A Static and Dynamic Investigation of Quantum Nonlinear Transport in Highly Dense and Mobile 2D Electron Systems

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A Static and Dynamic Investigation of Quantum Nonlinear Transport in Highly Dense and Mobile 2D Electron Systems

by

Scott Dietrich

A dissertation submitted to the Graduate Faculty in Physics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York

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Abstract

A Static and Dynamic Investigation of Quantum Nonlinear Transport in Highly Dense and Mobile 2D Electron Systems

by Scott Dietrich

Thesis Advisor: Sergey Vitkalov

Heterostructures made of semiconductor materials may be one of most versatile environments for the study of the physics of electron transport in two dimensions. These systems are highly customizable and demonstrate a wide range of interesting physical phenomena. In response to both microwave radiation and DC excitations, strongly nonlinear transport that gives rise to non-equilibrium electron states has been reported and investigated. We have studied GaAs quantum wells with a high density of high mobility two-dimensional electrons placed in a quantizing magnetic field. This study presents the observation of several nonlinear transport mechanisms produced by the quantum nature of these materials.

The quantum scattering rate, $1/\tau_q$, is an important parameter in these systems, defining the width of the quantized energy levels. Traditional methods of extracting $1/\tau_q$ involve studying the amplitude of Shubnikov-de Haas oscillations. We analyze the quantum positive magnetoresistance due to the cyclotron motion of electrons in a magnetic field. This method gives $1/\tau_q$ and has the additional benefit of providing access to the strength of electron-electron interactions, which is not possible by conventional techniques. The temperature dependence of the quantum scattering rate is found to be proportional to the square of the temperature and is in very good
agreement with theory that considers electron-electron interactions in 2D systems. In quantum wells with a small scattering rate – which corresponds to well-defined Landau levels – quantum oscillations of nonlinear resistance that are independent of magnetic field strength have been observed. These oscillations are periodic in applied bias current and are connected to quantum oscillations of resistance at zero bias: either Shubnikov-de Haas oscillations for single subband systems or magnetointersubband oscillations for two subband systems. The bias-induced oscillations can be explained by a spatial variation of electron density across the sample. The theoretical model predicts the period of these oscillations to depend on the total electron density, which has been confirmed by controlling the density through a voltage top-gate on the sample.

The peculiar nonlinear mechanism of quantal heating has garnered much attention recently. This bulk phenomenon is a quantum manifestation of Joule heating where an applied bias current causes selective flattening in the electron distribution function but conserves overall broadening. This produces a highly non-equilibrium distribution of electrons that drastically effects the transport properties of the system. Recent studies have proposed contributions from edge states and/or skipping orbitals. We have shown that these contributions are minimal by studying the transition to the zero differential conductance state and comparing results between Hall and Corbino geometries. This demonstrated quantal heating as the dominant nonlinear mechanism in these systems. To study the dynamics of quantal heating, we applied microwave radiation simultaneously from two sources at frequencies $f_1$ and $f_2$ and measured the response of the system at the difference frequency, $f = |f_1 - f_2|$. This provides direct access to the rate of inelastic scattering processes, $1/\tau_{\text{in}}$, that tend to bring the electron distribution back to thermal equilibrium. While conventional measurements of the temperature dependence indicate that $1/\tau_{\text{in}}$ is proportional to temperature, recent DC investigations and our new dynamic
measurements show either $T^2$ or $T^3$ dependence in different magnetic fields. Our microwave experiment is the first \textit{direct} access to the inelastic relaxation rate and confirms the non-linear temperature dependence.
Dedicated to my best friend and better half,
Lauren...
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\( e \)  charge of an electron = \( 1.602 \times 10^{-19} C \)
\( m \)  effective mass of an electron (0.067 \( m_e \) for an electron in GaAs)
\( k \)  Boltzmann constant (8.617 \( \times 10^{-5} eV/K \))
\( h \)  reduced Planck constant = \( 4.136 \times 10^{-15} eV \cdot s \)
\( T \)  temperature
\( \epsilon \)  electron energy
\( \epsilon_F \)  electron Fermi energy
\( n \)  electron density
\( f(\epsilon) \)  electron distribution function
\( \nu(\epsilon) \)  electron density of states
\( B \)  magnetic field
\( E \)  electric field
\( j \)  electric current density
\( V \)  voltage
\( I \)  current
\( R \)  resistance = \( V/I \)
\( r \)  differential resistance = \( dV/dI \)
\( \sigma \)  electric conductivity
\( \sigma_D \)  Drude conductivity
\( \rho \)  electric resistivity
\( \rho_D \)  Drude resistivity
\( \omega_c \)  cyclotron frequency
Symbols & Abbreviations

DC  Direct Current
AC  Alternating Current
RF  Radio Frequency
MW  Microwave
2D  Two-dimensional
2DEG 2D electron gas
DOS Density of states
LL  Landau level
SdH Shubnikov-de Haas
MISO Magnetointersubband Oscillations
SCBA Self-consistent Born Approximation
MR Magneto Resistance
Chapter 1

Introduction

Life on Earth, more or less confined to a thin layer on the surface, is host to boundless amounts of exciting activity. Humanity has spent centuries traversing the surface with a perpetual eye toward the sky, but has only now begun any true exploration in the dimension perpendicular to the Earth’s surface. In a reverse process, the scientific research on electronic systems has moved to smaller and smaller dimensions over the past few decades. The science and engineering of low dimensional systems has grown exponentially, driven by better fabrication techniques and a popular demand for smaller and more powerful electronic devices. The emergence and subsequent success of the semiconductor industry is a good example of the importance of these materials and its recent focus on flat materials where charge is confined to move in only two dimensions is a good indicator of future direction. An understanding of the physics of how charge is transported through these materials is therefore of great importance. Despite decades of extensive research in the area of these two-dimensional materials there remains a wide range of interesting physics that still exists for exploration.
Electronic systems made out of the so-called III-V compound semiconductors, such as GaAs and AlAs, may be one of most versatile materials for the study of the physics of electrons in low dimensions. Stacks of these materials can be made such that electrons are confined to move in a two-dimensional layer known as a quantum well. The continual development of techniques for extensive control of these materials has given them a high degree of customization. Through precise fabrication techniques, these systems can be used to study the effects of a wide range of variables: the width of the quantum well, the density of charge in the well, the disorder potential seen by the charges, and the mobility of the charge. All of these effects will change the physics of the system. For this reason, they are host to a number of fascinating physical phenomena and practical applications from the quantum Hall effect to optical absorption for solar cell technology. Thus, GaAs quantum wells provide an exciting sandbox for the exploration of interesting physics in two dimensions and continue to surprise with discoveries.

1.1 Brief synopsis of thesis

I start with the theoretical background of electron transport properties 2D electron systems. This includes a general description of the structure and fabrication of the samples used in the study. I extend the classical Drude model into the quantum regime in order to describe some of the fundamental quantum magnetoresistance phenomena that exist in these systems. After discussing the more well-known phenomena, I give an overview of the theory behind a peculiar quantum manifestation of Joule heating that was discovered by the team led by Sergey Vitkalov in 2007, deemed quantal heating. Chapter 3 contains a description of the experimental techniques and apparatuses used in the main investigation. In Chapter 4, I briefly discuss a few of the more pertinent experimental findings that have appeared in recent years.
Following this introduction to the field of transport phenomena in quantum wells, Chapters 5 - 9 present the experimental findings of this study. I first present a method for extracting the quantum lifetime of 2D electrons from positive quantum magnetoresistance. I then discuss results of an investigation of samples with varying quantum lifetimes which demonstrate oscillations of nonlinear resistance with respect to DC bias current. These oscillations are explained by a spatial redistribution of electrons across the sample. A similar mechanism is then observed in systems with two occupied subbands. Then, I present results that indicate quantal heating as the dominant contributor to nonlinearities in these systems rather than edge-state effects. This is accomplished by observing the threshold of the zero-differential conductance state in different sample geometry to separate bulk and edge-state contributions. I then discuss results of a dynamic investigation of quantal heating in a technique developed for the direct measurement of the inelastic scattering rate, which is aimed at solving a discrepancy in the temperature dependence of the inelastic scattering rate. Finally, I conclude by summarizing the findings of this study and propose future areas of interest based on the results.
Chapter 2

Theory

2.1 GaAs quantum wells

The samples presented in this study are high-mobility GaAs quantum wells grown by molecular beam epitaxy (MBE) on semi-insulating (001) GaAs substrates. Two AlAs/GaAs type-II superlattices grown on both sides of the well served as barriers. δ-doping of Silicon donors inside the superlattices on either side of the well provides electrons to the quantum well. The quantum well schematic can be seen in Figure 2.1. While these donors provide the high electron density to the two-dimensional electron gas (2DEG) inside the quantum well, they can also cause electrons in the 2DEG layer to scatter off the ionic potentials that are left behind. However, the benefits to electron transport from a larger 2D electron density obtained by the two-sided δ-doping approach outweigh the disorder effects [4]. The superlattice layers also provide significant screening of the donor potentials. Overall, this quantum well design provides a high mobility of 2D electrons inside the well at a high electron density [5]. Samples used in this study had electron densities around $10^{16}$ per $m^2$ and mobilities of 73 to 121 $m^2/Vs$ around 4.6K.
Figure 2.1: The schematic at the top presents the physical heterostructure formed by layers of GaAs (light blue) and AlAs (dark blue) with δ-Silicon doping (dashed-lines). The two graphs below show the electrical potential ($U$) and the electron density for the $\Gamma$ (black) and $X$ (blue) conduction bands.

The electrons in the 2DEG are confined within the quantum well in the $z$-direction, but are free in the x-y plane of the sample. Thus, we can consider the $z$-direction independently. A simple 1D examination of electrons in a potential well of finite height ($U_\Gamma$ seen in Figure 2.1) leads to bound states, called subbands, that are discussed in more detail in Section 2.5. The number and separation of the subbands is determined by the shape of the quantum well. The Fermi energy and, therefore, total electron density of the 2DEG will determine the number of subbands that are populated.

Figure 2.2 depicts several heterojunctions of AlAs and GaAs – all of which can be used as
quantum wells for the study of the 2DEG. For the samples used in this study and described above, (b-d) are a simplification of the actual quantum well seen in Figure 2.1. When stacking layers of these two materials to form the quantum well, band bending associated with electrons in the AlAs conduction band hop down to the GaAs conduction band and become trapped. By adding an additional layer of AlAs to the other side of the well – as in Figure 2.2(b) – we form a quantum well that resembles the textbook example of a finite potential well.

The change from Figure 2.2(b) to (c) represents a change in electron density via Silicon doping to increase the Fermi energy, $\epsilon_F$. As electrons are added to the 2DEG, the second subband starts to fill when $\epsilon_F$ reaches to bottom of the next subband, $\epsilon_2$. By widening the quantum well, less bound states are allowed. Also, band bending is considerably stronger both in comparison to the total well depth and because the additional distance allows conduction band energy to reach its bulk value as can be seen on the far right of Figure 2.2(a). This is seen in Figure 2.2(d). There are now less subbands bound in the quantum well and those remaining are less tightly-bound. Additionally, the increase of the potential barrier at the center of the well can overtake the lower subbands.

The single-subband system depicted in 2.2(b) is the basis for Chapters 5, 6, 8, and 9.
Oscillations of sample resistance caused by intersubband interactions in the two-subband system shown in 2.2(c) is the focus of the study presented in Chapter 7.

### 2.2 Classical Magnetoresistance

Classical contributions to changes in a material’s resistance due to a magnetic field can be classically explained using the Drude model of electrical conduction. In this model, charge carriers (electrons) move under the influence of any external forces until instantaneous collisions randomize their direction. It is a simple model developed only a year after the discovery of the electron, yet it still finds wide use in condensed matter physics to this day. In the steady state of the Drude model, the momentum of electrons gained due to external fields is balanced by the momentum lost during scattering events,

\[
\begin{align*}
\left[ \frac{dp}{dt} \right]_{\text{fields}} &= \left[ \frac{dp}{dt} \right]_{\text{scattering}} \\
\text{where } p &\text{ represents the average electron momentum. Relaxation due to scattering is approximated by the transport scattering time, } \tau_{tr}, \text{ which represents the average time between collisions.}
\end{align*}
\]

In a two-dimensional conductor with an applied perpendicular magnetic field \((B = B\hat{z})\), the change of momentum is given by the Lorentz force. Under these conditions, the above equation becomes

\[
\frac{P}{\tau_{tr}} = e \left( E + \frac{P}{m} \times B \right) .
\]
By introducing the current density $j = nep/m$ of charge carriers with charge $e$, mass $m$, and density $n$, we can simplify the above equation and obtain an expression for Ohm’s Law ($E = \rho j$):

$$E = \begin{pmatrix} \frac{m}{ne^2} & -\frac{m\omega_c}{ne^2} \\ \frac{m\omega_c}{ne^2} & \frac{m}{ne^2} \end{pmatrix} j$$

(2.3)

where $\omega_c = eB/m$ is the cyclotron frequency and the matrix defines the resistivity tensor $\rho$ in two dimensions. The diagonal elements represent the longitudinal resistivity $\rho_{xx}$ which is the inverse of the Drude conductivity, $\sigma_D = \frac{ne^2}{\tau \tau_{tr}}$. The off-diagonal elements give the longitudinal, or Hall, resistivity $\rho_{xy} = -\rho_{yx} = -\frac{B}{ne}$. The ratio of the longitudinal resistivity to magnetic field strength gives the Hall coefficient $R_H = -1/ne$ which is useful in obtaining the sign and density, $n$, of charge carriers.

By inverting the resistivity tensor, we find the components of the conductivity tensor,

$$\sigma_{xx} = \frac{\rho_{xx}}{\rho_{xx}^2 + \rho_{xy}^2} = \frac{\sigma_D}{1 + (\omega_c \tau_{tr})^2}$$

(2.4)

$$\sigma_{xy} = \frac{\rho_{xy}}{\rho_{xx}^2 + \rho_{xy}^2} = \frac{\sigma_D \cdot \omega_c \tau_{tr}}{1 + (\omega_c \tau_{tr})^2}$$

(2.5)

Notably, because $\rho_{xx}$ is independent of magnetic field while $\rho_{xy}$ grows linearly, at high magnetic fields $\rho_{xy} \gg \rho_{xx}$ and $\sigma_{xx} \sim \rho_{xx}$. This regime of a classically strong magnetic field is defined by $\omega_c \tau_{tr} \gg 1$. Here, the electric field is effectively oriented perpendicular to the applied current since the diagonal elements of the resistivity tensor are much smaller than the off-diagonal elements. Thus, all conduction along the direction of the current is tied to
scattering. The physical picture can be thought of thusly – all contributions to electron motion along the sample (longitudinal conductivity) are due to scattering events as they move across the sample. In an ideal system, the current is strictly perpendicular to both $\mathbf{B}$ and $\mathbf{E}$.

### 2.3 Transport Properties

While the Drude model remains a cornerstone of transport measurements, there are often times when a more detailed approach is necessary. For systems with non-uniform electron density or temperature, the Boltzmann equation becomes useful to examine transport properties. The aim of the Boltzmann equation calculate how changes in the electron distribution function lead to measurable effects in the transport properties of the system.

#### 2.3.1 Boltzmann equation [1]

The Boltzmann equation considers the time evolution of the local distribution of electrons, $f(\mathbf{r}, \mathbf{k})$, in the state $\mathbf{k}$ in the neighborhood of the point $\mathbf{r}$. The electron distribution function can change from three main effects: spatial diffusion, external fields, and scattering. The net rate of change of the distribution of electrons is given by a sum of these effects,

$$\frac{\partial f(\mathbf{r}, \mathbf{k})}{\partial t} = \left[ \frac{\partial f(\mathbf{r}, \mathbf{k})}{\partial t} \right]_{\text{diffusion}} + \left[ \frac{\partial f(\mathbf{r}, \mathbf{k})}{\partial t} \right]_{\text{fields}} + \left[ \frac{\partial f(\mathbf{r}, \mathbf{k})}{\partial t} \right]_{\text{scattering}}. \quad (2.6)$$

The Boltzmann equation gives the steady state of $f(\mathbf{r}, \mathbf{k})$ when $\partial f(\mathbf{r}, \mathbf{k})/\partial t$ is zero for all states $\mathbf{k}$ and points $\mathbf{r}$. Finding the steady state of a specific system involves finding each term on the right hand side of Equation 2.6.
In spatial diffusion, electrons move in and out of the neighborhood of \( \mathbf{r} \). Between collisions, electrons move in straight lines or paths dictated by external fields (discussed below). In pure stochastic motion, if the velocity of an electron in the state \( \mathbf{k} \) is denoted \( \mathbf{v}_k \), then the distance travelled in the time \( t \) is \( t \cdot \mathbf{v}_k \). Since the volume of phase space is invariant according to Liouville’s theorem, the number of electrons in the neighborhood of \( \mathbf{r} \) at time \( t \) is equal to the number of electrons a distance \( t \cdot \mathbf{v}_k \) away at time zero:

\[
 f(\mathbf{r}, \mathbf{k}, t) = f(\mathbf{r} - t \mathbf{v}_k, \mathbf{k}, 0). \tag{2.7}
\]

By the chain rule, this leads to a contribution from diffusion given by

\[
 \left[ \frac{\partial f(\mathbf{r}, \mathbf{k})}{\partial t} \right]_{\text{diffusion}} = -\mathbf{v}_k \cdot \nabla f(\mathbf{r}, \mathbf{k}). \tag{2.8}
\]

The forces associated with external fields will cause a change in the \( \mathbf{k} \)-vector of electrons. In the case of an applied electric field, the rate of change of the \( \mathbf{k} \)-vector \( \dot{\mathbf{k}} = -e\mathbf{E}/\hbar \). This \( \mathbf{k} \)-space velocity is analogous to the spatial velocity present in diffusion. This produces a similar expression by applying Liouville’s theorem, which equates the number of electrons in the \( \mathbf{k} \)-state at time \( t \) to the number of electrons in the \( \mathbf{k} - \dot{\mathbf{k}} t \) state at time zero,

\[
 f(\mathbf{r}, \mathbf{k}, t) = f(\mathbf{r}, \mathbf{k} - \dot{\mathbf{k}} t, 0), \tag{2.9}
\]

and consequently, the change in the electron distribution due to external fields is given by

\[
 \left[ \frac{\partial f(\mathbf{r}, \mathbf{k})}{\partial t} \right]_{\text{fields}} = -\dot{\mathbf{k}} \cdot \nabla_k f(\mathbf{r}, \mathbf{k}). \tag{2.10}
\]
Finally, electrons can be scattered from occupied state $k$ to an unoccupied state $k'$. The probability of these processes depends on the number of electrons in state $k$, $f(k)$, as well as the number of vacant states in the $k'$ state, $[1 - f(k')]$. This process will increase $f(k)$ while the reverse process of scattering from $k'$ to $k$ will reduce $f(k)$. Considering all possible scattering states, $k'$, will give the rate of change of $f(k)$ due to scattering which is given by

$$\left[ \frac{\partial f(\mathbf{r},k)}{\partial t} \right]_{\text{scattering}} = \int \{ f(k) \cdot [1 - f(k')] - f(k') \cdot [1 - f(k')] \} P(k,k') dk',$$  \hspace{1cm} (2.11)

where $P(k,k')$ is the bare probability of a scattering event, assuming the state $k$ is occupied and $k'$ is unoccupied. For linear transport, this expression is simplified using the relaxation time approximation

$$\left[ \frac{\partial f(\mathbf{r},k)}{\partial t} \right]_{\text{scattering}} = \frac{f(k) - f_T(k)}{\tau},$$  \hspace{1cm} (2.12)

where $\tau$ represents the average time between collisions that tend to bring the distribution back to thermal equilibrium given by the Fermi-distribution at temperature $T$, $f_T(k)$. A similar (but not identical) approximation can be made for nonlinear transport considering inelastic scattering which results in the above expression but where $\tau = \tau_in$ represents the average time between inelastic processes.

The net rate of change of the electron distribution is found by inserting Equations 2.8, 2.10, and 2.12 into the Boltzmann equation (2.6):

$$\frac{\partial f(\mathbf{r},k)}{\partial t} + \mathbf{v}_k \cdot \nabla f(\mathbf{r},k) + \mathbf{k} \cdot \nabla_k f(\mathbf{r},k) = \frac{f(k) - f_T(k)}{\tau}$$  \hspace{1cm} (2.13)
This expression can be used to compute the transport properties of a system by considering the evolution of $f(k)$ due to external fields, diffusion, and scattering.

### 2.3.2 DC conductivity

The Drude model expression for current density, $j = nev$, considers the average velocity of all $n$ electrons and in this simplification ignores the information provided by the distribution function. If we instead consider the number of electrons in the state $k$ with velocity $v_k$, the expression for the net current density can be described by

$$ j = 2 \int e v_k f(k) dk = \sigma E, \quad (2.14) $$

in response to a DC electric field, $E$, applied to an infinite medium at constant temperature, $T$. Here, the factor of 2 enters in the consideration of the electron spin.

One can use the Boltzmann equation to find the distribution function and eventually an expression for the conductivity, $\sigma$. The steady state of Equation 2.13 for a DC electric field is

$$ -\frac{e}{\hbar} E \cdot \nabla_k f_T = \frac{f - f_T}{\tau}, \quad (2.15) $$

which yields

$$ f = f_T - e\tau \left( \frac{\partial f}{\partial \epsilon} \right) E \cdot v \quad (2.16) $$

as the equilibrium distribution function. The second term on the right hand side is the correction to the electron distribution function due to the electric field. Here we have dropped the variable
k for clarity and used the energy-velocity relation for the \( k \) state, \( v_k = (1/\hbar) \partial \epsilon / \partial k \). Inserting this result into Equation 2.14, we find that

\[
j = -2e^2 \tau \int v_k E \cdot v_k \left( -\frac{\partial f}{\partial \epsilon} \right) dk + 2 \int e v_k f_T dk \tag{2.17}
\]

describes the net current density. Here the second term on the right hand side is zero and by converting the integral in the first term to one over energy space we find that

\[
j = \left[ -\int \frac{2e^2 m \tau}{\pi \hbar^2} v_k v_k \left( -\frac{\partial f}{\partial \epsilon} \right) d\epsilon \right] \cdot E = \sigma \cdot E, \tag{2.18}
\]

where we have used the 2D density of states (DOS), \( \nu_o = m/\pi \hbar^2 \), to convert the integral from \( dk \) to \( d\epsilon \). For an electric field along the \( x \)-direction, \( \langle v_x^2 \rangle = v^2/2 \) for two dimensions.

\[
\sigma = \int \left[ \frac{e^2 \tau m v^2}{2\pi \hbar^2} \right] \left( -\frac{\partial f}{\partial \epsilon} \right) d\epsilon \tag{2.19}
\]

In general, the variables inside the square bracket depend on energy so we can rewrite Equation 2.19 as

\[
\sigma = \int \sigma(\epsilon) \left( -\frac{\partial f}{\partial \epsilon} \right) d\epsilon \tag{2.20}
\]

For most classical transport, \( \sigma(\epsilon) \) is only weakly energy dependent in the region where the gradient is large (around the Fermi energy). Thus, \( \sigma(\epsilon_F) \) can be removed from the integral and by the fundamental theorem of calculus the integral produces the difference between the distribution function at the limits of zero and infinity, \( f(0) - f(\infty) = 1 \). We are left with the conductivity at the Fermi energy which is equal to \( \sigma_D \) as shown above. This shows that
temperature variations of $f(\epsilon)$ only weakly affect the transport properties of the material since there are little to now variations near the Fermi level. This is due mainly to the fact that $kT \ll \epsilon_F$. However, we see in this argument the potential for drastic changes to transport properties if there are, in fact, considerable variations of $\sigma(\epsilon)$ around $\epsilon_F$. This will be the basis for quantum nonlinear transport in these systems.

2.4 Quantum Magnetoresistance

It was shown in Section 2.2 that the longitudinal resistance, $\rho_{xx}$, in the Drude model does not depend on magnetic field, $B$, in a classically strong field ($\omega_c \tau_{tr} >> 1$). However, magnetoresistance has shown many interesting effects beyond this model. One source of monotonic positive magnetoresistance (MR) has recently been theoretically explained\cite{6}. Although the authors’ theory can explain many interesting phenomenon, I will quickly outline their semi-classical explanation for the positive magnetoresistance.

The trajectory of electrons in a magnetic field can be described by cyclotron orbits and this circular motion means that electrons are likely to return to the same scatterer. Since the cyclotron radius is inversely proportional to the field strength, the probability of an electron re-scattering increases with higher magnetic field. The leading order correction to the longitudinal Drude resistivity is

$$\Delta \rho_{xx} = 2\rho_D \left\{ \exp \left( \frac{2\pi}{\omega_c \tau_q} \right) + \exp \left( \frac{4\pi}{\omega_c \tau_q} \right) \left[ 1 - \frac{4\pi}{\omega_c \tau_q} \right]^2 \right\}$$  \hspace{1cm} (2.21)

in weak magnetic field limit, where $\tau_q$ is the quantum lifetime of the 2D electrons and $\rho_D = 1/\sigma_D$. The first exponential term is the leading correction and is the square of the Dingle
factor, $\delta = \exp(\pi/\omega_c \tau_q)$. The next term is the second correction and involves $\delta^4$. Higher order corrections are not shown here.

The quantum lifetime is the average time an electron spends in a given quantum state. Whereas $\tau_{tr}$ is dominated by contributions from large angle scattering such as strong impurity scattering, $\tau_q$ is very sensitive to all scattering events. It is the shortest timescale associated with the electron, since any slight change of momentum will change the quantum state of the electron. The ratio of the two scattering times, $\tau_{tr}/\tau_q$, gives the number of small angle scattering events needed to significantly change the electron momentum. It is often used as a dimensionless quantity to describe the purity of samples. $\tau_{tr}/\tau_q > 1$ is typical for GaAs quantum wells with superlattice barriers. Below, we will see that $\tau_{q}$ is an important parameter for quantum phenomena in 2D electron systems.

### 2.5 Quantization of energy levels

For the quantum wells described in Section 2.1, electrons are confined to a general potential, $V(z)$, and the electrons are free in the $x$ and $y$ directions. By considering the vector potential given by $\mathbf{B} = \nabla \times \mathbf{A}$ and using the Landau gauge, $\mathbf{A} = (0, Bx, 0)$, the Hamiltonian of this system can be written

$$H = \frac{p_x^2}{2m} + \frac{1}{2m} \left( p_y^2 - eBx \right)^2 + \frac{p_z^2}{2m} + V(z).$$  \hspace{1cm} (2.22)

We can set $p_y = \hbar k_y$ since it commutes with $H$ and with a bit of simplification we see that

$$H = \frac{p_x^2}{2m} + \frac{1}{2} \omega_c^2 \left( \frac{\hbar k_y}{m \omega_c} - x \right)^2 + \frac{p_z^2}{2m} + V(z),$$  \hspace{1cm} (2.23)
where $\omega_c$ is once again the cyclotron frequency. We can now treat the x and z components of the Hamiltonian separately. The first two terms on the right hand side describe a 1D quantum harmonic oscillator with its position off-set by $\hbar k_y/m\omega_c$. However, the off-set does not affect the energy eigenvalues of the system given by $\epsilon_N = (N + 1/2)\hbar \omega_c$. These discrete energy levels are known as Landau levels and the integer $N$ is the Landau level index.

The third and fourth terms on the right hand side describe the confinement to the quantum well. This also gives quantized energy levels that correspond to the subbands discussed in Section 2.1. The allowed subband energy values depend greatly on the potential $V(z)$, but in general the subband energies can be found by an numeric or graphical solution to the 1D Schrodinger equation and are labelled with the index $s$. Thus, the total energy an electron in the $N^{th}$ Landau level of the $s^{th}$ subband is $\epsilon = \epsilon_s + \epsilon_N = \epsilon_s + (N + 1/2)\hbar \omega_c$.

In the ideal case scenario described above, quantization of energy levels splits the zero-B density of states $\nu_o = m/\pi \hbar^2$ into perfectly discretized states. This leads to a series of delta functions corresponding to the Landau levels:

$$\nu(\epsilon) = (2\pi \lambda_B^2)^{-1} \sum_N \delta(\epsilon - \epsilon_s - \epsilon_N)$$

(2.24)

where $\lambda_B = \sqrt{\hbar/eB}$ is the characteristic magnetic length. It is important to note that the total number of states is conserved. Since the area under the density of states gives the total number of available states, electrons in the range $\epsilon_N \pm \hbar \omega_c/2$ are confined to the Landau levels when discretization occurs. Thus, the number of states in each Landau level (degeneracy) is given by the area under the curve in this range $\nu_o \times \hbar \omega_c = eB/\pi \hbar$.

Scattering processes from disorder and interactions cause a broadening of the delta function levels. Within the theory of the self-consistent Born approximation (SCBA) [7], the density of
states of the $N^{th}$ Landau level becomes

$$
\nu_{\text{SCBA}}^{(N)}(\epsilon) = \nu_o \frac{2\hbar \omega_c}{\pi \Gamma^2} \sqrt{1 - (\epsilon - \epsilon_s - \epsilon_N)^2}
$$

(2.25)

for a classically strong magnetic field ($\omega_c \tau_q >> 1$). $\Gamma = \sqrt{2\omega_c/\pi \tau_q}$ determines the width of the Landau level. The area under the SCBA density of states, $\nu_{\text{SCBA}}$, gives the same degeneracy as shown above. The broadened Landau levels produce a density of states which consists of a series of semicircles separated by gaps. When the levels are not significantly separated compared to broadening ($\omega_c \tau_q << 1$), there is a harmonically modulated density of states given by Equation 29 of [4]:

$$
\nu_G(\epsilon) = \nu_o \left[ 1 + 2 \sum_{k=1}^{\infty} (-\delta)^k \cos \left( \frac{2\pi k \epsilon}{\omega_c} \right) \right]
$$

(2.26)
where $\delta$ is the Dingle factor. This is also commonly denoted the Gaussian density of states. Each of the discussed quantized density of states is shown in Figure 2.3.

### 2.6 Quantum Oscillations

The magneto-oscillations in the density of states described above produce oscillations in many other properties of the 2DEG as $\omega_c$ or $\epsilon_F$ is changed. This includes oscillations in the magnetic susceptibility (the de Haas-van Alphen effect), specific heat, and resistivity (the Shubnikov-de Haas effect). Although there is a great interest in all of these phenomena, the main focus of this section will be on quantum oscillations of resistivity.

In general, the transport scattering time is renormalized by Landau quantization due to an increase in the density of states $[4]$:

$$\frac{1}{\tau_B(\epsilon)} = \frac{1}{\tau_{tr}} \frac{\nu(\epsilon)}{\nu_o}. \quad (2.27)$$

The combination of Landau quantization and the renormalization of the transport scattering rate leads to a net longitudinal conductivity given by

$$\sigma_{xx} = \int \left[ \frac{e^2 v_F^2 \nu(\epsilon) \tau_B(\epsilon)}{1 + \omega_c \tau_B^2(\epsilon)} \right] \left( -\frac{\partial f_T}{\partial \epsilon} \right) d\epsilon. \quad (2.28)$$

Where Equation 2.20 has been modified by $\nu(\epsilon)$ in the $k \rightarrow \epsilon$ integral transformation and the renormalization of $\tau \rightarrow \tau_B(\epsilon)$. Notably, the product of the two factors in the numerator actually cancels out the effect. Similarly for the Hall conductivity, we find:
\[ \sigma_{xy} = -\frac{enB}{h} + \int \left[ \frac{e^2v_F^2\nu(\epsilon)/\omega_c}{1 + \omega_c\tau_B(\epsilon)} \right] \left( -\frac{\partial f_T}{\partial \epsilon} \right) d\epsilon. \] (2.29)

which describes a frictionless drift of electrons in crossed electric and magnetic fields as well as an oscillating term.

### 2.6.1 Shubnikov-de Haas oscillations

At zero magnetic field and temperature, all states below \( \epsilon_F \) are completely populated. When a magnetic field is turned on, Landau levels form as was discussed in the previous section. As the magnetic field increases, the spacing between Landau levels, \( \hbar\omega_c \), increases and levels pass through the Fermi level. At fields where the \( \epsilon_F \) falls between levels resistivity is at a minimum while at fields where \( \epsilon_F \) falls at the center of a Landau level resistivity is at a maxima. These oscillations of magnetoresistance are known as Shubnikov-de Haas (SdH) oscillations [1].

For well-defined Landau levels, corrections to the conductivity lead to isolated peaks of height \( \sigma_{xx} \gg \sigma_D \). For overlapping Landau levels, the oscillatory correction to the resistivity can be described by

\[
\Delta \rho / \rho_D \simeq 4 \exp \left( -\frac{2\pi}{\omega_c\tau_q} \right) \cos \left( \frac{2\pi \epsilon_F}{\hbar\omega_c} \right) F \left( \frac{2\pi^2 kT}{\hbar\omega_c} \right). \] (2.30)

Again, the presence of the Dingle factor shows the importance of the quantum scattering time. Shubnikov-de Haas oscillations are periodic in \( 1/B \) and are thermally damped by the function \( F(x) = x / \sinh(x) \) [7]. An example of these oscillations can be seen Figure 2.4.

In experimental applications, the frequency and amplitude of SdH oscillations can give essential parameters of the 2DEG. The amplitude of oscillations contains information about \( \tau_q \).
Figure 2.4: This figure demonstrates the magnetoresistance of a 13nm wide quantum well at $T = 4.7K$ with $\tau_q = 4.8ps$ [8]. Data shows the presence of both the positive quantum magnetoresistance ($B = 0.1-0.3T$) described in Section 2.21 and Shubnikov-de Haas oscillations ($B > 0.3T$) discussed in Section 2.6.1.

as is evident in the exponential factor in Equation 2.30. Meanwhile, the period of oscillations contains the 2D electron density.

2.6.2 Magnetointersubband Oscillations

While the Shubnikov-de Haas oscillations are strongly temperature dependent, other oscillations of magnetoresistance have been observed which lack that thermal averaging. One of these magneto-oscillations comes from the interaction of two sets of Landau levels, which can occur in double quantum wells with a single populated subband in each or single quantum wells with two occupied subbands. If the electron density, and therefore $\epsilon_F$, is high enough in the energy diagram in Section 2.1 then electrons will begin to populate the second subband. This generates two sets of Landau levels offset by the energy gap between the subbands, $\Delta$. 
With increasing magnetic field, the sets of Landau levels in each band will line up when the subband gap is an integer multiple of the Landau level spacing ($\Delta = k \cdot \hbar \omega_c$ where $k$ is an integer). When this happens, a new channel for scattering opens up and resistivity increases due to intersubband scattering. When sets of Landau levels are perfectly misaligned, intersubband scattering is weak (overlapping levels) or nonexistent (well-separated levels). This leads to magneto-intersubband (MIS) oscillations.

The correction to $\sigma_D$ due to MIS oscillations in each subband, $\Delta \sigma^{(\alpha)}_{MISO}$, is the sum of the $\alpha = 1$ and 2 terms, given by [9, 10]

$$
\Delta \sigma^{(\alpha)}_{MISO} = \frac{2e^2 n_\alpha \tau_{1\alpha}^2}{m \tau_{12}} \left(1 - 2 \frac{\tau_\alpha}{\tau_{12}}\right) \delta_1 \delta_2 \cos \left(\frac{2\pi \Delta}{\hbar \omega_c}\right)
$$

(2.31)

where $n_\alpha$ and $1/\tau_\alpha$ are the electron density and the zero-B scattering rate in the $\alpha^{th}$ subband. Here $1/\tau_{12}$ is the zero-B intersubband scattering rate and $\delta_\alpha$ is the Dingle factor containing $\tau_\alpha$.

Although a general expression for the resistance oscillations is complicated for different types of disorder and strength of magnetic field, for systems with long-range disorder in the regime where $\tau_{tr} >> \tau_q$ with a classically strong magnetic field, $\Delta \rho_{MISO}$ takes on a simple form[9] given by

$$
\frac{\Delta \rho_{MISO}}{\rho_D} = \frac{2m \cdot \bar{\nu}_{12}}{e^2(n_1 + n_2)} \cdot \exp \left[\frac{-\pi}{\omega_c} \left(\frac{1}{\tau_q^{(1)}} + \frac{1}{\tau_q^{(2)}}\right)\right] \cdot \cos \left(\frac{2\pi \Delta}{\hbar \omega_c}\right)
$$

(2.32)

where $\bar{\nu}_{12}$ is an effective intersubband scattering rate. Here we note that the oscillations depend on the total density $n = n_1 + n_2$, are periodic in inverse magnetic field as with SdH oscillations, and contain two Dingle factors in the amplitude that results on a dependence on the total quantum scattering rate of both subbands $1/\tau_q^{(1,2)} = 1/\tau_q^{(1)} + 1/\tau_q^{(2)}$. 
Figure 2.5: Experimental data from GaAs single quantum well with AlAs/GaAs superlattice barriers. The inset of (a) displays the energy diagram where two subbands are populated. Here the Fermi energy, $\epsilon_F$, is above the energies of the two subbands, $\epsilon_1$ and $\epsilon_2$. (a) The longitudinal resistivity $\rho_{xx}(B)$ of the system at 4.2 K. (b) Normalized resistivity, $\rho_{xx}/\rho_o$, vs $1/B$ for two temperatures (4.2 and 12.5 K). (c) Amplitude of MISO, $\Delta\rho_{xx}/\rho_o$ vs $1/B$. Straight lines correspond to the amplitude of Equation 2.32.

Figure 2.5 demonstrates the resistance oscillations associated with magnetointersubband oscillations in a system with two occupied subbands. The displayed results have been reproduced and altered with permission by Bykov et. al. [11]. These oscillations are clearly periodic in inverse magnetic field, as depicted in Figure 2.5(b) for two temperatures. Here, the reduced amplitude at higher temperatures is due to a decrease in the quantum lifetime $\tau_q$ and not the thermal averaging seen in Shubnikov-de Haas oscillations. Accordingly, 2.5(c) shows that the amplitude decays exponentially with $1/B$ as predicted by Equation 2.32.

2.7 Quantal Heating

Another quantum phenomenon in 2D electron systems is a peculiar form of Joule heating that has been experimentally observed in high mobility GaAs quantum wells placed in a quantizing magnetic field [12, 13]. Joule heating is a well-known and ubiquitous phenomenon that transforms electrical energy into heat. Heating electrons by a DC electric field causes an increase in electron temperature while very weakly affecting the transport properties of the system.
However, the quantum nature of certain materials can alter this classical phenomenon, leading to drastic changes in electron transport. Applying a DC electric field to these materials creates a stratified electron distribution in energy space that differs significantly from the traditional Fermi-Dirac form. This selective flattening of the distribution function corresponds to the electronic density of states and conserves overall temperature (broadening of the electron distribution [12]). This quantal heating has been observed as a significant decrease in conductivity.

Due to conservation of total energy, $\epsilon_o$, and elastic electron-impurity scattering in presence of an electric field, the kinetic energy of an electron, $\epsilon_k$, depends on the stochastic motion of the electron position: $\epsilon_k = \epsilon_o - eE \cdot r$. Thus, diffusion in energy space is linked to the spatial diffusion.

The conductivity in a classically strong magnetic field ($\omega_c \tau_{tr} \gg 1$) is given by

$$\sigma(\epsilon) = \sigma_D \left[ \frac{\nu(\epsilon)}{\nu_o} \right]^2$$  \hspace{1cm} (2.33)

If we use this equation in the Einstein relation between conductivity and the spatial diffusion coefficient, $\sigma_{xx} = e^2 \nu(\epsilon) D(\epsilon)$, we find that $D(\epsilon) = \sigma_D \nu(\epsilon)/e^2 \nu_o^2$. This shows that the spatial diffusion coefficient is proportional to the quantized density of states, $\nu(\epsilon)$. As a consequence of total energy conservation, the spectral diffusion coefficient is linked to the spatial diffusion coefficient: $D_s(\epsilon) = (eE)^2 D(\epsilon) \sim \nu(\epsilon)$. Thus, spectral diffusion is strongest where there is a high density of states (inside Landau levels) and we should expect to see a selective flattening of the electron distribution function in these areas.

To calculate the spectral diffusion term in the Boltzmann equation for diffusion in energy space, we can utilize the diffusion equation for energy space,
\[ \frac{\partial f(\epsilon)}{\partial t} = \frac{1}{\nu(\epsilon)} \nabla_\epsilon [D_\epsilon(\epsilon)\nu(\epsilon)\nabla_\epsilon f(\epsilon)]. \] (2.34)

By adding this term to the Boltzmann equation with the spectral diffusion coefficient described above, we obtain the equation for the spectral diffusion

\[ -\frac{\partial f(\epsilon)}{\partial t} + E^2 \sigma_0 \nu_0 \nabla_\epsilon [\varphi^2 \nabla_\epsilon f(\epsilon)] = \frac{f(\epsilon) - f_T(\epsilon)}{\tau_{in}} \] (2.35)

Here, the left hand side of the equation describes the spectral diffusion of the electron distribution function, \( f(\epsilon) \), induced by elastic impurity scattering in the presence of an electric field, \( E \). The right hand side of the equation describes inelastic relaxation of the distribution function toward thermal equilibrium described by the Fermi-Dirac distribution at temperature \( T \), \( f_T(\epsilon) \). The validity of the relaxation time approximation that appears on the right hand side of Equation 2.35 is supported theoretically in the high temperature limit where the thermal energy greatly exceeds the Landau level spacing (\( kT \gg \hbar \omega_c \)). In this regime, the inelastic scattering rate, \( 1/\tau_{in} \) is considered independent of the energy \( \epsilon \) and electric field, \( E \).

In accordance with Equation 2.35, the spectral diffusion generates a spectral flow of electrons, \( J_\epsilon \), from low energy regions (occupied levels) to high energies (empty levels). In a steady state, the spectral flow is constant. Since it is proportional to the density of states and the gradient of the distribution function, \( J_\epsilon = D_\epsilon(\epsilon) \cdot \partial f / \partial \epsilon \sim \nu(\epsilon) \cdot \partial f / \partial \epsilon \), a selective flattening of the distribution function is expected. Because of this inverse relationship between the density of states and the gradient of the distribution function, a decreased value of \( \nu \) means that the gradient is increased in regions between Landau levels. Conversely, at the center of levels where \( \nu \) is a maximum the gradient is minimized to conserve the spectral flow.
Figure 2.6: The above curves demonstrate the longitudinal resistance, $R_{xx}$, of a Hall bar sample at different temperatures, $T$, and two DC biases, $I_{DC}$. It is reproduced with alterations with permission from a review paper by Sergey Vitkalov [14]. The blue curve presents a reference frame for the effect of ordinary heating (black) and quantal heating (red).

According to Equation 2.20 with $\sigma(\epsilon)$ is given by Equation 2.33, this modulation of the gradient of the electron distribution will cause a change in the total longitudinal conductivity, $\sigma_{xx}$. For energies where the gradient is minimized there is a decrease in the conductivity. For energies where the gradient is increased one might expect the conductivity to increase. However, these regions are associated with a strong depletion of conducting electronic states due to Landau quantization. Thus, the oscillations in the gradient of the distribution lead to a net overall decrease in the sample conductivity.
Figure 2.6 presents a concise picture of the effect of quantal heating. The blue curve demonstrates the magnetoresistance of a Hall bar sample at 2.16\textdegree\text{K} and serves as a point of reference for two important points: the amplitude of Shubnikov-de Haas oscillations and the magnetic field where quantization of levels begins, which is depicted by the arrow and marks the onset of quantum positive magnetoresistance. Thermally heating the sample to 4.2\textdegree\text{K} by external sources produces the black curve. This curve is nearly identical to the original blue curve except for a decrease in the amplitude of Shubnikov-de Haas oscillations which are strongly temperature dependent as given by Equation 2.30. However, the red curve is obtained by keeping the sample temperature around 2\textdegree\text{K} and applying a DC bias (Joule heating). If the effect of the Joule heating was simply changing the electron temperature, we would expect a curve similar to the black curve. Instead, we observe that the magnetoresistance changes significantly for all magnetic fields quantizing the electron spectrum. This demonstrates the presence of an inherently quantum form of Joule heating.

The theoretical model and experimental observation presented in this section describes the phenomenon of quantal heating. This mechanism is one of many quantum phenomena that affect electron transport in the highly dense and mobile systems as described above. It shows up in nearly all of the experiments discussed in this study, but is not always the main focus.
Chapter 3

Apparatus and Measurement

Techniques

In general, microscopic randomness tends to cover up most of the interesting quantum and nonlinear properties of electron systems. Electronic processes are dominated by inelastic processes at high temperatures, leading to very short quantum lifetimes and high relaxation rates. As seen in Chapter 2, Landau quantization requires a long quantum lifetime for well-separated levels to form while high relaxation rates can wash out nonlinear phenomena. Thus, fundamental research on these systems must be done at low temperatures. This section describes a few of the general experimental techniques and challenges of low temperature condensed matter physics. Specific details about the hardware and software involved in these techniques are saved for Appendix A.
3.1 $^3$He System

The liquid Helium-4 ($^4$He) cryostat remains a staple in low-temperature experiments and provides an ambient temperature of 4.2K. A common method for getting below this temperature is by pumping on the $^4$He, which provides access to temperatures down to 1K. To obtain even lower temperatures, pumping on $^3$He provides access to temperatures as low as 300mK.

Figure 3.1 demonstrates shows the inner vacuum chamber at the bottom of the probe. This unit contains the sample stage along with the $^3$He cryogenic system. $^3$He is stored in a closed container at room temperature that is connected by a small stainless steel capillary to the $^3$HePot and Charcoal Sorption Pump. $^4$He is pumped through small-diameter intake valve from the surrounding liquid Helium. A temperature of 1K can be reached by using the needle valve to adjust the pumping speed. When the temperature of the 1K plate drops below 2.5K, the $^3$He gas begins to condense in the $^3$He pot. The sorption heater can be turned on to release more $^3$He stored in the charcoal sorption pump for condensation. When this condensation has accumulated a significant portion of liquid $^3$He in the $^3$He pot, the sorption heater is switched off and the sorption pumping causes the temperature of liquid $^3$He to drop to 300mK.

3.2 Superconducting Magnet

The inner vacuum chamber is inserted into the superconducting magnet. Magnetic fields of -2 to +2 Tesla were utilized for the studies presented below. These fields are well within the 9 Tesla capabilities of the superconducting magnet made of twisted multi-filamentary NbTi wire. The coil is encased in a copper matrix for quench protection and bonded in place with epoxy to prevent training. Magnetic fields are applied perpendicular to the 2D electron gas.
3.3 Sample Mounting & Temperature Control

Samples are thermally anchored to a cold copper finger in vacuum along with a calibrated thermometer and 100-200Ω resistor, $R_H$. The cold copper figure is in contact with the 3He pot as shown in Figure 3.1. The sample is anchored to the cold copper finger. The DC and low frequency AC measurements described below are made through BeCu wires which are thermally anchored to the stage before they reach the sample. Heat is delivered to the stage by applying a DC current to $R_H$. The sample is mounted considerably closer to the thermometer than the...
heater to minimize the effect of any thermal gradients on the measurement of temperature. Sample temperatures of 0.3 to 30K are attainable through this apparatus.

A proportional-integral-derivative (PID) controller is used to maintain fixed temperatures. This system reads the current temperature via the calibrated resistor. It then dynamically applies a DC voltage to the heater \( R_H \) in order to maintain a fixed temperature.

### 3.4 Transport Measurements

Samples are studied in both the Hall bar and Corbino disk geometries. AuGe eutectic was used to provide electric contacts to the 2D electron gas. Studies of Corbino samples are inherently limited to two-point probe measurements. While this method includes contact resistance in its measurements. Luckily, there are many methods for creating reliable ohmic contacts to GaAs quantum wells and the contact resistances are generally only a few Ohms. Corbino samples are used for investigations where the two-terminal geometry is advantageous for the delivery of high frequency signals.

Most of the experiments presented in this study utilize the four-point probe method available in the Hall bar geometry. This method eliminates unwanted effects of contact resistance by using separate contacts for the application of current and measurement of voltage. The current contacts are sufficiently separated from the measured area by a typical distance of about 500µm, which is much greater than the typical inelastic relaxation length of the 2D electrons \( L_{in} = (D\tau_{in})^{1/2} \sim 1 - 5\mu m \). This ensures that the experiments are done at thermal equilibrium and the distribution of the electric current is uniform across the samples.
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3.4.1 DC and Lock-in Measurements

Figure 3.2 demonstrates the electronic configuration for samples in the Hall bar geometry. A DC voltage is applied over a large resistor, $R_{DC}$, and can be approximated as a constant DC current source. The voltage over $R_{DC}$ is measured to determine the DC bias current applied to the sample, $I_{DC}$. $R_{DC}$ is typically on the order of 10 to 110 kΩ depending on the desired range of bias currents. From a lock-in amplifier, an oscillating AC voltage is applied over a second large resistor, $R_{AC} = 1MΩ$, to ensure a constant AC current source, typically $1\mu A$ (1V). In general, two synchronized lock-in amplifiers are used to simultaneously measure the longitudinal voltage ($V_{xx}$) and Hall voltage ($V_{xy} = V_H$).

The differential resistance is the derivative of the voltage with respect to current and is found by dividing the measured AC voltage by the AC current,

$$ r = \frac{\partial V}{\partial I} = \frac{V_{AC}}{I_{AC}}. $$ (3.1)
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Here, $V_{AC}$ can be either the longitudinal or Hall voltage. Using a small AC excitation provides an accurate derivative since large AC excitations can average out interesting nonlinear effects. Measuring the differential resistance rather than the ohmic resistance ($R = V/I$) of samples utilizes the main advantage of the lock-in amplifier – the ability to extract a signal at a very specific frequency from an otherwise noisy environment. This results in considerably smoother experimental data and access to considerably smaller signals than those available with DC approaches.

For linear measurements of differential resistance (zero bias current), the differential resistance coincides with ohmic resistance. Thus, results presented below often use the $R_{xx}$ and $r_{xx}$ notation interchangeably when linear measurements are made.

### 3.4.2 RF and MW Measurements

Radio frequency (RF) and microwave (MW) excitations are delivered through 50Ω-impedance, stainless steel semi-rigid coaxial lines with silver-plated BeCu center conductors rated for DC-34GHz. The frequencies of radiation used in this study range from $10\text{MHz}$ to $20\text{GHz}$. Electromagnetic radiation in this range of frequencies is often termed microwaves for simplicity of discussion.

The incorporation of RF and microwave electronics into existing DC and low frequency AC measurements is depicted in Figure 3.3. DC, low frequency AC and high frequency RF/MW measurements are made through a single coaxial line. The high and low frequencies are kept independent by the use of a bias-tee. Additionally, RF/MW radiation is applied through a directional coupler to separate the radiation and the reflected signal that is detected at the MW analyzer. A terminal resistor $R_T = 50\Omega$ is placed at the end of the coaxial line to ensure
broadband impedance-matching of RF and MW radiation to the sample. A capacitor $C = 47\,\text{pF}$ is placed in series with $R_T$ to prevent DC and low frequency AC excitations from bypassing the sample through $R_T$.

With this arrangement two basic types of measurement can be made: changes of sample resistance can be measured with the lock-in amplifier by modulating the amplitude of RF/MW radiation; and oscillations of sample conductivity due to two interfering MW sources can be detected by the MW analyzer. These techniques are utilized in the study of the dynamics of quantal heating presented in Chapter 9.
Chapter 4

State of the Field

The nonlinear transport properties of highly mobile two-dimensional electrons placed in quantizing magnetic fields attract a great deal of attention both for its fundamental importance and remarkable properties. In response to both microwave radiation and DC excitations, strongly nonlinear electron transport [6, 12–47] that gives rise to unusual electron states [48–57] has been reported and investigated.

4.1 Quantum Lifetime

The electron quantum lifetime $\tau_q$ has been measured in many experiments [58] and is an important property of two dimensional systems [59] as seen in Chapter 2. The standard transport method to measure the electron lifetime is based on an extraction of the Dingle factor from temperature or magnetic field dependences of the amplitude of Shubnikov-de-Haas (SdH) oscillations. This method works well at low temperatures, at which the Dingle factor is nearly temperature independent. However, the application of this method to higher temperatures is considered to be problematic, since it involves a separation of unknown small temperature
variations of the Dingle factor from strong variations of SdH amplitude with the temperature. Moreover, theoretical investigations indicate that despite the fact that the electron-electron scattering affects the quantum lifetime it does not change the amplitude of the quantum resistance oscillations [60–64]. Thus the standard method is not applicable to study the electron-electron scattering, which – as shown in many parts of this study – is the dominant mechanism reducing the electron lifetime with temperature. Another limitation is the high sensitivity of the standard method to spatial fluctuations of the electron density due to long range variations of the bottom of the conducting band. These density variations induce spatial variations of the period of the SdH oscillations. It also decreases the total SdH amplitude, but does not change the electron lifetime. Finally, at high temperatures the SdH oscillations are absent and, thus, the standard method is simply inaccessible.

Recently, several transport methods have been introduced to access the temperature dependence of the quantum electron lifetime $\tau_q$. Electric field [32] and microwave [33] induced magnetoresistance oscillations show that the amplitude of the oscillations depends on temperature. At $T > 2K$ variations of the quantum scattering time $\tau_q$ are found to be temperature dependent. At temperature below 2K the dependence saturates, indicating an electric-field-induced overheating. The overheating may create very peculiar electron distributions [12] and is a restriction of this method. The quantum electron lifetime $\tau_q$ was recently accessed through the amplitude of magnetophonon resistance oscillations [65, 66]. However, the method depends on the rate of electron-phonon scattering and, therefore, requires high temperatures.

In other transport experiments [11, 67, 68] which are done on multi-subband electron systems, the temperature dependence of the quantum lifetime $\tau_q$ has been extracted from quantum contributions to the magnetoresistance [6, 9, 69]. Despite the fact that the magnetoresistance
may be affected by other scattering mechanisms (such as classical magnetoresistance, memory effects, etc.) in multi-subband electron systems the magnetoresistance demonstrates magnetointersubband oscillations (MISO) [69]. These oscillations set the scale of the quantum contributions and thereby facilitate the interpretation of experimental data.

### 4.2 Magnetointersubband Oscillations

Magnetotransport phenomena in high-mobility semiconductor structures are commonly studied with only one populated subband ($\epsilon_1$), because the electron mobility decreases with filling the second subband ($\epsilon_2$) due to inter-subband scattering [70]. The latter also gives rise to magnetointersubband oscillations (MISO) of the dissipative resistance [71]. As discussed in Section 2.6.2, electron systems with two populated subbands have resistance maxima at magnetic fields, $B$, satisfying the relation [69, 72, 73]: $\Delta = k \cdot \hbar \omega_c$, where $\Delta = \epsilon_2 - \epsilon_1$ is the energy separation of the bottoms of the subbands, $\omega_c$ is the cyclotron frequency, and $k$ is a positive integer. In contrast to Shubnikov de Haas oscillations the magnetointersubband oscillations exist at high temperature $kT > \hbar \omega_c$. An interference of these oscillations with phonon-induced oscillations has been reported [74].

At small quantizing magnetic fields a finite electric current induces several additional nonlinear phenomena. At low temperatures small currents considerably decrease the resistance. The dominant mechanism inducing the resistance drop is a peculiar Joule heating (quantal heating), which produces a non-uniform spectral diffusion of electrons over the quantized spectrum. The spectral diffusion is stabilized by inelastic processes ("inelastic" mechanism) [43]. The heating has been recently observed and studied [12, 13, 30, 35]. At higher currents electron transitions between Landau levels occur due to an elastic electron scattering on impurities in the presence of
an electric field [17, 46]. The transitions increase the resistance, which was observed in electron systems both with a single occupied subband [23, 50, 75] and with multi-subband occupation [76–79]. In the latter case, an interference of the magnetointersubband quantum oscillations (MISO) with the current induced inter-level scattering was reported.

### 4.3 Zero Differential Resistance State

Recent interest in a comprehensive study of the nonlinear magnetotransport in two dimensional electron systems has been stimulated by an observation of the Zener tunneling of highly mobile 2D electrons between Landau levels, which is induced by Hall electric field in GaAs/AlGaAs heterojunctions [17]. The effect was originally found in Hall bar geometry and appeared as oscillations of magnetoresistance $r_{xx}(B)$ induced by DC electric current $I_{DC}$. Positions of the oscillations in magnetic field $B$ obeyed the following relation: 

$$
\gamma R_e e E_H = l \hbar \omega_c,
$$

where $\gamma \approx 2$, $l$ is an integer, $\omega_c$ is the cyclotron frequency, $R_e$ is the cyclotron radius and $E_H$ is Hall electric field. Later, the Zener oscillations of the magnetoresistance $r_{xx}$ were found in highly doped GaAs quantum wells [23], in double quantum wells [29] and in a hole gas [75]. Very recently the Zener oscillations have been detected in the differential conductance of Corbino discs, where the Hall electric field $E_H$ is absent [80].

Another intriguing nonlinear phenomenon that is observed in 2D electron systems placed in crossed electric and quantizing magnetic fields, is the electronic state with zero differential resistance (ZDR state) [51]. Experimental data has demonstrated that in the Hall bar geometry the initial decrease of the longitudinal differential resistance $r_{xx}$ with applied DC current $I_{DC}$ terminates at $I_{DC} = I_{th}$ corresponding to $r_{xx} = 0$. At $I_{DC} > I_{th}$ the differential resistance remains zero in a broad range of electric currents $I_{DC} > I_{th}$ that significantly exceed the
threshold value, $I_{th}$. The initial drop of the resistance is associated with quantal heating induced by the spectral diffusion of 2D electrons in crossed electric and magnetic fields [11, 13, 30, 43].

The transition into the ZDR state is attributed to the local instability of the electric current at $I_{DC} > I_{th}$ [56]. The local instability is considered to be the origin of another spectacular phenomenon - the zero resistance state observed in highly mobile 2D electron systems under a microwave irradiation [48, 49, 81]. However, an uncertainty of the microwave field distribution in studied samples limits the quantitative comparison of the nonlinear response with theories.

Recently, a strong nonlinear response of 2D electrons was observed in a geometry where nonlocal electron transport – associated with the propagation of the edge states and/or skipping orbits [82–89] – plays the dominant role [90]. The observation of the nonlocal nonlinear response has raised a question regarding the possibility of a significant contribution from edge states and/or skipping orbits to the nonlinear transport of 2D electrons observed in the Hall bar geometry [32, 52–54, 76, 78, 79, 91–95] and, thus, the applicability of the currently accepted theoretical approach [43] to the observed nonlinearity. It is worth noting that a separation of the local and the nonlocal contributions to the electronic conductance is a challenging problem in the Hall bar geometry.

4.4 Quantal Heating

Joule heating is a remarkable physical phenomenon which transforms electric energy into heat. Recently it was shown that the quantum properties of matter significantly affect the heating [12, 43], giving rise to a thermal stratification (quantization) of the electron distribution in energy space [30]. This effect, called quantal heating, does not exist in classical electron systems. Its theoretical description and experimental observation can be found in detail in Section
2.7. The most essential property of quantal heating is the conservation of the total number of quantum states participating in the electron transport and, thus, the conservation of the overall broadening of the electron distribution \([12, 30]\). In contrast to classical Joule heating, quantal heating leads to outstanding nonlinear transport properties of highly mobile 2D electrons, driving them into exotic nonlinear states in which voltage (current) does not depend on current \([51]\) (voltage) \([80]\). Quantal heating also provides significant contributions to nonlinear effects at high driving frequencies - an important topic in contemporary research \([4]\).

Joule heating of 2D electrons in quantizing magnetic fields was observed in the pioneer work on the 2D electron transport \([96]\). The heating decreased the amplitude of the Shubnikov de Haas (SdH) oscillations of the conductivity providing a way to measure the electron inelastic relaxation. Assuming that the distribution of overheated electrons is described by an electron temperature \([97–110]\), the inelastic relaxation rate of 2D electrons in GaAs heterojunctions was found to be proportional to the temperature at a lattice temperature of few Kelvin \([97, 103, 108]\). This widely accepted result has been examined and a substantial inconsistency was found between inelastic relaxation times in zero and quantizing magnetic fields \([108]\). The SdH result was further challenged in recent investigations of Joule heating \([12]\). In these investigations the temperature approximation of overheated electrons has been relaxed. Using a spectral diffusion equation \([43]\), the nonequilibrium electron distribution was evaluated numerically, revealing significant deviations from the Fermi-Dirac form. The temperature dependence of the inelastic relaxation rate obtained by this method was found to be considerably different from the expected: \(T^2\) at \(kT \gg \hbar \omega_c\) and \(T^3\) at \(kT \sim \hbar \omega_c\) \([12]\). These findings raise a concern regarding the validity of the inelastic relaxation time \(\tau_{SdH}(T)\) obtained by the SdH method.
Chapter 5

Quantum Lifetime and Positive Magnetoresistance

As discussed in Chapter 2, the quantum scattering time, $\tau_q$, is an important parameter describing transport in 2D quantum systems. In this chapter, I will describe an application of transport measurements on quantum wells with a single occupied subband to study the quantum positive magnetoresistance [6] at different temperatures. Both positive and negative magnetoresistance has been observed in 2D electron systems [111–115], however the quantum contribution to the magnetoresistance has not been identified in those works. In a very recent experiment on a two-subband electron system, the quantum positive magnetoresistance [9] and the classical magnetoresistance were separated [116].

As described in Section 2.21, the positive magnetoresistance is induced by the quantized (periodic) motion of electrons in magnetic fields. Due to the circular motion, a scattered electron may return to the same impurity repeatedly, enhancing the total scattering amplitude. The stronger the magnetic field, the larger the probability for the electron to return to the same
impurity. Thus, the scattering rate increases with the magnetic field, giving rise to the positive magnetoresistance [6].

We have found good agreement between our experiment and the theory in a broad range of magnetic fields. Comparison with the theory yields the quantum lifetime of 2D electrons, $\tau_q$. At small temperatures, $T$ the inverse of the electron lifetime (quantum scattering rate) is found to be proportional to $T^2$ and deviates from the $T^2$ behavior above 15 K. The temperature dependence agrees very well with the conventional theory of electron-electron ($e-e$) scattering at zero magnetic field in broad range of temperatures from 0.3 K to 20 K. The comparison yields the electron screening vector, $q_s$, which is close to the Thomas-Fermi value, $q_{TF}$, for these electron systems. Good agreement is also found between contributions of the electron-electron scattering to the quantum scattering rate $1/\tau_q$ and the rate of inelastic electron relaxation $1/\tau_{inl}$, which is obtained on the same sample by a different method [12].

5.1 Experimental Setup

The samples used in this study are high-mobility GaAs quantum wells grown by molecular beam epitaxy on semi-insulating (001) GaAs substrates. The width of the GaAs quantum well is 13 nm. Two AlAs/GaAs type-II superlattices grown on both sides of the well served as barriers, providing a high mobility of 2D electrons inside the well at a high electron density [5]. One sample was studied with electron density $n = 8.2 \times 10^{15} m^{-2}$ and mobility $\mu = 93 m^2/Vs$. Another sample with comparable parameters shows similar results. In this paper we show results for the first sample.

The studied 2D electron system is etched in the shape of a Hall bar. The width and the length of the measured part of the sample are $d = 50\mu m$ and $L = 250\mu m$. A 12 Hz alternating
current is applied through current contacts formed in the 2D electron layer. The longitudinal AC voltage $V_{xx}$ is measured between potential contacts displaced 250$\mu m$ along each side of the sample. The Hall voltage $V_{xy}$ is measured between potential contacts displaced 50$\mu m$ across the electric current.

### 5.2 Results and Discussion

Figure 5.1 shows the magnetoresistance of 2D electrons taken at different temperatures. All curves demonstrate a similar behavior. At small magnetic fields ($B < 0.1T$) the curves show
5. Quantum Lifetime and Positive Magnetoresistance

Extremely weak (unrecognizable in the present scale) dependencies on the magnetic field. At higher magnetic fields, the resistance increases. Not shown in Figure 1 is the trace at the lowest temperature $T=0.3\,K$, which indicates that the resistance increase correlates with the quantization of the electron spectrum. Namely, the positive magnetoresistance (taken at $T > 2\,K$) starts at the magnetic field at which the quantum (SdH) oscillations are first observed at $T = 0.3\,K$. The positive magnetoresistance and its temperature dependence are the main targets of the experiments. Below we compare the resistance increase with the interference enhancement of impurity scattering in the quantizing magnetic fields [6]. At even higher magnetic fields the magnetoresistance shows quantum oscillations, which depend strongly on the temperature. The oscillations are beyond the scope of this paper.

Figure 5.2 shows the positive magnetoresistance in better detail and demonstrates a comparison with theory [6]. The theory considers 2D electrons, which are moving in magnetic field and scattered by a rigid disordered potential. Due to circular electron motion in magnetic field, electrons scattered by an impurity may return to the impurity again and again, enhancing the total scattering amplitude. The stronger the magnetic field, the more probable it is for the electron to return to the same impurity. A quantitative account of the interference of quantum amplitudes is demonstrated by the following equation [6]:

$$ R(B) = R_0 \cdot \left\{ 1 + 2 \left[ e^{-\alpha} + e^{-2\alpha}(1 - \alpha)^2 \right] \right\}, $$

(5.1)

where $\alpha = 2\pi/(\omega_c \tau_q)$, $\omega_c$ is cyclotron frequency and $R_0$ is the resistance at zero magnetic field.

Although the theory is developed for a broad range of temperatures, it considers the elastic impurity scattering to be dominant in the electron dynamics. Equation 5.1 was derived in the
Figure 5.2: Comparison between experiment (solid lines) and theory (open circles) at different temperatures as labeled. Insert shows the data plotted vs inverse magnetic field $1/B$, indicating exponential growth of the magnetoresistance at small magnetic fields.

absence of electron-electron and electron-phonon interactions ignoring possible temperature effects on the magnetoresistance. Presented in Figure 5.1 data show considerable variations of the magnetoresistance with the temperature. In accordance with Equation 5.1 the electron lifetime $\tau_q$ is the leading parameters affecting the shape of magnetoresistance. In comparison with the theory we consider the time $\tau_q(T)$ to be the dominant temperature dependent variable. While the exponential terms will likely contain a simple combination of quantum disorder scattering rate and electron-electron scattering rate, there is a possibility that prefactor $\alpha$ in the last term of Equation 5.1 depends only on disorder scattering time. In the paper the parameter $\alpha$ is used
as a single fitting parameter for all terms in Equation 5.1. The possible overestimation of the electron lifetime is below 3%.

The best correspondence with the theory is obtained for the following fitting function:

$$R(B, T) = R_0(T) + 2R_D \left[ e^{-\alpha} + e^{-2\alpha}(1 - \alpha)^2 \right], \quad (5.2)$$

where the resistance at zero magnetic field $R_0(T)$ and a "Drude" resistance $R_D$ are additional fitting parameters. The resistance $R_0$ takes into account all scattering events responsible for the conductivity, whereas resistance $R_D$ accounts only the scattering, which is responsible for the quantum positive magnetoresistance. Thus the fitting procedure excludes effects of electron-phonon scattering, which reduce the electron transport scattering time $\tau_{tr}$ at zero magnetic field and, most likely, do not change the quantum interference enhancement of the impurity scattering in strong magnetic fields [6]. The parameter $R_0$ absorbs also all other scattering processes, which do not contribute to the quantum positive magnetoresistance. Figure 5.2 demonstrates the comparison with the theory at three different temperatures. Good agreement is obtained in a broad range of magnetic fields. The insert shows the magnetoresistance plotted against inverse magnetic field $1/B$, indicating the exponential growth of the resistance, which is consistent with Equation 5.1 at $\alpha \gg 1$. The exponentially strong enhancement of the scattering rate at the small magnetic fields provides an immunity of the utilized procedure with respect to possible smooth resistance variations of yet unidentified origin.

Presented in Figure 5.2, comparison with the theory yields the quantum scattering time (electron lifetime) $\tau_q$. Figure 5.3(a) shows the temperature dependence of the quantum scattering rate $1/\tau_q$. The dependence is plotted vs the square of the temperature. The plot indicates
5. Quantum Lifetime and Positive Magnetoresistance

\[ \frac{1}{\tau_q} (GHz) = 201 + 1.05 \cdot T^2 \]  

Figure 5.3: (a) Temperature dependence of the quantum scattering rate \( \frac{1}{\tau_q} \) (open circles) plotted vs \( T^2 \). The dashed red line presents a linear fit of the temperature dependence at \( T < 15K \). Solid line presents expected temperature dependence due to electron-electron scattering (see Equation 5.4). The insert shows parameters \( R_0 \) (open circles) and \( R_D = 27\Omega \) (black squares), which are used to compare data shown in Figure 5.1 with Equation 5.2. (b) Temperature dependence of transport scattering rate \( \frac{1}{\tau_r} \).

quadratic variations of the quantum scattering rate \( \frac{1}{\tau_q} \) with the temperatures below 15 K. Shown in the Figure the dashed straight line approximates the \( T^2 \) dependence:

\[ \frac{1}{\tau_q} (GHz) = 201 + 1.05 \cdot T^2 \]  \hspace{1cm} (5.3)

The \( T^2 \) dependence suggests that the electron-electron scattering is the dominant mechanism, reducing the electron lifetime. In accordance with conventional theory [117–119] at zero magnetic field the \( e-e \) scattering rate is

\[ \frac{1}{\tau_{ee}} = \frac{\pi (kT)^2}{4\hbar E_F} \cdot ln \left( \frac{\hbar q_s v_F}{kT} \right), \]  

\hspace{1cm} (5.4)

where \( k \) is Boltzmann’s constant, \( E_F \) and \( v_F \) are the Fermi energy and velocity and \( q_s \) is the screening wave vector. A comparison of the temperature dependence of the \( \frac{1}{\tau_q} \) with the theory
is shown in Figure 5.3(a) by the solid black line. The comparison utilizes the screening wave vector $q_s$ as the only fitting parameter. The screening wave vector is found to be $q_s = 1.82 \times 10^8 \, 1/m$, which is very close to the Thomas-Fermi screening wave vector in 2D, given by $q_{TF} = 2me^2/(\epsilon \hbar^2) = 1.96 \times 10^8 \, 1/m$, where $\epsilon = 12.9$ is the GaAs lattice dielectric function. Thus variations of the electron lifetime with the temperature are in good agreement with the theory of electron-electron scattering at zero magnetic field [117–119].

The current accuracy of the experiment at $T < 15$ K does not allow to distinguish the exact $T^2$ dependence of the quantum scattering rate $1/\tau_q$ from the one given by Equation 5.4. The $T^2$ dependence is predicted theoretically for the rate of inelastic relaxation $1/\tau_m$ of non-equilibrium electron distribution at low temperatures $T < \hbar \omega_c (\omega_c \tau_{tr})^{1/2}$ (see Equation 37 and Equation 42 in Reference[43]). Observed temperature dependence of the inelastic relaxation rate [12] is in agreement with the theory. The $T^2$ behavior of the inelastic relaxation is the result of a modification of electron screening in strong magnetic fields at a distance $d \sim (D/\omega_c)^{1/2}$, where $D = (v_F)^2/(2\omega_c^2 \tau_{tr})$ is the diffusion coefficient in strong magnetic fields. At this scale there is a change in the dynamics of electron propagation from a ballistic motion at short distances $r < d$ to a ”ballistic diffusion” at long distances $r > d$ [43]. Shown in Figure 5.3(a) agreement between experiments and theory at zero magnetic field (see Equation 5.4) indicates that possible variations of electron lifetime $\tau_q$ due to the change of the electron dynamics in weak quantizing magnetic fields are small.

The insert in Figure 5.3(a) presents dependences of the parameters $R_0$ and $R_D$ on the temperature. The parameter $R_0$ is very close to the actual resistance $R_{xx}(T)$ at zero magnetic field. The parameter $R_D$ was found to be temperature independent fluctuating at $R_D = 27\Omega$. Data shown in Figure 5.3(a) are obtained at fixed $R_D = 27\Omega$ [120].
On a qualitative level the temperature dependence of the electron-electron scattering rate is consistent with results obtained by other research groups. All recent experiments demonstrate the $T^2$ dependence of the $e$-$e$ scattering rate: $1/\tau_{ee} = \lambda T^2 / E_F$, where coefficient $\lambda \sim 1$ varies between research groups, methods and/or samples. Physical parameters $(E_F, v_F, q_s)$ affecting the $e$-$e$ scattering rate in Equation 5.4 may vary from sample to sample. In addition Fermi liquid corrections to the $e$-$e$ scattering rate may be essential. In this sense the variation of the parameter $\lambda$ between samples and different methods are expected. Below we compare the obtained electron-electron scattering rate $1/\tau_{ee}$ with the rate of the inelastic relaxation $1/\tau_{in}$ obtained by a different method on the same sample [12]. We also compare these results with theory. In the comparison theoretical expressions for $e$-$e$ scattering rate have the same physical parameters and one should expect a correlation between electron-electron scattering rates obtained by different experimental methods and corresponding theoretical estimations.

In accordance with Equation 5.3 and 5.4 the rate of the electron-electron scattering at low temperatures is $1/\tau_{ee} = 1.05 \cdot T^2$ GHz. Measured on the same sample the inelastic relaxation time $\tau_{in}$ is shown in Figure 6a of Reference [12]. The inelastic relaxation rate follows the $T^2$ dependence. The particular value of the rate depends on the form of the density of states (DOS) used for a comparison with the theory [43]. For Gaussian DOS the inelastic rate is found to be $1/\tau_{in}^G = (0.56 \pm 0.05) \cdot T^2$ GHz whereas for SCBA density of state the rate is $1/\tau_{in}^{SCBA} = (0.96 \pm 0.15) \cdot T^2$ GHz. A theoretical evaluation of the inelastic relaxation rate $1/\tau_{in}$ uses Equation 42 and Equation 37 in Reference [43]. At magnetic field $B=0.15T$ and $q_s = 1.82 \cdot 10^8 \ 1/m$ the estimated value $1/\tau_{in}^{th} = 0.88 \cdot T^2$ GHz. Thus the experimental and theoretical values of the inelastic relaxation rate $1/\tau_{in}$ are consistently smaller than the rate of electron-electron collisions $1/\tau_{ee}$ limiting the electron lifetime.
Figure 5.3(b) shows the temperature dependence of transport scattering rate $1/\tau_{tr}$. The dependence is obtained from the resistivity at zero magnetic field. At all temperatures the transport scattering rate $1/\tau_{tr}$ is much smaller than the quantum scattering rate $1/\tau_q$. The functional form of the temperature dependence of the transport scattering rate $1/\tau_{tr}$ is different from the $T^2$ dependence of the quantum scattering rate $1/\tau_q$, shown in Figure 5.3(a). The main reason for the difference is that electron-electron collisions preserve the total momentum of electron system and do not contribute directly to the resistance. Electron collisions, nevertheless, transfer an electron from one quantum state to another states, decreasing the lifetime of the electron in a given quantum state.

It is accepted that the electron-phonon interaction is the dominant mechanism of the temperature dependence of the transport scattering time $\tau_{tr}$ in GaAs quantum wells [65, 121]. Figure 5.3(b) shows that at $T = 20\, \text{K}$ the phonon contribution to the electron scattering rate is about 15 GHz. This value is significantly smaller than the rate of electron-electron scattering $1/\tau_{ee}(T = 20\, \text{K}) \approx 370\, \text{GHz}$ shown in Figure 5.3(a). The comparison emphasizes once again the dominant contribution of the $e-e$ scattering to the electron lifetime at low temperatures.

5.3 Limitations of the method

The comparison presented above indicates a very good agreement between experiment and theory [6]. Extracted from analysis, the quantum scattering rate $1/\tau_q$ is consistent with measurements by other methods [12]. The temperature variations of the electron-electron scattering rate are in accord with the conventional theory [117–119].

Below we discuss reasons for complimentary agreement between experiment and the theory of the positive magnetoresistance as well as possible limitations of the method. A difficulty
of the practical implementation of the method to general systems is a contribution of other mechanisms to the magnetoresistance, which are beyond the theory [6]. The theory considers the disordered potential in Self-Consistent Born Approximation (SCBA) [122]. Within the SCBA the scattering events are uncorrelated.

Different theories, accounting for correlations in electron scattering (non-markovian or memory effects) indicate a wide-ranging variety of possible behavior of the magnetoresistance [123–125]. A quantitative account of all possible effects may create difficulties for applications of the presented method, since some parameters significant for these theories may not be known a priori. A good practical indication of the small contribution of such effects to the electron transport is the absence of the magnetoresistance at small magnetic fields, at which Landau levels are not formed yet ($\omega_c \tau_q \ll 1$). In accordance with classical (Drude) theory, which ignores correlations in electron scattering, the magnetoresistance must be absent in one valley conductors [1]. This is the case for our samples (see Figure 5.4). Indeed, the magnetoresistance is very small at $B < 0.07$ Tesla at which the electron spectrum is not quantized.

In contrast, a giant negative magnetoresistance has been recently found in high mobility electron systems [81, 126–129]. The origin of this effect is not completely understood. The presence of two types of disorder (non-Gaussian disorder [125]) is considered as a possible explanation of the phenomenon [129]. Additional investigations are required to separate different mechanisms of the magnetoresistance, which are expected in a general case.

Below we estimate contributions of effects of the correlated scattering and correlated disorder to the magnetoresistance in our samples. As a first step we have to evaluate the correlation length of the disorder $\xi$. In high mobility samples the electrically charged donors are displaced from conducting layer by a distance $l$. Inside the conducting layer the displaced electric charges
5. Quantum Lifetime and Positive Magnetoresistance

create a weak and smooth fluctuating electric potential with correlation length $\xi \approx l$. The potential induces the small angle scattering of electrons. An electron needs many scattering events to relax (randomize) its original momentum. As a result the transport scattering time $\tau_{tr}$, describing the relaxation of the electron momentum, is much longer than the quantum scattering time $\tau_q$, which is an average time between two successive scattering events. Due to the scattering the direction of electron momentum performs a diffusive like motion with a typical step $\theta_0 \sim h/(p_F \xi) \ll 1$, where $p_F$ is electron momentum. During the transport scattering time $\tau_{tr}$ an electron scatters about $N = \tau_{tr}/\tau_q$ times by the disorder potential and changes its direction by $(\Delta \theta)^2 = \theta_0^2 \cdot N \sim 1$. Thus $\tau_q/\tau_{tr} \approx \theta_0^2 = (h/(p_F \xi))^2 [6]$. The ratio of these two times provides an estimation for the correlation length of the disordered potential $\xi$. In our samples the quantum scattering time at $T = 2K$ is about 5 ps (see Figure 5.3(b)), whereas the sample conductivity at zero magnetic fields yields transport scattering time of $\tau_{tr} = 32$ ps. Thus the disorder correlation length is about $\xi \sim (h/p_F) \cdot (\tau_{tr}/\tau_q)^{1/2} = 11$ nm.

Figure 5.4 presents effects of correlated disorder and correlated scattering on the magnetoresistance in our sample. In accordance with the theory a smooth disorder provides two distinct contributions to the resistivity [125]:

$$\frac{\Delta \rho_{xx}}{\rho_0} = -\left(\frac{\xi}{R_c}\right)^2 + \frac{2\zeta(3/2)}{\pi} \left(\frac{\xi}{l_{tr}}\right)^3 \left(\frac{l_{tr}}{R_c}\right)^{9/2},$$  \hspace{1cm} (5.5)

where $l_{tr}$ is mean free path of electrons moving in the smooth disorder and $R_c$ is cyclotron radius. In Equation 5.5 the first term is due to a bending of electron trajectories within the correlation length $\xi$. The second term is associated with a classical memory effect due to the circular motion of electrons in a magnetic field [130]. In Figure 5.4 the line (1) shows the contributions of correlated disorder to classical magnetoresistance, which is plotted in accordance with
Equation 5.5 for correlation length $\xi = 11 \text{ nm}$. A thick solid line (2) shows experimental results at $T = 4.72\text{K}$. The thin solid line (3) is a difference between upper (2) and lower (1) curves, demonstrating the magnetoresistance without contribution of the classical memory effects. Below $B = 0.3\text{T}$ curves (2) and (3) are indistinguishable, indicating very small contribution of the correlated disorder to the resistivity.

At $B=0.15\text{T}$ the magnetic length $\lambda = 66 \text{ nm}$ is considerably longer the correlation length of the disorder $\xi = 11 \text{ nm}$ and SCBA works well for most part of scattering events. Thus the
theory [6], accounting for the interference contribution of returning paths near this magnetic field, provides the leading contribution to the magnetoresistance.

5.4 Conclusion

This study demonstrates a simple transport method to access the electron lifetime $\tau_q$ of two-dimensional electrons in quantizing magnetic fields in a broad temperature range. For two-dimensional electrons in GaAs quantum wells, the temperature variations of the quantum scattering rate $1/\tau_q$ are found to be proportional to the square of the temperature at $T < 15$ Kelvin and are in very good agreement with the theory taking into account electron-electron interactions in 2D systems.
Chapter 6

Quantum Oscillations of Nonlinear Resistance

The nonlinear transport properties of highly mobile two-dimensional electrons placed in quantizing magnetic fields attract a great deal of attention both for its fundamental importance and remarkable properties. As discussed in Chapter 4, many theoretical and experimental investigations have reported strongly nonlinear electron transport that gives rise to unusual electron states. The discussion in the following chapter presents a new phenomena and follows the published results from 2012 [131] and 2013 [132].

In this chapter, I will show that at higher magnetic fields there is an additional nonlinear mechanism which induces substantial oscillations of the longitudinal resistance in response to the applied electric current. The period of the oscillations does not depend on the magnetic field. The oscillations are observed at low temperatures and strong magnetic fields, at which Shubnikov-de Haas (SdH) oscillations are well-developed. The current-induced oscillations correlate with the SdH oscillations and are periodic in inverse magnetic fields. The oscillations are
absent at smaller magnetic fields at which the SdH oscillations are also small or absent. The oscillations are found in samples with long quantum electron lifetime $\tau_q = 4\, \text{ps}$ and are not observed in systems with broad Landau levels ($\tau_q = 1\, \text{ps}$).

The proposed theoretical model considers the oscillations as a result of the electrostatic re-distribution of the electron density, which induces an electric field and, thus, an electric current in the systems. The electron re-distribution occurs across the sample and results in a spatial variation of the number of occupied Landau levels. The model indicates that the resistance oscillates with the DC bias current with a period that does not depend on magnetic field or temperature.

The above model also considers variations of the electron population of the Landau levels as the origin of the phenomenon. However, in contrast to the SdH oscillations, the variations of the electron density $\delta n$ are related to the applied electric current (Hall voltage) and are spatially non-uniform across the conducting system\cite{131}. The model indicates that the period of the current-induced oscillations is proportional to the electron density $n$. It follows from the fact that the redistribution of the electron density $\delta n(r)$ creates the electric field, inducing the electric current $I_{DC}$. In strong magnetic fields $B$ the electric field is almost perpendicular to the current and produces the Hall voltage $V_H = I_{DC} \cdot R_{xy} = I_{DC} \cdot B/ne$, where $R_{xy}$ is Hall resistance and $e$ is electron charge. At the same density profile $\delta n(r)$ and, thus, at the same Hall voltage $V_H$, stronger electric current $I_{DC}$ flows through the 2D conducting system with proportionally higher electron density $n$: $I_{DC} = (eV_H/B) \cdot n$.

This paper presents the results of an investigation of the current-induced oscillations in electron systems with a variable electron density. At a fixed density $n$ and magnetic field $B$, the resistance oscillates with the electric current $I_{DC}$ and demonstrates a period that is proportional
to the electron density. At fixed magnetic field the differential resistance is found to be oscillating periodically with both the electron density \( n \) and the Hall voltage \( V_H \). The observed periods are independent of \( n \) and \( V_H \). The results indicate the leading role of the current-induced electrostatic potential in the observed nonlinear phenomenon and strongly supports the proposed model. Below I present obtained results.

6.1 Experimental Setup

The samples used in this study are high-mobility GaAs quantum wells grown by molecular beam epitaxy on semi-insulating (001) GaAs substrates. The width of the GaAs quantum well is 13 nm. Two AlAs/GaAs type-II superlattices grown on both sides of the well served as barriers, providing a high mobility of 2D electrons inside the well at a high electron density\([5]\). This is an important property of our samples and is discussed below in more detail. Two samples (N1, N2) were studied with electron density \( n_{1,2} = 8.2 \times 10^{15} \text{ m}^{-2} \), mobility \( \mu_{1,2} = 93 \text{ m}^2/\text{Vs} \) and quantum lifetime \( \tau_q = 4 \text{ ps} \). Another two samples (N3, N4) had similar electron density \( n_3 = 8.2 \times 10^{15} \text{ m}^{-2} \), \( n_4 = 12.2 \times 10^{15} \text{ m}^{-2} \), and mobility \( \mu_3 = 86 \text{ m}^2/\text{Vs} \), \( \mu_4 = 89 \text{ m}^2/\text{Vs} \), but much shorter quantum lifetime \( \tau_q = 1 \text{ ps} \).

The studied 2D electron systems are etched in the shape of a Hall bar. The width and the length of the measured part of the samples are \( d = 50 \mu m \) and \( L = 250 \mu m \). To measure the resistance we have used the four probes method. Direct electric current \( I_{DC} \) (DC bias) is applied simultaneously with 12 Hz AC excitation \( I_{AC} \) through the same current contacts (\( x \)-direction). The longitudinal and AC,(DC) voltage \( V_{xx}^{AC} \), \( V_{xx}^{DC} \) is measured between potential contacts displaced 250\( \mu m \) along each side of the sample. The Hall voltage \( V_H \) is measured between potential contacts displaced 50\( \mu m \) across the electric current in \( y \)-direction.
**Figure 6.1:** Dependence of dissipative differential resistance $R_{xx}$ on magnetic field $B$ and electric current $I_{DC}$. Left panel presents data for sample N1 with quantum scattering time $\tau_q = 4$ ps at temperature $T = 4.77$ K: (a) 2D plot $R_{xx}$ vs $B$ and $I_{DC}$; (b) $R_{xx}$ vs $B$ at $I_{DC} = 0$ µA; (c) $R_{xx}$ vs $I_{DC}$ at $B = 0.6$ T; (d) $R_{xx}$ vs $I_{DC}$ at $B = 2.0$ T; (e) $R_{xx}$ vs $I_{DC}$ at $B = 1.87$ T. Right panel presents data for sample N3 with quantum scattering time $\tau_q = 1$ ps at temperature $T = 4.2$ K: (a1) 2D plot $R_{xx}$ vs $B$ and $I_{DC}$; (b1) $R_{xx}$ vs $B$ at $I_{DC} = 0$ µA; (c1) $R_{xx}$ vs $I_{DC}$ at $B = 0.6$ T; (d1) $R_{xx}$ vs $I_{DC}$ at $B = 2.0$ T; (e1) $R_{xx}$ vs $I_{DC}$ at $B = 1.87$ T.

Measurements were carried out for different temperatures in the range of 0.3 to 10 Kelvin in a He-3 insert in a superconducting solenoid. Samples and a calibrated thermometer were mounted on a cold copper finger in vacuum. Magnetic fields were applied perpendicular to the 2D electron layers.
6. Quantum Oscillations of Nonlinear Resistance

Figure 6.2: Figures A1, B1 and C1 present dependence of differential resistance of sample N1 on magnetic field and DC bias taken at different temperatures as labeled. Bright(dark) spots indicate high(low) resistance. Figures A2, B2 and C2 present horizontal cuts of the plots shown in A1, B1 and C1 at magnetic field $B = 2.19$ T. Figures A3, B3 and C3 present horizontal cuts of the plots shown in A1, B1 and C1 taken at magnetic field $B = 2.25$ T.

6.2 Results

Figure 6.1 presents the magnetoresistance of two 2D electron systems with approximately the same electron density but with different electron lifetime $\tau_q$. The left panel (a)-(e) shows data taken at temperature $T = 4.77$ Kelvin for sample N1 with $\tau_q = 4$ ps. Figure 6.1(a) demonstrates an overall behavior of the differential resistance at different DC currents from -80 to 80 $\mu$A and magnetic fields from 1 to 2.25 Tesla. Taken at zero DC bias ($I_{DC} = 0$ $\mu$A) vertical cut of the 2D plot corresponds to the linear response of the system. The cut, extended to zero magnetic field, is shown in figure 6.1(b). The figure demonstrates well-known Shubnikov de Haas (SdH)
oscillations of the resistance. These oscillations are periodic in the inverse magnetic field $1/B$. Figure 6.1(c) demonstrates a horizontal cut of the 2D plot, which is taken at magnetic field $B = 0.6\ T$. At this magnetic field the SdH oscillations are absent as shown in Figure 6.1(b). The strong decrease of the resistance with the DC bias is due to quantal heating, which is studied for this sample in detail in [12] (see also [35]). Figure 6.1(d) presents another dependence of the resistance on the DC bias. The dependence is taken at a maximum of SdH oscillations and corresponds to a horizontal cut of the 2D plot at B=2 Tesla. Figure 6.1(d) shows oscillations of the resistance with the DC bias. Figure 6.1(e) shows a DC bias dependence of the resistance taken at minimum of SdH oscillations at $B = 1.87\ T$. Figure 6.1(e) demonstrates oscillations, which are complementary to the oscillations shown in figure 6.1(d). Sample N2 exhibits similar oscillations (not shown). The oscillations presented in figures 6.1(a),(d) and (e) are the main subject of this paper although the interesting inversion of the phase of SdH oscillations with DC bias has also been independently observed [95].

The right panel of figure 6.1 presents data obtained for sample N3 with similar electron density but with considerably shorter quantum scattering time $\tau_q = 1\ ps$. The data are taken at temperature $T = 4.2\ K$. Figures 6.1(a1)-(e1) demonstrate dependencies taken at the same conditions as the dependencies presented in figures 6.1(a)-(e). Due to the shorter time $\tau_q$ the Landau levels in the sample N3 are considerably broader than the quantum levels in the sample N1 and overlap substantially at $B = 0.6\ T$ (see Figure 2 in [12]). In result shown in Figure 6.1(c1) resistance variations are considerably smaller the one shown in Figure 6.1(c) [12, 43]. Figures 6.1(a1), (d1) and (e1) exhibit qualitatively different behavior: sample N3 does not show any oscillations with the DC bias.

Figure 6.2 demonstrates the effect of temperature on these oscillations. Figure 6.2 A1,
B1 and C1 present dependence of the differential resistance on magnetic field and DC bias taken at different temperatures as shown. The amplitude and shape of the oscillations depend on the temperature but the positions of the oscillations are essentially the same at different temperatures. At the lowest temperature $T = 0.3 \, K$ spin splitting of the Landau levels is observed. The splitting makes correlations between different curves less obvious. Figures 6.2 A2, B2 and C2 present horizontal cuts of the corresponding 2D plots 6.2 A1, B1 and C1 taken at magnetic field $B = 2.19$ Tesla. At this magnetic field a maximum of SdH oscillations, which correspond to a spin polarized Landau level, is observed at $T = 0.3 \, K$. The cuts indicate maximums at around $+35$ and $-35 \, \mu A$ for all three temperatures. Arrows mark the maximums.

The magnitude of the oscillations increases as the temperature decrease. Figures 6.2 A3, B3 and C3 present horizontal cuts taken at magnetic field $B = 2.25 \, T$. These cuts correspond to a SdH resistance maximum at high temperatures, which evolves into a minimum at lowest temperature $T = 0.3 \, K$ at which the spin splitting is larger the temperature. Two cuts taken at $T = 7.98 \, K$ and at the $T = 4.75 \, K$ (Figure 6.2 A3 and B3) demonstrate good correlation. At lowest temperature the resistance demonstrates minimum at zero DC bias and maximums at $+35$ and $-35 \, \mu A$ are not observed.

Figure 6.3 presents dependence of the differential resistance on the inverse magnetic field and DC bias for sample N1. The plot emphasizes the periodicity of the observed oscillations with respect to both the DC bias and the inverse magnetic field $1/B$. The figure indicates that the positions of the oscillations with respect to the DC bias do not change considerably with almost two times variation of the magnetic field.

Figure 6.4(a) presents vertical cuts of Figure 6.3 taken at different DC biases, which are close to maximums and minimums shown on Figure 6.1(d). The figure demonstrates that the
Figure 6.3: Dependence of the dissipative resistance on inverse magnetic field and DC bias. $T=4.77 \, K$. Sample N1

1/$B$ periodic oscillations at $I_{DC} = -32.5$ and -71.1 $\mu A$ are in phase whereas the oscillations at $I_{DC} = -12.5$ and -53.8 $\mu A$ are $180^0$ shifted with respect to SdH oscillations at zero DC bias. Figure 6.4(b) presents a Fourier spectrum of the oscillations at $I_{DC} = 0$ and -32.5 $\mu A$. The inset shows a dependence of the amplitude of the first harmonic of the oscillations on the DC bias. The experiment indicates a reduction of the oscillations with the DC bias increase.

Figures 6.3 and 6.4 demonstrates strong correlation of the DC biased-induced oscillations with the quantum oscillations at zero DC bias (SdH oscillations). This is an indication that these oscillations have a common origin. Below we consider a model, in which the oscillations are induced by spatial variations of the number of occupied Landau levels across the Hall bar sample.
6. Quantum Oscillations of Nonlinear Resistance

Shubnikov de Haas oscillations occur due to quantization of electron spectrum in a magnetic field [58]. With an increase of the magnetic field energy gaps between Landau levels increase and the top occupied Landau level intersects the Fermi energy $E_F$. At this condition resistivity of the electron systems is at a maximum. When the Fermi energy is between two Landau levels
the resistivity is at a minimum. Thus the resistance oscillates with variations of the number of
the Landau levels occupied by electrons.

We propose that the DC bias-induced oscillations also occur due to a variation of the electron
filling factor but, in contrast to SdH oscillations, the variation appears across the sample and is
related to a spatial change of electron density $\delta n$. If the change is comparable with the number
of electron states in a Landau level $n_0 = m/(\pi \hbar^2) \cdot \hbar \omega_c$, then one should expect a variation of the
electron resistivity. As shown below the spatial variation of the resistivity leads to oscillations
of the sample resistance.

A simple electrostatic estimation demonstrates that in a vacuum the variation of electron
density $\delta n \sim n_0$ creates a voltage, which is on several orders of magnitude stronger than the
one observed in the experiment. The estimation dictates, therefore, the presence of a strong
screening of electric charges $\epsilon \delta n$ in the samples. The proposed model assumes that the screening
is due to X-electrons, which are located near the conducting 2D layer.

Figure 6.5 shows a schematic diagram of our samples. The conducting GaAs quantum well
is sandwiched between two layers of AlAs/GaAs superlattices (SL) of the second kind [5]. The
main purpose of the X-electrons is to enhance the electron mobility by screening the charged
impurities near the conducting 2D layer. The parameters of the superlattices are adjusted to
set the system close to a metal-insulator transition. At this condition the barely-conducting
SL layers efficiently screen electric charges and do not contribute considerably to the overall
conductivity of the structure.

To estimate parameters relevant to the screening of the electron density $\delta n$ we consider the
superlattice as a metallic sheet placed at a distance $d$ from the conducting layer. A spatial
variation of the electron density $\delta n$ induces a variation of the voltage $V(r)$ across the layer:
Figure 6.5: Schematic diagram of a GaAs quantum well with AlAs/GaAs short-period super-lattice barriers. The two lower plots show the Fermi energy level $\epsilon_F$, the edges of the conduction band $U_\Gamma$ and $U_X$ and the density distributions of $\Gamma$ and $X$ electrons.

$$e\delta n(r) = CV(r),$$

where $C = \epsilon\epsilon_0/d$ is capacitance of the structure per unit area, $\epsilon = 12$ is lattice permittivity, and $\epsilon_0$ is permittivity of free space. A typical electric potential in the present experiments is $V = 60$ mV at $B = 2$ T. This yields $d \sim \epsilon\epsilon_0 V/(\epsilon n_0) = 39$ nm. This distance is comparable with the thickness of the superlattice, 27-80 nm.

Electric contacts connect the GaAs and the SL layers. Thus the system is considered as a set of parallel conductors. At zero magnetic field the distribution of the electric potential driving the current is the same in all layers due to the same shape of the conductors. That is to say at $B=0$ the potential difference between different layers is absent. In the poorly conducting
Figure 6.6: Dependence of the electric potential on position $y$ in the direction perpendicular to the electric current in strong magnetic field. Line $V^{2D}_H$ describes the potential in GaAs quantum well, in which strong Hall effect is developed. Line $V^{SL}_H$ describes the potential in the highly resistive superlattice layer, in which the Hall voltage is negligibly small due to the negligibly small current in the layer.

SL layers the electric current is several order of magnitude smaller than the one in the highly conducting GaAs quantum well.

The layers have a different distribution of the electric potential in a strong magnetic field, at which $\omega_c\tau^{2D}_{tr} \gg 1$ and $\omega_c\tau^{SL}_{tr} \ll 1$, where $\tau^{2D}_{tr}$ and $\tau^{SL}_{tr}$ are transport times in the GaAs and in the SL layers. The distribution is shown in Figure 6.6(a) for a small total current (linear response). At $\omega\tau^{2D}_{tr} \gg 1$ the electric field in the GaAs layer is almost perpendicular to the current due to the strong Hall effect. In contrast the very small electric current in the SL layer induces a Hall
voltage, which is negligible. The Hall voltages are shown in the Figure 6.6(a). Figure 6.6(b) presents distribution of electric charges in the structure. Electric charges are accumulated near the edges of the 2D highly conducting GaAs layer, inducing the Hall electric field $E_H$. The charges are partially screened by charges accumulated in the conducting SL layers.

Due to the small Hall voltage $E_{H}^{SL}$ and the absence of the electric current across the system the change of the electric potential $\phi^{SL}(y)$ in the SL layer is negligibly small. Below we consider the potential $\phi^{SL}$ as a constant. Due to a finite screening length $\lambda_s$ in the SL layer the charge accumulation occurs at a distance $d \sim \lambda_s$. Below we approximate the charge distribution by a charged capacitor with an effective distance $d_{eff}$ between conducting plates.

A simplified model of the observed oscillations is presented below. The model considers a long 2D Hall bar with a width $L_y$ [133, 134]. Electric current is in $x$-direction and the Hall electric field is in $y$-direction. In a long conductor the electric field $\mathbf{E} = (E_x, E_y)$ is independent on $x$, due to the uniformity of the system in $x$ direction:

$$\frac{\partial E_x}{\partial x} = \frac{\partial E_y}{\partial x} = 0 \quad (6.1)$$

For a steady current Maxwell equations yield:

$$\frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x} \quad (6.2)$$

Equation 7.4 and Equation 7.5 indicate, that the $x$ component of the electric field is the same at any location: $E_x = E = \text{const}$.

Boundary conditions and the continuity equation require that the density of the electric current in $y$ direction is zero: $J_y = 0$ and therefore,
6. Quantum Oscillations of Nonlinear Resistance

\[ E_x = \rho_{xx} J_x \quad E_y = \rho_{yx} J_x \]  

(6.3)

where \( \rho_{xx} \) and \( \rho_{yx} \) are longitudinal and Hall components of the resistivity tensor [1]. We approximate the SdH oscillations of the resistivity by a simple expression [7]:

\[ \rho_{xx}[n(y)] = \rho_D \left[ 1 - \alpha \cdot \cos \left( \frac{2\pi n}{n_0} \right) \right] \]  

(6.4)

where \( \rho_D \) is Drude resistivity and \( \alpha \) describes the amplitude of the quantum oscillations. At a SdH maximum (minimum) filling factor \( \nu = n/n_0 \) is half integer (integer).

An electrostatic evaluation of the voltage between conducting layers, shown in Figure 6.6(b), yields:

\[ \phi^{2D}(y) = \phi^{SL} + \frac{e\delta n(y)d_{eff}}{2\epsilon\epsilon_0} \]  

(6.5)

where \( \phi^{2D} \) and \( \phi^{SL} \) are electric potentials of the GaAs (2DEG) and superlattice (SL) layers, and \( \epsilon \) is permittivity of the SL layer. Expressing the electron density \( \delta n \) in terms of electric potential \( \phi^{2D} \) from Equation 7.8 and substituting the relation into Equation 6.4 one can find dependence of the resistivity on the electric potential: \( \rho_{xx}(\phi^{2D}) \).

The relation \( E_y = -d\phi^{2D}/dy \) together with Equation (7.6) yields:

\[ - \frac{d\phi^{2D}}{dy} \rho_{xx}(\phi^{2D}) = \rho_{yx}E \]  

(6.6)

Separation of the variables \( \phi^{2D} \) and \( y \) and subsequent integration of Equation 9.1 between two sides of the 2D conductor (y-direction) with corresponding electric potentials \( \phi_1 \) and \( \phi_2 \)

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[1] This reference is not visible in the image. Please provide the correct reference number or citation. (Note: The reference number is 1 for Drude resistivity, 7 for SdH oscillations, 9 for separation of variables, and 7.6 and 7.8 for electric potential equations.)
yield the following result:

\[
\rho_D \left( \phi_2 - \phi_1 - \frac{\alpha}{\beta} \left\{ \sin[\beta(\phi_2 - \phi_1)] \right\} \times \cos \left[ \beta(\phi_2 + \phi_1) + \theta_0 \right] \right) = \rho_{xy} E L_y \tag{6.7}
\]

where \( \beta = \frac{2\pi \epsilon_0 e}{(ed_{\text{eff}}) n_0} \),

\[\theta_0 = 2\pi n/n_0 - 2\beta \phi^{SL}\]

where \( L_y \) is a width of the sample. Taking into account that longitudinal voltage is \( V_{xx} = E L_x \), where \( L_x \) is a distance between the potential contacts, and the Hall voltage \( V_H = \phi_2 - \phi_1 = -\int E_y dy = -\rho_{yx} I \) (see Equation 7.6), the following relation is obtained:

\[
V_{xx} = R_D \left( I - \frac{\alpha}{\beta \rho_{xy}} \left\{ \sin (\beta \rho_{xy} I) \cdot \cos [\beta(\phi_2 + \phi_1) + \theta_0] \right\} \right), \tag{6.8}
\]

where \( R_D = L_x \rho_D / L_y \) is Drude resistance.

Equation 6.13 is simplified further for filling factors corresponding to a minimum or a maximum of SdH oscillations. In this case the voltage \( \phi^{2D}(\delta y) - \phi^{SL} \) is expected to be an asymmetric function of the relative position \( \delta y = y - y_0 \) with respect to the center of the sample \( y_0 : \phi_1 - \phi^{SL} = -(\phi_2 - \phi^{SL}) \) [135]. An example of the asymmetric distribution of the electric potential is shown in Figure 6.6(a) for small currents. In this case \( \phi_1 + \phi_2 = 2\phi^{SL} \) and the argument of the cosine in Equation 6.13 becomes independent on the electric current. For the integer (a SdH minimum) and half-integer (a SdH maximum) filling factors the differential resistance \( r_{xx} = dV_{xx}/dI \) is found to be
6. Quantum Oscillations of Nonlinear Resistance

\[ r_{xx} = R_D \left[ 1 - \alpha \cdot \cos \left( \frac{2\pi I}{I_0} \right) \cdot \cos \left( \frac{2\pi n}{n_0} \right) \right] \]  
(6.9)

\[ I_0 = \frac{e^2 \epsilon d_{eff}}{\pi h \epsilon \epsilon_0} \]  
(6.10)

Equation 7.12 demonstrates periodic oscillations of the differential resistance with both the electric current \( I \) and the inverse magnetic field (\( n_0 \sim B \)). The period of the current induced oscillations \( I_0 \) does not depend on the magnetic field and temperature in accordance with the experiment. The phase difference between oscillations starting at the SdH maxima and minima is \( \pi \), which is in agreement with Figure 6.1(d,e). The period of the oscillations shown above \( I_0 \approx 35 \, \mu\text{A} \) indicates that the screening occurs at an effective distance \( d_{eff} \approx 36 \, \text{nm} \). The distance is comparable with thicknesses of the SL layers: 27 and 76 nm.

The \( 1/B \) periodic oscillations of the resistance \( R_{xx} \) at \( I = i \cdot I_0 \) \( (i = 1, 2...) \) are in-phase with SdH oscillations \( (I = 0 \, \text{A}) \) whereas a phase of the oscillations at \( I = (i - 1/2) \cdot I_0 \) is shifted by \( \pi \) with respect to the phase of the SdH oscillations. This is in agreement with the results presented in Figure 6.4(a).

Figures 6.1(d,e), 6.4(b) show that the amplitude of the oscillations depends on the current: the oscillations are weaker at a higher current. This behavior is beyond the simplified model presented above. There are several possible mechanisms which may affect the amplitude of the quantum oscillations. One of the possibilities is the Joule heating. The heating may significantly decrease the amplitude of quantum oscillations [12, 30] reducing the magnitude of the spatial variations of the local resistivity.

The current induced oscillations are absent in samples N3 and N4. These samples have the same electron densities and mobilities as samples N1 and N2 but four times shorter quantum
scattering time $\tau_q$. We suggest that the observed significant difference in the $\tau_q$ and the absence of the oscillations is result of a less effective screening in the SL layers of the samples N3, N4. A weaker screening is expected in conducting superlattices, which are closer to the metal-insulator transition. In this case the screening of an electric charge occurs at a larger distance $\lambda_s$ due to smaller density of conducting states. Thus the effective thickness $d_{eff} \sim \lambda_s$ and therefore the period $I_0$ can be significantly larger in weaker conducting SL layers.

6.4 Sample Gating: Test of Model

The expression in Equation 9.1 indicates a linear relation between the period $I_0$ and electron density $n$. This linear relation can be understood considering that the origin of the oscillations is the redistribution of conducting electrons $\delta n$, which provides the electric voltage driving the electric current $I_{DC}$. In a strong magnetic field the voltage is mainly the Hall voltage $V_H = (B/ne) \cdot I_{DC}$ [1]. At the same electron re-distribution $\delta n$ and, thus, at the same Hall voltage $V_H$ a proportionally higher electric current $I_{DC} = (eV_H/B) \cdot n$ flows through a system with a higher electron density $n$. It makes the DC period to be proportional to the electron density $n$. Below we compare experimental results with this model.

For this study, a fifth sample (N5) is equipped with a metallic gate placed on the top of the structure at a distance $d_{gate} = 126 \text{ nm}$ from the center of the quantum well. An application of a negative voltage between the gate and the 2D electron system provides a manual variation of the electron density in situ during experiments. The screening layers are electrically connected to the highly conducting electrons inside the quantum well through the overdoped areas of electric contacts to the 2D electron system [131]. The electrostatic depopulation of the structure with the gate voltage is described below. With an application of the negative gate voltage the
Figure 6.7: Approximation of the structure by an “effective” capacitor [131]. The 2DEG is sandwiched between two screening superlattices (SL) placed at an effective distance $d_{\text{eff}}$ from the 2D electron gas. The SL layers screen electric charges $\delta n(y)$ induced by applied DC bias inside the conducting 2DEG. Placed at a distance $d_{\text{gate}}$ gate controls the averaged density $n$ across the structure. An antisymmetric application of the DC bias to current leads (not shown) produces an antisymmetric distribution of the Hall potential $\phi^{2D}(y)$ across 2DEG (y-direction). The potential $\phi^{2D}(y)$ is induced by an antisymmetric redistribution of the electron density $\delta n(y)$ with a net variation $\Delta n = \int \delta n(y) dy = 0$. In a general case the electron redistribution $\delta n(y)$ and the Hall potential $\phi^{2D}(y)$ can be quite complex [134]. At small DC biases the electric potential $\phi^{2D}$ is shown in the upper part of the plot.

The screening layer (SL) located between the gate and the 2DEG is depopulated first since this layer is closest to the gate. The depopulation of this layer corresponds to the first region in Figure 6.8(c): $0 > V_g > -0.5$ V, where the density of conducting electrons is nearly independent of the gate voltage. After complete depopulation of this screening layer, the highly conducting 2D layer (2DEG) begins to depopulate. This corresponds to the second region in Figure 6.8(c): $-0.5 > V_g > -1.8$ V, where the density of conducting electrons varies linearly in the range from $2.2$ to $8.0 \times 10^{15}$ m$^{-2}$ with the gate voltage. The remaining screening layer is fully populated in both regions, providing a partial screening of the impurity potential for all gate voltages used in the experiment.
Special care was taken to ensure that longitudinal DC bias and AC voltage were applied symmetrically to the 2D electron gas. In this case the total (integral across the sample) electron density $n$ determined by the gate voltage $V_g$ does not vary with the DC bias. To accomplish this task both current leads to the sample were simultaneously brought to equal and opposite potentials with respect to the ground. The antisymmetric application of the DC bias keeps the electric potential $\phi^{2D}$, shown in Figure 6.7, close to the ground in the middle of the sample.

Figure 6.8 demonstrates longitudinal ($R_{xx}$) and Hall ($R_{xy}$) resistances of the sample at small currents (linear response). Quantum oscillations of the longitudinal resistance $R_{xx}$ with applied gate voltage $V_g$ are shown in Figure 6.8(a). The figure demonstrates a periodic variation of the resistance with gate voltage below -0.8V. At $V_g > -0.8V$, the oscillations are aperiodic. Figure 1(b) shows Hall resistance $R_{xy} = B/ne$ of the electron system for magnetic fields up to 1.0 Tesla at several gate voltages in the range of 0.0 to -1.8 Volts. The slope of the Hall resistance $\alpha_B = 1/ne$ yields values of the electron density as a function of gate voltage shown by open squares in Figure 6.8(c). The electron density was also obtained from the dependence of the Hall resistance $R_{xy} = B/ne$ on the gate voltage $V_g$ measured at a fixed magnetic field $B = 0.89T$. In Figure 6.8(c) the density is shown by the solid line. Both methods yield mutually consistent results, which are also in accord with the rate of the electron density variations with the gate voltage evaluated from the period of the quantum oscillations shown in Figure 6.8(a) at $V_g < -0.8V$. The period is proportional to the degeneracy of Landau levels $n_0 = (m/\pi \hbar^2) \cdot \hbar \omega_c$: $\Delta V_g^0 = en_0/C = (e/C) \cdot eB/\pi \hbar$, where $C$ is a capacitance of the structure per unit square and $\omega_c$ is cyclotron frequency [7]. This period is found to be $\Delta V_g^0 = 0.196V$ yielding the capacitance $C = 773 \mu F/m^2$. Shown in Figure 6.8(c), the dependence $n$ vs $V_g$ has slope $dn/dV_g = C/e = 5m^{-2}/V$ at $V_g < -0.8V$ yielding capacitance $C = 810 \mu F/m^2$, which is comparable with the one
Figure 6.8: (a) Longitudinal resistance $R_{xx}$ shown versus gate voltage $V_g$ at $B = 1.95T$. Open circles present the experimental data, while the solid line is a spline interpolation; (b) Hall resistance $R_{xy}$ shown versus magnetic field $B$ for varying $V_g$ as labeled. (c) Dependence of the electron density on the gate at $T = 5K$. Open squares present electron density obtained from the slope of the magnetic field dependence of the Hall resistance shown in Figure (b). Solid line presents the electron density obtained from the Hall resistance measured at a fixed magnetic field $B=0.89T$ and varying gate voltage.
obtained above from Figure 6.8(a). Thus different methods provide similar results, validating our evaluation of the electron density $n$ from the transport measurements.

Figure 6.8(c) demonstrates two distinctly different regions. The first covers the range of $0 > V_g > -0.5$ Volts where the density of conducting electrons is nearly independent of the gate voltage. In this region the screening layer located between the gate and the 2DEG depopulates. The second region encompasses lower gate voltages $V_g < -0.8V$, where the electron density drops.
linearly with $V_g$, indicating the electron depopulation of the conducting quantum well with the gate voltage. The first region is the initial subject of discussion.

Figure 6.9(a) shows an overall view of the first region as a contour plot of the longitudinal differential resistance $R_{xx}$ vs the gate voltage $V_g$ and the DC bias $I_{DC}$. Figure 6.9(b) presents several horizontal slices of the contour map taken at different gate voltages as labeled. The figures demonstrate that while the central peak maximum ($I_{DC} = 0$) stays constant over this range of gate voltages, there exist two important changes in the structure of the $I_{DC}$ curves: the central peak broadens and the outlying maxima move toward higher DC biases, eventually leaving the region of study.

Interestingly enough, both changes can be explained by the electron depopulation of the screening layer. The electron depopulation should typically decrease the screening ability of the layer, increasing the electrostatic impurity potential and, thus, the electron-impurity scattering rate. Shown in Figure 6.9 widening of the central peak with the gate voltage decrease is in complete accord with this conclusion. The figure indicates a considerable increase of the quantum scattering rate ($1/\tau_q$) of 2D conducting electrons, which at low temperatures is dominated by the elastic electron-impurity collisions [131]. Recent quantitative measurements, which are done on similar samples, indicate an exponential increase of the quantum scattering rate with the gate voltage decrease [136].

The movement of the outlying maxima toward the higher DC biases at lower gate voltages is also in the agreement with the electron screening decrease. Namely in accordance with proposed model [131] the positions of the maxima are proportional to the effective screening length $d_{eff}$ in the structure (see Equation 9.1). With a decrease of the screening, the length $d_{eff}$ should increase and the maxima should move to higher biases. Thus, the first region
Figure 6.10: (a) Contour plot showing longitudinal differential resistance as a function of electron density and DC bias. (b) Vertical cut of the contour plot showing longitudinal differential resistance versus electron density corresponding to the red dotted line in (a) taken at $I_{\text{DC}} = 0$. The differential resistance oscillations demonstrate high periodicity as compared to Figure 6.8(a).

demonstrates nonlinear response which is in qualitative agreement with the proposed model of the phenomenon.

Figure 6.10(a) demonstrates the nonlinear response of two dimensional electrons in the second region, where the gate voltage changes the density of the conducting electron substantially. The figure presents a contour plot of the longitudinal differential resistance on electron density and DC bias. The electron density has been re-calculated from the gate voltage using Figure 6.8(c). One of the most notable changes is the apparent periodicity of the central peaks at zero bias, which, in contrast to Figure 6.8(a), is observed in the entire region of variations of the gate voltage. The periodicity can be seen with more clarity in Figure 6.10(b) demonstrating a
period of $0.9 \times 10^{15}\text{m}^{-2}$ for the quantum oscillations.

Figure 6.10(a) also demonstrates a tendency for the secondary peaks to move apart with increasing electron density $n$. Slices of the contour plot were taken at minima and maxima of the oscillations at $I_{DC} = 0$ and are presented in the upper and lower portions of Figure 6.11 respectively. Open circles indicate the locations of the secondary maxima that were studied.

Figure 6.12 shows the location of the differential resistance maxima plotted against corresponding electron densities. The straight solid lines represent the linear fitting for the positions of the maxima, which are done in accordance with expression given by Equation 9.1. The indicated slopes $m$ of these lines yield an averaged screening length $d_{eff} = 55\text{nm}$, which is longer the one obtained at $V_g = 0V$: $d_{eff} = 34\text{nm}$, when the screening layers are in a full action. The screening length obtained for zero gate voltage is comparable with previously obtained values for non-gated samples ($36\text{nm}$) [131]. The result indicates a weak effect of the top gate on the electrostatic screening in the system. This is in accord with the large distance $d_{gate} = 126\text{nm}$ between the gate and the conducting layer shown in Figure 6.7.

The linear drop of the DC period $I_0$ indicates a week variation of the remaining screening with the electron density at $V_g < -0.8 V$. It could be understood considering the electrostatic depopulation of the structure at $V_g < -0.8V$. In the second region a negative gate voltage decreases the density of highly conducting 2D electrons (2DEG) inside the quantum well. Due to the energy independent density of 2D electron states, the Thomas-Fermi screening length, which controls the screening at large distances, does not depend on the electron density. Thus the depopulation of the conducting quantum well should not change the screening considerably. The remaining screening layer is fully populated in both regions, providing a partial screening of the impurity potential for all gate voltages.
Figure 6.11: Longitudinal differential resistance versus DC bias shown for various electron densities labeled from the top down to the bottom. Top (bottom) panel presents data obtained at minima (maxima) of SdH oscillations at $I_{DC} = 0$. Open circles indicate the resistance maxima used for the analysis in Figure 6.12.

The Hall voltage $V_H$ controls the DC bias induced electrostatic depopulation in the 2D system and does not affect the averaged electron density $n$ depending on the gate voltage $V_g$. Moreover, the voltage $V_H$ does not affect the Landau level degeneracy $n_0$ and, therefore, does not change the periodicity determined by variations of the averaged electron density $n$ (see Equation
Thus one should expect that plotted vs $V_H$ and $n$ differential resistance oscillations will demonstrate periods which are independent of these variables. Figure 6.13 shows the contour plot of the resistance $R_{xx}$ vs $V_H$ and $n$. The periods of the differential resistance oscillations do not depend on the electron density and the applied Hall voltage.
6. Quantum Oscillations of Nonlinear Resistance

Figure 6.13: Dependence of dissipative resistance on Hall voltage $V_H$ and electron density at $B = 1.95 \, T$ and $T = 5 \, K$.

6.5 Conclusion

Oscillations of differential resistance are observed in response to both electric current and magnetic field, which is applied perpendicular to 2D electrons in GaAs quantum wells. The oscillations are periodic with the current and with the inverse magnetic field. The period of the current induced oscillations does not depend on magnetic field and temperature. The SdH oscillations are a part of the set at zero DC bias. The proposed model considers spatial variations of the electron filling factor, which are induced by applied DC bias, as the origin of the resistance
oscillations. The present experiment, thus, indicates a feasibility of the significant re-population of Landau levels by the electric current.

Nonlinear quantum oscillations of dissipative differential resistance are observed in response to DC bias applied to highly mobile two dimensional system with variable electron density $n$ placed in strong magnetic fields. The period of the oscillations $I_0$ is found to be proportional to the electron density indicating the DC bias induced electrostatic redistribution of the conducting electrons to be the dominant nonlinear mechanism of the oscillations. The density dependence of the period agrees well with the behavior of the current-induced oscillations at different magnetic fields providing strong support to the proposed origin of the phenomenon [131]. At high magnetic fields and low temperatures, the observed nonlinear mechanism is comparable with the quantal Joule heating and considerably enhances the overall nonlinear response at maxima of the quantum oscillations. Presented experiments indicate the feasibility of significant re-population of the quantum levels by applied electric current in two dimensional electron systems.
Chapter 7

Magnetic Field & Temperature

Independent Resistance Oscillations

The previous chapter discussed one kind of current-induced resistance oscillations in electron systems with a single band occupation [131]. These oscillations occur in electric fields that are significantly smaller than the one required for the current-induced Landau-Zener transitions between Landau levels [17]. The period of these current-induced oscillations is found to be independent of the magnetic field. The oscillations are considered to be a result of spatial variations of the electron filling factor (electron density $\delta n$) with the applied electric field.

In this chapter, I will discuss the observation of current-induced resistance oscillations of the dissipative resistance in electron systems with two populated subbands. Two kinds of oscillations are detected. At small magnetic fields we observed resistance oscillations with a period proportional to the magnetic field. We found that these oscillations are related to the current-induced Landau-Zener transitions between Landau levels [17, 76, 78]. At higher magnetic fields another type of the resistance oscillations emerges with a period that is independent of the
magnetic field. In the paper these oscillations are studied at high temperatures at which only MIS-oscillations are present. This chapter follows the results and discussion presented in a 2013 publication [94].

Despite a similarity between the current-induced oscillations with the B-independent period found in single subband systems [131] and the oscillations reported in this chapter, there is at least one distinct feature to distinguish the two. Namely, the oscillations in the two-subband systems occur at high temperatures $kT \gg \hbar \omega_c$ and, therefore, the total number of the electron states carrying the electric current (inside the energy interval $kT$) does not oscillate with the Fermi energy (in other words with the total electron density $n$). In this regime Shubnikov-de Haas oscillations are damped and in single subband systems the current-induced oscillations are absent [131]. Thus even if both kinds of observed oscillations have a common origin, the oscillations reported in this paper are not directly (simply) related to the spatial variations of the electron density $\delta n$ induced by the electric current. Another interesting feature is the phase of these oscillations. The oscillations appears to be quasi-periodic with respect to the applied current but with an apparent $\pi$-phase shift with respect to the zero bias. Below we present our findings and provide an interpretation of the obtained results.

7.1 Experimental Setup

The samples used in this study are high-mobility GaAs quantum wells grown by molecular beam epitaxy on semi-insulating (001) GaAs substrates. The width of the GaAs quantum well is 13 nm. Two AlAs/GaAs type-II superlattices grown on both sides of the well served as barriers, providing a high mobility of 2D electrons inside the well at a high electron density [5]. Two
samples were studied with electron density \( n_{1,2} = 8.09 \times 10^{15} \, \text{m}^{-2} \) and mobility \( \mu_1 = 121 \, \text{m}^2/\text{Vs} \) and \( \mu_2 = 73 \, \text{m}^2/\text{Vs} \).

The studied 2D electron systems are etched in the shape of a Hall bar. The width and the length of the measured part of the samples are \( d = 50 \, \mu\text{m} \) and \( L = 450 \, \mu\text{m} \). To measure the resistance we use the four point probe method. Direct electric current \( I_{DC} \) (DC bias) is applied simultaneously with 12 Hz AC excitation \( I_{AC} \) through the same current contacts (\( x \)-direction). The longitudinal AC (DC) voltage \( V_{xx}^{AC} \) (\( V_{xx}^{DC} \)) is measured between potential contacts displaced 450 \( \mu\text{m} \) along each side of the sample. The Hall voltage \( V_H \) is measured between potential contacts displaced 50 \( \mu\text{m} \) across the electric current in \( y \)-direction.
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Figure 7.2: Dependence of differential resistance $R_{xx}$ on magnetic field and averaged density of electric current $J$; (b) Dependence of the resistance on the current density $J$ at fixed magnetic field as labeled. Index $j = \pm 1, \pm 2...$ numerates Landau-Zener transitions inside lowest subband, which obey Equation 7.1. $T=5.1$ K. Sample N1.

### 7.2 Results

Figure 7.1 presents the dependence of the dissipative resistance on the magnetic field at temperature $T = 4.35$ K. At this temperature $kT > \hbar \omega_c$ and Shubnikov-de Haas oscillations are suppressed at $B < 0.5$ T. The maximums of the observed magneto-intersubband oscillations (MISO) are due to the enhancement of elastic electron scattering, which occurs when the Landau levels in two subbands are lined up with each other (state P in Figure 7.1). At this condition
Figure 7.3: Dependence of resistance $R_{xx}$ on magnetic field and current density $J$. Labels +A, +B and +C indicate different maximums induced by DC bias. $T=2.1$ K. Sample N1.

elastic electron transitions occur between the subbands, increasing the total electron scattering rate and, thus, the resistance. Minima of the oscillations occur when the Landau levels in one subband are between the levels of another subband. In this condition the elastic electron scattering between subbands is suppressed (state M in Figure 7.1) [69].

Figure 7.2(a) presents differential resistance $R_{xx}$ at different averaged density of the electric current $J = I_{DC}/(d = 50\mu m)$ and small magnetic fields. The differential resistance oscillates with the DC bias. An example of the oscillations is shown in Figure 7.2(b) at fixed magnetic field $B = 0.12$ Tesla. The dependence is a horizontal cut of the 2D plot and is shown by the dashed line in Figure 7.2(a). The position of a resistance maximum $j$ is proportional to the
magnetic field and satisfies the following relation:

\[ 2eE_jR_{c}^{(1)} = j \cdot \hbar \omega_c, \]  

where \( E_j \) is the electric field (mostly the Hall electric field in the sample) corresponding to the maximum \( j \), \( R_{c}^{(1)} \) is the cyclotron radius of electrons in the first subband (the lowest subband) and \( j = 0, 1, 2... \) is an integer. Equation 7.1 describes Landau-Zener transitions between Landau levels in the first subband [17].

At a higher resolution the data shows oscillations of the magnitude of the maximums \( j = \pm 1 \) with the magnetic field at \( B > 0.1 \, T \). The oscillations are periodic in inverse magnetic field and are in-phase with the intersubband oscillations at zero DC bias (\( j = 0 \)). Similar oscillations are observed for the minimum between \( j = 0 \) and \( j = \pm 1 \) maximums. These oscillations are shifted by phase \( \pi \) with respect to the oscillations of the maximums \( j = 0, \pm 1 \). The observed oscillations appear as an interplay between the DC bias induced Landau-Zener transitions between Landau levels inside the lowest subband and the intersubband transitions, which are periodic in inverse magnetic field \( 1/B \). At higher DC biases (\( |j| > 1 \)) the amplitude modulation with the \( 1/B \) periodicity disappears. In particular no amplitude modulation is found for \( j = \pm 2, 3 \) maximums.

Figure 7.3 presents a typical nonlinear response at a high magnetic field. The response is symmetric with respect to applied DC bias and is shown for the positive bias. There are several distinct features, which appear with the DC bias. The features are labeled in the Figure. Firstly, we discuss the evolution of the resistance with the DC bias at the minimum of a MIS oscillation (state \( M \) in Figure 7.1). When the DC bias is applied, the resistance falls down and, then, develops a shoulder labeled by symbol +A. The initial drop of the resistance is mostly due
to the quantal heating. Further increase of the DC current leads to formation of a maximum labeled by symbol +C.

When the DC bias is applied to state P (see Figure 7.1), corresponding to the maximum of a MIS oscillation, the resistance drops much more abruptly and significantly in comparison with the previous case. At low temperatures the resistance drop reaches zero and forms zero resistance state (ZDRS) \[51, 54, 55, 77\]. Further increase of the DC bias leads to the formation of a maximum labeled by symbol +B.
Figure 7.5: Evolution of differential resistance with magnetic field and current density in broad range of magnetic fields. White straight lines indicate Landau-Zener transitions which obey Equation 7.1. Upper panel presents horizontal cut through MISO maximum at $B = 0.548$ T (gray line) and cut through MISO minimum at $B = 0.532$ T (black line). Sign +(-) indicates regions of current density $J$, inside which the current-induced oscillations have 0 (180) degree phase shift with respect to MIS-oscillations at $J = 0$ A/m. Right panel presents two vertical cuts of the 2D plot taken at current densities as labeled. Magnetic field dependence at $J = 3.03$ A/m indicates strong reduction of the resistance oscillations at $B < B_c$ inside the region corresponding to Landau-Zener transitions. $T = 5$ K. Sample N2.

An evolution of the discussed features with the magnetic field is shown in Figure 7.4(a). The Figure demonstrates that the positions of all features ($\pm A$, $\pm B$, $\pm C$) are essentially independent of the magnetic field. Figure 7.4(b) presents horizontal cuts of the 2D plot through a maximum ($B = 0.418$ T) and a minimum ($B = 0.408$ T) of the inter-subband quantum oscillations.

Figure 9.1 presents an overall behavior of the quantum oscillations in a broad range of
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Figure 7.6: Positions of resistance maximums and different magnetic fields and current density. Two kind of oscillations are observed: in magnetic fields at and below $B_c$, which satisfy Equation 7.2, the maximums correspond to Landau-Zener transitions in lowest subband that obey Equation 7.1. Solid straight lines at $j = \pm 1, 2,$ and 3 represent the equation. At $B > B_c$ the resistance maximums follow the vertical solid lines representing features $\pm A, \pm B$ and $\pm C$ shown on Figure 7.3,7.4,9.1. The crossover between two kind of oscillations occurs at $B = B_c$ presented by line $j = \pm 1$. Sample N1.

magnetic fields and DC biases. The data was obtained from sample N2. The Figure shows the crossover of the intraband Landau-Zener transitions, obeying Equation 7.1, and the oscillations marked as $\pm A, \pm B, \pm C$, which have the MISO periodicity. The apparent crossover occurs near the Landau-Zener transition corresponding to $j = \pm 1$. Namely the oscillations with $1/B$ MISO
periodicity occurs at magnetic fields $B_c$ corresponding to

$$\hbar \omega_c \geq 2eE_1R_c^{(1)}.$$  \hspace{1cm} (7.2)

At smaller magnetic fields ($B < B_c$) the oscillations are significantly reduced. Two vertical cuts of the 2D plot taken at different currents are shown in the right panel of Figure 9.1. The curve taken at $J=3.03 \text{ A/m}$ shows the strong reduction of the oscillations at $B < B_c$ in a comparison with the MISO at $J=0\text{A}$. Thus the main intraband Landau-Zener transition ($j = \pm 1$) forms a boundary below which the current-induced oscillations with $1/B$ intersubband periodicity are strongly damped.

The upper panel of Figure 9.1 shows two horizontal cuts of the 2D plot. The black solid line presents the dependence of the resistance $R_{xx}$ on DC bias taken at $B = 0.532 \text{ T}$ corresponding to a minimum of MISO. The grey line presents the dependence taken at $B = 0.548 \text{ T}$ corresponding to a MISO maximum. The two curves intersect at 8 points. These intersections marks the regions at which the oscillations with MISO periodicity changes their phase by $\pi$. At the intersections the oscillations are nearly vanished. Sign "+" indicates the region between two intersections in which the oscillations are in-phase with the MISO, whereas sign "-" indicates the regions in which the oscillations are shifted by phase $\pi$ with respect to the MISO.

Figure 7.6 presents an accurate position of the resistance maximums with $1/B$ periodicity at different currents and magnetic fields for sample N1. The Figure indicates clearly that at $B = B_c$ ($j = \pm 1$) the resistance maximums follow the main Landau-Zener transition $j = \pm 1$ whereas at $B > B_c$ the maximums are nearly independent of magnetic field (features $\pm A$, $\pm B$, $\pm C$). The solid lines $j = \pm 1$ mark the boundary between the two kinds of oscillations. The lines obey Equation 7.1 at $j = \pm 1$ with the cyclotron radius $R_c^1$ corresponding to the lowest subband.
The complete theory of the current-induced oscillations of the resistance of 2D electron system with two populated subbands is not available for a general case. The case of a bilayer electron system with two closely spaced and almost equally populated electronic subbands has been studied recently [76, 78]. These results are in qualitative agreement with the present data at small magnetic fields $B < B_c$.

At high magnetic fields $B > B_c$, Figures 9.1 and 7.6 present a new kind of current-induced quantum oscillations. A striking feature of these oscillations is the independence of the position of these oscillations on magnetic field. An interesting property of these oscillations is the region in which the oscillations occur. Figures 9.1, 7.6 show that these oscillations start at the line corresponding to Landau-Zener transitions at $j = \pm 1$ in the lowest subband and propagate to higher magnetic fields. Another interesting property is an apparent quasi-periodicity of the oscillations with applied current. Namely the features $\pm A$, $\pm B$, $\pm C$ are displaced by about the same value of the electric current density from each other: $\delta J \sim 1.27 A/m$. The phase of the oscillations is shifted by $\pi$ with respect to zero DC bias. It seems strange that the MIS-oscillations ($J=0 A/m$) are not a part of this periodic set.

Figure 7.7 demonstrates the $1/B$ periodicity and the phase of the current-induced oscillations at different DC biases as labeled. The Figure indicates that oscillations at $J = 1.97 A/m$ (B+ feature) are in phase with MISO, whereas oscillations at $J = 0.575 A/m$ (A+ feature) are shifted by $\pi$ with respect to MISO. Figure 7.7 shows also the strong reduction of the oscillations at $J = 0.971 A/m$. At this current the oscillations change phase by $\pi$. The current corresponds to the intersection of two curves shown in upper panel of Figure 9.1.

The $1/B$ periodicity of the oscillations and the magnetic field independence of the electric current $I_{DC}$, inducing the oscillations at $B > B_c$, indicates a similarity of these quantum
oscillations with the current-induced quantum oscillations reported recently in Reference [131].

Below we consider a model, which is, in many respects, analogous to one described in Reference [131]. The model reproduces the main properties of the observed quantum oscillations.

### 7.3 Model and Discussion

Current-induced quantum oscillations with 1/B periodicity were recently observed in 2D electron systems with a single occupied subband [131]. The oscillations occur in a strong magnetic field at which Shubnikov-de Haas oscillations (SdH) are well developed [58]. With respect to the
electric current, the oscillations are periodic with a period that is independent of the magnetic
field. The proposed model considers the oscillations as the result of a variation of the electron
filling factor with the DC bias. In contrast to SdH oscillations, the variation appears across
the sample and is related to a spatial change of the electron density $\delta n$. If the change $\delta n$ is
comparable with the number of electron states in a Landau level $n_0 = m/(\pi \hbar^2) \cdot \hbar \omega_c$, then one
should expect a variation of the electron resistivity. The spatial variation of the resistivity leads
to oscillations of the sample resistance [131].

MIS-oscillations are due to a periodic enhancement of the inter-subband scattering, when
Landau levels in two subbands are lined up as shown in Figure 7.1. MISO have maxima in
magnetic fields $B$ satisfying the relation [69, 72, 73]: $\Delta_{12} = l \cdot \hbar \omega_c$, where $\Delta_{12} = E_2 - E_1$ is
the energy separation of the bottoms of the subbands and $l$ is an integer. In contrast to SdH
oscillations, the MIS-oscillations exist at high temperature $kT > \hbar \omega_c$ and are insensitive to
variations of the Fermi energy and/or electron density $n$ for non-interacting 2D carriers.

For interacting electron systems the situation is different. Recent direct experiment indicates
that gap $E_0$ between conducting and valence bands of 2D electron systems formed in GaAs
quantum wells depends considerably on the electron density $n$ [137]. This observation opens
a way to consider the dependence of the energy separation between two subbands $\Delta_{12}$ on the
electron density as a mechanism leading to the current-induced quantum oscillations in magnetic
fields $B > B_c$. Indeed the experiment Reference [137] demonstrated about one percent change
of the gap $E_0$ at a Hall voltage $V_H = 75 \text{ mV}$ in magnetic field $B = 0.3 \text{ T}$. The Hall voltage
is comparable with the one observed in our experiment: $V_H \approx 50 \text{ mV}$ at $B = 0.35 \text{ T}$ and $J = 4 \text{ A/m}$. At $B = 0.35 \text{ T}$ the phase of the MISO $2\pi \Delta_{12}/\hbar \omega_c \approx 2\pi \cdot 30$ requires about 3 percent
change of the inter-subband energy separation $\Delta_{12}$ to make an additional MIS-oscillation cycle.
The comparison indicates the feasibility of the proposed mechanism, taking into account that in our samples the GaAs quantum well is sandwiched between conducting layers, which enhance significantly the electron screening and, therefore, the variations of the electron density $\delta n$ with the DC bias [131].

In the model described below we assume that the DC bias-induced variation of the electron density $\delta n(r)$ changes the energy separation $\Delta_{12}(n)$ between two subbands across samples. Since relative variations of the electron density is small $\delta n/n \ll 1$, we will consider only the linear term of the dependence $\Delta_{12}(n)$:

$$\Delta_{12}(n) = \Delta_0 + \gamma \delta n(r),$$

where $\Delta_0$ is the energy separation at zero DC bias and the parameter $\gamma$ is a constant. The following consideration is qualitatively similar to the model described in detail in Reference [131]. Below we describe the main parts of the model, omitting some details.

The conducting 2D electron system in the GaAs quantum well is sandwiched between two layers of AlAs/GaAs superlattices (SL) of the second kind [5]. The parameters of the superlattices are adjusted to set the system close to a metal-insulator transition. At this condition, the barely-conducting SL layers efficiently screen electric charges but do not contribute considerably to the overall conductivity of the structure. Electric contacts connect the GaAs and the SL layers. Thus the system is considered as a set of parallel conductors. At zero magnetic field the distribution of the electric potential driving the current is the same in all layers due to the same shape of the conductors. That is to say at $B=0$ the potential difference between different layers is absent. In the poorly conducting SL layers the electric current is several order of magnitude smaller than the one in the highly conducting GaAs quantum well.
Figure 7.8: Dependence of the electric potential on position $y$ in the direction perpendicular to the electric current in strong magnetic field. The line labelled $V_{H}^{2D}$ describes the potential in the GaAs quantum well, in which a strong Hall effect is developed. The line labelled $V_{H}^{SL}$ describes the potential in the highly resistive superlattice layer, in which the Hall voltage is negligible due to the negligibly small current in the layer.

The layers have a different distribution of the electric potential in a strong magnetic field, at which $\omega_c \tau_{tr}^{2D} \gg 1$ and $\omega_c \tau_{tr}^{SL} \ll 1$, where $\tau_{tr}^{2D}$ and $\tau_{tr}^{SL}$ are transport times in the GaAs and in the SL layers. At $\omega c\tau_{tr}^{2D} \gg 1$ the electric field in the GaAs layer is almost perpendicular to the current due to the strong Hall effect. In contrast the very small electric current in the SL layer induces a negligible Hall voltage. The Hall voltages are shown in Figure 7.8 (a) for small currents (linear response). Figure 7.8(b) presents distribution of electric charges in the structure. Electric charges are accumulated near the edges of the 2D highly conducting GaAs layer, inducing the Hall electric field $E_H$. The charges are partially screened by charges accumulated in the conducting SL layers.

Due to the small Hall voltage $V_{H}^{SL}$ and the absence of the electric current across the system the change of the electric potential $\phi^{SL}(y)$ in the SL layer is negligibly small. Below we consider
the potential $\phi_{SL}$ as a constant. Due to a finite screening length $\lambda_s$ in the SL layer the charge accumulation occurs at a distance $d \sim \lambda_s$. Below we approximate the charge distribution by a charged capacitor with an effective distance $d_{eff}$ between conducting plates.

The proposed model considers a long 2D Hall bar with a width $L_y$ [133, 134]. Electric current is in $x$-direction and the Hall electric field is in $y$-direction. In a long conductor the electric field $\mathbf{E} = (E_x, E_y)$ is independent of $x$, due to the uniformity of the system in the $x$ direction:

$$\frac{\partial E_x}{\partial x} = \frac{\partial E_y}{\partial x} = 0 \quad (7.4)$$

For a steady current Maxwell equations yield:

$$\frac{\partial E_x}{\partial y} = \frac{\partial E_y}{\partial x} \quad (7.5)$$

Equation 7.4 and Equation 7.5 indicate, that the $x$ component of the electric field is the same at any location: $E_x = E =$const.

Boundary conditions and the continuity equation require that the density of the electric current in $y$ direction is zero: $J_y = 0$ and therefore,

$$E_x = \rho_{xx} J_x \quad E_y = \rho_{yx} J_x \quad (7.6)$$

where $\rho_{xx}$ and $\rho_{yx}$ are longitudinal and Hall components of the resistivity tensor [1]. We approximate the MIS-oscillations of the resistivity by a simple expression [69]:

$$\rho_{xx}[n(y)] = \rho_D \left[ 1 + A_{mis} \cdot \cos \left( \frac{2\pi \Delta_{12}}{\hbar \omega_c} \right) \right] \quad (7.7)$$
where $\rho_D$ is Drude resistivity, and $A_{mis}$ describes the amplitude of the intersubband quantum oscillations. The amplitude is different from the amplitude of Shubnikov-de Haas oscillations, since the two phenomena have a different origin \[69\].

An electrostatic evaluation of the voltage between conducting layers, shown in Figure 7.8(b), yields:

$$\phi^{2D}(y) = \phi^{SL} + \frac{e\delta n(y)d_{eff}}{2\epsilon\epsilon_0}$$  \(7.8\)

where $\phi^{2D}$ and $\phi^{SL}$ are electric potentials of the GaAs (2DEG) and superlattice (SL) layers, and $\epsilon$ is permittivity of the SL layer. Expressing the electron density $\delta n$ in terms of electric potential $\phi^{2D}$ from Equation 7.8 and substituting the relation into Equation 7.3 and then into Equation 7.7 one can find dependence of the resistivity on the electric potential: $\rho_{xx}(\phi^{2D})$.

The relation $E_y = -d\phi^{2D}/dy$ together with Equation 7.6 yields:

$$-\frac{d\phi^{2D}}{dy}\rho_{xx}(\phi^{2D}) = \rho_{yx}E$$  \(7.9\)

Separation of the variables $\phi^{2D}$ and $y$ and subsequent integration of Equation 7.9 between two sides of the 2D conductor ($y$-direction) with corresponding electric potentials $\phi_1$ and $\phi_2$ yield the following result:
\begin{equation}
\rho_D \left( \phi_2 - \phi_1 + \frac{2A_{\text{mis}}}{\beta} \left\{ \sin \left[ \frac{\beta}{2} (\phi_2 - \phi_1) \right] \right. \right.
\times \cos \left[ \frac{\beta}{2} (\phi_2 + \phi_1) + \theta_0 \right] \left. \right) = \rho_{xy} E L_y \right)
\end{equation}

\begin{equation}
\beta = 4\pi \varepsilon_0 \epsilon \gamma / (e d_{\text{eff}} \hbar \omega_c),
\end{equation}

\begin{equation}
\theta_0 = 2\pi \Delta_0 / \hbar \omega_c - \beta \phi_{SL}
\end{equation}

where $L_y$ is the width of the sample. Taking into account that longitudinal voltage is $V_{xx} = E L_x$, where $L_x$ is a distance between the potential contacts, and the Hall voltage $V_H = \phi_2 - \phi_1 = - \int E_y dy = - \rho_{yx} I$ (see Equation 7.6), the following relation is obtained:

\begin{equation}
V_{xx} = R_D \left( I - \frac{2A_{\text{mis}}}{\beta \rho_{xy}} \left\{ \sin \left( \frac{\beta \rho_{xy} I}{2} \right) \cdot \cos \left[ \frac{\beta}{2} (\phi_2 + \phi_1) + \theta_0 \right] \right\} \right),
\end{equation}

where $R_D = L_x \rho_D / L_y$ is the Drude resistance.

Equation 7.11 is simplified further for two cases corresponding to a minimum and a maximum of MIS-oscillations. In these cases the voltage $\phi^{2D}(\delta y) - \phi^{SL}$ is expected to be an asymmetric function of the relative position $\delta y = y - y_0$ with respect to the center of the sample $y_0$ (as shown in Figure 7.8) and, thus, $\phi_1 - \phi^{SL} = -(\phi_2 - \phi^{SL})$ and the argument of the cosine in Equation 7.11 becomes to be independent on the electric current. In these cases the differential resistance $r_{xx} = dV_{xx}/dI$ is found to be

\begin{equation}
\begin{aligned}
r_{xx} &= R_D \left[ 1 + A_{\text{mis}} \cdot \cos \left( 2\pi \frac{I}{I_{\text{mis}}} \right) \cdot \cos \left( \frac{2\pi \Delta_0}{\hbar \omega_c} \right) \right].
\end{aligned}
\end{equation}
where the electric current $I_{mis} = e^3 \hbar d_{eff} n / \epsilon_0 \epsilon n m \gamma$ determines the period of the DC bias-induced oscillations. The current is proportional to the effective screening length $d_{eff}$ and inversely proportional to the parameter $\gamma$ relating variations of the sub-band energy separation $\Delta_{12}$ with variations of the electron density $n$ in Equation 7.3.

Equation 7.12 demonstrates oscillations of the differential resistance with the electric current. The period of the oscillations $I_{mis}$ does not depend on the magnetic field in accordance with the experiment. A similar periodicity of the resistance is found in electron systems with a single populated sub-band [131]. In this case the period of the oscillations $I_0 = (e^3 d_{eff} n) / (\pi \hbar \epsilon \epsilon_0)$ is also independent of the magnetic field and proportional to the screening length $d_{eff}$ (see Equation 9 in Reference [131]). In both cases the observed dependence on the screening length $d_{eff}$ follows from the fact that an electron system with an effective screening (small $d_{eff}$) requires strong variations of the electron density $\delta n$ in the conducting layer to produce the same electric field (current). Thus a smaller electric current is required to depopulate a Landau level or to change the inter-band energy separation $\Delta_{12}$ in the systems with stronger screening.

The independence of the characteristic currents $I_{mis}$ and $I_0$ on the magnetic field is a direct consequence of the origin of the observed phenomena. In the case of electron systems with a single band populated the resistance oscillations are induced by a variation of electron density $\delta N_{sdH}$, which is on the order of the total number of electron states in a Landau level $n_0$: $\delta N_{sdH} \approx n_0 = eB / \pi h \sim B$ and, thus, is proportional to the magnetic field. The variation of electron density $\delta N_{sdH}$ produces Hall voltage $V_H$, which, due to the principle of the linear superposition of electric fields, is proportional to the density variation: $V_H = F[\delta N_{sdH}]$, where $F[x]$ is a linear functional, $A \cdot F[x] = F[Ax]$. Characteristic electric current $I_0$ obeys $I_0 = V_H / \rho_{xy} = (1 / \rho_{xy}) \cdot F[\delta N_{sdH}] = F[\delta N_{sdH} / \rho_{xy}]$. Due to the independence of the argument
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\( (\delta N_{SdH} \sim B)/(\rho_{xy} \sim B) \) on the magnetic field \( B \) the current \( I_0 \) does not depend on the magnetic field either.

In electron systems with two populated subbands the resistance oscillations are induced by variations of the inter-subband separation \( \Delta_{12} \) on the order of \( \hbar \omega_c: \gamma \delta N_{mis} = \hbar \omega_c \sim B \). We note that in this case the characteristic scale of the electron density variations is also proportional to the magnetic field. Arguments, which are similar to one used above, yield \( I_{mis} = F[(\hbar \omega_c)/(\gamma \cdot \rho_{xy})] \) and, as in the previous case, the characteristic electric current does not depend on the magnetic field.

The Equation 7.12 indicates that the amplitude of the MIS-oscillations is strongly modulated by the DC bias. In particular at \( I = I_{mis}/4 \) the amplitude is zero. At this node the 1/B periodic oscillations change phase by \( \pi \). The strong amplitude modulation with the DC bias and the \( \pi \) phase shift at a node agree with the experiment.

Following from Equation 7.12 the positions of the nodes and anti-nodes of the oscillations with respect to the electric current \( I \) do not agree with the experiment. In accordance with Equation 7.12 the nodes occurs at the averaged density of the electric current \( (J = I/L_y) \)

\[
J_k = \frac{I_{mis}}{4L_y} \cdot k \quad (7.13)
\]

\[
k = 2i - 1,
\]

\[
i = 1,2,3...
\]

where \( k \) is a node index. Upper panel of Figure 9.1 shows nodes at 0.22, 0.93, 2.41 and 3.91 A/m. Thus the relative positions of the nodes observed in the experiment do not follow the
node positions (or index k) in Equation 7.14a. Below we show that the disagreement is reduced significantly taking into account the Joule heating.

The model discussed above does not take into account the DC heating of the 2D electrons. The Joule heating in systems with a discrete spectrum (quantal heating) has a peculiar form providing strong impact on the electron transport [12]. In electron systems with two subbands occupied the quantal heating inverts the MIS-oscillations [29, 35]. A quantitative account of the heating will be done in this paper in a simplified form, taking into account an analytical approximation of the heating which is valid for two subbands with equal electron population. As shown below, the approach yields the positions of the nodes which agree with the experiment.

The expression for the resistivity of 2D electron systems with two equally populated subbands in crossed electric and quantizing magnetic field reads [35]

\[
\rho_{xx} = \rho_D \left\{ 1 + e^{\frac{2\pi}{\omega_c \tau_q}} \frac{1 - 3Q}{1 + Q} \left[ 1 + \cos \left( \frac{2\pi \Delta_{12}}{\hbar \omega_c} \right) \right] \right\}, \quad (7.14a)
\]

\[
Q = \frac{2\pi^3 J^2}{e^2 \mu c^2} \cdot \frac{\tau_{in}}{\tau_{tr}}, \quad (7.14b)
\]

where \( \tau_q \) is quantum scattering time, \( \tau_{in} \) and \( \tau_{tr} \) are inelastic and transport scattering times. To account for the heating we replace Equation 7.7 by Equation 7.14b and evaluate differential Equation 7.9 numerically with fitting parameters approximating the experimental data. Due to a quite rough approximation of the heating, the fitting parameters may deviate significantly from actual physical values. To find the fitting parameter corresponding to the inelastic scattering time we use the fact that the second term of Equation 7.14b is zero at \( Q = 1/3 \) [35]. Assuming that at a small DC bias and low temperatures the quantal heating dominates [12, 35], we related the first node shown in Figure 9.1 at \( J = 0.22 \ A/m \) to the condition \( Q = 1/3 \). This yields \( \tau_{in} = 1.8 \ ns \) at \( B = 0.53 \ T \). Using this value we solved Equation 7.9 numerically. The
result is shown in Figure 7.9 (a). At small DC bias \( J \approx 0.17 \, \text{A/m} \) the Figure demonstrates the oscillation node, induced by the heating with a small contribution from the variation of the band separation \( \Delta_{12} \). Other nodes occur at considerably higher DC biases and are shifted with respect to the nodes shown in Figure 7.9(b), which obtained by the numerical evaluation, ignoring the quantal heating \((Q = 0)\).

At \( Q > 1/3 \) the heating not only shift the nodes but also inverts the oscillations induced by the variation of the band separation. Namely, shown in Figure 7.9(a) the maximum at \( J = 1.75 \, \text{A/m} \) is a result of the DC bias-induced evolution of the MISO maximum at \( J=0\text{A/m} \). Without the heating the MISO maximum evolves into a minimum at \( J = 1.65 \, \text{A/m} \) shown in Figure 7.9(b). Thus the heating inverts minimums to maximums and vice versa. The inversion is directly related to the sign change of the second term in Equation 7.14b at \( Q = 1/3 \).

The heating and the variation of the band separation affect differently the maximums and minimums of MIS-oscillations. Conversely, quantal heating decreases the resistance at any magnetic field. A variation of the resistance, induced by the change of the band separation, depends on the magnetic field. At a maximum (state P in Figure 7.1), a variation of \( \Delta_{12} \) destroys the level alignment decreasing the inter-band scattering and, thus, the resistance. At a minimum (state M in Figure 7.1), a variation of \( \Delta_{12} \) improves the level alignment and increases the inter-band scattering and the resistance. Thus at a MISO maximum both the heating and the variations of the band separation decreases the resistance whereas at a MISO minimum two mechanisms work against each other. As a result the drop of the resistance at a MISO maximum is considerably stronger than the one at a MISO minimum. In fact, the shoulder (feature +A in Figure 7.3) is a result of the competition between two mechanisms at a MISO minimum whereas ZDRS states, developed from MISO maximums, is a strong indication of the
Figure 7.9: (a) Numerical simulation of the dependence of differential resistance on DC bias at $B=0.53$ T. Fitting parameters used in the numerical simulation: $\tau_m=1.8$ ns, $\tau_q=2.5$ ps and $\tau_r=45$ ps; electron density $n=8.09\times10^{15}$ 1/m$^2$; effective screening length $d_{ef}=30$ nm; parameter $\gamma=1.10^{-37}$ Jm$^2$ (see Equation 7.3). (b) Numerical simulation of the dependence of differential resistance on DC bias with the same fitting parameters as in (a) but without DC heating: $\tau_m=0$ ns (Q=0). Filled (open) circles present evolution of a MISO maximum (minimum) with the DC bias joint decrease of the resistance due to both mechanisms. The behavior is reproduced in the proposed model. Indeed, Figure 7.9(a) shows that the initial drop of the MISO maximum is considerably stronger than the decrease of the MISO minimum with the DC bias.

Figure 7.10 presents a comparison of the positions of oscillation nodes, obtained in the model, with the experiment. For the purpose of a comparison, the node positions are plotted versus the index $k$, which is defined in Equation 7.14a. Without the heating, nodes of oscillations obey Equation 7.14a. Filled triangles demonstrate this behavior. When the heating is on (filled squares), the first node ($k=1$) is due mostly to the heating. The following nodes ($k=3, 5, \text{and } 7$) are due mostly to the variation of the band separation. As shown in the Figure the positions
of the nodes correlate well with the experimental values (open circles) taken from the upper panel of Figure 9.1.

The quantal heating produces an additional node of the DC bias-induced oscillations. It changes the systematic placement of the node positions described by Equation 7.14a. In the case of a strong quantal heating (as in Figure 7.9) the additional node occurs at the very beginning of the resistance evolution. Expected from Equation 7.14a node counting can be largely restored by a reduction of the node index by two, which is the difference between consecutive indexes \( k \) in Equation 7.14a. The corresponding transformation is shown in Figure 7.10: the dashed line is the shift by two units to the right of the solid line representing index \( k \) in Equation 7.14a.
7. Conclusion

Quantum oscillations of nonlinear resistance, which occur in response to electric current and magnetic field applied perpendicular to GaAs quantum wells with two populated subbands, are investigated. At small magnetic fields, the current-induced oscillations are found to be related to Landau-Zener transitions between Landau levels inside the lowest subband. The period of these oscillations is proportional to the magnetic field. At high magnetic fields, a different kind of quantum oscillations are observed. With respect to the DC bias, these resistance oscillations are quasi-periodic with a period that is independent of the magnetic field. At a fixed electric current, the oscillations are periodic in inverse magnetic field. The period is independent of the DC bias. The proposed model considers these oscillations as a result of a joint effect between the Joule heating in the systems with discrete spectrum and the spatial variations of the energy separation between two subbands, which is induced by the electric current. The obtained results indicate the feasibility of considerable modification of the electron spectrum by applied electric current in two dimensional electron systems.
Chapter 8

Bulk vs. Edge Contributions to Nonlinear Resistance

As discussed in Chapter 2 and 4, a fascinating form of Joule heating has been reported through an observation of a strong decrease in sample resistance with applied DC bias. This quantal heating is a bulk phenomenon as explained in Section 2.7. However, a strong nonlinear response of two dimensional electrons was observed in a geometry in which a nonlocal electron transport – associated with the propagation of the edge states or/and skipping orbits [82–89] – may play the dominant role [90]. The observation of the nonlocal nonlinear response has raised a question regarding the possibility of a significant contribution from the edge states and/or skipping orbits to the nonlinear transport of 2D electrons observed in the Hall bar geometry [32, 52–54, 76, 78, 79, 91–95] and, thus, the applicability of the currently accepted theoretical approach [43] to the observed nonlinearity. I should stress that in the Hall bar geometry a separation between the local and the nonlocal contributions to the electron conductance is a challenging problem.
A convenient geometry in which the nonlocal contributions of the edge states and/or skipping orbits to the electron conductance can be significantly suppressed is the Corbino geometry. In this geometry the edge states are localized near the edges of the inner and outer contacts and do not propagate through the Corbino ring. Thus experiments in the Corbino geometry provide the information on the bulk nonlinear response. A comparison of the nonlinear response of Corbino discs with the response of Hall bar samples may shed a light on the amount of the nonlocal contributions to the nonlinear resistance in the Hall bar geometry. Below I present the investigation of the nonlinear response of Corbino discs and compare it with experiments on Hall bar samples, following published results and discussion [138].

8.1 Experimental Setup

This study focuses on Corbino discs with inner radius \( r_1 = 0.9 \text{ mm} \) and outer radius \( r_2 = 1 \text{ mm} \). The Corbino discs were fabricated from selectively doped heterojunction GaAs/AlAs. The heterojunction was a single GaAs quantum well sandwiched between AlAs/GaAs superlattice barriers [5]. The width of the quantum well was 13 nm. The structure was grown by molecular beam epitaxy on (100) GaAs substrate. AuGe eutectic was used to provide electric contacts to the 2D electron gas. The contacts were made by thermal diffusion after the AuGe deposition and photo-lithography. Differential conductance \( g_{12} = I_{AC}/V_{AC} \) were measured using AC current \( I_{AC} \) with frequency from 10 Hz to 1 kHz. An AC voltage \( V_{AC} \) was applied between contacts 1 and 2, shown on the insert to Figure 8.1. The amplitude of the voltage was kept fixed and was below 1 mV during experiments. The measurements were taken at temperatures \( T = 1.6K \) and \( T = 4.2K \) in magnetic fields \( B < 1 \text{ T} \). Three samples with electron density \( n = 8 \times 10^{15} \).
8. Bulk vs. Edge Contributions to Nonlinear Resistance

Figure 8.1: (Color online.) Dependence of conductance $g_{12}$ of 2D electron Corbino disc on magnetic field at temperature $T = 1.6\,K$ at different DC electric fields as labeled. Insert shows the electric scheme for measurements of differential conductance $g_{12}$.

$m^{-2}$ and mobility $\mu = 150\, m^2/Vs$ at $T = 4.2\,K$ were studied and have demonstrated the same results. The paper presents data for one of these samples.

8.2 Results and Discussion

Figures 8.1 and 8.2 present dependence of the differential conductance $g_{12}(B)$ of 2D electrons in the Corbino disc on the magnetic field $B$ taken at $T = 1.6\,K$ at different electric fields as labeled. For the studied samples the width of the conducting o-ring was much less than the averaged radius of the o-ring: $\Delta r = r_2 - r_1 \ll (r_2 + r_1)/2$. Due to this property the DC electric...
Figure 8.2: Dependence of conductance $g_{12}$ of 2D electron Corbino disc on magnetic field at temperature $T = 1.6\,\text{K}$ at different DC electric fields as labeled. Arrows indicate the positions of the maximum $B_l$ at $l=1$ in different electric fields. Insert presents the dependence of $B_{l}^2$ on DC electric field $E_{DC}$. The solid line corresponds to relation $\gamma eE_{DC}R_c = \hbar \omega_c$. At $\gamma = 2$ the electron effective mass $m_e \approx 0.070$, which is in accord with other experiments [139].

Field between contacts was nearly independent of the radius $r$ and equal to $E_{DC} = V_{12}/\Delta r$. At $E_{DC}=0$ the magnetoconductance $g_{12}(B)$ demonstrates Shubnikov-de Haas (SdH) oscillations in magnetic fields exceeding $0.3T$ as shown in Figure 8.1. An application of the electric field $E_{DC} = 250 \,\text{V/m}$ decreases the amplitude of the quantum oscillations significantly and at strong magnetic fields the conductance of the structure approaches values that are very close to zero. Shown in Figure 8.2, further increase of the DC electric field produces additional peaks in the dependence $g_{12}(B)$, which are labeled by arrows. As shown recently, these maximums result
Figure 8.3: (a) Dependence of differential conductance $g_{12}$ on DC electric field $E_{DC}$ at different magnetic fields as labeled. Arrow indicates maximum corresponding to Zener transition at $l=1$. $T = 1.6K$; (b) Dependence of electric current $I_{DC}$ on DC voltage $V_{DC}$ at temperature $T = 1.6K$ in different magnetic fields as labeled. Placed in the upper left corner insert shows suggested N-shaped dependence $J_{DC}(E_{DC})$ indicating two electric fields $E_1$ and $E_2$ corresponding to the same value $J_{DC}$. Placed in the lower right corner insert shows possible distribution of the electric field corresponding to the electron state with zero differential conductance in a 2D Corbino disc.

from Zener tunneling between Landau levels, which is induced by applied electric field $E_{DC}$ [80]. Positions of the maximums obey the following relation: $\gamma R_e eE_{DC} = l \hbar \omega_c$, shown in the insert to Figure 8.2.

Figure 8.3 presents the dependencies of $g_{12}(E_{DC})$ for different magnetic fields as labeled and the temperature $T = 1.6K$. At magnetic field $B = 0.261T$ the initial drop of the differential conductance with the $E_{DC}$ is due to the intra-level quantal heating [12, 43]. The increase of the differential conductance at higher electric field is related to inter-level electron transitions [17, 140]. In Figure 8.3(a) the maximum marked by the arrow corresponds to Zener tunneling between Landau levels at $l=1$. At higher magnetic field $B = 0.847T$ the differential conductance demonstrates similar behavior at small electric fields but at higher DC biases the conductance
retains values near zero $g_{12} \approx 0$ in a broad range of the electric fields $E_{DC}$. This is the Zero Differential Conductance State (ZDCS). Figure 8.4(a) reveals that the transition into the ZDC state is associated with one or few sharp ”spikes” of the differential conductance into the region with negative values. As shown in the figure the state with $g_{12}=0$ does not occur at $T = 4.2K$.

Figure 8.3(b) presents $V-I$ dependencies of the 2D Corbino disc at temperature $T = 1.6K$ for two different magnetic fields as labeled. The figure shows that when the 2D electron systems enters the state with zero differential conductance, the electric current $I_{DC}$ saturates and becomes independent of the electric field $E_{DC}$. A comparison between the dependencies $g_{12}(E_{DC})$ and $I_{DC}(V_{DC})$ taken at temperature $T = 1.6K$ and magnetic field $B = 0.847T$ indicates that the electric current $I_{DC}$ reaches a saturation value $I_s$ at electric field $E_{DC} > E_{th}$.

Similar to the case of the Hall bar geometry [51] we consider that in the studied Corbino discs, the $g_{12} = 0$ state occurs due to a local instability of the electric field $E_{DC}$ [56]. The dominant nonlinear mechanism, leading to the instability, is a peculiar Joule heating (quantal heating), which occurs in systems with a discrete spectrum [12, 43]. The instability develops at the conditions of a negative differential conductivity corresponding to the negative slope of the N-shaped $V-I$ dependence shown in the insert to Figure 8.3(b). Shown in Figure 8.4(a) regions with the negative differential resistance further supports this interpretation. In the case of the N-shaped $V-I$ dependence, a spatially uniform distribution of the electric field is not stable and typically should evolve into a structure containing domains of a weak $E_1$ and a strong $E_2$ as shown in the insert to Figure 8.3(b) [141]. At these conditions both moving and static domains may occur. In the first case in a conductor with a fixed voltage applied there are oscillations of the electric current. This is known as Gunn effect [142]. In the case of static domains the constant electric current saturates with the applied voltage [143]. There is
a similarity between nonlinear transport in Gunn diodes \[142\] and in the 2D electron systems presented in this paper. We note however that despite the similarity the nonlinear mechanisms leading to the local instability of the electric field \(E_{DC}\) are different in these two systems.

### 8.3 Conclusion

The presented nonlinear response of Corbino discs is obtained in the regime where the edge states and/or skipping orbits are localized near the contacts and do not participate in the electron transport through the systems. It is important to compare the obtained results with the nonlinear response of Hall bar samples, where the electron transport near the edge may provide significant contributions \[90\]. Below we compare the threshold electric field \(E_{th} = 96\ V/m\) corresponding to the transition into the state with zero differential conductance shown in Figure 8.4a with the Hall electric field corresponding to the transition into the state with zero
differential resistance (ZDRS) in a Hall bar sample fabricated from the same quantum well. Figure 8.4(b) presents the dependence of the differential resistance of the Hall bar sample on the applied DC bias $I_{DC}$ taken at the same experimental conditions. The transition to the ZDR state occurs at Hall electric field $E^H_{th} = 118 \, V/m$, corresponding to the threshold DC bias $I_{DC} = 9.3 \, \mu A$. The comparison demonstrates quite similar values of the electric fields, at which both ZDRS and ZDCS transitions occur. Furthermore we note that samples with comparable physical parameters demonstrate comparable threshold fields. In particular shown in Figure 2a of Reference[51] the threshold electric current $I_{th} = 6.7 \, \mu A$ corresponds to the ZDRS transition obtained at $B = 0.784T$, $T = 1.94K$ on sample N1 with electron density $n = 8.2 \times 10^{15} \, m^{-2}$ and mobility $\mu = 85 \, m^2/Vs$. Taking into account that the Hall resistance of the sample N1 at $B = 0.784T$ is $R_H = B/ne = 597 \, \Omega$, one can evaluate the Hall electric field $E_H$ corresponding to the current $I_{th}$: $E^H_{th} = R_H \cdot I_{th}/W = 80 \, V/m$, where $W = 50 \, \mu m$ is the width of the sample N1.

The sample demonstrates similar value of the threshold electric field. Thus in the studied Hall bar samples the edge states and/or skipping orbits do not provide a considerable contribution to the nonlinear response and, thus, the accepted model of the nonlinearity [12, 43] holds for these systems.

In summary the paper presents experimental study of the effect of DC electric field on the conductance of Corbino discs of highly mobile two dimensional electrons placed in crossed electric and quantizing magnetic fields. Experimental data shows that at low temperature the differential conductance of the Corbino discs reaches zero value in a broad range of applied DC voltages. It indicates the presence of the zero differential conductance state in which the electric current does not depend on the voltage. The results are in accord with the data obtained in the Hall bar geometry indicating that the nonlinearity leading to the ZDC and ZDR states occurs inside 2D electron systems. It provides significant support for the model of the local nonlinearity
based on the quantal Joule heating in systems with discrete or modulated spectrum. Finally both the zero differential conductance and zero differential resistance states are observed in systems with a modest electron mobility broadening significantly the class of electron systems in which the quantal heating is essential.
Chapter 9

Dynamics of Quantal Heating

As explained in Chapter 4, there remains a controversy around the temperature dependence of the inelastic scattering rate, $1/\tau_{in}$. Measurements of the effect of Joule heating on the amplitude of Shubnikov-de Haas oscillations have concluded that $1/\tau_{in}$ is proportional to temperature. However, these findings assume that the distribution of overheated electrons can be described by the Fermi-Dirac distribution, $f_T(\epsilon)$. Omitting this assumption, the DC overheated electron distribution was found to be significantly different from the Fermi-Dirac form, leading to the temperature dependence of $1/\tau_{in}$ which is proportional to the square or cube of temperature [12].

In this chapter, I present an experimental method which accesses the temporal evolution of electron transport under Joule heating. The method provides a direct measurement of the inelastic relaxation time $\tau_{in}$. At high temperatures the time is found to be in a good quantitative agreement with the inelastic time $\tau_{in}^{DC}$ obtained in DC experiments on quantal heating [12]. At low temperatures a disagreement between these two times is observed. The temperature dependence of the inelastic time is found to be significantly different from the one obtained by
9. Dynamics of Quantal Heating

SdH method [97, 103, 108]. The discussion and results below follow a publication that is under review during the writing of this study.

9.1 Experimental Setup

The quantum wells used in this study were etched into the shape of a Corbino disc with inner radius \( r_1 = 0.9 \text{ mm} \) and outer radius \( r_2 = 1 \text{ mm} \). Samples are fabricated from a selectively doped single GaAs quantum well sandwiched between AlAs/GaAs superlattice barriers. The width of the well was 13 \text{ nm}. The structure was grown by molecular beam epitaxy on a (100) GaAs substrate. AuGe eutectic was used to provide electric contacts to the 2D electron gas. The 2D electron system with electron density \( n = 8 \times 10^{15} \text{ m}^{-2} \) and mobility \( \mu = 112 \text{ m}^2/\text{Vs} \) at \( T = 4.8 \text{K} \) was studied at different temperatures from 2.4K to 6K in magnetic fields up to 1T.

Figure 9.1 shows the experimental setup. Two microwave sources supply the radiation to the sample through a semi-rigid coax at two different frequencies \( (f_1, f_2) \). The interference between these sources forms microwave radiation with amplitude modulation at the difference (beat) frequency \( f = f_1 - f_2 \). The modulated microwave induces oscillations of Joule heating and, thus, the sample resistance \( \delta R_f \) at the frequency \( f \). Application of a DC current \( I_{DC} \) to the structure produces voltage oscillations \( \delta V_f = \delta R_f I_{DC} \), which propagate back to a microwave analyzer through the same coax. The analyzer detects the voltage oscillations at frequency \( f \) (f-signal). In addition, the setup contains a bias-tee which provides measurements in the DC domain. These measurements are essential for a calibration of the microwave setup.

In experiments frequency \( f_1 = 8 \text{ GHz} \) was fixed while frequency \( f_2 \) was scanned from 5.5 to 7.999 GHz. To take into account variations of the microwave power \( P_2 \) delivered to the sample in the course of the frequency scan, a DC measurement of the resistance variation induced by
9. Dynamics of Quantal Heating

Figure 9.1: Experimental setup for the difference frequency method. Two microwave sources (SRC) at frequencies $f_1$ and $f_2$ are sent to the sample through a broadband directional couplers. The reflected signal is measured by the microwave analyzer at the difference frequency of the two sources $f = f_2 - f_1$. The incorporated RC circuit ($R=50$ Ohm, $C=47$ pF) provides broadband matching. A low pass filter (LPF) blocks the high frequency signals ($f_1, f_2$) from the analyzer. Bias current ($I_{DC}$) as well as low frequency lock-in measurements ($I_{AC}$) are incorporated into the same universal measurement line through a bias-tee.

the same applied microwave power $P_2$ is done. At a small applied power the induced resistance variation is proportional to $P_2$, thereby providing the power calibration. A similar calibration is done for the receiver channel at frequency $f$ and is based on the reciprocal property of the microwave setup.

9.2 Results

Figure 9.2 presents the magnetic field dependence of the resistance of the sample, $R$, with neither DC bias nor microwaves applied (thin solid line). As expected in the Corbino geometry the resistance shows the classical parabolic increase with the magnetic field $B$. The thick solid line presents the nonlinear response of the sample ($f$-signal) measured at difference frequency $f = 1$ MHz. The nonlinear response is very weak at small magnetic fields $B < 0.1 T$. At these fields the Landau level separation $\hbar \omega_c$ is much smaller the level width $\Gamma$ and both the quantization of the electron spectrum and quantal heating are absent [12]. Above $0.1 T$ the Landau quantization
occurs and quantal heating starts to grow, reaching maximums at about 0.3 and 0.45T. At higher magnetic fields the $f$-signal drops due to a decrease of the cyclotron radius of electron orbits leading to significant reduction of the spatial and, thus, spectral diffusions [43]. At $T = 4.8K$ and $B > 0.5T$, SdH oscillations are visible in both the resistance and the $f$-signal. The frequency dependence of the $f$-signal was studied at magnetic field $B=0.333$ T corresponding to a maximum of the sample conductivity at low temperatures (not shown).

Figure 9.3 presents the dependence of the $f$-signal and differential resistance on the DC voltage $V_{DC}$ at different frequencies $f$ as labeled. At small DC biases the $f$-signal is proportional to $V_{DC}$ while the differential resistance $r_{xx} \sim V_{DC}^2$. These data agree with the relation $j = \sigma_0 E + \alpha E^3$ between the current density $j$ and the electric field $E$, which is expected for small...
fields. Here $\sigma_0$ is the ohmic conductivity (linear response). In this perturbative regime, the nonlinear current density $j_\omega (\sim f$-signal) at angular frequency $\omega = 2\pi f$ should be proportional to applied DC ($E_0$) and MW ($E_1, E_2$) electric fields: $j_\omega = 3\alpha E_0 E_1 E_2$. The observed microwave power dependence (not shown) of the $f$-signal is in complete agreement with the expected behavior at small microwave power. The $f$-signal demonstrates an additional interesting features at higher DC biases, which were beyond the scope of the present work.

The frequency dependence of the nonlinear response can be understood from an analysis of the spectral diffusion equation for the electron distribution function $f(\epsilon)$ [12, 43]:

$$
- \frac{\partial f(\epsilon)}{\partial t} + E^2 \frac{\sigma_D}{\nu(\epsilon)} \partial_\epsilon \left[ \tilde{\nu}^2(\epsilon) \partial_\epsilon f(\epsilon) \right] = \frac{f(\epsilon) - f_T(\epsilon)}{\tau_m} 
$$

(9.1)

Here, $\sigma_D(B)$ is the Drude conductivity in a magnetic field $B$, $\tilde{\nu} = \nu/\nu_0$ is ratio of the density of electron states (DOS) $\nu(\epsilon)$ to the DOS at zero magnetic field $\nu_0$ and $f_T$ represents the Fermi-Dirac distribution at a temperature $T$. Below we consider the case of a low difference frequency $\omega = \omega_1 - \omega_2 \ll \omega_1, \omega_2$ corresponding to the experiments ($\omega_i = 2\pi f_i$). At small electric field $E(t) = E_0 + E_1 \exp(\nu \omega_1 t) + E_2 \exp(\nu \omega_2 t)$ the distribution function can be written as $f(\epsilon) = f_T + \delta f_\omega$, where the oscillating distribution $\delta f_\omega \sim E_1 E_2 \exp[i(\omega_1 - \omega_2)t]$ is the leading contribution to the $f$-signal. A substitution of this function into Equation 9.1 yields the following solution for the electron distribution oscillating at difference frequency $\omega$:

$$
\delta f_\omega(\epsilon) = \frac{2E_1 E_2 \exp(i\omega t)}{i\omega + 1/\tau_m} \cdot \frac{\sigma_D}{\nu(\epsilon)} \partial_\epsilon \left[ \tilde{\nu}^2(\epsilon) \partial_\epsilon f_T(\epsilon) \right] 
$$

(9.2)
The oscillating electron distribution results in oscillations of the electric current density at frequency $\omega$:

$$j_\omega = E_0 \int \sigma_\epsilon [-\partial_\epsilon (\delta f_\omega)]d\epsilon = \frac{2E_0 E_1 E_2 exp(i\omega t)}{i\omega + 1/\tau_{in}} \cdot \Sigma$$  \hspace{1cm} (9.3)

Here $\sigma_\epsilon$ is the conductivity at energy $\epsilon$ and $\Sigma = -\sigma_D \int \sigma_\epsilon \partial_\epsilon [\partial_\epsilon (\nu^2 \partial_\epsilon f_T)/\nu]d\epsilon$. Equation 9.3 indicates that at high difference frequency $\omega \gg 1/\tau_{in}$ the $f$-signal is inversely proportional to frequency. In this regime, microwave radiation is ”on” for a short time $\Delta t \sim 1/\omega$, which is not enough to considerably change the electron distribution.

Figure 9.4 presents the frequency dependence of the $f$-signal at different temperatures.
as labeled. The observed $f$-signal is nearly frequency independent at low frequencies and is inversely proportional to the frequency in the high frequency limit. The solid lines represent the frequency dependence expected from Equation 9.3: $j_\omega = A/|1 + i\omega \tau_{in}|$ with amplitude $A$ and time $\tau_{in}$ as fitting parameters. The figure indicates a good agreement between the data and the frequency dependence described by Equation 9.3. The insert to the figure presents the temperature dependence of the inelastic scattering time $\tau_{in}$ obtained from the fit. The high temperature behavior of the inelastic time is consistent with $1/T^2$ decrease indicating the dominant contribution of electron-electron interactions to the inelastic electron relaxation. At low temperatures a deviation from the $1/T^2$ behavior is found, indicating a suppression of the $e$-$e$ contribution. The suppression is expected at low temperatures, when $kT < \hbar \omega_c$. At this condition the $e$-$e$ scattering is ineffective, since the scattering conserves the total electron energy.

### 9.3 Discussion

Below we compare the inelastic time $\tau_{in}$ with the time $\tau^{DC}_{in}$ obtained in the DC domain [12]. Figure 9.5(a) presents the dependence of the normalized conductivity [144] of the sample $\sigma/\sigma_D$ on the applied electric field $E \approx V_{DC}/(r_2 - r_1)$ and numerical simulations of the DC response [12], yielding the inelastic relaxation time $\tau^{DC}_{in}$. Figure 9.5(b) shows temperature dependences of the time $\tau^{DC}_{in}$ and the inelastic time obtained from the dynamics of the nonlinear response ($f$-signal). At high temperatures both times are close to each other. At lower temperatures there is a considerable difference between these two times. The DC-domain inelastic time $\tau^{DC}_{in}(T)$ follows $1/T^3$ decrease, while the time $\tau_{in}$ is mostly proportional to $1/T^2$ with a tendency to $1/T^3$ at low temperatures. The observed difference may be related to effects of an electron redistribution,
induced by the DC bias, which are relevant in the DC domain at low temperatures \([92, 132]\). The redistribution mechanism is different from quantal heating and may not be active in the microwave experiments.

The linear temperature dependence of the inelastic scattering rate \(1/\tau_{SdH}^{in}\) observed in SdH experiments at liquid \(^4\)He temperatures \([97, 103, 108]\) has been attributed to a crossover \([103]\) between Bloch-Grunaizen (BG) and equipartition regimes \([107, 121]\). The dependence is found to be in an agreement with the theory \([145–148]\) and direct measurements of the inelastic rate at zero magnetic field\([149]\). In contrast, thermopower measurements of the electron temperature
Figure 9.5: (a) Solid lines present the dependence of the normalized conductivity $\sigma/\sigma_D$ on electric field $E$ at different temperatures: 5.5, 4.8, 4.1, 3.6, 3.1 and 2.4K from top to bottom. Symbols present simulation of the DC conductivity based on the numerical solution of Equation 9.1 using Gaussian approximation for the electron density of states [12]; (b) Temperature dependences of the inelastic scattering time obtained in high frequency experiments ($f$-signal shown in Figure 9.4) and from the response in the DC-domain shown in (a). $B = 0.333T$.

shows a strong $T^3$ increase of the inelastic relaxation rate at zero magnetic field, coexisting with the $1/\tau_{in}^{SdH} \sim T$ on the same sample and at the same temperatures[108]. The authors have attributed the discrepancy to a difference in the electron-phonon scattering rate at zero and a strong magnetic fields. Our direct measurements as well as the results obtained in DC domain [12] indicate the presence of both $T^2$ and $T^3$ terms in the inelastic relaxation rate in quantizing magnetic fields. Within an order of magnitude the cubic term agrees with the one seen in the thermopower experiments [108]. We attribute the $T^2$ term to $e-e$ scattering [43] and the $T^3$ dependence to the electron-phonon scattering due to unscreened deformation potential
in BG regime [148, 150]. The considerable disagreement with the SdH results indicates, thus, an incompleteness of the accepted interpretation of DC biased SdH oscillations [97, 103, 108].

9.4 Conclusion

In conclusion, the dynamics of the nonlinear microwave response of 2D electrons is studied at different temperatures in GaAs quantum well placed in quantizing magnetic fields. The dynamical response provides the direct measurement of the inelastic electron relaxation. When temperature $T$ exceeds the Landau level separation the relaxation rate $1/\tau_{\text{in}}$ is found to be proportional to $T^2$, indicating the electron-electron interaction as the dominant mechanism limiting the nonlinearity. At lower temperatures the rate tends to be proportional to $T^3$, indicating a reduction of the $e$-$e$ contribution and the important role of the electron-phonon scattering in the inelastic relaxation. The temperature dependence of the relaxation time is found to be significantly different from the one obtained from DC biased SdH oscillations, indicating a difficulty with the widely accepted interpretation of this phenomenon [97, 103, 108].
Chapter 10

Conclusions

10.1 Summary

The focus of this study has been the quantum phenomena that give rise to nonlinear transport properties of highly dense and mobile two-dimensional electron systems. GaAs quantum wells with one and two occupied subbands were studied with electron densities around $10^{16}$ per $m^2$ and mobilities of 73 to 121 $m^2/Vs$ around 4.6K. Several timescales have been shown to drastically affect the presence of nonlinear quantum mechanisms in these systems.

We demonstrate a simple transport method to access the electron lifetime $\tau_q$ in a broad temperature range that was previously unattainable. The method is based on the analysis of the quantum positive magnetoresistance caused by enhanced scattering due to cyclotron motion. For these systems, the temperature variations of the quantum scattering rate $1/\tau_q$ are found to be proportional to the square of the temperature for temperatures up to 15 Kelvin and are in very good agreement with the theory taking into account electron-electron interactions in 2D systems.
In quantum wells with a long quantum lifetime $\tau_q$, quantum oscillations of nonlinear resistance that are independent of magnetic field strength have been observed. These oscillations are periodic in applied bias current and are intimately connected to quantum oscillations of resistance at zero bias: Shubnikov-de Haas (SdH) for single subband systems and magnetointersubband (MIS) oscillations for two subband systems. The nonlinear resistance oscillations inherit the temperature dependence of their parent oscillations. Those associated with SdH oscillations are also temperature dependent while those associated with the MIS oscillations are temperature independent. The bias-induced oscillations can be explained by a spatial variation of electron density across the sample caused by the Hall electric field. The proposed theoretical model predicts the period of these oscillations to depend on the total electron density, which has been confirmed by controlling the density through a voltage top-gate on the sample.

A new mechanism for nonlinear transport has garnered much attention recently. The mechanism is a quantum manifestation of Joule heating where an applied bias current causes selective flattening in the electron distribution function but conserves overall broadening, producing a highly non-equilibrium distribution of electrons that drastically effects the transport properties of the system. Hence, this Joule heating effected by the quantized energy spectrum has been named quantal heating. It is observed through a significant decrease in sample conductivity for small bias currents. Competing explanations of these experimental results have proposed contributions from edge states and/or skipping orbitals to be the main cause of this effect. We have shown that these contributions are minimal by studying the transition to the zero differential conductance state and comparing results between Hall and Corbino geometries. In the Corbino geometry, edge states are confined to the inner and outer radii and therefore do not contribute to the radial transport. The zero differential conductance (resistance) state exists as a regime where voltage (current) does not depend on current (voltage). The onset of this state at a bias
field $E_{th}$ (current $I_{th}$) is nearly identical for both geometries and demonstrates that the edge states contribute much more weakly to the nonlinear transport. As a result, it is clear that the bulk mechanism of quantal heating is the dominant nonlinear mechanism in these systems.

The mechanism of quantal heating depends on the rate of inelastic processes that bring the electron distribution back to thermal equilibrium. To study the dynamics of these relaxation processes, we applied microwave radiation simultaneously from two sources at frequencies $f_1$ and $f_2$ and measure the response of the system at the difference frequency, $f = |f_1 - f_2|$. The functional dependence of this $f$-signal provides direct access to the rate of inelastic scattering processes, $1/\tau_{in}$, in a wide range of temperatures. While conventional measurements of the temperature dependence indicate that $1/\tau_{in}$ should be proportional to temperature, recent DC investigations and this new direct measurement show either $T^2$ or $T^3$ dependence in different magnetic fields. This microwave experiment is the first direct access to the inelastic relaxation rate and confirms the temperature dependence obtained through the analysis of quantal heating.

10.2 Future Studies

There are likely many undiscovered and equally interesting nonlinear mechanisms in these systems. While the above experiments provide several valuable results, there remain several possibilities for continued work on these phenomena.

The quantum oscillations discussed in Chapters 6 and 7 are not fully explained. The proposed model can explain the oscillations accurately, but a full picture of their magnitudes is not fully realized. Qualitatively, the decrease in their magnitudes with increased bias current can be attributed to the phenomenon of quantal heating. However, this explanation is really only applicable for the initial drop and the first oscillation and probably would not accurately describe
the continual decrease with bias. A comprehensive theoretical picture of this phenomenon is thus an open area. Additionally, it is not known to what extent these oscillations are independent of magnetic field. For nonlinear resistance oscillations tied to Shubnikov-de Haas oscillations, does the field-independence extend into the quantum Hall regime? This study observed oscillations as field-independence up to $2T$ for temperatures greater around $4K$. The detailed temperature and high magnetic field behavior of these oscillations was not studied in detail.

The magnetointersubband oscillations (MISO) in systems with two occupied subbands discussed in Chapter 7 are notable for their amplitude being temperature-dependent only indirectly through the temperature dependence of the quantum lifetime and thus their presence at high temperatures. Also explained in Chapter 7 is the difficulty of extracting the quantum lifetime unless the quantum lifetimes in each band are approximately equal. Otherwise, it is only possible to extract the total quantum scattering rate $1/\tau = 1/\tau_1 + 1/\tau_2$. However, it is possible to engineer heterostructures to have three subbands where the lower two subbands are much closer to one-another than the third – as seen in Figure 2.2. In these systems, the quantum lifetimes of the lower two subband should be nearly identical because of relatively identical electron densities. Thus, the system exhibits three sets of MISO with only two different $\tau_q$. This should allow for the extraction of the quantum lifetime of all three subbands from the resistance oscillations caused by intersubband scattering.

In Chapter 9, we discuss a powerful new method for the investigation of the inelastic processes in two-dimensional systems in response to microwave radiation. Although this method has been employed to demonstrate the temperature dependence of the inelastic scattering rate $1/\tau_{in}$, a detailed investigation should be done to understand the evolution of $1/\tau_{in}(T)$ as magnetic field is increased. Previous DC results demonstrated a transition between $T^2$ and $T^3$
behavior that suggests a weakening of contributions to relaxation from electron - electron interactions. Data from high magnetic field even suggested a return to $T^2$ behavior. Although elements of the existing theory can predict some of these changes, a full investigation is needed to understand the mechanisms behind the temperature dependence of the inelastic scattering rate.

Initial studies focused on the linear regime of bias currents have provided direct evidence in favor of square and cubic temperature dependence of the inelastic scattering rate. However, the bias voltage dependence of the $f$-signal demonstrates interesting features, which are shown in Figure 9.3 but are beyond the scope of that investigation. Although the biases used in the study are in the linear regime and are thusly far below the regime of the electron spatial redistribution (density variation) discussed for the quantum oscillations (Chapters 6 and 7), the $f$-signal does appear to oscillate. Could these oscillations be a manifestation of this mechanism? Since the $f$-signal represents an amplitude we observe sharp cusps corresponding to the zero point of the oscillations. However, at high frequencies these zeroes shift and become curved. The nature of these interesting features were beyond the scope of the study and remain unexplored.
Appendix A

Hardware and Software

- Stanford Research Systems Model SR830 DSP Lock-in Amplifier
- Stanford Research Systems SIM960 Analog PID Controller
- Stanford Research Systems SIM921 AC Resistance Bridge
- Stanford Research Systems SIM922A Diode Monitor
- Stanford Research Systems Model SG384 DC to 4.05 GHz Signal Generator
- Agilent 34401A Multimeter
- Agilent E4407B ESA-E Series Spectrum Analyzer

All data acquisition devices were controlled through GPIB connection by LabView software.
Bibliography


[120] Direct comparison with Equation 9.1 (or with Equation 5.2 at $R_D = R_0$) yields $1/\tau_q(GHz) = 234 + 1.33 \cdot T^2(K^2)$ and $q_s = 3.4 \cdot 10^8 \cdot 1/m$. At $R_D = R_0$ agreement between Equation 9.1 and the resistance $R_{xx}(B)$ is observed in 2-3 times narrower interval of magnetic fields.


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