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Abstract

The dynamics of fluids in which the constituent particles carry nonabelian charges can be described succinctly in terms of group-valued variables via a generalization of the co-adjoint orbit action for particles. This formalism, which is particularly suitable for incorporating anomalies, has previously been used for the chiral magnetic and chiral vorticity effects. Here we consider the similar effect for the isospin which corresponds to an angular asymmetry for neutral pions.

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Fluid dynamics can be formulated in terms of group-valued variables [1]. The basic idea behind this is to construct a Lagrangian generalizing the usual co-adjoint orbit action which describes particle motion as the transport of various conserved quantum numbers, the latter being the simultaneously diagonalizable generators of an appropriate group. Thus, for the transport of mass and spin, one would consider the Poincaré group, or the de Sitter group with a suitable contraction leading to the Poincaré group, see for example [2]. Likewise, the color group, say $SU(3)$, would describe the color flow in the quark-gluon plasma. One of the advantages of this formulation, as pointed out recently, is that one can incorporate the effect of anomalies very easily via a generalization of the usual Wess-Zumino term [3].

The existence of such an effective action for anomalies is in complete conformity with the 't Hooft argument for the usual Wess-Zumino term [4]. Consider a situation where all the flavor symmetries of the quarks are gauged with anomalies canceled by a set of spectator fermions. Then if the quark degrees of freedom are realized in a different phase with a different set of variables taking the place of the quark fields, there should be a term given in terms of the new variables which can reproduce the anomalies so that the anomaly cancellation with the spectator fermions is still preserved in the new phase. This is the basic *raison d'être* for the Wess-Zumino term. The original argument by 't Hooft was applied to the phase with confinement and chiral symmetry breaking so that mesons and baryons took the place of the quark degrees of freedom, but the same reasoning can be used for the fluid phase, with the quarks replaced by the fluid degrees of freedom such as flavor flow velocities and densities. The group theoretical formulation of fluid dynamics is especially appropriate for this context, because the original Wess-Zumino action for anomalies is given in terms of the group-valued pseudoscalar meson fields and so, for the fluid phase, essentially the same expression can be used, except for the group-valued fields being reinterpreted in terms of flavor flow velocities and densities. In this way, one can obtain the chiral magnetic effect (CME) and the chiral vorticity effect from a purely symmetry-based approach. We note that beyond the original papers on the CME [5], it has been discussed from many different points of view, from a combination of hydrodynamics and thermodynamics [6], from holography [7], using chiral Lagrangians [8], from fluid-gravity correspondence [9], in different dimensions [10] and from the lattice formulaton [11]. The chiral vorticity effect has also been discussed in the literature, see [12]. A recent phase space formulation of anomalies is suggestively close to the fluid description we use, although the similarities need further exploration [13].

A priori, it may seem that processes mediated by the anomalies cannot be very important since the unconfined fluid phase of quarks and gluons is obtained only for a very short time, of the order of strong interaction scales while flavor processes, typically, are on a much longer time-scale. However, some flavor processes can be enhanced. The high value of the magnetic field generated by the passing ions in a slightly off-center collision acts as an enhancement factor. Thus an anomaly mediated process, such as the electromagnetic current induced via the term responsible for the $\pi^0 \to 2\gamma$, can be significant, because one of the contributing factors is the magnetic field. This was the main reason for considering

the CME in the context of charge asymmetry in heavy ion collisions. However, we should point out that this enhancement may not be adequate enough to produce observable effects. There are alternate, and arguably compelling, explanations of the charge asymmetry [14], due to correlations imposed by conservation laws, initial state fluctuations, etc. It could very well be that the observed charge asymmetry is due to one of these alternate possibilities, although the relevance of the CME for heavy ion collisions is not entirely a settled matter. Overall, the issue of anomalies in fluid dynamics still remains a matter of research interest and the present paper is set in this larger context.

The content of this paper can be summarized as follows. We will explore the fluid action based on group theory in more detail. The focus of [3], as well as many of the earlier papers, was on one of the anomalies, namely, the fluid version of the term that leads to the standard $\pi^0 \to 2\gamma$ decay. However, the standard model has other flavor anomalies and some of these other ones can also have interesting consequences. The main result of our analysis in this paper is to point out that, similar to the CME, there is also an isospin asymmetry which can be manifested as an an angular asymmetry in the emission of neutral pions. There is some enhancement for the latter as well, albeit, not to the same extent as the CME.

We start with a very brief review of the formalism from [1, 3]. The key observation is that for the motion of particles which carry a nonabelian charge corresponding to a Lie group G, the action is given by [15, 16]

$$S = \int dt \left[\frac{1}{2} m \, \dot{x}^2 + A_i^a Q^a \dot{x}_i\right] - i \int dt \sum_{s=1}^{r} w_s \, \mathrm{Tr}\left(q_s g^{-1} \dot{g}\right) \tag{1}$$

where we have written the nonrelativistic action for the usual kinetic term in S. (It can be made relativistic without affecting the results which follow.) Further, g is an element of the group G, taken to be a matrix in the fundamental representation, with t^a being an orthonormal basis in the same representation, $\mathrm{Tr}(t^a t^b) = \frac{1}{2}\delta^{ab}$. q_s are the diagonal generators in the same basis, i.e., they are the generators of the Cartan subalgebra; thus the range of the summation is from 1 to the rank of the group, denoted by r. $A_i = -i t^a A_i^a$ is the nonabelian gauge field and $Q^a = \sum_s w_s \, \mathrm{Tr}(g \, q_s \, g^{-1} \, t^a)$.

The last term in (1) is the co-adjoint orbit action which describes the dynamics of the gauge charges and which, upon quantization, gives the Hilbert space corresponding to one unitary irreducible representation (UIR) of G corresponding to the highest weight $\mathbf{w} = \{w_s\} = (w_1, w_2, \cdots, w_r)$. Q^a then become operators realizing the charge algebra

$$\left[Q^a, Q^b\right] = i f^{abc} Q^c. \tag{2}$$

Under $g \to g \exp\left(i q_s \theta^s\right)$, the change in the action is given by $\Delta S = \frac{1}{2} \sum_s w_s \int dt \, \dot{\theta}^s$. One can choose $\theta^s(t)$ such that $[g \exp\left(i q_s \theta^s\right)]$ traces out a closed loop in G over the range of t. The single-valuedness of e^{iS} over such a closed path in G leads to quantization conditions on $\mathbf{w}$ corresponding to the UIRs of G; consistent quantization is obtained only for the UIRs of G.

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The generalization of the co-adjoint orbit action for a number of particles would be

$$S = -i \int dt \sum_\lambda \mathbf{w}_\lambda \cdot \mathrm{Tr}\left(\mathbf{q}\, g_\lambda^{-1}\, \dot{g}_\lambda\right) \tag{3}$$

where $\lambda = 1, 2, \cdots, N$ labels the particles and $\mathbf{w} \cdot \mathbf{q} = \sum_s w_s h_s$. For the fluid viewed as composed of individual particles transporting nonabelian charges, we may take the continuum limit of this action, as is usually done in the Lagrange approach to fluids, by replacing the indexing label λ by $\mathbf{x}$, the position of the particle, and with the corresponding changes $\sum_\lambda \to \int d^3\mathbf{x}/v$, $w_\lambda/v \to \rho(\mathbf{x})$. Here v is a small volume over which the coarse-graining is done to get a continuum description. This procedure leads to

$$S = -i \int d^4x \sum_s \rho_s \, \mathrm{Tr}(q_s\, g^{-1}\, \dot{g}) \tag{4}$$

where $g = g(\mathbf{x}, t)$. Since the charge density ρ_s can be taken as the time-component of the four-vector current, the relativistic version of (4) is straightforwardly given by

$$S = -i \int d^4x \sum_s j_s^\mu \, \mathrm{Tr}\left(q_s\, g^{-1}\, D_\mu g\right) \tag{5}$$

where $D_\mu g = \partial_\mu g + A_\mu g$, $A_\mu = -it^a A_\mu^a$. The inclusion of terms corresponding to the usual terms of the fluid action is also straightforward. We get

$$S = -i \int d^4x \sum_s j_s^\mu \, \mathrm{Tr}\left(q_s g^{-1} D_\mu g\right) - \int d^4x \; F(n_1, n_2, \ldots) + S_{YM}(A) \tag{6}$$

where, for each value of s, $j_s^\mu j_{s\mu} = n_s^2$. The function $F(n_1, n_2, \cdots)$ contains information about the pressure and the enthalpy. We have also introduced the standard Yang-Mills action for the gauge field.

The equations of motion for (6) do give the appropriate nonabelian magnetohydrodynamics. Further, the canonical quantization of this action leads to the expected current algebra, with the following equal-time rules for the charge density,

$$\left[\rho^a(\mathbf{x}, t), \, \rho^b(\mathbf{y}, t)\right] = i f^{abc} \rho^c(\mathbf{x}, t) \, \delta^3(\mathbf{x} - \mathbf{y}). \tag{7}$$

The velocity for the transport of the nonabelian charge can be introduced via $j^\mu = n\, u^\mu$, $u^2 = 1$, for each value of s. Also, from (6), the current which couples to the gauge field A_μ^a is given by

$$J^{a\mu} = \sum_s j_s^\mu \, \mathrm{Tr}(g\, q_s\, g^{-1}\, t^a) = \sum_s u_s^\mu\, n_s\, Q_s^a \tag{8}$$

which is in the Eckart form [17], again, for each value of s.

We can also interpret the dynamical variables as follows. The charge density, considered as a matrix in the fundamental representation, transforms as $\rho \to h^{-1}\, \rho\, h$, $h \in G$, $\rho = \rho^a t^a$. We can thus pick a specific transformation g which diagonalizes ρ,

$$\rho = g\, \rho_{diag}\, g^{-1}, \qquad \rho_{diag} = \sum_s n_s\, q_s \tag{9}$$

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so that $\rho^a = \sum_s n_s \, Tr \left(g \, q_s \, g^{-1} \, t^a \right)$. Thus $g(\mathbf{x}, t)$ is part of the charge density and n_s are the eigenvalues of ρ. The eigenvalues n_s are gauge-invariant and their flow is given by u_s^μ.

We can now consider the specialization of the action (6) to the fluid or plasma phase of the standard model. We will consider the quark-gluon plasma phase for three flavors of quarks, u, d, s. In other words, we consider a phase with thermalized u, d, s quarks, so that they must be described by fluid variables while the heavier quarks are described by the field corresponding to each species. We will also consider all flavor symmetries to be gauged; once the action has been written down, we can specialize to the case of the flavor gauge fields being those of the electroweak theory. We will also neglect the quark masses so that we have the full flavor symmetry $U(3)_L \times U(3)_R$. Thus the group G to be used in (6) is

$$G = SU(3)_c \times U(3)_L \times U(3)_R$$

with individual flows corresponding to the charges. In this discussion our focus is on the flavor transport, so we will drop $SU(3)_c$ from the equations to follow. Thus the diagonal elements of the algebra are the λ_3, λ_8 and $\lambda_0 = \mathbb{1}$ of $U(3)_L$ and $U(3)_R$. The fluid action is then given by

$$
\begin{aligned}
S \; = \; \int d^4x \bigg[& -i \, j_3^\mu \mathrm{Tr} \left(\frac{\lambda_3}{2} g_L^{-1} D_\mu g_L \right) - i \, j_8^\mu \mathrm{Tr} \left(\frac{\lambda_8}{2} g_L^{-1} D_\mu g_L \right) - i \, j_0^\mu \mathrm{Tr} \left(g_L^{-1} D_\mu g_L \right) \\
& -i \, k_3^\mu \mathrm{Tr} \left(\frac{\lambda_3}{2} g_R^{-1} D_\mu g_R \right) - i \, k_8^\mu \mathrm{Tr} \left(\frac{\lambda_8}{2} g_R^{-1} D_\mu g_R \right) - i \, k_0^\mu \mathrm{Tr} \left(g_R^{-1} D_\mu g_R \right) \\
& -F(n_0, n_3, n_8, m_0, m_3, m_8) + S_{YM}(A) \\
& + \Gamma_{WZ}(A_L, A_R, g_L g_R^\dagger) - \Gamma_{WZ}(A_L, A_R, \mathbb{1}) \bigg]
\end{aligned}
\tag{10}
$$

where $j_{0,3,8}^\mu$ apply to $U(3)_L$, $k_{0,3,8}^\mu$ apply to $U(3)_R$, $g_L \in U(3)_L$, $g_R \in U(3)_R$ and $n_l^2 = j_l^2$, $m_l^2 = k_l^2$ with $l = 0, 3, 8$. The last term is the usual gauged WZ term $\Gamma_{WZ}(A_L, A_R, U)$ given in terms of A_L, A_R and the meson fields $U \in U(3)$ [18, 19], but, for our purpose, U is replaced by $g_L g_R^\dagger$. We have also subtracted $\Gamma_{WZ}(A_L, A_R, \mathbb{1})$ which is necessary to bring the analysis to the Bardeen form of the anomalies [19]. Recall that the Bardeen form is the one which not only preserves the vector gauge symmetries, but also gives a manifestly vector-gauge-invariant form to the remaining axial anomalies. This form is what is appropriate for the analysis we need. In this sense, the action (10) is a modification of the action given in [3]. Explicitly $\Gamma_{WZ}(A_L, A_R, U)$ is given by [18, 19][1]

$$
\begin{aligned}
\Gamma_{WZ}(A_l, A_R, U) = C \int & \mathrm{Tr} \left(dU U^{-1} \right)^5 \\
& + 5C \int \mathrm{Tr} \left(A_L dA_L + dA_L A_L + A_L^3 \right) dU U^{-1} \\
& + 5C \int \mathrm{Tr} \left(A_R dA_R + dA_R A_R + A_R^3 \right) U^{-1} dU
\end{aligned}
$$

---

[1]There are some sign differences with the expression used in [3]. This removes some sign inconsistencies we had (which did not affect the results in [3]) and the present choice is consistent with [19].

$$-\frac{5}{2}C\int \mathrm{Tr}\left[\left(A_L dU U^{-1}\right)^2 - \left(A_R U^{-1} dU\right)^2\right]$$

$$-5C\int \mathrm{Tr}\left[A_L\left(dU U^{-1}\right)^3 + A_R\left(U^{-1}dU\right)^3\right]$$

$$-5C\int \mathrm{Tr}\left(dA_L dU A_R U^{-1} - dA_R dU^{-1} A_L U\right)$$

$$-5C\int \mathrm{Tr}\left(A_R U^{-1} A_L U(U^{-1}dU)^2 - A_L U A_R U^{-1}(dU U^{-1})^2\right)$$

$$+5C\int \mathrm{Tr}\left((dA_R A_R + A_R dA_R)U^{-1}A_L U - (dA_L A_L + A_L dA_L)U A_R U^{-1}\right)$$

$$+5C\int \mathrm{Tr}\left(A_L U A_R U^{-1}A_L dU U^{-1} + A_R U^{-1}A_L U A_R U^{-1}dU\right)$$

$$+5C\int \mathrm{Tr}\left(A_R^3 U^{-1}A_L U - A_L^3 U A_R U^{-1} + \frac{1}{2}U A_R U^{-1}A_L U A_R U^{-1}A_L\right)$$

$$(11)$$

where $C = -i(N/240\pi^2)$, N being the number of colors ($= 3$ for us). As observed in [3] the action given above incorporates all the flavor anomalies in fluid dynamics. It is straightforward to verify that (11) does indeed lead to the usual chiral magnetic effect. We can now specialize to the problem at hand. We are interested in terms involving the Z^0 field and the electromagnetic field. Taking the electromagnetic field to be the magnetic field of the passing ions, which is then the enhancement factor, the Z^0-dependent term can lead to the anomlay-induced weak neutral current. For this calculation, we may neglect all gauge fields except A_μ (the electromagnetic field) and Z_μ. This means that we can take the left and right gauge fields to be

$$
\begin{aligned}
A_{L\mu} &= -ieQA_\mu - i\frac{g}{\cos\theta_W}(I_3 - Q\sin^2\theta_W)\,Z_\mu \\
&= (-i)\,(\alpha A_\mu + \beta Z_\mu) \quad (12) \\
A_{R\mu} &= -ieQA_\mu - i\frac{g}{\cos\theta_W}(-Q\sin^2\theta_W)Z_\mu \\
&= (-i)\alpha\,(A_\mu - \tan\theta_W Z_\mu) \quad (13)
\end{aligned}
$$

where we have used $e = g\sin\theta_W$ and defined

$$\alpha = eQ, \qquad \beta = \frac{g}{\cos\theta_W}I_3 - \alpha\tan\theta_W \qquad (14)$$

The matrices I_3 and Q are the usual ones,

$$I_3 = \begin{pmatrix} \frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{pmatrix}, \qquad Q = \begin{pmatrix} \frac{2}{3} & 0 & 0 \\ 0 & -\frac{1}{3} & 0 \\ 0 & 0 & -\frac{1}{3} \end{pmatrix} \qquad (15)$$

Evidently, $Q = I_3 + \mathbb{1}/6$. The terms of interest to us are those involving the electromagnetic field and terms with one power of Z. The terms in the action (11) involving just the electromagnetic field are [20]

$$S_{int}^{(1)} = -5\,C\int \left[A\,dA\,\mathrm{Tr}\left(2\,\alpha^2(dU\,U^{-1} + U^{-1}dU) - \alpha\,U\,\alpha\,dU^{-1} + \alpha\,dU\,\alpha\,U^{-1}\right)\right.$$

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$$-iA\,\mathrm{Tr}\left(\alpha\,(dU\,U^{-1})^3 + \alpha(U^{-1}dU)^3\right)]\tag{16}$$

These terms lead to the usual chiral magnetic effect as discussed in [3]. The terms with one power of Z in Γ_{WZ} are collected together as

$$\begin{aligned}
S_{int}^{(2)} &= 5\,C\int \mathrm{Tr}\left\{iZ(\beta - \tan\theta_W U\alpha U^{-1})(D_A UU^{-1})^3\right.\\
&\quad\left. + ZdA(-4\alpha\beta + 4\tan\theta_W U\alpha^2 U^{-1} + \{\tan\theta_W \alpha - \beta, U\alpha U^{-1}\})D_A UU^{-1}\right\}
\end{aligned}\tag{17}$$

where $D_A UU^{-1}$ is the derivative covariant with respect to the electromagnetic field,

$$D_A UU^{-1} \equiv dUU^{-1} - iA(\alpha - U\alpha U^{-1}).\tag{18}$$

Explicitly, in local coordinates, the interaction term (17) is

$$\begin{aligned}
S_{int} &= -\frac{iN}{48\pi^2}\epsilon^{\mu\nu\gamma\delta}\int d^4x\, Z_\mu\mathrm{Tr}\left\{i(\beta - \tan\theta_W U\alpha U^{-1})(D_A UU^{-1})_\nu(D_A UU^{-1})_\gamma(D_A UU^{-1})_\delta\right.\\
&\quad\left. + \partial_\nu A_\gamma(-4\alpha\beta + 4\tan\theta_W U\alpha^2 U^{-1} + \{\tan\theta_W \alpha - \beta, U\alpha U^{-1}\})(D_A UU^{-1})_\delta\right\}
\end{aligned}\tag{19}$$

It is useful to consider the reduction of this expression for the case of two flavors, as this is adequate for illustrating the result on neutral currents. For this, we take

$$U = e^{i\theta}\begin{pmatrix} V & 0 \\ 0 & 1 \end{pmatrix}\tag{20}$$

where V is a 2×2 $SU(2)$ matrix. We may take it to be of the form $h_L h_R^\dagger$ where h_L and h_R are elements of $SU(2)$. They can be related to the flow velocities by using the equations of motion for the full action (10). The derivatives now simplify as

$$dU\,U^{-1} = i\,d\theta\,\mathbb{1} + \begin{pmatrix} dV\,V^{-1} & 0 \\ 0 & 0 \end{pmatrix}, \quad D_A U\,U^{-1} = i\,d\theta\,\mathbb{1} + \begin{pmatrix} D_A V\,V^{-1} & 0 \\ 0 & 0 \end{pmatrix}$$

$$D_A V\,V^{-1} = dV\,V^{-1} - ieA\left(\frac{\tau_3}{2} - V\frac{\tau_3}{2}V^{-1}\right)\tag{21}$$

where τ_3 is the diagonal Pauli matrix. The A-dependent terms in (16) then simplify as

$$\begin{aligned}
S_{int}^{(1)} &= -20\,Ci\,e^2\int A\,dA\,d\theta\\
&\quad -\frac{5}{2}Ci\,e^2\int \mathrm{Tr}\left[\tau_3 V\tau_3 V^{-1} - \mathbb{1}\right]A\,dA\,d\theta - \frac{5}{2}Ce^2\int A\,dA\,\mathrm{Tr}\left[\tau_3(dV\,V^{-1} + V^{-1}dV)\right]\\
&\quad -\frac{5}{2}Ce\int A\,d\theta\,\mathrm{Tr}\left[\tau_3(dV\,V^{-1})^2 + \tau_3(V^{-1}dV)^2\right]\\
&\quad +\frac{5}{2}Ci\,e\int A\,\mathrm{Tr}\left[\tau_3(dV\,V^{-1})^3 + \tau_3(V^{-1}dV)^3\right] + \frac{5}{3}Ci\,e\int A\,\mathrm{Tr}\left[(dV\,V^{-1})^3\right]
\end{aligned}\tag{22}$$

If we set $h_L = h_R$ or $V = \mathbb{1}$, which is adequate for the basic chiral magnetic effect,

$$S_{int}^{(1)} = -20\,C\,ie^2\int A\,dA\,d\theta\tag{23}$$

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The electromagnetic current which follows from this is

$$J^{em} = -\frac{Ne^2}{6\pi^2} dA\, d\theta$$

$$J^{em\,\mu} = -\frac{e^2}{4\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha}\partial_\beta\theta \qquad (24)$$

where the second line gives the component-form for $N = 3$. We can do a similar simplification of the terms involving Z to get

$$S^{(2)}_{int} = 5\, Ci \left(-8e^2 \cot 2\theta_W\right) \int Z\, dA\, d\theta$$

$$- 5Cie^2 \cot 2\theta_W \int \mathrm{Tr}(\tau_3 V \tau_3 V^{-1} - \mathbb{1})\, Z\, dA\, d\theta$$

$$- \frac{5}{2} Ce \int Z\, d\theta \left[\cot\theta_W \mathrm{Tr}\left[\tau_3 (DV\, V^{-1})^2\right] - \tan\theta_W \mathrm{Tr}\left[\tau_3 (V^{-1} DV)^2\right]\right]$$

$$+ \frac{5}{2} Ce^2 \int Z\, dA \left[(-\cot\theta_W + \tan\theta_W)\, \mathrm{Tr}(\tau_3 DV\, V^{-1}) + 2\tan\theta_W \mathrm{Tr}(\tau_3 V^{-1} DV)\right]$$

$$+ \frac{5}{2} Cie \int Z \left[\cot\theta_W \mathrm{Tr}\left[\tau_3 (DV\, V^{-1})^3\right] - \tan\theta_W \mathrm{Tr}\left[\tau_3 (V^{-1} DV)^3\right]\right]$$

$$- \frac{5}{3} Cie \tan\theta_W \int Z\, \mathrm{Tr}(DV\, V^{-1})^3 \qquad (25)$$

Again, if we set $V = \mathbb{1}$, this reduces to

$$S^{(2)}_{int} = -\frac{Ne^2}{6\pi^2}(\cot 2\theta_W) \int Z\, dA\, d\theta \qquad (26)$$

The standard coupling of Z to the neutral current is of the form $(g/\cos\theta_W) Z_\mu J^{Z\,\mu}$, so that, we can identify the anomaly-induced neutral current as

$$J^Z = -\frac{Ne}{12\pi^2}(\cos 2\theta_W)\, dA\, d\theta$$

$$J^{Z\,\mu} = -\frac{e}{8\pi^2}(\cos 2\theta_W)\, \epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha}\partial_\beta\theta \qquad (27)$$

Since the current for the third component of the weak isospin is $J^3 = J^Z + \sin^2\theta_W\,(J^{em}/e)$, we get

$$J^{3\,\mu} = -\frac{e}{8\pi^2} \epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha}\partial_\beta\theta \qquad (28)$$

In the quark-gluon fluid, we may replace $\dot\theta$ in terms of the chemical potentials for the left and right axial charges as by $\dot\theta = \frac{1}{2}(\mu_L - \mu_R)$, and so the spatial component of this current can be written as

$$J^{3\,i} = \frac{e}{8\pi^2}(\mu_L - \mu_R)\, B^i \qquad (29)$$

Thus there is an induced weak isospin asymmetry possible.

We now turn to the interpretation of this induced current. In the region where we have the quark-gluon fluid, there is a current in the direction of the magnetic field. This

corresponds to a flow of the constituent particles of the medium. Continuity requires that such a current should exist just outside of the fluid region where the degrees of freedom are the hadrons. In other words, we expect this current to translate into the hadronic version of the weak isospin current just outside of the fluid region. Since pions are the most significant component of the hadrons, the current of interest is $J^{3\mu} = -\frac{1}{2}f_\pi \partial^\mu \Pi^0 + \cdots$. This shows that if we consider Π^0 as a classical field configuration, then the result (28) can be interpreted as saying that a gradient in the pion field is generated by the anomaly; it is given by

$$\partial^\mu \Pi^0 = \frac{e}{4\pi^2 f_\pi} \epsilon^{\mu\nu\alpha\beta} F_{\nu\alpha}\partial_\beta \theta$$

$$\partial^i \Pi^0 = -\frac{e}{4\pi^2 f_\pi} (\mu_L - \mu_R) B^i \tag{30}$$

Effectively, this implies an asymmetry in the distribution of neutral pions, in the direction of the magnetic field of the passing heavy nuclei. More explicitly, since pions are detected via the 2γ final states, consider the effective Lagrangian for this decay,

$$S_{eff} = -\frac{\alpha_e}{4\pi f_\pi} \int d^4x \ \epsilon^{\mu\nu\alpha\beta}\partial_\mu \Pi^0 \ F_{\nu\alpha}A_\beta \tag{31}$$

This shows that a classical pion field may be thought of as an antenna for the radiation of correlated photons. If we use (30), we may even write (31) as

$$S_{eff} = -\frac{\alpha_e e}{8\pi^3 f_\pi^2} \int \left[\bar{F}^{\mu\nu} F_{\mu\nu} A_\alpha \partial^\alpha \theta + 2 \bar{F}^{\mu\nu} F_{\alpha\mu}A_\nu \partial^\alpha \theta \right]$$

$$= -\frac{\alpha_e e}{8\pi^3 f_\pi^2}(\mu_L - \mu_R) \int \epsilon^{ijk}\bar{B}_i F_{0j}A_k \tag{32}$$

where we have indicated the magnetic field generated by the passing ions with an overbar; the other two fields correspond to the radiated photons. The magnitude of this effect remains small, of the order of the CME, with a further suppression due to the pion decay, via f_π in (30). (Here we are not counting the additional factor of α_e/f_π due to (31), since it is there for any observed pion decay.)

Another effect of the action (26) is to consider it as an interaction term generating a virtual Z^0 which can then decay into leptons. Effectively, this amounts to replacing Z_μ by $(g/\cos\theta_W)(J^Z/M_Z^2)$, so that

$$S_{int}^{(2)} \approx -\frac{Ne^2}{12\pi^2}(\cot 2\theta_W)\frac{g}{M_Z^2 \cos\theta_W} \int d^4x \ J_\mu^Z F_{\nu\alpha}\partial_\beta\theta \ \epsilon^{\mu\nu\alpha\beta}$$

$$\approx \frac{Ne^2 g}{12\pi^2}\frac{(\cot 2\theta_W)}{M_Z^2 \cos\theta_W}(\mu_L - \mu_R) \int d^4x \ J_i^Z \bar{B}_i \tag{33}$$

$$J_i^Z = \frac{1}{2}\left(\bar{\nu}_{eL}\gamma_i\nu_e - \bar{e}_L\gamma_i e_L\right) + \sin^2\theta_W(\bar{e}\gamma_i e) + \cdots$$

This can show up as an asymmetry in the lepton distribution, particularly for neutrinos, although experimentally it would be too small for detection. Part of this effect will also act as

a modification of the charged particle asymmetry. The effect is much smaller than the chiral magnetic effect, because of the suppression by M_Z^{-2}. With more energetic collisions, as at the LHC, the transient magnetic field is higher, which gives somewhat better enhancement; one can also get some kinematic enhancement from the current for highly energetic leptons. These can mitigate the effect of M_Z^{-2} to some extent.

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