) used ER-10C and Puria (2003) used ER-7C microphone, we used a three-port ER-10A; differences in the termination of the ear canal are expected to modify middle ear transmission estimates. When the middle ear attenuation was estimated assuming that the ear canal was open during reverse transmission (Puria and Rosowski, 1996), the estimated attenuation was found to range between 4 and 36 dB (Puria and Rosowski, 1996). The frequencies of the maximum and minimum also changed when the ear canal was open and when it was closed by an ER-10C (Puria and Rosowski, 1996). This finding highlights the importance of the
ear canal termination during reverse transmission and partly explains the differences between their estimations of middle ear attenuations and our estimations of outer-middle ear transmission. Furthermore, the invasive measurements meant that a hole was drilled in the vestibule to place a hydropressure transducer to measure the pressure and another hole was drilled into the scala tympani anterior to the round window to place the inner ear sound source (Puria and Rosowski, 1996; Puria, 2003). Such invasive procedures may have changed the cochlear impedance (Allen, 1986) and impact estimations of middle ear attenuation.

4.1.1 Middle Ear Muscle Activation

The levels and phases of $f_1$ and $f_a$ were extracted from averaged ear canal recordings collected at each $f_b$ level. Since the levels of stimuli $f_1$ and $f_a$ were constant for all $f_b$ levels, the estimated level and phases in the ear canal should have not changed unless the middle ear muscle was activated or the position of the probe changed. The $f_1$ level differences and $f_a$ level differences at different $f_b$ levels did not exceed 5 dB SPL in the frequency range that the OMET was estimated. The comparison of the estimates of probe fit before and after data collection at each $f_b$ level suggested that there was probe slippage during the recordings because such changes were highly correlated to the $f_1$ and $f_a$ level changes. Since the primary level changes due to middle ear muscle activation can be smaller than 1 dB (Henin et al., 2014), the probe slippage prevented us from detecting the middle ear muscle activation through observing primary level changes.

The $f_1$ phase differences and $f_a$ phase differences at different $f_b$ levels did not exceed 0.1 and 0.3 Rad, respectively, in the frequency range that the OMET was estimated; such phase changes are small and within the errors of measurement and were probably not a result of middle ear muscle activation (Henin et al., 2014).
4.2 Fractional-Order Model

Parameter fitting of the model was performed through a step by step algorithm to minimize the sum of squared errors between both magnitudes and phases of the input impedance of the ear canal estimated from the model and power reflectance measurements. The normalized root mean squared log errors of the impedance magnitudes were between $0.36 - 2.98\%$. The normalized root mean squared errors of the impedance magnitudes and phases were between $4.03 - 19.11\%$ and $6.29 - 26.26\%$, respectively.

A lumped element model was fit to individual temporal bone measurements of stapes velocity over pressure at the TM for 16 cadavers (O’Connor and Puria, 2008). Mean values of stapes velocity over umbo velocity and umbo velocity over incus velocity measurements by Aibara et al. (2001) and Willi et al. (2002), respectively, were used to assist in the fitting procedure (O’Connor and Puria, 2008). The normalized root mean squared errors of the fits between $0.2 - 6$ kHz were between $9.1 - 38.1\%$ and $6.9 - 36.7\%$, for magnitude and phase estimates of stapes velocity to TM pressure, respectively (O’Connor and Puria, 2008). The magnitude errors in our model for two of the participants were lower than the magnitude errors for all the individuals in O’Connor and Puria’s model; highest magnitude error in our model belonged to NDP2, which was still lower than the magnitude errors for 62% of the individual temporal bones in O’Connor and Puria’s model (2008). The lowest phase error using our model was lower than the phase errors for all individual temporal bones by O’Connor and Puria; and the highest phase error using our model was lower than 18% of the temporal bones in O’Connor and Puria (2008).

The performance of the model was evaluated by comparing the outer-middle ear transmission estimates from the model and DPOAEs. The normalized root mean squared errors between the OMET estimates from the model and DPOAEs were between $7 - 15\%$. The OMET estimates from the model and DPOAEs were in the same level range for all participants. However, more maxima and minima were observed for the OMET estimates
using DPOAEs in comparison with OMET estimates from the model. The cochlear function might have impacted the OMET estimates using DPOAEs and played a role in such discrepancies.

Quantitative comparison/evaluation of our fractional-order lumped element model and the traditional (integer-order) lumped element models is feasible only when specific data, extracted from those elements for which the viscoelasticity was incorporated, is available. What our modeling approach offers that is not available in any traditional lumped element modeling, is:

- The ability to taking into account the history including non-local (in time) interactions between the middle ear subsystems.

- The possibility of fine-tuning through fitting the fractional orders of viscoelastic elements rather than increasing the number of elements (in traditional models), which leads to excessive cost (computation and time) of parameter fitting in the traditional models; therefore, reducing the robustness and accuracy of the model in much higher-dimensional parameter space.

4.2.1 Forward Transfer Function

The estimated forward transmission from the model ranges from $-19$ to $24$ dB between $0.2 - 10$ kHz for all six participants. When Puria (2003) and Nakajima et al. (2009) estimated the forward middle ear transmission in five and six human cadavers temporal bones, respectively, the forward transmission ranged from approximately $-25$ to $26$ dB between $1 - 10$ kHz; our estimates of forward transmission are in the range of the estimates by Puria (2003) and Nakajima et al. (2009). The estimated main resonance frequencies of forward transfer functions occurred between $3 - 5.1$ kHz. The outer ear forward transmission (outer ear gains) for all six participants are depicted in Fig. 4.1. As can be seen, low-frequency resonances occur between $3 - 5$ kHz, where the main resonance
frequencies of the outer-middle ear forward transfer functions occurred, which suggests that the high-frequency resonances in forward transfer function occur as a result of outer ear resonances. Three of the individuals had another resonance frequency between 1 – 1.3 kHz; the forward transfer functions for the rest of the participants had a bump between 0.9 – 2 kHz. These results are comparable to the forward middle ear transmission estimates from temporal bones that had one to two resonances between 0.6 – 2.5 kHz (Puria, 2003; Nakajima et al., 2009). The high-frequency resonances in our estimates of the forward outer-middle ear transmission was not observed in the middle ear forward transmission estimates from cadavers (Puria, 2003; Nakajima et al., 2009), which is expected since the ear canal was not included in these invasive measurements.

The outer ear pressure gains for the six participants are compared with measurements/models from Zwislocki (1962), Shaw (1974), Shaw and Stinson (1981), and Kringlebotn (1988) in Fig. 4.1. As can be seen, our estimated gain and gains estimated from Zwislocki (1962), Shaw (1974), Shaw and Stinson (1981), and Kringlebotn (1988) all have a low-frequency resonance and one to two high-frequency resonances above 10 kHz. Furthermore, the outer ear resonance frequencies differ depending on the length of the ear canal and the amount of reflection at the TM.
Figure 4.1: Estimated outer ear gains from the proposed model (colored dashed lines), by Kringlebotn (1988) (solid black line), Zwislocki (1962) (dashed line), Shaw and Stinson (1981) (dotted line), Shaw (1974) (circled line). The original figure is taken from Kringlebotn (1988).

Measurements of $\frac{V_{st}}{P_{TM}}$ (i.e., forward middle ear velocity transfer function) in cadavers by O’Connor and Puria (2006) and O’Connor et al. (2008) are compared with $\frac{V_{st}}{P_{TM}}$ estimates from the proposed model for the six participants in Fig. 4.2. As can be seen, the main resonance frequencies of the model occurs between 1 – 2 kHz, which are in the same frequency range that the low-frequency-resonances/bumps were observed in the forward outer-middle ear transfer function, which suggests that the presence of the bumps/resonances at low frequencies in the forward outer-middle ear transfer functions were due to the middle ear resonances.

Both estimations are close in level. However, there are slight differences in the magnitudes and phases mainly in the mid-frequency region, which might be due to differences between living human ears and cadavers and also due to changes in the cochlear impedance in invasive measurements in cadavers (Allen, 1986).
Figure 4.2: Estimated $\frac{V_{st}}{P_{TM}}$ by the proposed model (colored dashed lines) and measured $\frac{V_{st}}{P_{TM}}$ by O’Connor and Puria (2006) on cadavers.

The estimated forward transmissions in guinea pigs (Dancer and Franke, 1980; Magnan et al., 1997), gerbils (Dong and Olson, 2006), and cats (Nedzelnitsky, 1980) using invasive measurements were higher in level than the estimates of forward transmission in our model for humans. Inter-species differences may resulted in such differences; the larger area and lever ratio in guinea pigs, cats, and gerbils (reviewed in Puria et al. (1997)) may result in higher forward transmission in comparison with humans.

4.2.2 Reverse Transfer Function

The estimated reverse outer-middle ear attenuation from the model ranged from 31 to 60 dB between $0.2 – 10 \text{kHz}$ for the six participants, which were similar to the findings by Zwicker and Harris (1990). They estimated the reverse attenuation to range between 30 and 60 dB by performing a psychoacoustic study (explained in section 1.2.2).

The absolute value of the reverse transfer functions in all of our participants had a main resonance frequency between $3 – 6 \text{kHz}$, which was higher than the low-frequency resonances of the ear canals and also higher than the main resonance frequencies in the forward outer-middle ear transfer functions. Since the termination impedances were
different in forward and reverse transmissions, the frequencies and magnitudes of the resonances and anti-resonances in reverse transfer functions were expected to be different from those in the forward transfer functions. The reverse outer-middle ear transfer functions from different individuals had very similar patterns, which might be due to the same termination impedance (probe-microphone) in reverse transmission. Since the termination impedance in the forward direction was the cochlear impedance and was different among individuals, more differences among the patterns of forward transfer functions for different individuals were expected and observed.

Comparison between reverse middle ear transfer functions from the model and from invasive measurements is depicted in Fig. 4.3. The reverse middle ear attenuation ranged between $25 - 70$ dB in measurements from five temporal bones of human cadavers (Puria, 2003); as seen, our estimates of middle ear reverse attenuation is in this range. Differences between middle ear reverse transmission from the model and from the invasive measurements can be observed in both magnitudes and phases (Fig. 4.3). Because the middle ear termination in our model was the ear canal and ER-10A probe-microphone (in reverse transmission), and the middle ear termination in the invasive measurements was only ER-7C, differences between middle ear reverse transmission estimates (both magnitude and phase) from the model and from invasive measurements are expected. Comparison between the phases of the model and invasive measurements shows that the group delays are larger in invasive measurements. Because the TM delay might be larger in reverse transmission (Dong and Olson, 2006), steeper phase slopes are expected for invasive measurements; the TM delays in forward and reverse transmissions were assumed to be equal in the model. The middle ear cavity impedance change during invasive measurements might be another source of discrepancies between reverse transmission estimates from the model and from invasive measurements. During invasive measurements, the middle ear cavity was widely opened by drilling a hole into the
hypotympanum and then it was filled with saline to keep the bones in position (Puria, 2003); both of these procedures impacted the middle ear cavity impedance.

![Middle ear reverse transfer function](image)

**Figure 4.3:** Estimated $\frac{P_{sv}}{P_{TM}}$ by the proposed model (colored dashed lines) and measured $\frac{P_{sv}}{P_{TM}}$ by Puria (2003) on cadavers. The original figure is taken from Puria (2003).

### 4.2.3 Tympanic Membrane Input Impedance

A comparison of the input impedance of the TM with the measurements on cadaver ears by Voss et al. (2000) is shown in Fig. 4.4. As can be seen, the patterns of impedance magnitudes and phases from the model are similar to those of invasive measurements. The magnitudes are decreasing in level by similar slopes until a minimum and then either increasing toward a maximum or stay almost flat. Although the phase and magnitude patterns are similar, the impedance magnitudes from the model are slightly lower in level than the invasive measurements, which might be due to the differences between live human ears and cadaver ears.
4.2.4 Tympanic Membrane Delay

Tympanic membrane delays by the proposed model ranged between 10 and 43 $\mu$s with the mean value of 19.2 $\mu$s for the six participants.

TM group delay was calculated from the linear part of the mean of the middle ear forward transfer function phases estimated from measurements in human cadaver temporal bones (Aibara et al., 2001; O’Connor and Puria, 2006; O’Connor et al., 2008; O’Connor and Puria, 2008; Nakajima et al., 2009). The group delays were calculated to be 62 $\mu$s in 12 temporal bones (TBs) (Aibara et al., 2001), 134.4 $\mu$s in 4 TBs (O’Connor and Puria, 2006), 76.1 $\mu$s in 12 TBs (O’Connor et al., 2008), and 83 $\mu$s in 6 TBs (Nakajima et al., 2009).

The estimated group delays in O’Connor and Puria (2006) and O’Connor et al. (2008) were calculated by O’Connor and Puria (2008) for the 4 temporal bones (set A) and the 12 temporal bones (set B). The ear canal to TM distance was between 2 – 3 mm (O’Connor and Puria, 2006; O’Connor et al., 2008), which resulted in approximately 7.3$\mu$s delay (the time for a wave with a speed of 344.43 m/s to travel 2.5 mm); subtracting this value from
the middle ear group delays resulted in 127.1 and 54.9\(\mu s\) group delays for set A and B, respectively. To estimate the TM delay from these estimations, the delay from the umbo to the stapes should be subtracted from the group delays. The delays between the umbo and stapes are estimated to be around 51.6 and 37.3\(\mu s\) for set A and B, respectively (O’Connor and Puria, 2008). After subtracting the umbo to stapes delays from the middle ear group delays, the TM delays were estimated to be 75.5 and 17.6\(\mu s\) for set A and B, respectively (O’Connor and Puria, 2008). Therefore, to estimate the TM delay, it is critical to subtract the umbo to stapes delay and the ear canal to TM delay from the middle ear group delay. Hence, differences between our estimates of TM delay and middle ear group delays by Aibara et al. (2001), O’Connor and Puria (2006), O’Connor et al. (2008), O’Connor and Puria (2008), and Nakajima et al. (2009) are expected. The estimated TM delay in our model range between 10 – 43\(\mu s\), which is shorter than the TM delay by O’Connor and Puria (2008), which range between 17.6 – 75.5\(\mu s\). Because the TM delays by O’Connor and Puria (2008) were estimated using the mean of the phases of forward middle ear gains, differences between our estimates of TM delay, which was estimated for each individual, with those by O’Connor and Puria (2008) are expected. Another reason for the differences between the estimates of the TM delay might be due to invasive measurement and differences between TM in cadavers and alive humans. It is shown that the stiffness of living tissues is higher than the dead tissues (Perlman et al., 1984). The higher the stiffness, the faster the speed of the wave would be through the membrane. Therefore, the TM delay is expected to be shorter in living ears than in cadavers, which is in agreement with our finding.

4.2.5 Ear Canal Length

The ear canal length for one of the males (2.34 cm) was larger than the ear canal lengths for all the females (1.5, 2, 2.15, and 2.21 cm) but the ear canal length for one male
(1.78 cm) was smaller than three of the females’ ear canal lengths. Although the average ear canal length for females is smaller than males (Chan and Geisler, 1990), shorter ear canals for individual males have been observed (Zemplenyi et al., 1985). The probe position during reflectance measurements might have impacted the effective ear canal length and might be one source of differences between the estimated ear canal lengths.

4.2.6 Clinical Implications

The proposed modeling approach can be employed to model infants' ears. Comparison between model parameters in adults and infants help explain some of the differences between the anatomy and functions of the adults and infants' ears (Abdala, 1996; Abdala and Keefe, 2006).

Quantification of the outer and middle ear impact on sound transmission is vital to better assess cochlear health using OAEs. The proposed model provides estimates of forward and reverse transmissions by the outer and middle ear. Therefore, the model enables us to calculate the amount of sound going into the cochlea and the amount of sound that is generated in the cochlea before traveling back and entering to the middle ear. Hence, the model would help to separate the impact of the outer and middle ear from that of the cochlea on otoacoustic emissions, potentially improving design of hearing diagnostic tools.

Because the model was used for normal hearing adults in this work, the potentials of the model for clinical applications are needed to be investigated by employing and evaluating the model for pathological ears. Reflectance measurements can be obtained from pathological ears to determine the model parameters. Knowing the frequency-dependant impact of the outer and middle ear (from the model) on sound transmission to/from the inner ear may design hearing aids more suitable for cochlear or middle ear pathologies. For example, the model can provide cochlear compression estimates less
impacted by middle ear characteristics, which could potentially improve the way people set the frequency- and level-dependant compression gain in the hearing aids for people with sensorineural hearing loss.

4.3 Future Work

Incorporation of fractional calculus in mathematical modeling of outer and middle ear opens up new possibilities and control in capturing the complex dynamics of hearing system. While our approach was employed for normal hearing adults, in the future, this modeling technique can be used for pathological ears to come up with high-fidelity simulation tools in clinical applications. In either normal or pathological cases, there are still many open questions to be addressed in the future:

- Developing artifact rejection algorithm in order to estimate the outer-middle ear transmission in participants with lower emissions, or middle ear or cochlear pathologies.

- In order to capture multi-scale and multi-physical properties of the tympanic membrane using only one element instead of distributed transmission line, it is possible to consider the fractional-order to be variable in time. That translates into the frequency-dependant fractional-order.

- The study of consequences of physiological changes and pathologies in the outer or middle ear by perturbing the model parameters. DPOAE data from participants with middle-ear pathologies such as ossicular discontinuity, ossicular fixation, and otitis media in addition to participants with cerumen can be used for setting the parameters of such models.

- The study of middle ear muscle activation mechanics by the model. In addition, the model parameters can be set to simulate negative middle ear pressure change,
which would assist in diagnosing middle ear or cochlear pathologies in presence of negative middle ear pressure.

- Study the differences between living ears and cadavers ears or between invasive and noninvasive measurements in non-humans. Data from living ears can be collected before either invasive measurements or measurements from cadavers and the parameters of the model can be fixed to match each data set estimations and then used for the comparison.

### 4.4 Conclusions

- Sum of forward and reverse outer-middle ear transmission was estimated noninvasively using DPOAE generator components in humans. The estimated outer-middle ear transmission was negative with attenuations ranging from 18 to 39 dB between 1 – 3.3 kHz.

- A fractional-order lumped element model was proposed to simulate the viscoelasticity of the biological tissues of the middle ear. The model was fit successfully to the impedance magnitude and phase estimates from power reflectance measurements from six individuals. The model was validated by comparing its estimates of outer-middle ear transmission with the estimates using DPOAE generator components.

- The forward gain and reverse attenuation computed from the model ranged from -19 to 24 dB and from 31 to 60 dB between 0.2 – 10 kHz, respectively, for six participants. The resonance frequencies in the forward and reverse directions from the model were found to be different due to different termination impedances.

- The outer-middle ear transmission estimates from the fractional-order lumped element model were comparable to the estimates using DPOAE generator components.
Incorporating fractional elements into the traditional lumped element models yields new possibilities for fine tuning in complex biological systems, where the Newtonian and Hook's laws fail to fully capture the viscoelastic response of the biomaterials in human ear.
Appendices

A DPOAE and Its Components

A.1 NDP1

Figure 4.5: [NDP1] $2f_1 - f_2$ (solid lines) and $2f_a - f_b$ (dashed lines) composite DPOAEs, generator components, and reflection components depicted in panel (a), (b), and (c), respectively.
A.2 NDP2

Figure 4.6: [NDP2] Two-tone (dashed lines) and three-tone (solid lines) $2f_1 - f_2$ composite DPOAEs, generator components, and reflection components depicted in panel (a), (b), and (c), respectively.
A.3 NDP3

Figure 4.7: [NDP3] Two-tone (dashed lines) and three-tone (solid lines) $2f_1 - f_2$ composite DPOAEs, generator components, and reflection components depicted in panel (a), (b), and (c), respectively.
Figure 4.8: [NDP5] Two-tone (dashed lines) and three-tone (solid lines) $2f_1 - f_2$ composite DPOAEs, generator components, and reflection components depicted in panel (a), (b), and (c), respectively.
A.5 NDP6

Figure 4.9: [NDP6] Two-tone (dashed lines) and three-tone (solid lines) $2f_1 - f_2$ composite DPOAEs, generator components, and reflection components depicted in panel (a), (b), and (c), respectively.
A.6 NDP11

Figure 4.10: [NDP11] Two-tone (dashed lines) and three-tone (solid lines) $2f_1 - f_2$ composite DPOAEs, generator components, and reflection components depicted in panel (a), (b), and (c), respectively.
B Probe Fit

B.1 NDP1

Figure 4.11: [NDP1] $L_1$ and $L_a$ primaries estimated at the entrance of the ear canal at different $L_b$s (depicted in the legend) in the three-tone condition.

Figure 4.12: [NDP1] The patterns observed when subtracting $L_1$ at $L_b$s of 35, 40, 45, and 50 dB SPL from $L_1$ when $L_b = 55$ dB SPL (panel (a)). Panel (b) depicts same differences for $L_a$. 
Figure 4.13: [NDP1] Differences between $L_1$ ear canal calibrations when $L_b = 55$ and $L_b = 40$ (panel (a)) or 45 dB SPL (panel (b)). The difference between ear canal calibrations mean at $L_b = 55$ and before 40 or 45 dB SPL are depicted by light green lines; the difference with after 40 or 45 dB SPL are depicted by dark green lines. The differences between $f_1$ levels for $L_b = 55$ and $L_b = 40$ or 45 dB SPL are shown in red.

Figure 4.14: [NDP1] Differences between $L_a$ ear canal calibrations when $L_b = 55$ and $L_b = 40$ (panel (a)) or 45 dB SPL (panel (b)). The difference between ear canal calibrations mean at $L_b = 55$ and before 40 or 45 dB SPL are depicted by light green lines; the difference with after 40 or 45 dB SPL are depicted by dark green lines. The differences between $f_1$ levels for $L_b = 55$ and $L_b = 40$ or 45 dB SPL are shown in red.
Figure 4.15: [NDP1] The $f_1$ phase differences (panel (a)) and $f_a$ phase differences (panel (b)) between phases when $L_b = 55$ dB SPL and phases when $L_b = 35, 40, 45$ and 50 dB SPL.
C Model Parameter Fitting

C.1 NDP1

Figure 4.16: [NDP1] Imaginary parts of the impedance magnitude estimates from the simplified model and from reflectance measurements.

Figure 4.17: [NDP1] Impedance magnitude estimates from the simplified model (dashed black line) and from the reflectance measurements (green line) after fixing $R_s$ and $K_s$. 
Figure 4.18: [NDP1] Impedance magnitude estimates from the simplified model (dashed black line) and from the reflectance measurements (green line) after fixing $m_s$.

Figure 4.19: [NDP1] Impedance magnitude and phase estimates from the simplified model (dashed black lines) and from the reflectance measurements (green lines).
Figure 4.20: [NDP1] Impedance magnitude estimates from the model (black dashed lines) and from reflectance measurements (green lines), after fixing the values of the fractional capacitors.

Figure 4.21: [NDP1] Impedance magnitude and phase estimates from the model (black dashed lines) and from the reflectance measurements (green lines), after setting the cochlear and ossicles’ masses and resistors.
Figure 4.22: \([NDP1]\) Impedance magnitude and phase estimates from the model (black dashed lines) and from reflectance measurements (green lines) after setting the fractional-orders of the fractional capacitors and the middle ear cavity parameters.

Figure 4.23: \([NDP1]\) Impedance magnitude and phase estimates from the transformed model (black dashed lines) and from reflectance measurements (green lines).
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