2022

How Groupy is a Group?

Rohit J. Parikh
CUNY Graduate Center

How Groupy is a Group?

Rohit Parikh
City University of New York

August 25, 2022

Why do people and animals cooperate?

Figure: A beehive.

Figure: Chinese troops

Figure: Black lives matter

Our three cases

1. Bees
2. Chinese troops
3. BLM demonstrators

The reasons are different in the three cases.

1. Bees cooperate because they are all sisters who have the same DNA. From a purely evolutionary point of view the survival of one is the survival of all.
2. Chinese troops cooperate because they are obeying a single commander.
3. Black lives people cooperate because they have a single goal, and they have communicated to be together at one time and place.

But communication is necessary. Even a master must tell his servants what he wants them to do. BLM people must rely on email or phone calls to coordinate about the time and place.

With bees, however, the “communication” is via millions of years of evolution which is now part of their DNA.

Decision theory for groups

When an individual is about to make a decision she knows what she knows, she (typically) has an idea of what her options are, and she does not need to communicate with herself. I might write a note to my future self but I do not usually write notes to my current self.

But when a (human) group is considering alternatives of action, it needs a certain amount of internal communication. Some facts may be known to everyone, may be common knowledge, or may be known only to a select few in a subgroup. In any case some knowledge structure is crucial.

Similarly, when an action is to be taken, each member of the group needs to do her part. But she also needs to know what her part is and when she should perform it.

This is why an orchestra needs a conductor. Skilled musicians will not perform well without one, although a string quartet does not need one nor does a small group playing Indian classical music.

These last are similar to Margaret Gilbert's two people going for a walk.

In Margaret Gilbert's, example two people are going for a walk together. If they are together, they can talk to each other or perhaps one can read the other's movements and follow suit. The issue of communication will be simple. The bodily movements will constitute the needed communication.

But complex issues will arise with large groups – like armies. And they may arise also with small groups which are separated by distance. If I am to pick you up at the airport so that we can join a family picnic, you and I need to communicate when and where I am to meet you.

But what about bees?

Do they communicate?

Dance Language of the Honey Bee

Social behavior in bees has a number of advantages. One of the most important of these is the ability to quickly mobilize a large number of foragers to gather floral resources that may only be available for a short period of time. The ability to communicate location with such precision is one of the most interesting behaviors of a very interesting insect.

The recruitment of foragers from a hive begins when a scout bee returns to the hive engorged with nectar from a newly found nectar source. She begins by spending 30-45 seconds regurgitating and distributing nectar to bees waiting in the hive. Once her generosity has garnered an audience, the dancing begins. ....

In all cases the quality and quantity of the food source determines the liveliness of the dances. If the nectar source is of excellent quality, nearly all foragers will dance enthusiastically and at length each time they return from foraging. Food sources of lower quality will produce fewer, shorter, and less vigorous dances; recruiting fewer new foragers.

Proposal for a mesaure of coordination

For this talk we will put aside communication and simply propose a measure of the amount of coordination. How much coordination is there among bees? Among Chinese troops? Among BLM activists? There should be a theory of a) the amount of information among members of the group and b) the basic similarity among agents which allows more coordination with less communication. Schelling's Focal Points make a good example.

Consider n agents $\{1, ..., n\}$. Each agent has a (finite) action space (or belief space) A_i. A profile of actions (or beliefs) is a tuple $(a_1, ..., a_n)$ where each $a_i \in A_i$.

We consider the set $\mathcal{X}$ of all theoretically possible profiles and $\mathcal{X} = \Pi_1^n(A_i) : i = 1...n$ We also assume a set $\mathcal{P}$ of *actually possible* profiles. If $p \in \mathcal{P}$ then it is possible that the agents act in ways a_i so that $p = (a_1, .., a_n)$ that is to say $a_i = (p)_i$

If some profile q is in $\mathcal{X}$ but not in $\mathcal{P}$ then we assume that the agents do not jointly act in such a way that the action of agent i is $(q)_i$.

Suppose that ten people are doing a group dance and at some stage they have to put one foot forward.

Then the theoretically possible profiles are where everyone puts one foot forward either right or left just as he likes..

There are 2^{10} theoretically posssible profiles.

But either a) everyone puts the right foot forward or b) everyone puts the left foot forward. So there are only two *actually possible* profiles.

The degree of coordination C is defined to be $lg(\alpha)$ where $\alpha = (|\mathcal{X}|/|\mathcal{P}|)$

$|X|$ will generally be cardinalitiy of the set X and lg will be log to the base 2.

In this case C is $lg(2^{10}/2) = lg(2^9) = 9$

Another example. Suppose 1, 2, 3 are three agents and there are actions a,b for each. a stands for "take care of the baby", b for "go to the beach". Agent 1 would prefer b but will do a if neither 2 nor 3 is taking care of the baby.
We could but need not think of this as a sequential game with player 1 playing last.

Then $\mathcal{P} = \{(a, b, b), (b, b, a), (b, a, b), (b, a, a)\}$

(b,b,b) is not in $\mathcal{P}$. For 1 will do a if the other two are doing b. Nor is (a,b,a). 1 prefers b to a, and will not do a if 3 is already doing a.

Of the eight members of $\mathcal{X}$ Four are in $\mathcal{P}$. Four are not.

So α is two and the degree of coordination $C = 1$. If $\mathcal{P} = \mathcal{X}$ then α would be 1 and C would be zero. If 1 did not care what 2 and 3 were doing then C would be zero.

The number C indicates the extent to which the actions of the three agents are *coordinated*. The highest degree of coordination arises when $\mathcal{P}$ is a singleton.

In this preliminary draft we will not ask *why* $\mathcal{P}$ is a singleton. There could be many reasons and we will not - yet - go into the various reasons.

Subgroups

Suppose that there are two subgroups of the set of agents and for simplicity we will let them be $G_1 = \{1, ..., m\}$ and $G_2 = \{m+1, ..., n\}$. Given a profile P of the whole group G we let $P_1 = (p_1, .., p_m)$ and $P_2 = (p_{m+1}, ..., p_n)$ Abusing notation we will write $P = P_1 + P_2$.

Let $\mathcal{P}_1$ be the set of possible profiles of G_1 and $\mathcal{P}_2$ be the set of possible profiles of G_2. If $\mathcal{P} = \{P + Q | P \in \mathcal{P}_1 \texttt{and Q} \in \mathcal{P}_2\}$ then we will say that G_1 and G_2 are *independent*. This will clearly happen iff $|\mathcal{P}| = |\mathcal{P}_1| \times |\mathcal{P}_2|$

We can also define the *degree* of coordination to be $lg(|(\mathcal{P}_1| \times |\mathcal{P}_2)|/|\mathcal{P}|)$. It will be 0 iff the two groups are independent.

Another example. There are two kingdoms $\mathcal{K}$ and $\mathcal{K}'$. There are three fruits available, apples, bananas and cherries. King k likes apples and bananas but hates cherries. King k' likes babanas and cherries but hates apples.

The king may choose whatever fruit he chooses among the ones he likes but citizens are required to eat the same fruit as the king. So the profile for $\mathcal{K}$ consists of $\{(a, ..., a), (b, .., b)\}$. For $\mathcal{K}'$ it is $\{(b, ..., b), (c, ...c)\}$.

Since the profiles are independent, there are four profiles for the entire group, $\mathcal{K} + \mathcal{K}'$. C is zero.

Suppose now that king k' is more powerful and demands that the people of kingdom $\mathcal{K}$ eat the same fruit as the people of kingdom $\mathcal{K}'$ when possible.

Now kingdom $\mathcal{K}$ can still eat applies when $\mathcal{K}'$ eats cherries (since they do not have cherries) but are forbidden to eat apples when $\mathcal{K}'$ eats bananas. $\mathcal{P}_1$ and $\mathcal{P}_2$ remain the same as before and have size 2 each, but the profile $(a, ..., a, b, ..., b)$ vanishes.

$\mathcal{P}$ has size 3, $\alpha = 1.5$ and C rises from 0 to 0.585.

The demand of king k' clearly requires communication. And we conjecture that communication increases the value of C. If there is no communication, then C must of necessity be 0.

Restrictions

A restriction r on $\mathcal{P}$ is a non-empty subset $\mathcal{Q}$ of $\mathcal{P}$. We will write $\mathcal{Q} = r(\mathcal{P})$. We may also think of r not as a particular restriction but as a map from profiles to (sub)sets of profiles.

A system $\mathcal{S}$ of commands from king k' to kingdom $\mathcal{K}$ is a map from $\mathcal{P}'$ to the set $\mathcal{R}$ of restrictions.

The intuition is that when $\mathcal{K}'$ has a certain profile p then it sends a restriction to $\mathcal{K}$ which shrinks the possible profiles for $\mathcal{K}$. An identity command at ease leaves $\mathcal{P}$ as it is.

Assuming that at least one of the commands is the identity, the profile $\mathcal{P}$ stays as it is. But the joint profile of $\mathcal{K}$ and $\mathcal{K}'$ may shrink, raising the value of C.

We will say that r is at least as strict as r' if for all sets of profiles $\mathcal{P}$, $r(\mathcal{P}) \subseteq r'(\mathcal{P})$. It is more strict if for at least some sets of profiles $\mathcal{P}$, $r(\mathcal{P}) \subset r'(\mathcal{P})$. The system of commands $\mathcal{S}$ is more strict than $\mathcal{S}'$ if all the restrictions in $\mathcal{S}$ are at least as strict as those of $\mathcal{S}'$ and at least one is more strict.

Observation If $\mathcal{S}$ is more strict than $\mathcal{S}'$ then there is more coordination with $\mathcal{S}$ than with $\mathcal{S}'$.

There is obviously an algebra of restrictions. For instance there could be a restriction on the speed at which you can drive and also a restriction on the amount of alcohol you can imbibe before driving. These two restrictions commute and the algebra will be simpler.

Suppose that there are two religions in the same community. We will call them H and M without intending any particular meaning. They also have holidays, D for H and R for M.

Most members of H observe D, but not all. Most members of M observe R but not all. Also some members of H observe R and some members of M observe D. Assume that in each community, 70% observe their own holiday and 20% observe the holiday of the other community. Also 5% in each community observe both holidays and that leaves 15% in each community who observe neither. Assuming the communities to be of equal sizes, we can calculate the degree of coordination within each community as well as the coordination within the whole society.

We can also calculate the coordination between the two communities.

Let us suppose that given a profile P it results in payoffs $u_i = u_{p,i}$ to member i of the group. We would like to suggest that ideally each group should coordinate its actions so as to maximize the welfare of the whole group. But how do we define the welfare W of the whole group?

We could let $W(p) = \Sigma_1^n u_{p,i}$ But we would also like the various u_i to be reasonably equal. The sum does not enforce equality.

Rawls might define $W(p) = n \times \mu_i(u_{p,i})$, where $u_{p,i}$ is the minimum. But that ignores every u which is not the minimum.

From Rawls' point of view, a society of three in which the incomes are 101,101,101 is **better** than a society with incomes of 100,200,300 since the minimum in the first society is higher.

Limitations of rationality

Knowledge in Society

> *The following, possibly apocryphal story about the mathematician Norbert Wiener, well known for his absent mindedness, illustrates something even more subtle. At one time the Wieners were moving and in the morning as he was going to work, Mrs. Wiener said to him, "Now don't come home to this address in the evening." And she gave him a piece of paper with the new address. However, in the evening Wiener found himself standing in front of the old address and not knowing what to do – he had already lost the slip of paper with the new address. He went to a little girl standing by and said, "Little girl, do you know where the Wieners have moved to?" The little girl replied, "Daddy, Mom knew what would happen so she sent me to fetch you."*

The moral of the story, for *us*, is that communication works only if the memory of all parties involved is reliable.

Wimmer and Perner on beliefs about beliefs

Wimmer and Perner are concerned primarily with the perception by children of other people's *mindsets*. The following quote from [WM] is a story about Maxi which they told a group of children:

> Mother returns from her shopping trip. She bought chocolate for a cake. Maxi may help her put away the things. He asks her, "Where should I put the chocolate?" "In the blue cupboard," says the mother.

The story continues

Later, with Maxi gone out to play, the mother transfers the chocolate from the blue cupboard to the green cupboard. Maxi then comes back from the playground, hungry, and he wants to get some chocolate.

In Wimmer and Perner's experiment, little children who were told the Maxi story were then asked the *belief* question, "Where will Maxi look for the chocolate?"

Children at the age of three or less invariably got the answer wrong and assumed that Maxi would look for the chocolate in the *green* cupboard where *they* knew it was.

Even children aged four or five had only a one-third chance of correctly answering this question or an analgous question involving Maxi and his brother (who also wants the chocolate and whom Maxi wants to deceive).

Children aged six or more were by contrast quite successful in realizing that Maxi would think the chocolate would be in the *blue* cupboard – where he had put it and that if he wanted to deceive his brother, he would lead him towards the green cupboard.

Thus it seems that representation of other people's mindset comes fairly late in childhood, well after they have learned to deal with notions of belief and belief based action for themselves and for others who share their own view of reality. In [St] Chris Steinsvold investigates modal logics which are intended to represent the states of mind of young children. See also [SP].

Older children are not much better. In an experiment in my daughter's seventh grade class, I found that they were unable to deal with the muddy children puzzle beyond the first one or two levels.

When there are shortcomings of rationality then the degree of coordination will be less since some unintended profiles will creep in.

The muddy children puzzle

In this by now well-known puzzle, a number of children are playing in the mud and some of them get their foreheads dirty. At this the father comes on the scene and announces, "at least one of you has got her forehead dirty."

Scenario 1: Suppose there is only one child, say Amy, who is dirty. Then she will realize that her own forehead must be dirty since she can see that the others are clean.

Scenario 2: Suppose now that there are two dirty children, Sarah and Amy, who are asked in turn, "Do you know if your forehead is dirty?" Now when Sarah is asked, she can see Amy's dirty forehead and she replies, "I don't know." However, when Amy is asked, she is able to reason, "If *my* forehead were clean, Sarah would have known that hers must be dirty since all the others are clean. But Sarah did not know. So my forehead must be dirty."

Scenario 1: Suppose there is only one child, say Amy, who is dirty. Then she will realize that her own forehead must be dirty since she can see that the others are clean.

Scenario 2: Suppose now that there are two dirty children, Sarah and Amy, who are asked in turn, "Do you know if your forehead is dirty?" Now when Sarah is asked, she can see Amy's dirty forehead and she replies, "I don't know." However, when Amy is asked, she is able to reason, "If *my* forehead were clean, Sarah would have known that hers must be dirty since all the others are clean. But Sarah did not know. So my forehead must be dirty."

This reasoning on Amy's part requires a representation by Amy of Sarah's state of mind, and clearly Amy must be at least six for this to work. However, Sarah herself must have some reasoning ability and Amy must know that she has such abilities. It is not enough for Amy to know Sarah's view of reality, she must also represent Sarah's logical abilities in her own mind.

In particular, suppose that there are three dirty children – Jennifer, Sarah, and Amy – who are asked in turn whether they know if they are dirty, and with Amy being asked last.

If Sarah is only three, Amy would not be justified in concluding from Sarah's "I don't know" that in that case, Amy herself must be dirty.

Amy would need to know that *if* Amy were clean, Sarah would have carried out a representation in her own mind of Jennifer's state of mind and concluded from Jennifer's "I don't know" that Sarah must herself be dirty. But if Sarah is only three, Amy cannot rely on such reasoning on Sarah's part.

As the number of dirty children increases, there is a need for higher and higher levels of "I know that he knows that she knows that...." Common knowledge is at the end of this road and has been offered as the explanation of co-ordinated behaviour ([Lew, HM, CM, Chw]).

For instance Halpern and Moses in [HM] show that the co-ordinated attack problem requires common knowledge between the two generals, and that given the means of communication they have, such common knowledge is impossible to attain. Clark and Marshall [CM2] indicate similar difficulties with the referent of "the movie playing at the Roxy today."

Conclusion

We have indicated some complex issues about the identity and workings of a group. While groups of humans, formed consciously, may involve some prior deliberation, other groups arise for other reasons. And even humans may not deliberate rationally.

Much work remains to be done.

Thanks very much for listening!