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Cooperative Target Tracking in Balanced Circular and Elliptical Formations

Lili Ma and Naira Hovakimyan

Abstract—This paper extends our earlier results on cooperative target tracking in cyclic pursuit using a group of mobile robots by further prescribing the formation radius and achieving an elliptical formation pattern. Prescribing the formation radius of a balanced circular formation is achieved by adjusting a parameter in the existing control input to each robot. The new elliptical formation pattern is obtained via a transformation matrix. Both single-integrator and double-integrator robot models are considered. The effectiveness of the proposed schemes is demonstrated by simulation examples. **Key Words:** Cooperative target tracking, balanced circular formation, prescribed formation radius, elliptical formation.

I. INTRODUCTION

Coordinated and formation control has become an active research area for many years. Coordinated systems are particularly interesting to the robotics community because they allow simple individual agents (robots) to achieve complex tasks in ways that are scalable, extensible, and robust to failures of individual agents [1]. In addition to the studies of fundamental formation schemes such as collective motion [2], cyclic pursuit [3], [4], and circular motion [5]–[7], research efforts have also been devoted to applying these research findings in suitable applications [8].

One application is cooperative target tracking, where a group of robots are coordinated to track static or moving target(s). It is believed that sending multiple coordinated robots with simple sensors can lead to a better target tracking performance than sending only one robot with complex sensors [9]–[11]. Cooperative enclosing of a static target was achieved where group members surround the target [12]–[14]. Cooperative control for tracking and enclosing of a moving target were also reported [15]–[24]. Work on cooperative tracking of multiple targets [25]–[27] and enclosing a target in the 3D space [28] also draw researchers' attention. Different formation patterns were adopted into cooperative tracking, including logarithmic spiral [4], [24], balanced circular [29], uniform spacing [30], and triangular [31] formations.

Cooperative target tracking while simultaneously maintaining formation has been achieved from slightly different perspectives. One way is to add “formation” on top of an existing “tracking” scheme [32], [33]. Another way is to add “tracking” to an existing “formation scheme” so that the center of the formation pattern tracks the trajectory of

the target. It is commonly expected that coordinated control strategies that are generally designed with respect to a static center will also show good performance for moving targets.

Our work in [24] added tracking to an existing formation scheme [3], [4] and achieved cooperative target tracking in three different concentric formations, i.e., rendezvous, logarithmic spiral formation, and balanced circular formation. This paper further specifies the formation radius of the balanced circular formation, based on which elliptical formations of different orientations and shapes can be obtained. Notice that the formation radius of the balanced circular formation in [24] was not under control, which may limit its application in realistic situations. For example, if the formation radius is too small, it would be more likely for agents to collide with their neighbors. If the formation radius is too large, the information exchange between agents may not be guaranteed due to limited communication range. In this paper, this formation radius can now be specified by adjusting a parameter in the controller. Prescribing the formation radius makes the cooperative target tracking more useful and meaningful. Further, elliptical formations are achieved by further applying a transformation matrix, yielding more versatile formation patterns. Simulation results and descriptions are provided to elaborate how elliptical formations of different orientations and shapes can be achieved by properly selection of the control parameters. The elliptical formation can be convenient in an scenario where the robots need to pass through a narrow corridor. Simply reducing the radius of a circular formation might not work well since inter-vehicle may occur.

For robot models, both single- and double-integrator robot models are considered. The single-integrator model is the simplest model to characterize the motion of a mobile robot. This model is widely used in multi-robot coordination control problems such as consensus and formation control [34]. The double-integrator model is more representative in comparison with a single-integrator model that cannot for example model motion in a plane in which acceleration is the control input [35]. Systems with double-integrator model are of great interest due to their simple and descriptive features [36], and have been used to describe the dynamics of agents [37], mobile robots [38], [39], aircraft [40], and spacecraft [41]. For communication topology, the cyclic pursuit strategy is used, where an agent, suppose agent i , only pursues its “next” agent, i.e., agent $i+1$ (modulo n) for totally n agents.

The paper is organized as follows. Section II presents cooperative target tracking in balanced circular formation with prescribed formation radius for both the single-integrator and the double-integrator robot models and their extensions

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to elliptical formations. Simulation results are provided in Sec. III. Section IV concludes the paper.

II. MAIN RESULTS

Consider n mobile robots in the plane, where agent i pursues the agent $i + 1$. Let $\mathbf{x}_i(t) = [x_i(t), y_i(t)]^\top \in \mathbb{R}^2$ be the position of the agent i at time $t \geq 0$ for $i = 1, 2, \dots, n$. Let $\mathbf{x}_t(t)$ denote the moving target's trajectory as a function of time; $\dot{\mathbf{x}}_t(t) = [v_x(t), v_y(t)]^\top$ be the target's velocity; and $\ddot{\mathbf{x}}_t(t) = [\dot{v}_x(t), \dot{v}_y(t)]^\top$ be the target's acceleration. This section describes extensions to our work in [24], by further specifying the formation radius for two robot models (single-integrator and double-integrator).

A. Specifying Formation Radius of Balanced Circular Formation for Single-Integrator Model

If the kinematics of each agent is described by

$$\dot{\mathbf{x}}_i(t) = \mathbf{u}_i(t), \quad (1)$$

then by applying the control law [24]

$$\mathbf{u}_i(t) = \underbrace{R(\vartheta)(\mathbf{x}_{i+1} - \mathbf{x}_i)}_{\mathbf{u}_{ic}(t)} - \underbrace{k_c(\mathbf{x}_i - \mathbf{x}_t)}_{\mathbf{u}_{it}(t)} + \dot{\mathbf{x}}_t, \quad (2)$$

with

$$\begin{aligned} \frac{\pi}{n} < |\vartheta| < \frac{2\pi}{n}, \quad k_c = 2 \sin\left(\frac{\pi}{n}\right) \sin(\vartheta - \frac{\pi}{n}), \\ R(\vartheta) = \begin{bmatrix} \cos \vartheta & \sin \vartheta \\ -\sin \vartheta & \cos \vartheta \end{bmatrix}, \end{aligned} \quad (3)$$

the agents' positions starting at any initial conditions exponentially converge to an evenly-spaced circular formation centered at the target's trajectory. In controller (2), $\mathbf{u}_{ic}(t)$ and $\mathbf{u}_{it}(t)$ denote the *coordination* and the *tracking* components, respectively. In terms of information exchange, agent i only "seeks" its "next" agent: agent $i + 1$ (modulo n).

The target's position and velocity information will be estimated by each agent using its onboard sensor and/or estimation. A vision-based estimation scheme was described in our previous works and is not repeated here [24], [42]. The angle ϑ is used to extend the classic linear cyclic pursuit scenario to one in which each agent pursues the leading neighbor along the line of sight rotated by the common offset angle $\vartheta \in [-\pi, \pi]$ [3].

The control law (2) commands a group of robots to cooperatively track a moving target without being able to specify the formation radius. In other words, the formation radius is under no control. Among the three possible formation patterns, i.e., rendezvous to a single point, convergence to an evenly-spaced circular formation, and convergence to an evenly-spaced logarithmic spiral formation, we are particularly interested in specifying the formation radius of the evenly-spaced circular formation, since it suits well to the cooperative target tracking scenario.

Let d_0 denote the desired inter-agent distance. It was shown in [4] that by adjusting the angle ϑ_i according to

$$\vartheta_i = \frac{\pi}{n} + k_r (d_0 - \|\mathbf{x}_{i+1}(t) - \mathbf{x}_i(t)\|), \quad k_r > 0, \quad (4)$$

the control law

$$\mathbf{u}_i(t) = R(\vartheta_i)(\mathbf{x}_{i+1}(t) - \mathbf{x}_i(t)) \quad (5)$$

ensures that the formation where all agents move on a circle of radius

$$r_0 = \frac{d_0}{2 \sin\left(\frac{\pi}{n}\right)} \quad (6)$$

is a locally stable relative equilibrium for system (1) and (5). Applying this idea, the control law (2) is extended to achieve cooperative target tracking in balanced circular formation by further specifying the formation radius.

Proposition 1: Consider the single-integrator robot model in (1). The following control law

$$\mathbf{u}_i(t) = \underbrace{R(\vartheta_i)(\mathbf{x}_{i+1} - \mathbf{x}_i)}_{\mathbf{u}_{ic}(t)} - \underbrace{k_c(\mathbf{x}_i - \mathbf{x}_t)}_{\mathbf{u}_{it}(t)} + \dot{\mathbf{x}}_t, \quad (7)$$

with

$$k_c = 2 \sin\left(\frac{\pi}{n}\right) \sin\left(\vartheta_0 - \frac{\pi}{n}\right), \quad (8)$$

where the angle ϑ_0 is selected to be

$$\frac{\pi}{n} < |\vartheta_0| < \frac{2\pi}{n} \quad (9)$$

and the angle ϑ_i is adjusted according to (4), and

$$R(\vartheta_i) = \begin{bmatrix} \cos \vartheta_i & \sin \vartheta_i \\ -\sin \vartheta_i & \cos \vartheta_i \end{bmatrix} \quad (10)$$

ensures that the agents' positions starting at any initial conditions exponentially converge to a balanced circular formation centered at the target's trajectory $\mathbf{x}_t(t)$ with a formation radius r_0 as specified in (6).

Proof: The proof follows from Theorem III.7 in [4] and Theorem 1 in [24]. The controller in (7) can be rewritten as:

$$\mathbf{u}_i(t) = R(\vartheta_i)[(\mathbf{x}_{i+1} - \mathbf{x}_t) - (\mathbf{x}_i - \mathbf{x}_t)] - k_c(\mathbf{x}_i - \mathbf{x}_t) + \dot{\mathbf{x}}_t. \quad (11)$$

The overall system consisting of the robot model (1) and controller (7) can be rewritten in compact form as:

$$\dot{\mathbf{x}}(t) = \underbrace{[L \otimes R(\vartheta_i) - k_c I_{2n}]}_{A(\vartheta_i)}(\mathbf{x}(t) - I_{2n} \mathbf{x}_t(t)) + I_{2n} \dot{\mathbf{x}}_t(t), \quad (12)$$

where $\mathbf{x}(t) = [\mathbf{x}_1^\top(t), \mathbf{x}_2^\top(t), \dots, \mathbf{x}_n^\top(t)]^\top$ and L is the circular matrix

$$L = \begin{bmatrix} -1 & 1 & 0 & \dots & 0 \\ 0 & -1 & 1 & \dots & 0 \\ \vdots & & & \ddots & \\ 1 & 0 & 0 & \dots & -1 \end{bmatrix}, \quad I_{2n} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ \vdots & \vdots \\ 1 & 0 \\ 0 & 1 \end{bmatrix}_{2n \times 2}. \quad (13)$$

By letting

$$\mathbf{e}_i(t) = \mathbf{x}_i(t) - \mathbf{x}_t(t), \quad \mathbf{e}(t) = \mathbf{x}(t) - I_{2n} \mathbf{x}_t(t), \quad (14)$$

(12) becomes

$$\dot{\mathbf{x}}(t) - I_{2n} \dot{\mathbf{x}}_t(t) = \underbrace{[L \otimes R(\vartheta_i) - k_c I_{2n}]}_{A(\vartheta_i)}(\mathbf{x}(t) - I_{2n} \mathbf{x}_t(t)), \quad (15)$$

yielding the following error dynamics:

$$\dot{e}(t) = A(\vartheta_i) e(t). \quad (16)$$

Denote $e_i(t)$ by its polar form as $e_i(t) = \rho_i(t) \angle \theta_i(t)$. Further define the separation angles between each two adjacent agents relative to the moving target as $\phi_i(t) = \theta_{i+1}(t) - \theta_i(t)$. According to the Theorem III.7 in [4], the following equilibrium point in its polar form

$$\begin{bmatrix} \rho_1(t) \angle \theta_1(t) \\ \rho_2(t) \angle \theta_2(t) \\ \vdots \\ \rho_{n-1}(t) \angle \theta_{n-1}(t) \\ \rho_n(t) \angle \theta_n(t) \end{bmatrix} = r_0 \begin{bmatrix} \angle 2\pi/n \\ \angle 2\pi/n \\ \vdots \\ \angle 2\pi/n \\ \angle \frac{-2\pi(n-1)}{n} \end{bmatrix} \quad (17)$$

is a locally stable relative equilibrium for system (1) and (7). The equilibrium in (17) shows that $e(t)$ in (16) converges to a balanced circular formation with evenly-spaced separation angles and fixed/constant formation radius $r_0 = d_0/(2 \sin(\frac{\pi}{n}))$ as given in (6). Since $e_i(t) = x_i(t) - x_t(t)$, as defined in (14), it is immediate to see that the center of formation will converge exponentially to the trajectory of the moving target $x_t(t)$. $\square$

In (7), the two control components are dedicated for two different purposes. The tracking control $u_{it}(t)$ ensures that the center of formation follows the target's position $x_i(t)$ to achieve tracking of the moving target. The formation control $u_{ic}(t)$ aims at achieving the balanced circular formation. Particularly, the pre-specified formation radius r_0 is obtained by adjusting the angle $\vartheta_i(t)$ for each agent. Intuitively, if the formation radius is smaller than the desired value r_0 (i.e., when agents are "close to each other"), all agents need to spiral out. Conversely, if the formation radius is larger than r_0 (i.e., when agents are "far away from each other"), they will spiral in [4]. Notice that $\vartheta_i(t)$ can be different for each agent at the same time instant t .

B. Specifying Formation Radius of Balanced Circular Formation for Double-Integrator Model

Consider the double-integrator model

$$\ddot{x}_i(t) = u_i(t). \quad (18)$$

The following control law applied to (18)

$$u_i(t) = \underbrace{k_d[R(\vartheta_i)(x_{i+1} - x_i) - \dot{x}_i]}_{u_{ic}(t)} + \underbrace{R(\vartheta_i)(\dot{x}_{i+1} - \dot{x}_i) - k_d k_c(x_i - x_t) - k_c(\dot{x}_i - \dot{x}_t) + k_d \ddot{x}_t + \ddot{x}_t}_{u_{it}(t)}, \quad (19)$$

with $k_d > 0$, ϑ , k_c , and $R(\vartheta)$ given in (3), guarantees cooperative tracking of a moving target in cyclic pursuit, if $-k_d$ is not an eigenvalue of $A(\vartheta)$ [24]. Notice that the target's acceleration is further assumed to be available to each agent.

We now extend the results described in Sec. II-A on cooperative target tracking with a prescribed formation radius for single-integrator robot model to double-integrator robot model. We have the following results:

Proposition 2: Consider the double-integrator robot model in (18). Applying the following control law

$$u_i(t) = \underbrace{k_d[R(\vartheta_i)(x_{i+1} - x_i) - \dot{x}_i]}_{u_{ic}(t)} + \underbrace{R(\vartheta_i)(\dot{x}_{i+1} - \dot{x}_i) - k_d k_c(x_i - x_t) - k_c(\dot{x}_i - \dot{x}_t) + k_d \ddot{x}_t + \ddot{x}_t}_{u_{it}(t)}, \quad (20)$$

with ϑ_i given in (4), k_c in (8), and $R(\vartheta_i)$ in (10), the n -agent system is globally uniformly ultimately bounded in balanced circular formation centered at the target's trajectory $x_t(t)$ with radius $r_0 = d_0/(2 \sin(\frac{\pi}{n}))$, if $-k_d$ is not an eigenvalue of $A(\vartheta_i) = L \otimes R(\vartheta_i) - k_c I_{2n}$ and $k_d > 0$.

Proof: For the double-integrator robot model in (18) with the controller in (20), we have

$$\begin{aligned} \ddot{e}_i(t) &= \ddot{x}_i(t) - \ddot{x}_t(t) \\ &= k_d [R(\vartheta_i)(x_{i+1} - x_i) - k_c(x_i - x_t)] \\ &\quad + R(\vartheta_i)(\dot{x}_{i+1} - \dot{x}_i) - k_c(\dot{x}_i - \dot{x}_t) \\ &\quad - k_d(\dot{x}_i - \dot{x}_t). \end{aligned} \quad (21)$$

Referring to (14) and further denoting:

$$\dot{e}(t) = \dot{x}(t) - I_{2n} \dot{x}_t(t), \quad \ddot{e}(t) = \ddot{x}(t) - I_{2n} \ddot{x}_t(t), \quad (22)$$

the vector form of (21) can be written as:

$$\begin{bmatrix} \dot{e}(t) \\ \ddot{e}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{0} & I_{2n} \\ k_d A(\vartheta_i) & A(\vartheta_i) - k_d I_{2n} \end{bmatrix} \begin{bmatrix} e(t) \\ \dot{e}(t) \end{bmatrix}. \quad (23)$$

According to Theorem IV.2 of [4], for k_c given in (8) and if $-k_d$ is not an eigenvalue of $A(\vartheta_i)$, the error system (23) converges to an evenly-spaced circular formation. Since $e(t) = x(t) - I_{2n} x_t(t)$ as defined in (14), it is immediate to see that the center of formation will converge exponentially to the trajectory of the moving target $x_t(t)$. Thus, cooperative target tracking using the multi-robot system is achieved in an evenly-spaced circular formation.

We now show that formation radius can be specified as in (6). Rewrite the double-integrator system as:

$$\dot{x}_i(t) = v_i(t), \quad \ddot{x}_i(t) = u_i(t). \quad (24)$$

Let $v_i^d(t)$ denote the desired velocity of agent i as:

$$v_i^d(t) = R(\vartheta_i)(x_{i+1} - x_i) - k_c(x_i - x_t) + \dot{x}_t. \quad (25)$$

The controller (20) results in the following first-order system

$$\dot{v}_i(t) + k_d v_i(t) = \dot{v}_i^d(t) + k_d v_i^d(t) + \zeta_i(t), \quad (26)$$

where $\zeta_i(t)$ denotes the component due to the derivative of $R(\vartheta_i)$ in the form of

$$\zeta_i(t) = -k_r \|\dot{x}_{i+1}(t) - \dot{x}_i(t)\| R(\frac{\pi}{2} - \vartheta_i)(x_{i+1}(t) - x_i(t)). \quad (27)$$

Denote

$$\tilde{v}_i(t) = v_i(t) - v_i^d(t). \quad (28)$$

Equation (26) can be written as

$$\dot{\tilde{v}}_i(t) = -k_d \tilde{v}_i(t) + \zeta_i(t). \quad (29)$$

With the selection of the following Lyapunov function

$$V_i(t) = \frac{1}{2} \tilde{\mathbf{v}}_i^\top(t) \tilde{\mathbf{v}}_i(t), \quad (30)$$

its derivative $\dot{V}_i(t)$ can be written as:

$$\dot{V}_i(t) \leq -k_d V_i(t) + \delta_i^2 \leq 0, \forall \|\tilde{\mathbf{v}}_i(t)\| \geq \sqrt{\frac{2}{k_d}} \delta_i \triangleq \epsilon_i, \quad (31)$$

where $\delta_i = \|\tilde{\mathbf{v}}^\top(t) \boldsymbol{\zeta}_i(t)\|$.

The inequality in (31) shows that for $k_d > 0$, the formation performance $\mathbf{v}_i(t)$ approaches $\mathbf{v}_i^d(t)$ arbitrarily close with an upper bound ϵ_i . In other words, $\|\mathbf{v}_i(t) - \mathbf{v}_i^d(t)\| \leq \epsilon_i$ for $i = 1, 2, \dots, n$. Therefore, the formation behavior of the double-integrator system can be controlled to approach that of the single-integrator system arbitrarily close. Consider the tracking and coordination performances together. By applying the controller (20) to the system (18), the n -agent system is globally uniformly ultimately bounded in a balanced circular formation centered at the target's trajectory $\mathbf{x}_t(t)$ with a formation radius $r_0 = d_0/(2 \sin(\frac{\pi}{n}))$. $\square$

C. Formations in Elliptical Formations

This section describes transformation of the agents' circular trajectories into elliptical trajectories around the target for both the single-integrator and the double-integrator robot models. Adopting the idea in [4], consider the application of the following non-singular similarity transformation to the rotation matrix $R(\vartheta)$:

$$T = \begin{bmatrix} \frac{1}{2} & 0 \\ z_0 \cos \phi & z_0 \sin \phi \end{bmatrix}, \quad (32)$$

where the parameters z_0 and ϕ can be adjusted to obtain different orientations and shapes of the elliptical formations, as to be illustrated in Sec. III. The trajectories' of the agents are then transformed into

$$T \mathbf{x}_i(t), \quad i \in \{1, 2, \dots, n\}, \quad (33)$$

yielding elliptical trajectories centered at the moving target.

More particularly, cooperative target tracking in elliptical formations can be achieved using the following control laws:

- For Single-Integrator Robot Model: The control law in (7) is now replaced by:

$$\mathbf{u}_i(t) = \underbrace{TR(\vartheta_i)T^{-1}(\mathbf{x}_{i+1} - \mathbf{x}_i)}_{\mathbf{u}_{ic}(t)} - \underbrace{k_c(\mathbf{x}_i - \mathbf{x}_t)}_{\mathbf{u}_{it}(t)} + \dot{\mathbf{x}}_t. \quad (34)$$

- For Double-Integrator Robot Model: The control law in (20) is now replaced by:

$$\begin{aligned} \mathbf{u}_i(t) = & k_d [TR(\vartheta_i)T^{-1}(\mathbf{x}_{i+1} - \mathbf{x}_i) - \dot{\mathbf{x}}_i] \\ & + TR(\vartheta_i)T^{-1}(\dot{\mathbf{x}}_{i+1} - \dot{\mathbf{x}}_i) \\ & - k_d k_c(\mathbf{x}_i - \mathbf{x}_t) - k_c(\dot{\mathbf{x}}_i - \dot{\mathbf{x}}_t) \\ & + k_d \dot{\mathbf{x}}_t + \ddot{\mathbf{x}}_t. \end{aligned} \quad (35)$$

For the above two control laws (34) and (35), the parameter ϑ_i is given in (4); k_c is given in (8); the rotation matrix $R(\vartheta_i)$ is shown in (10); and the non-singular transformation matrix T is shown in (32). The two parameters z_0 and ϕ are set the

same for all agents. For both single-integrator and double-integrator robot models, since the elliptical formation control is achieved by a similar transformation on the rotation matrix, the stability results hold.

III. SIMULATION RESULTS

Simulation examples are given to present two aspects. Firstly, the formation radius of the balanced circular formation can now be prescribed, making the cooperative target tracking more useful and more under control. Secondly, elliptical formations of different orientations and shapes are achieved. Upon formation, agents scatter on an ellipse centered at the target. Results are presented for both the single-integrator and the double-integrator robot models. For the double-integrator robot model, the nonholonomic robot described in [43] is used.

A. Circular Formation with Specified Radius

Example 1: Cooperative target tracking in balanced circular formation is now able to further specify the formation radius. We start by considering the single-integrator robot model (1). To focus more on the formation performance, the target is assumed to be either a stationary target or a target moving with linear velocity. The following parameters are used: $r_0 = 15$ (m), $\vartheta_0 = 1.8 \frac{\pi}{n}$, $k_r = 0.08$.

When the target is stationary, results are shown in the first column of Fig. 1. When the target moves on a straight line, results are shown in the second column of Fig. 1. For both cases, the trajectories of the agents and the target are first presented, followed by the separation angles between each two adjacent agents (to show that all agents spread evenly around the circle), and the average formation radius with respect to the target (to show that the desired formation radius can be achieved). The number of agents is selected to be $n = 6$ and $n = 8$, respectively, to show that the proposed schemes work for different numbers of agents.

Figure 1(a) and (b) show the trajectories of all agents (in thin solid curves of different colors), the trajectory of the center of all agents (in bold dashed black), and the trajectory of the target (in bold red). At the end of the simulation, the positions of all agents are denoted by big dots, showing that all agents spread evenly around a circle centered at the target. It can be seen that the trajectory of the center of all agents gradually approaches the position/trajectory of the target, thus demonstrating successful cooperative target tracking.

For formation, the balanced circular formation includes two aspects. One is that all agents should spread evenly around a circle, emphasizing on the feature of being "balanced". The other is regarding the average formation radius, which is the average of all relative distances between the agents and the formation center. This average formation radius can now be commanded to approach a specified value. Figure 1(c) and (d) show the separation angles between each two adjacent agents, which approach $360^\circ/6 = 60^\circ$ for $n = 6$ and $360^\circ/8 = 45^\circ$ for $n = 8$. Thus, all agents spread evenly around a circle. Figure 1(e) and (f) show the average formation radius, which approaches the specified value of

(a) Trajectories

(b) Trajectories

(c) Separation Angles

(d) Separation Angles

(e) Formation Radius

(f) Formation Radius

Fig. 1. Cooperative target tracking with prescribed formation radius for **single-integrator** robot model (1). Plots in the 1st row are for a stationary target. Plots in the 2nd row are for a target moving with linear velocity.

$r_0 = 15$ (m). Notice that the formation radius was not under control in our previous work [24]. Being able to further specify the formation radius makes the cooperative target tracking more under control and thus more useful.

Example 2: Results for cooperative target tracking with prescribed formation radius for the double-integrator robot model (18) are shown in Fig. 2. The selected parameters are: $r_0 = 15$ (m), $\vartheta_0 = 1.1 \frac{\pi}{n}$, $k_r = 0.08$, $k_d = 1$. The initial positions of all agents are the same as those in Fig. 1. The target's velocity is assumed to be either piecewise-constant or time-varying. The number of agents is selected to be either 6 or 8. It can be seen that successful cooperative target tracking has been achieved with specified formation radius.

B. Achieving Elliptical Formations

Example 3: Using the double-integrator model (18), this example shows cooperative target tracking in the elliptical formation. Results are given in Fig. 3. The control parameters for the elliptical formations are selected to be $z_0 = 0.5$ and $\phi = 0.7 \pi$. Figure 3(a) plots the trajectories of all agents, the trajectory of the center of all agents, and the trajectory of the target. It can be seen that the formation center tracks the moving target fairly well. Several snapshots of the agents' positions are given in Fig. 3(b), where the same color is used for the same agent. Figure 3(b) illustrates the process of

(a) Trajectories

(b) Trajectories

(c) Separation Angles

(d) Separation Angles

(e) Formation Radius

(f) Formation Radius

Fig. 2. Cooperative target tracking with prescribed formation radius for **double-integrator** robot model (18). Plots in the 1st row are for a target moving with piecewise-constant velocity. Plots in the 2nd row are for a target moving with time-varying velocity.

simultaneous formation and target tracking. Figure 3(c)~(f) plot four snapshots showing how the elliptical formation is obtained over time. Clearly, all agents are driven to an ellipse, whose center resides in the moving target. It can also be seen that formation has been achieved at an earlier stage of the whole process, i.e., around $t = 4.5$ (sec.) for a total simulation duration of 60 seconds.

Example 4: Orientation and shape of the elliptical pattern can be controlled via the parameters z_0 and ϕ in (32). For a fixed ϕ value, changing z_0 results in a different orientation of the ellipse. Choose $\phi = 0.3 \pi$. Cooperative target tracking for $z_0 = \{0.5, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, 1.2\}$ is shown in Fig. 4(a)~(h). The target is assumed to have a linear velocity to focus on the formation pattern. It can be seen that as the value of z_0 varies, the orientation of the ellipse, indicated by the cyan angle, changes accordingly. An enlarged view of the change of the ellipse's orientation is shown in Fig. 4(i), where all the eight ellipses in Fig. 4(a)~(h) are plotted together. Orientations of these ellipses vary from 45° to 68° roughly.

Example 5: This example shows the effect of changing ϕ when z_0 is fixed, for example, when z_0 is selected to be $z_0 = 0.5$. Cooperative target tracking for several different values of ϕ is shown in Fig. 5 for $n = 12$ and $\phi =$

Fig. 3. Cooperative target tracking in an **elliptical** formation for **double-integrator** robot model (18). The target's velocity is time-varying. Total simulation time is 60 seconds.

$\{0.3\pi, 0.4\pi, 0.5\pi, 0.6\pi, 0.7\pi, 0.8\pi\}$. To focus more on the shape of the elliptical formation, the target is assumed to be stationary. It can be seen that the roundness of the ellipse changes as ϕ varies. When $\phi = 0.5\pi$, the “ellipse” is actually a circle. This is because the corresponding matrix T in (32) becomes

$$T = \begin{bmatrix} 0.5 & 0 \\ 0 & 0.5 \end{bmatrix}$$

for the selected values of $z_0 = 0.5$ and $\phi = 0.5\pi$. Using this transformation matrix, the balanced circular formation, based on which T is applied, is only scaled instead of changing its shape. From Fig. 5, it is clear that the roundness of the ellipse increases when ϕ increases from 0.3π to 0.5π ; it then decreases as ϕ further increases from 0.5π to 0.8π . In summary, the shape and the orientation of the elliptical formation are the results of the two parameters ϕ and z_0 .

Example 6: The simulation results presented so far assume perfect knowledge of the target's information, including its position, velocity and/or acceleration. In reality, these information need to be either measured or estimated by each agent using its local sensors/estimators. Assume that each agent is equipped with an onboard camera. A vision-based estimation scheme was described in our previous work (Sec. III.A in [24]), estimating a moving target's velocity and acceleration via visual measurements of the relative bearing and subtended angles. The local measurements obtained by each agent, together with each agent's own position in the global coordinate system, result in the state estimation of the target in the global coordinate system as well. This example presents simulation results when 0.1% uniform noise is injected into the visual measurements via the Matlab function `randn()`. Simulation results are shown in Fig. 6, where plots in the first row are for a (unknown) piecewise-constant target motion and plots in the second row are for a (unknown) time-varying target motion. Figure 6(b) and (d) show that the estimated quantities (in thin curves of different

Fig. 4. Elliptical formations of different **orientations** are achieved by changing z_0 for $\phi = 0.3\pi$.

colors) converge to their true values (plotted in bold red) for all agents. The team successfully tracks the moving target

Fig. 5. Elliptical formations of different **shapes** are achieved by varying ϕ for $z_0 = 0.5$.

in elliptical formations. Certainly, increasing the amount of measurement noises will significantly degrade the tracking and formation performance and eventually cause a failure.

IV. CONCLUSIONS AND FUTURE INVESTIGATIONS

This paper discusses cooperative tracking of a moving ground target using a group of mobile robots under the cyclic pursuit communication topology. First, a balanced circular formation with prescribed formation radius is achieved. This is an extension to our previous work where the formation radius was not specified. Prescribing the formation radius makes the cooperative target tracking more useful. Further, by applying a transformation matrix, cooperative target tracking in the elliptical formation is achieved for both the single-integrator and double-integrator robot models. Four controllers are proposed: two controllers for the single-integrator robot model (in balanced circular formation with prescribed formation radius and in elliptical formation) and two controllers for the double-integrator model (balanced circular and elliptical). Extensive simulation examples are present to show the effectiveness of the proposed controllers for targets that are stationary, moving with constant, piecewise-constant, or time-varying velocities; how the orientation and shape of the elliptical formation can be controlled; the scalability

Fig. 6. Cooperative target tracking with vision-based **estimation** of the target's information.

of the proposed schemes; and the performance of both tracking and formation under vision-based measurements and estimation. Future research will investigate appropriate schemes to avoid inter-vehicle/obstacle collisions [44], [45].

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