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Süleyman Hilmi Kal
Central Bank of the Republic of Turkey

Ilhami Gunduz
NYC College of Technology

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Global Capital Flows, Time Varying Fundamentals and Transitional Exchange Rate Dynamics: An MS-VAR Approach

Küresel Sermaye Akımları, Zamana Göre Değişen Temeller ve Geçişken Kur Dinamikleri: Bir MS-VAR Yaklaşımı

Süleyman Hilmi KAL[1], İlhami GÜNDÜZ[2]

ABSTRACT

This paper studies whether dynamic relationship between exchange rate and economic and financial fundamentals vary depending on exchange rate is overvalued and undervalued with respect to its fundamental value. To achieve this, we implement two-state Markov Switching Vector Auto Regression (MSVAR) model with time varying transition probabilities to investigate whether the relationship among exchange rate, interest rate and inflation dynamics depend on overvaluation and undervaluation of exchange rates for the pre-crises period between years 1972-2009. We govern the transition between the undervalued and overvalued states by using Sharpe Ratios of debt and equity investments of the currency to assess whether risk adjusted returns induce overvaluation or undervaluation of the currencies. We employ this model to the bilateral exchange rate, which is defined between US Dollar and four highly traded currencies (AUD, CAD, JPY, and UKS). We provide evidence that the relationship among these variables varies in terms of on magnitude, direction and statistical significance in between the overvalued and undervalued regimes. Furthermore, we show that risk adjusted excess debt and equity returns influence the overvaluation and the undervaluation of the currencies.

Keywords: Capital flows, Markov regime switching, Sharpe ratio
JEL Classification: D51, F31, O16

[1]Dr., Central Bank of the Republic of Turkey, Ankara, Turkey
[2]Phd Student, The Graduate Center, Cuny, New York, USA

ORCID: S.H.K. 0000-0003-3698-3515;
İ.G. 0000-0002-9985-7754

Corresponding author/Sorumlu yazar:
Süleyman Hilmi KAL,
Central Bank of the Republic of Turkey,
Ankara, Turkey
E-mail/E-posta: suleyman.kal@tcmb.gov.tr

ÖZ

Bu çalışma döviz kurları ile ekonomik ve finansal değişkenler arasındaki dinamik ilişkinin kurun temel değerine göre daha değerli ve az değerli olmasına bağlı olarak nasıl değiştiğini

araştırmaktadır. Bu maksatla, kriz öncesi 1972-2009 yılları arasında, döviz kuru, faiz oranı ve enflasyon dinamikleri arasındaki ilişkinin kurun daha değerli ve az değerli olmasına bağlı olup olmadığını incelemek için zamanla değişen geçiş olasılıklarını kullanan MSVAR yöntemi uygulanmıştır. Riske uyarlanmış getirinin, para birimlerinin daha değerli veya az değerli olmasına neden olup olmadığını belirlemek için, borç ve öz kaynak yatırımlarının Sharpe oranları kullanılarak daha değerli ve az değerli durumlar arasındaki geçişler sağlanmıştır. Bu model en fazla işlem hacmine sahip olan, Amerikan Doları ve 4 para birimi (AUD, CAD, JPY ve UKS) arasındaki ikili döviz kurları için uygulanmıştır. Bu değişkenler arasındaki ilişkinin daha değerli ve az değerli rejimler arasında yön, büyüklük ve istatiksel anlamlılık açısından değiştiği belgelenmiştir. Ayrıca, riske uyarlanmış hisse senedi ve borç senedi getirilerinin kurların daha değerli olmasını ve az değerli olmasını etkilediği gösterilmiştir.

Anahtar kelimeler: Sermaye akımları, Markov switching modeli, Sharpe oranı
JEL Sınıflaması: D51, F31, O16

1. Introduction

There are some anomalies in international economics. Two of them are related to foreign exchange rates. The first anomaly or puzzle is the relationship between exchange rates and interest rates, which is characterized by Uncovered Interest Rate Parity (UIP) condition. UIP predicts that high yielding currencies should be depreciated. However, there is overwhelming empirical evidence against UIP.

Studies on this topic go back to Fisher (1930). He tests whether investors equate the returns on short-term assets of different currencies by converting them to the same currency or not. Since then, many researchers have obtained systematic deviations from parity. Fama (1984) finds negative coefficient between interest rate differential and forward premium by running an Ordinary Least Square (OLS) regression between the spot exchange rates and 30-day forward exchange rates of nine major currencies. In contrast to UIP, he concludes that an increase in the interest differential is associated with a decline in expected depreciation. He explains this anomaly with the following two factors: the first is that the variance of the risk premium is greater than the variance of expected depreciation and the interest rate differential, and the second is that the covariance between the expected depreciation and the risk premium is negative.

For the post Bretton Woods period especially, numerous researchers (MayHodrick, 1987; Froot, 1995; Engel, 1994) report strong rejection of UIP by

doing empirical studies. However, favorable empirical evidence has appeared (Bekaert and Hodrick, 2001; Chinn and Meredith, 2001). UIP holds better on long horizons and deviations from UIP occurs more in the shorter periods. These findings pave the way to the more nuanced and sophisticated studies of Lothian and Wu (2005) who conclude that results in longer horizons are inconsistent with expectation hypothesis contrary to the theory in post 1970 period by using a data span of two centuries and running rolling regression for long periods and shorter sub periods. Bekaert, Wei and Xing (2002), on the other hand, obtain mixed statistical evidence against UIP. They conclude that UIP is currency depended rather than being horizon depended. Alquist and Chinn (2006) find that interest rate parity holds better in short periods while net exports does well in predicting the exchange rates during shorter periods. In contrast to other markets, LIBOR markets have minimal frictions which could lead to rejecting UIP. Omar, Haan, Scholtes (2012) test UIP using LIBOR rates for a wide range of maturities. They find that UIP holds for short-term maturities when market-specific heterogeneity is controlled for. Furthermore, their estimation results show that the speed of adjustment to the long-run equilibrium is proportional to the maturity of the underlying instrument. F. Boschen and Smith (2012) show that the UIP anomalies apparent in six major currency pairs have diminished over our 1995-2010 sample period. They further show that the observed decline in deviations from UIP is associated with the substantially higher transaction volume now present in the foreign exchange markets.

The second anomaly is the Purchasing Power Parity (PPP), which describes the relationship between price differentials and exchange rates. PPP suggests that currencies of the high inflation countries should depreciate. However, empirical evidence for PPP is at best mixed as that of UIP. Fundamental exchange rate models based on PPP failed to outperform the Random Walk models as reported by the survey of Meese and Rogoff (1983). There has been a vast literature on PPP (Froot and Rogoff, 1995; Sarno and Taylor, 2002; Taylor, 2006). Nowadays, a new wave of studies is emerging to capture the deviations from the PPP such as Kanas (2006) who revisits Taylor's (2002) study by extending his data set and using the regime switching approach. He finds that PPP holds for most of the

period with some deviations. Lee and Yoon (2007) use a similar approach in which they employ the Hamiltonian regime switching model. They find that PPP holds locally within the regime. Various studies have generally been unable to reject the hypothesis that the real exchange rate follows a random walk.

A brief summary of the recent studies with better tests, statistical techniques and different sample periods indicates a more nuanced relationship among exchange rates, interest rate and price differentials (UIP, PPP). As it can be noticed from the above survey, most of the studies in UIP and PPP examine exchange rate-interest rate and exchange rate-inflation rate dynamics separately in isolation, yet very few empirical researches have focused on modeling UIP and PPP jointly. Johansen and Juselius (1992), Juselius (1991, 1995), MacDonald and Marsh (1997, 1999), Juselius and MacDonald (2003), Bjorland and Hungnes (2005) are some exceptions. It is also evident from the brief survey that both UIP and PPP do not always hold and show time and circumstance varying characteristics ranging from short-run to long-run, or high inflation to low inflation. Variations of UIP and PPP conditions motivated to search for non-linear methods. Kanas (2006) and Lee and Yoon (2007) propose various non-linear approaches. All these approaches report that there is a very strong evidence of relationship between UIP and PPP.

In this study, we have two purposes. Our first aim is to capture non-linear co-dynamics of UIP and PPP depending on overvaluation and undervaluation of the currencies with respect to their fundamental values. Our second goal is to explore the effect of risk adjusted excess returns on global capital flows. To do so, we implement two-state Markov Switching Vector Autoregressive Model (MSVAR) with time varying transition probability in which Switching between the states are linked to risk adjusted excess returns (Sharpe Ratio) of domestic debt and equity markets to capture overvaluation and undervaluation of exchange rates. Therefore, we implement the model on the quarterly data between 1972:1 and 2009:4 of the four floating currencies: Australian Dollar (AUD), Canadian Dollar (CAD), Japanese Yen (JPY), and the UK Sterling (UKS). We use these currencies since they have the highest cross border trade volume. Our results provide

evidence of time varying joint dynamics of UIP and PPP by producing time varying coefficients between exchange rate, interest rate and inflation differentials in an MSVAR setting. We hope that this study will shed light on some unresolved issues, which are still considered as puzzles in international finance and economics.

This paper contributes to the existing literature on several fronts. First, with a new perspective, we showed that the relationship between exchange rates, inflation and interest rates depend on undervaluation and overvaluation of currencies. Second, we also showed that overvaluation and undervaluation of the currencies depend on their risk adjusted returns of equity and debt investments. The rest of the paper is organized as follows. Section 2 describes the model, section 3 discusses the data, section 4 explains how we test Markov Switching model, and section 5 presents and interprets the empirical results. Section 6 summarizes and concludes this study.

2. Model

Two-state Markov Regime Switching Vector Autoregression (MSVAR) model with time varying transition probabilities is employed in this paper. Quandt (1958), Godlfeld and Quandt (1973) who use the Markov regime switching models in econometrics are pioneers in this literature. However, the method is later used and popularized by Hamilton's (1990).

In Markov models, observable variable (y_t) depends on state variables which are not directly observed. Here s_t is used for state variable definition. States are defined as state 0 and state1. In this context, states are defined depending on the overvaluation and undervaluation of the exchange rate with respect to its value determined by economic fundamentals.

State variables are unobserved and evolve following a Markovian regime change. Here, (s_t), state variable governs the distribution of each period. Thus, there are two distributions each with different means and variance of y_t which are based on the respected state. We define state 0 as an overvalued state and state 1 as an undervalued state. So y_t follows a state dependent behavior:

$$\left(y_t/s_t\right) \sim N\left(\mu_{s_t}, \sigma_{s_t}^2\right) \tag{1}$$

Thus, in case $s_t = 0$ the observed changes of y_t is a random draw from distribution $\left(Y_{t/s_t}\right) \sim N\left(\mu_0, \sigma_0^2\right)$ and in case $s_t = 1$, the observed changes of y_t is a random draw from distribution $\left(Y_{t/s_t}\right) \sim N\left(\mu_1, \sigma_1^2\right)$.

Following Abiad (2002), the probability density of y_t conditional on state variable is formulated as follows:

$$f(y_t|s_t) = \frac{1}{\sqrt{2\pi\sigma_{s_t}}} \exp\left\{\frac{-(y_t - \mu_{s_t})^2}{2\,\sigma_{s_t}^2}\right\} \tag{2}$$

Transition between the states is only based on the previous state. Transition probability is defined as the probability of switching from one state to another state. For example, the transition from State (t - 1) to State (t) is shown as:

$$P(s_t = i|\, s_{t-1} = j) = P_{ij} \tag{3}$$

Where P is probability, $i = 0,1$ and $j = 0,1$.
Transition is formulated as follows:

$$P(s_t = 1|\, s_{t-1} = 1) = P_{11} \tag{4}$$

$$P(s_t = 0|\, s_{t-1} = 1) = P_{01} = 1 - P_{11} \tag{5}$$

$$P(s_t = 0|\, s_{t-1} = 0) = P_{00} \tag{6}$$

$$P(s_t = 1|\, s_{t-1} = 0) = P_{10} = 1 - P_{00} \tag{7}$$

Transition probabilities matrix is a (*2x2*) and it is called transition matrix Γ. It is written Γ as follow:

$$\Gamma = \begin{bmatrix} P_{00} & P_{10} \\ P_{01} & P_{11} \end{bmatrix} \quad \text{and} \quad \sum_{j=0}^{1} P_{ij} = 1, \quad 0 \le P_{ij} \le 1$$

Transition probabilities can be constant, as Hamilton suggested in his very well cited work (Hamilton, 1989), or it can be time varying, as developed by Diebold, Weinbach and Lee (1994). In the Hamiltonian framework, the probabilities of switching between the states are fixed, exogenous and do not vary over time. These features of the model limit the explanatory power of the Markov process. Allowing transition probabilities to change over time, depending on a vector of variables, enriches the Markov process by enabling it to model the underlying process of transitional dynamics explicitly. Hamilton's constant transition model is later evolved by Diebold, Weinbach and Lee (1994) by addition of time-varying transition probabilities which are estimated with logistic functions of vector of Sharpe Ratios (x_{t-1}) as follows:

$$p_t^{11} = P(s_t = 1 \mid s_{t-1} = 1, x_{t-1}; \beta_1) = \frac{exp\ x_{t-1}\beta_1}{1+exp\ x_{t-1}\beta_1} \tag{8}$$

$$p_t^{01} = P(s_t = 0 \mid s_{t-1} = 1, x_{t-1}; \beta_1) = 1 - \frac{exp\ x_{t-1}\beta_1}{1+exp\ x_{t-1}\beta_1} \tag{9}$$

$$p_t^{00} = P(s_t = 0 \mid s_{t-1} = 0, x_{t-1}; \beta_0) = \frac{exp\ x_{t-1}\beta_0}{1+exp\ x_{t-1}\beta_0} \tag{10}$$

$$p_t^{10} = P(s_t = 1 \mid s_{t-1} = 0, x_{t-1}; \beta_0) = 1 - \frac{exp\ x_{t-1}\beta_0}{1+exp\ x_{t-1}\beta_0} \tag{11}$$

Equations (8) to (11) are transition probabilities, which are time varying logistic functions. These transition probabilities are function of Sharpe Ratios. Sharpe Ratio is the risk adjusted excess returns in debt and equity markets. Sharpe Ratio is as proxy variable to measure the capital flows because the risk adjusted rate of excess return important criteria for forex trade. Thus, Sharpe ratio with x_{t-1} is formulated as follow:

$$x_{t-1} = \frac{E[R_{kt}] - R_f}{\sigma_k} \tag{12}$$

Where R_f is risk-free interest rate and σ is the standard deviation of expected return of the investment strategy, $E[R_{KT}]$ is the expected rate of return from

investments in the domestic debt market or domestic equity market.[1] We describe these two markets as follows:

Domestic Debt Market (DDM):

$$E[R_{DDM}] = \text{-E}[\frac{e_{t+1}-e_t}{e_t}] + [r_t - r_t^*] \tag{13}$$

Expression is the expected return of the currency for appreciation of domestic currency and the second expression, $[\,r_t - r_t^*\,]$, is the interest rate differential between domestic and foreign nominal interest rates.

Equity Market (EM):

$$E[R_{EM}] = \text{-E}\left[\frac{e_{t+1}-e_t}{e_t}\right] + \left[\frac{SMY_{t+1}-SMY_t}{SMY_t}\right] \tag{14}$$

The first expression on the right hand side of equation (14), $\{\text{-E}\left[\frac{e_{t+1}-e_t}{e_t}\right]\}$, is the expected return due to appreciation of domestic currency and the second expression, $[\frac{SMY_{t+1}-SMY_t}{SMY_t}]$, is the stock market yield differential between domestic and foreign currencies (US Dollar).

Since Sims (1980) study, Vector Autoregressive model has become one of the major tools of empirical studies. Krolzig (1997) introduces the regime changes to vector autoregressive model. An MSVAR model provides framework of modeling multivariate representation of related variables non-linearly. Following Martinez Peria (2002), we model exchange rate, interest rate and price differentials jointly in the following way:

$$\varDelta e_t = c_{S_t}^e + \alpha_{S_t}^e[\varDelta e_{t-1}] + \theta_{S_t}^e[\varDelta rd_{t-1}] + \lambda_{S_t}^e[\varDelta \pi d_{t-1}] + \varepsilon_t \tag{15}$$

$$rd_t = c_{S_t}^r + \alpha_{S_t}^r[\varDelta e_{t-1}] + \theta_{S_t}^r[\varDelta rd_{t-1}] + \lambda_{S_t}^r[\varDelta \pi d_{t-1}] + \omega_t \tag{16}$$

$$\pi d_t = c_{S_t}^\pi + \alpha_{S_t}^\pi[\varDelta e_{t-1}] + \theta_{S_t}^\pi[\varDelta rd_{t-1}] + \lambda_{S_t}^\pi[\varDelta \pi d_{t-1}] + v_t \tag{17}$$

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[1] k is Domestic Debt Market or Equity Market

In the above, an MSVAR system not only coefficients but also constant terms $(c_{S_t}^e, c_{S_t}^r, c_{S_t}^\pi)$ are functions of state variables. That is, as we move from one state to other coefficients vary. Disturbance terms $(\varepsilon_t, \omega_t, v_t)$ are i.i.d, N (0, 1). In order to obtain likelihood function of the model as a product of the likelihood functions of each individual equation, we transform VAR system by using a Cholesky decomposition to get the diagonal variance-covariance matrix of the model.[2] This transformation is harmless (Martinez Peria, 2002). Hence, we write the transformed VAR system as follows:

$$\Delta e_t = c_{S_t}^e + \alpha_{S_t}^e [\Delta e_{t-1}] + \theta_{S_t}^e [\Delta rd_{t-1}] + \lambda_{S_t}^e [\pi d_{t-1}] + \eta_t \qquad (18)$$

$$rd_t = c_{S_t}^r + \gamma_{S_t}^r [\Delta e_t] + \alpha_{S_t}^r [\Delta e_{t-1}] + \theta_{S_t}^r [rd_{t-1}] + \lambda_{S_t}^r [\pi d_{t-1}] + \zeta_t \qquad (19)$$

$$\pi d_t = c_{S_t}^\pi + \gamma_{S_t}^\pi [\Delta e_t] + \alpha_{S_t}^\pi [\Delta e_{t-1}] + \kappa_{S_t}^\pi [rd_t] + \theta_{S_t}^\pi [rd_{t-1}] + \lambda_{S_t}^\pi [\pi d_{t-1}] + \Omega_t \qquad (20)$$

We use Maximum Likelihood Method (MLE) to estimate equations 18, 19 and 20. We report estimated coefficient values of transformed VAR model, equations 18, 19, and 20, in Tables 2, 3, and 4. We define the variables - change in exchange rate, interest rate differential, and inflation rate differential - in the above VAR system as follows:

$$\Delta e_t = \frac{e_t - e_{t-1}}{e_{t-1}} \qquad (21)$$

$$rd_t = r_t - r_t^* \qquad (22)$$

$$\pi d_t = \Delta \pi_t - \Delta \pi_t^* \qquad (23)$$

Where Δe_t is change in exchange rate, rd_t is interest rate differential, r_t is nominal interest rate, πd_t is inflation rate differential, and $\Delta \pi d_t$ is percentage change of inflation rate.[3]

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[2] We transform VAR system for computational purposes.

[3] The variable which has an asterisk stands for foreign country's variable, whereas non asterisk variable stands for domestic variable.

We define the deviation of the observed exchange rate from its fundamentally determined value as follows:

$$d_t = \Delta S_t - \Delta f_t \tag{24}$$

Where d_t is deviation parameter, ΔS_t is observed change in nominal exchange rate, and Δf_t is the fundamentally determined change in exchange rate following the sticky price exchange rate model with stock prices. We formulate Δf_t as follows:

$$\Delta f_t = \alpha_0 + \alpha_1(\Delta m_t - \Delta m_t^*) - \alpha_2(\Delta y_t - \Delta y_t^*) + \alpha_3(r_t - r_t^*) + \alpha_4(\Delta \pi_t - \Delta \pi_t^*) + \alpha_5(SM - SMY_t^*) \tag{25}$$

Where α_0 is an intercept, m_t is a money supply, y_t is a gross domestic product, r_t is a nominal interest rate, π_t is an inflation rate, and SMY_t is a stock market yield.

As mentioned above, we define state 0 as an overvalued state and state 1 as an undervalued state. We identify both overvalued state and undervalued state by comparing the deviation parameter value with zero. The overvalued state is identified when deviation parameter is below zero, whereas the undervalued state is identified when deviation parameter is above zero. On the other hand, overvalued (undervalued) state is identified when the fundamentally determined change in exchange rate is greater (smaller) than the observed change in the exchange rate. We assume that the risk adjusted excess return in each period changes the expectations of the global investors and in turn influences the position of the exchange rate relative to its fundamentally determined value.

In this study, we apply time-varying transition probabilities of switching model to above VAR platform. We utilize risk adjusted excess returns in debt and equity markets as the variables governing transition probabilities. Mechanics of the transition dynamics work by depending on the value of the coefficient of the Sharpe ratios. If the coefficient of the Sharpe ratios is positive, the transition dynamics works in the following way. As Sharpe ratios get higher, this induces numerator and denominator of the logistic function to converge to each other and the logistic function approaches to 1 (100%). As logistic function gets closer

to 1, the probability of staying in the same state (P_{11}, P_{00}) gets larger and probability of transition between the states gets smaller due to sum of the probability of staying in the same state and the probability of transition to the other state is equal to 1 ($P_{01}=1-P_{11}$ and $P_{10}=1-P_{00}$). On the other hand, if the coefficient of Sharpe Ratio is negative, then the opposite of what is described above happens. That is probability of staying in the same regime decreases, whereas probability of transition between the regimes increases.

3. The Data

In this paper, we investigated the relationship between exchange rates, inflation and interest rates depending on overvaluation and undervaluation of the exchange rates with respect to their fundamental value. Our sample period is selected pre-great financial crises period, since the period after the financial crises is marked some unconventional monetary practices which may be subject of a different study. We use seasonally adjusted quarterly data between 1972:1 and 2009:4 of four bilateral nominal exchange rates of four countries: Australia, Canada, Japan, and the United Kingdom.[4] Indeed, we use data on five macroeconomic variables-money supply, income, inflation rates, long term (15 year) interest rate, and equity prices from these four countries and on the United States (US). We extract data from the International Financial Statistics (IFS) published by International Monetary Fund. In order to measure money supply, we use seasonally adjusted country specific data: M2+CDS is used for Japan, M2 is used for US, gross M2 is used for Canada, M3 is used for Australia, and M4 for UK.[5] To measure income, we use Gross Domestic Product (GDP) chain volume with country specified reference date prices: 2002 reference prices is used for both Australia and Canada, whereas 2000 reference prices used for both UK and US, and nominal GDP is used for Japan. In order to measure long-term interest rates, we use 15-year Treasury bill rates for all currencies. We use Equity prices as

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[4] Bilateral exchange rate is defined as price of US dollar in terms of each foreign currency.

[5] We extract UK data from statistical data base of Bank of England since there are some discontinuities for UK monetary aggregate data in IFS database.

the bases for stock market yield for each currency.[6] We use Consumer Price Index (CPI) to measure price level in each economy. We calculate the inflation rate for each quarter by using CPI.

4. Testing Markov Switching Approach

Markov process is crucial to get links between the regimes and the economic fundamentals. Indeed, we want to test whether the exchange rate follows a single regime with the same mean in both states versus different means in two states. However, applying a test to make this comparison is not straightforward. Since the constant coefficient model with one regime does not have transition probabilities, whereas the Markov model has it. Therefore, we adopt the well-known Wald test to compare these two approaches.[7] In this study, we adopt Wald test as in Frommel et al (2005) and DeGrauwe (2001). Null hypothesis of this test checks whether means (μ) of the two states are equal to each other versus they are different from each other.

The null and alternative hypotheses are tested as follows:

H_0: $\mu_0 = \mu_1$
H_1: $\mu_0 \neq \mu_1$

We perform Wald test statistic by using following formula:

$$\frac{\widehat{\mu_0} - \widehat{\mu_1}}{var(\widehat{\mu_0}) + var(\widehat{\mu_1}) - 2cov(\widehat{\mu_0}, \widehat{\mu_1})} \approx X^2 \qquad (26)$$

5. Discussion of Results

Table 1 shows Sharpe Ratios of each currency. As stated before, we use the Sharpe Ratios as a proxy variable to measure the capital flows. We govern the

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[6] End of quarter share prices used for each country.

[7] See Engel and Hamilton (1983).

transition between the undervalued and overvalued states by using the Sharpe Ratios of debt and equity investments of the currency to assess whether the risk adjusted returns induce overvaluation or undervaluation of the currencies. The results of the Sharpe ratios are interesting. For AUD, higher risk adjusted returns at debt and equity markets increases the likelihood of switching from an undervalued state to overvalued state. For CAD, higher risk adjusted returns in debt markets increases the likelihood of switching from an overvalued state to undervalued state. Hence, it increases the likelihood of staying in the undervalued state. However higher risk adjusted returns in the equity markets have the opposite effect on CAD. For JPY, Sharpe ratios do not have any statistically meaningful effect in both states. For UKS, Sharpe Ratios are meaningful in the undervalued state although it is not statistically significant in the overvalued state.

Table 1: Estimated Coefficients of Time Varying Transition Probabilities of Domestic Debt Market and Equity Market in State 0 and State 1

	Australia	t-stat	Canada	t-stat	Japan	t-stat	UK	t-stat
$\beta\text{-}DDM_0$	13,7236	33,0626	4,5518	41,4	5,9039	33,2782	-1,5711	-7,4832
βDDM_1	-4,1054	-4,5461	-2,2071	-4,0275	-6,7252	-9,6025	-3,4697	-25,7497
$\beta\text{-}EM_0$	0,1247	-0,7138	-0,2227	-2,0349	0,3493	0,5715	1,4837	1,8726
$\beta\text{-}EM_1$	-1,53	-1,0836	0,4164	0,8354	-0,0447	-0,0638	2,9204	55,9406

Note: Variable is significant at 10% significance level according to t-test result. Betas are the coefficients of governing variables x_{t-1} (Sharpe Ratio = $(SR_{(t)} = \frac{E[R_{st}] - R_{ft}}{\sigma_s}$)) of time varying transition probability which is formulated as logistic function $(p_t^{11} \quad \frac{exp \, x_{t-1}\beta_1}{1+exp \, x_{t-1}\beta_1})$. According to this, if the beta is positive, higher risk adjusted return increases the probability of staying in the same regime and decreases the probability of transition to the other regime $(p_t^{01} = 1 - p_t^{11})$.

Table 2 shows the estimated coefficients of exchange rate equation (Equation 18) in the both overvalued and the undervalued states. In the overvalued state: higher interest rate differential depreciates AUD and JPY, whereas it appreciates UKS. Nonetheless, it has no statistically significant effect on CAD in the same state. In the undervalued state, the interest rate differential appreciates JPY although it does not have any statistically significant effect on other currencies. This result provides a very important insight to the effect of interest rate differential on the

currencies and provides evidence that the relationship between interest rates and exchange rates are varying depending on currencies are overvalued or undervalued with respect to their fundamentally determined values. Moreover, not only the interest rate differential but also inflation differential has a state dependent effect on the currencies. It causes depreciation of CAD in both of the states and JPY in the overvalued state. However, it does not have any statistically meaningful effect on the other currencies in both of the states. It is also important to notice that all of the currencies are more or less high volatilities in the undervalued state.

Table 2: Estimated Coefficients of MSVAR on Exchange Rate Equations in State 0 and State 1

		Australia	t-stat	Canada	t-stat	Japan	t-stat	UK	t-stat
State 0	Cons	-0,0381	-1,2899	0,0839	1,6222	0,9357	0,1685	0,052	1,8329
	Δe_{t-1}	1,0168	35,696	0,9181	2,4784	0,9475	3,5328	0,938	0,1305
	rd_{t-1}	-0,0029	0,0104	0,0903	0,1569	111,2542	1,6444	-0,7911	-5,7875
	πd_{t-1}	-0,9339	-2,6879	-0,0915	-0,0555	60,7364	0,3927	-0,0574	-0,1363
State 1	Cons	-0,0101	-0,1036	-0,0047	-0,1175	-1,4919	-0,6088	0,0357	0,5789
	Δe_{t-1}	1,0522	7,0131	1,0225	34,6092	1,0325	5,7034	0,9343	0,0301
	rd_{t-1}	0,0588	0,1281	0,2214	0,6693	-55,0322	-1,5475	0,2315	0,7515
	πd_{t-1}	0,9146	1,2426	0,9705	1,0048	-63,278	-0,6815	0,007	0,0098
Volatility of	η_0	0,0009	4,2051	0,0012	3,0858	50,9295	3,9781	0,0002	2,548
	η_1	0,0012	1,1513	0,0004	2,2621	20,438	4,7762	0,0011	3,3683

Note: Variable is significant at 5% significance level according to t-test result. There are two sets of estimated coefficients of MSVAR in two state processes. State 0 is the overvalued state in which the observed change in exchange rate is less than the change suggested by the economic fundamentals. State 1is the undervalued state in which the observed change in exchange rate is more than the change suggested by the economic fundamentals.

Table 3 shows the effect of exchange rate, lagged of exchange rate and inflation differential on interest rate differential. According to this, exchange rate and lagged value of exchange rate of UKS in an overvalued state has a significant effect on the interest rate differential. None of other variables have statistically meaningful effect on the interest rate differential almost for all other currencies in the neither of the states.

Table 3: Estimated Coefficients of MSVAR on Interest Rate Equations in State 0 and State 1

		Australia	t-stat	Canada	t-stat	Japan	t-stat	UK	t-stat
State 0	Cons	0,0023	0,2463	0,0115	0,8483	-0,0099	-0,883	-0,002	-0,2031
	Δe_t	-0,0311	-0,4842	-0,0045	-0,0659	-0,0003	-0,7127	-0,127	-1,0082
	Δe_{t-1}	0,0274	0,4122	-0,0024	-0,0346	0,0004	0,8413	0,136	1,0999
	rd_{t-1}	0,9408	4,6875	0,8659	9,4421	0,7898	5,6636	0,822	6,7632
	πd_{t-1}	0,3722	2,8689	-0,0002	-0,0009	0,2708	0,7266	0,1349	0,6961
State 1	Cons	0,0104	1,4613	-0,0019	-0,1339	-0,0026	-0,5916	-0,007	-0,3099
	Δe_t	0,0342	0,3789	0,0303	0,3129	-0,0001	-0,5422	-0,031	-0,3607
	Δe_{t-1}	-0,0406	-0,4282	-0,0288	-0,2859	0,0001	0,5515	0,0502	0,6002
	rd_{t-1}	0,833	9,5461	0,9185	8,9417	0,894	7,5054	0,7768	6,6757
	πd_{t-1}	-0,0306	-0,0993	0,0191	0,0749	0,2239	1,5969	0,239	0,9702
Volatility of	η_0	0,0001	4,8674	0,0001	4,0113	0,0002	4,6569	0,0001	3,7089
	η_1	0,0001	3,0141	0,0000	3,7223	0,0001	0,5515	0,0002	3,7207

Note: Variable is significant at 5% significance level according to t-test result. There are two sets of estimated coefficients of MSVAR in two state processes. State 0 is the overvalued state in which the observed change in exchange rate is less than the change suggested by the economic fundamentals. State 1 is the undervalued state in which the observed change in exchange rate is more than the change suggested by the economic fundamentals.

Table 4 shows the effect of exchange rate, lag of exchange rate, interest rate differential and lag of price differential on the price differential. The findings here provide some interesting relationships. According to these findings, higher exchange rate increases the price differential of both AUD and CAD when these currencies are overvalued. It has the same effect for UKS price differential when UKS is undervalued. Again of AUD, when it is overvalued higher interest rate differential decreases prices differential and lag of interest rate differential increases price differential when it is undervalued. For CAD again higher interest rate differential causes higher price differential when CAD in overvalued. Another interesting result of this table is that in both of the states, higher price differentials are followed by lower ones or vice versa.

Table 4: Estimated Coefficients of MSVAR on Inflation Differential Equations in State 0 and State 1

		Australia	t-stat	Canada	t-stat	Japan	t-stat	UK	t-stat
State 0	Cons	-0,0066	-0,4843	-0,008	-0,6859	-0,0092	-2,125	000439	1,9421
	rd_t	0,0402	0,217	0,0431	0,766	0,0002	0,9877	-0,089	-0,3809
	Δe_t	-0,0292	-0,1521	-0,036	-0,7105	-0,0001	-0,822	0,0224	0,1004
	Δe_{t-1}	0,6623	1,3458	0,0057	0,0374	0,1597	1,0687	0,232	0,6353
	rd_{t-1}	-0,5213	-1,1322	0,0737	0,6105	-0,0861	-0,588	-0,285	-0,8118
	πd_{t-1}	-0,053	-0,1464	0,0477	0,6927	-0,0954	-0,6529	0,2053	0,8929
State 1	Cons	-0,0019	-0,0677	0,0432	1,04	-0,0033	-0,7504	0,0025	0,117
	rd_t	-0,0617	-0,1864	0,1143	0,3494	0,0001	0,3884	0,008	0,1165
	Δe_t	0,0622	0,187	-0,148	0,3494	-0,0001	-0,3813	-0,013	-0,2121
	Δe_{t-1}	-0,3373	-0,1892	0,1478	0,1334	0,2568	1,8011	0,2729	2,2027
	rd_{t-1}	0,4677	0,3217	-0,289	-0,310	-0,1173	-0,9059	-0,095	-0,6513
	πd_{t-1}	-0,3435	-0,9555	0,0709	0,0744	-0,1322	-0,7919	0,1768	0,8339
Volatility of	η_0	0,0002	5,159	0,0000	4,0803	0,0000	3,577	0,0001	3,7732
	η_1	0,0004	1,181	0,0002	3,0255	0,0001	5,7055	0,0001	3,084

Note: Variable is significant at 5% significance level according to t-test result. There are two sets of estimated coefficients of MSVAR in two state processes. State 0 is the overvalued state in which the observed change in exchange rate is less than the change suggested by the economic fundamentals. State 1is the undervalued state in which the observed change in exchange rate is more than the change suggested by the economic fundamentals.

We report Wald test results for all the equations in Tables 5-7. According to this, coefficients of interest rate differential and constants are different across the states for AUD, JPY and CAD in exchange rate equation. Furthermore, lag of inflation differential has different coefficients for AUD, CAD and UKP in Interest rate differential equation. Lag of interest rate differential has different coefficients for CAD, AUD, and UKP in price differential equation.

Table 5: Wald Test Results

	Australia	Canada	Japan	UK
Mean	0,0219	1,9161	0,0004	0,1819
Cons	6,9181***	0,2026	5,1071***	0,9088
Δe_{t-1}	0,2385	1,3681	0,0001	5,0299***
r_t	1,1177	2,4439*	5,6401***	0,9804
ip_t	1,2658	1,9141	0,8911	0,0365

Note: Variable is significant at 5% significance level according to t-test result.

Table 6: Wald Test Results

	Australia	Canada	Japan	UK
Mean	0,21	0,0011	0,0150	0,0013
Cons	0,32	0,0000	0,0000	0,0000
Δe_t	123	0,0000	0,0000	0,0001
Δe_{t-1}	0,03	0,0000	0,0000	0,0008
r_{t-1}	0,02	1,5955	0,0000	0,0021
ip_{t-1}	975	2467	0,0008	268,7558

**Note:** Variable is significant at 5% significance level according to t–test result.

Table 7: Wald Test Results

|	Australia	Canada	Japan	UK
Mean	0,0265	0,0005	0,0175	0,0058
Cons	0,0000	0,0000	0,0000	0,0000
Δe_t	0,0118	0,0011	0,0000	0,4160
Δe_{t-1}	0,0003	0,0000	0,0000	0,0000
r_t	0,0110	0,0002	0,0000	0,0037
r_{t-1}	5033	1268	0,0034	400,17
ip_{t-1}	0,2604	34	0,0003	0,8893

Note: Variable is significant at 5% significance level according to t–test result.

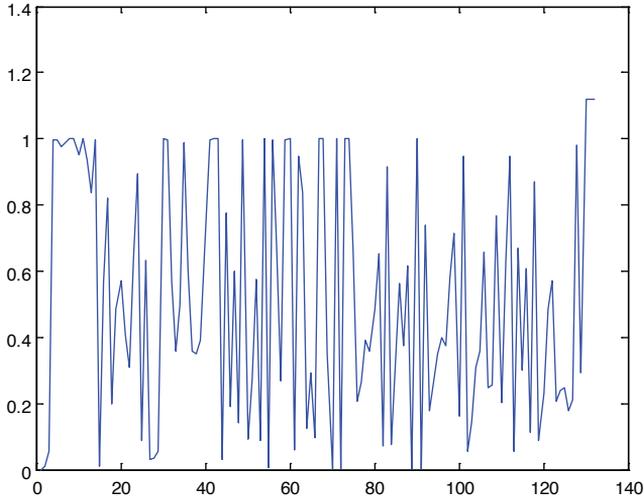### Figure 1: AUD Smoothed State Transition Probability of Being in State 1
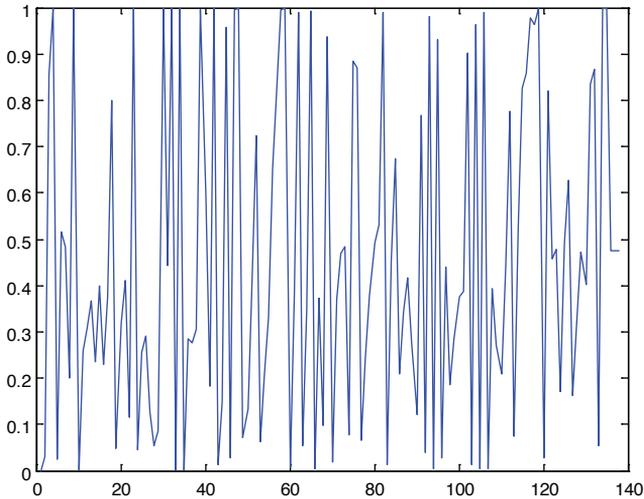
Figure 2: CAD Smoothed State Transition Probability of Being in State 1
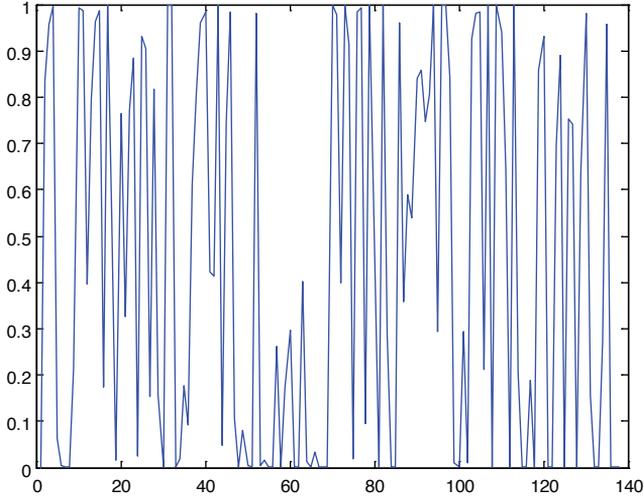
Figure 3: JPY Smoothed State Transition Probability of Being in State 1
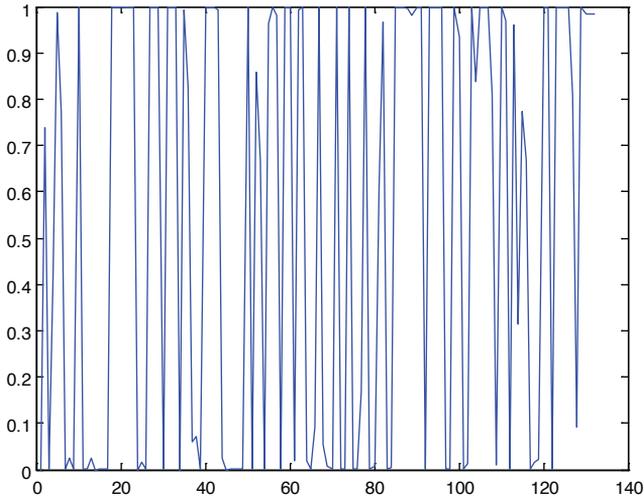
Figure 4: UKS Smoothed State Transition Probability of Being in State 1

6. Conclusion

In this study we investigated whether the PPP, UIP holds when the exchange rate is overvalued and undervalued relative to its fundamentally determined value. To this end, we adopted a two-state an MSVAR model with time varying transition probabilities to bilateral exchange rate that is defined between US Dollar and highly traded four Currencies -Australian Dollar(AUD), Canadian Dollar(CAD), Japanese Yen(JPY) and UK Pound(UKS)- between 1972:1 and 2009:4. Our results provided some interesting evidence. We found that magnitude, direction (sign) and statistical significance of the relationship among exchange rates, interest rates and price differential vary depending on the position of exchange rate relative to its fundamentally determined value.

The time varying transition probabilities of our model is governed by risk adjusted debt and equity investments in each currency to explore any effect of global flows, which seeks highest return in a global scale in the world of liberal capital markets, on the overvaluation and the undervaluation of the currencies.

Our results provide evidence that risk adjusted returns have considerable effect on the overvaluation and undervaluation of the currencies. The results of this study provide evidence that the relationship between exchange rate, interest rate and inflation depends on overvaluation and undervaluation of currencies. This issue has far reaching implications for monetary and fiscal policy.

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