Another One Busts The Dust

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ANOTHER ONE BUSTS THE DUST

BY

KEIKO HIRANAKA
This manuscript has been read and accepted for the Graduate Faculty in Physics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

ANOTHER ONE BUSTS THE DUST

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Keiko Hiranaka

Brown dwarfs are substellar objects with core temperatures insufficient for sustained hydrogen fusion. Their physical properties such as mass, temperature, and radius are similar to those of gas giant planets, so studying brown dwarfs may also benefit exoplanet studies.

There is a population of 'red' L dwarfs, which have redder $J - K$ colors in the near-infrared than normal objects. Red L dwarfs include young, low-gravity objects, which are systematically red, and red field-gravity objects. The observed reddening in L dwarfs is not well explained by current atmosphere models.

We present an analysis of red L dwarfs using our model which includes small, sub-micron size dust particles in the upper atmosphere in addition to the cloud decks common to all L dwarfs which consist of larger particles. We hypothesize that the red NIR colors of some L dwarfs could be explained by a "dust haze" of small particles. We developed a model which combines Mie theory and Hansen particle size distributions to reproduce the extinction due to the proposed dust haze. We apply our analysis to 20 young L dwarfs and 36 red field L dwarfs. We constrain the properties of the dust haze including particle size distribution and column density using Markov-Chain Monte Carlo methods. We find that sub-micron range silicate grains reproduce the observed reddening. We also find that Hansen particle size distributions reproduce the shape of the observed reddening better than power law particle size distributions. Current brown dwarf atmosphere models include large grain ($\sim 10 \, \mu m$) dust clouds but not ISM-size small dust grains. Our results provide a proof of concept and motivate a combination of large and small dust grains in brown dwarf atmosphere models.

Finally, we discuss our future prospects and possible application of the dust haze analysis for atmospheric models of brown dwarfs and exoplanets, and variability in brown dwarfs.
In memory of Akira Hiranaka and Potechi
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I would like to thank my advisor Prof. Kelle Cruz for guiding me through my Ph.D study and for being patient with me and believing in me. She has been an awesome advisor I often bragged about to my friends.

I also would like to thank my other advisor Dr. Mark Marley for giving me insights and inspirations. It was a great experience to have visited him at NASA Ames Research Center even for a week or so.

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I would like to thank the superintendent of my last apartment building, Nicky, for driving me to the ER and literally saving my life when my appendix was rupturing.

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Last but not least, I would like to thank all my friends and family (including the people already mentioned) for the love and support, especially those who made it to my thesis defense for laughing at my well-thought-out hilarious jokes but not dying from laughing too hard.
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1.1.1 Artist’s rendition by Robert Hurt of the Infrared Processing and Analysis Center (DwarfArchives.org), edited by Kelle Cruz. It shows a comparison between the Sun, M, L, and T type brown dwarfs, and Jupiter. The masses and sizes of brown dwarfs are similar to those of gas giant planets like Jupiter.

1.1.2 Figure from Faherty et al. (19). NIR spectral sequence of early L dwarfs is shown. Black spectra are field-gravity objects and red spectra are low-gravity objects. Characteristic molecular absorption bands are specified.

1.3.1 Figure from Cardelli, Clayton, and Mathis (13). Comparison between the extinction law and three lines of sight with largely separated $R$ values. The effect of varying $R$ on the extinction curves is apparent particularly at the shorter wavelengths (higher $1/\lambda$).

1.3.2 Figure from Baldassare (8). An example of an effectively de-reddened low-gravity L dwarf. The red is the spectrum of the field standard object, the gray is the low-gravity object prior to de-reddening, the black is the same low-gravity object after de-reddening. All spectra are normalized to the peak of J band.

1.3.3 Figure from Baldassare (8). Flux ratio of de-reddened flux to original flux. This is a graphical depiction of the extinction law and is equivalent to Figure 1.3.1.

1.4.1 Comparison of Rayleigh scattering and Mie scattering. Rayleigh scattering is strongly wavelength-dependent and scatters light in all directions. Mie scattering is not strong wavelength-dependent and has strong forward scattering.

2.8.1 A conceptual representation of our dust haze model. The regular clouds of large particles ($\sim 10 \, \mu m$, grey cloud symbol) exist in both normal and red L dwarfs. An additional haze of small particles (green layer) is present in the red L dwarf atmosphere, which causes the observed reddening. We do not know yet the specific location of the dust haze but it has to be at an altitude where the temperature is low enough so that the dust grains do not emit in the near IR.
2.8.2 Top panel shows spectra of a red L0 dwarf spex prism 0141-4633 (red) and field standard L0 dwarf 2M0345 (black). Both spectra are normalized by the mean flux. The young object has excess flux longward of 1.5 µm. Bottom panel shows the observed reddening for the same object. The observed reddening was derived by dividing the spectrum of the standard object by the spectrum of the red object and taking the log of the flux ratio. The overall shape of the observed reddening resembles a power law curve.

2.8.3 Top panel shows forsterite extinction coefficients according to Mie theory averaged over Hansen particle size distributions with various combinations of mean effective radius ($a$) and effective variance ($b$). Different colors correspond to different effective radii. Different line style correspond to different effective variance. The shapes of the extinction curves for smaller particles (0.1 – 0.4 µm) resemble the observed reddening but larger particles (1.0 µm) do not. Bottom right panel shows forsterite extinction coefficients averaged over power law particle size distributions with indices of -3 and -3.5. The extinction curves are too flat and do not reproduce the observed reddening. Bottom right panel shows forsterite extinction coefficients averaged over Gaussian particle size distributions mean radius of 0.3 and 0.5 µm.

2.8.4 Comparison of different particle size distributions. The gray lines are the power law distributions with power indices of -3 and -3.5. The magenta is a Gaussian distribution with characteristic grain size of 0.5 µm. The green lines are Hansen distributions for $a = 0.2$ µm and $b = 0.1, 0.5$. The Hansen distributions are a variation of the gamma distribution and they are wider than the Gaussian, but narrower than the power law distributions. The Hansen distributions can reproduce the shapes of the observed reddening better than the other distributions.

2.8.5 Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.

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2.8.173 Improvement on $\chi^2$ due to the dust haze prescription. The ratio of $\chi^2_{\text{before}}$ to $\chi^2_{\text{after}}$ is plotted against $\Delta(J-K)$ color. $\chi^2_{\text{before}}$ is $\chi^2$ between standard and red L dwarf spectra, and $\chi^2_{\text{after}}$ is $\chi^2$ between standard and corrected red L dwarf spectra. Green markers denote low-gravity L dwarfs and magenta markers denote field-aged red L dwarfs. In all objects, $\chi^2$ value is reduced after the correction.

2.8.174 A scatter plot of mean effective radius $a$ [$\mu$m] against $\Delta(J-K)$ color. Green markers denote low-gravity L dwarfs and magenta markers denote field-aged red L dwarfs both ranging between L0 – L5. Circles denote objects with PDFs with clear peaks. Diamonds denote objects with PDFs for $b$ hitting the limit. Thin diamonds denote objects with PDFs for $a$ hitting the limit. Squares denote objects with PDFs for both $a$ and $b$ hitting the limits. Our sample includes objects with $\Delta(J-K) < 0.1$. There is no noticeable trend in radius in relation to color. There is also no visible difference between young and red field L dwarfs.

2.8.175 A scatter plot of column density $N$ [$10^8$ cm$^{-2}$] against $\Delta(J-K)$ color.

3.2.1 Figure from Radigan et al. (45). The J-band light curves of 2M2139 (data points) and the best-fitting model (red curve, $P = 7.721$ hr) are shown. The light curves are from September 21 and 23, 2009.

3.2.2 Figure from Radigan et al. (45). In each panel, simultaneous model fits to both the NIR spectrum of 2M2139 (black line) and photometric variability (black filled circles) are shown. The top and bottom panels show one of the two best-fitting model combinations.

3.3.1 Figure from Marley (41). Comparison of clouds in the atmospheres of young giant planets like HR 8799c, T dwarfs, and Jupiter. Clouds in the atmospheres of young gas giants may be similar to clouds in young brown dwarf atmospheres. At cooler temperatures mature giant planets like Jupiter, a wider variety of grain species may be present.

3.3.2 This is a color-magnitude diagram of brown dwarfs and planetary mass companions from Faherty et al. (21). Small filled circles are field-gravity brown dwarfs, big circles with black bordering are low-gravity brown dwarfs, and triangles are planets. Colors separate spectral classes. The low-gravity brown dwarfs and planets have redder $J-K$ colors than the field-gravity objects.
CHAPTER 1

Introduction

1.1. Introduction to Brown Dwarfs and L dwarfs

Brown dwarfs are substellar objects with intermediate masses between stars and planets (less than 0.075 solar mass (9)). Figure 1.1.1 illustrates a comparison of brown dwarfs to the Sun and Jupiter in size, mass, and temperature. Brown dwarfs form in molecular clouds like stars, but do not accumulate enough mass to sustain hydrogen fusion in their cores. As a result, their temperatures keep cooling and their surface gravities increase as brown dwarfs age and contract. Typical effective temperatures of brown dwarfs are 2000–3000 K at the beginning of their lives but low mass brown dwarfs can get below 1000 K. Brown dwarfs share physical properties with warm exoplanets: they both have radii similar to Jupiter and cool temperatures. Young brown dwarfs may have extended clouds or dust haze that shape their emergent spectra, and their atmospheric properties are likely to be similar to those of young gas giant planets.

Brown dwarfs are categorized based on their spectra into spectral types M (late-M), L, and T. Kirkpatrick et al. (28) established optical classifications for L dwarfs, which are cooler than the coolest M dwarfs and

![Figure 1.1.1. Artist’s rendition by Robert Hurt of the Infrared Processing and Analysis Center (DwarfArchives.org), edited by Kelle Cruz. It shows a comparison between the Sun, M, L, and T type brown dwarfs, and Jupiter. The masses and sizes of brown dwarfs are similar to those of gas giant planets like Jupiter.](image)
1.2. Current Atmospheric Models of Brown Dwarfs

Atmospheric models typically combine hydrodynamics, radiative and convective energy transport, and gas phase chemistry to model the atmospheres of stars. Typical free parameters include the effective temperature $T_{\text{eff}}$, surface gravity $g$, radius $R$ or mass $M$, and element abundance (which is usually assumed to be the solar abundance). Magnetic fields are presently neglected. Furthermore, models generally include assumptions such as hydrostatic equilibrium, mixing length parameter, and chemical equilibrium. At cool temperatures, condensate clouds form in stellar and substellar atmospheres. Condensates are harder to model than gases because clouds can take a variety of forms, dimensions, positions, compositions, and

Figure 1.1.2. Figure from Faherty et al. (19). NIR spectral sequence of early L dwarfs is shown. Black spectra are field-gravity objects and red spectra are low-gravity objects. Characteristic molecular absorption bands are specified.

are lacking the oxide bands (TiO and VO) that dominate the far optical spectra of M dwarfs. The effective temperatures of L dwarfs are between 1300 K and 2500 K. As shown in Figure 1.1.2, the NIR spectra of early L dwarfs are characterized by strong bands of $\text{H}_2\text{O}$, FeH, CO, and neutral atomic lines of Na, Fe, K, Al, and Ca (Kirkpatrick (29)).
grain properties, resulting in too many parameters to model efficiently. Clouds also feedback onto the entire atmosphere and affect the chemistry and physics, which makes them even harder to produce a self-consistent model.

Many independent groups have attempted to model the atmospheres of brown dwarfs with discrete cloud decks. Clouds in current atmosphere models are treated empirically by varying a few free parameters. Such parameters include a particle size distribution and conditions at the cloud bottom and top. These atmosphere models reproduce the general trend of L dwarf spectra but they all fail to give a reasonable explanation for the observed reddening in the NIR. In the following subsections, I describe and compare different dust treatments used in atmosphere models of brown dwarfs.

### 1.2.1. Tsuji model. (51)

The free parameters in the Tsuji models include the effective temperature $T_{\text{eff}}$, log $g$, micro turbulent velocity, and critical temperature $T_{\text{cr}}$. The condensed dust grains precipitate at the condensation temperature $T_{\text{cond}}$, which is determined by thermal stability for given $T_{\text{eff}}$ and log $g$. In the Tsuji models, dust forms where the temperature is lower than $T_{\text{cond}}$ and higher than $T_{\text{cr}}$. $T_{\text{cr}}$ controls the thickness of the cloud. The cloud will be thin if $T_{\text{cr}}$ is only slightly lower than $T_{\text{cond}}$, and it will be thick if $T_{\text{cr}}$ is much lower than $T_{\text{cond}}$. The Tsuji models assume small submicron-size dust grains with radius $a = 0.01 \, \mu m$.

### 1.2.2. Ackerman & Marley model. (1)

In the Ackerman & Marley cloud model, the sedimentation efficiency factor $f_{\text{sed}}$ and the vertical eddy diffusion coefficient $K$ are free parameters in addition to the effective temperature $T_{\text{eff}}$ and log $g$. Like $T_{\text{cr}}$ in the Tsuji models, $f_{\text{sed}}$ controls the thickness of the clouds. Large $f_{\text{sed}}$ corresponds to rapid grain growth, which leads to efficient sedimentation and thin clouds. On the other hand, small $f_{\text{sed}}$ corresponds to slow grain growth and thick clouds. The Ackerman & Marley cloud model computes a broad lognormal particle size distribution for each condensate at each level in the atmosphere to depict the measured bimodal size distribution of terrestrial cloud droplets. The standard deviation $\sigma$ in the lognormal size distribution is another free parameter. The particle radius ranges between $\sim 10 - 100 \, \mu m$.

### 1.2.3. Allard & Homeier model (BT-settl model). (3; 4)

The free parameters for the BT-settl models are the surface gravity log $g$, and the effective temperature $T_{\text{eff}}$. The mixing time scale and micro-turbulent velocity are assumed to be constant according to the Mixing Length Technique inside the convectively unstable region. The cloud model is solved layer by layer from the innermost to the outermost. The initial element abundances at the innermost layer is set to solar values. For each layer, the chemical equilibrium (described in Allard et al. (2)) is solved, followed by comparing condensation, sedimentation, coalescence, and convective time scales to compute and adjust the grain size
and number density as well as the elemental abundances. Chemical abundance is solved again for the new elemental abundances. These steps are repeated until the fraction of sedimented grains no longer changes. The BT-settl models assume an ISM size distribution $n(r) = r^{-3.5}$. They predict the grain sizes to be $\lesssim 3 \mu m$.

### 1.2.4. Helling & Woitke model. (53)

The free parameters for the Helling & Woitke models are the gas temperature $T$, pressure $p$, and convective velocity $v$. The convective velocity limits the maximum grain size $a_{\text{max}}$, which is found to be $\sim 1 \mu m$ in the upper regions and $\sim 100 \mu m$ in the lower regions. The Helling & Woitke models assume an inhomogeneous, depth-dependent dust grain size distribution. They determine dust properties such as grain sizes, grain material composition, total grain volume, and remaining gas-phase element abundances by solving conservation equations of dust moments $L_j = \int V^{1/3} f(V) dV$ where $L_j$ is the $j$th moment of the grain size distribution, $V$ is the dust particle’s volume, and $f(V)$ is the grain size distribution function.

### 1.3. Small Grains in Brown Dwarf Atmospheres Motivated By Interstellar Reddening Law

Interstellar reddening is the extinction of starlight caused by the interstellar dust. The interstellar grains, which have radii less than $1 \mu m$, absorb and scatter blue light more effectively than red light. As a result, distant stars look redder than they actually are.

The interstellar extinction (reddening) law is an empirically derived relationship between absolute extinction $A(\lambda)/A(V)$ and the ratio of visual extinction $A(V)$ to selective extinction $E(B-V) = A(B) - A(V)$. The interstellar extinction law depends on only one parameter $R = A(V)/E(B-V)$. The universal extinction law was derived by Shultz & Wiemer (48) and Sneden et al. (49) and the value of $R$ was measured to be 3.1. Cardelli, Clayton, and Mathis (12) gave an expression for a mean interstellar extinction curve in the linear form $A(\lambda)/A(V) = a(x) + b(x)/R$, where $A(\lambda)$ is the total extinction at wavelength $\lambda$ and $x = 1/\lambda$. The coefficients $a(x)$ and $b(x)$ were determined for different wavelengths by Cardelli, Clayton, and Mathis (13) and O’Donnell (44) using the least squares fitting. Figure 1.3.1 shows three extinction curves with varying $R$ values.

Baldassare (8) found that the extinction curves according to the interstellar extinction law successfully de-redden the spectra of low-gravity L dwarfs. These low-gravity objects have redder near-infrared (NIR) spectra than older, field-gravity objects. This observed reddening is normally attributed to clouds and dust in their atmospheres. The spectra of low-gravity L dwarfs look like the spectra of older L dwarfs after the interstellar reddening law has been applied.
Figure 1.3.1. Figure from Cardelli, Clayton, and Mathis (13). Comparison between the extinction law and three lines of sight with largely separated $R$ values. The effect of varying $R$ on the extinction curves is apparent particularly at the shorter wavelengths (higher $1/\lambda$).

Baldassare (8) applied the interstellar reddening law to the low-resolution Spex Prism spectra of 17 low-gravity L dwarfs with spectral types ranging from L0 to L7. The de-reddened spectra of the low-gravity objects were compared to a series of standard L dwarf spectra found in Kirkpatrick et al. (31). When applying the interstellar reddening law, the value of $E(B-V)$ was varied in order to find a ‘best fit’ between the low-gravity object and the standard object. Figure 1.3.3 shows extinction curves with varying $E(B-V)$ values. The best fit was determined by eye, and was defined as the fit for which the general spectral features fit most closely.
1.4. Extinction of Light by Particles

Photons radiated by astronomical objects get scattered and absorbed by particles between the objects and the observer. Photons with scattering angles less than 90 degrees are forward scattered, and photons with scattering angles greater than 90 degree are back scattered (see Figure 1.4.1). Extinction is the sum of absorption and scattering. Some of the scattering processes relevant in astronomy include Rayleigh scattering and Mie scattering.

As shown in Figure 1.3.2, the interstellar reddening law effectively de-redds the red NIR spectra of young, low-gravity L dwarfs. Because brown dwarfs are so close to the Sun, we do not expect significant interstellar reddening. Therefore, the reddening must be caused by dust in the brown dwarf atmosphere. The fact that the interstellar reddening law works leads us to the hypothesis that there may be small, ISM-size dust grains in the atmospheres of brown dwarfs.

Figure 1.3.2. Figure from Baldassare (8). An example of an effectively de-reddened low-gravity L dwarf. The red is the spectrum of the field standard object, the gray is the low-gravity object prior to de-reddening, the black is the same low-gravity object after de-reddening. All spectra are normalized to the peak of J band.
1.4. EXTINCTION OF LIGHT BY PARTICLES

1.4.1. Rayleigh Scattering. Rayleigh scattering is the scattering of light by particles much smaller than the wavelength of the light ($\ll \lambda$) or molecules in the atmospheres. It is strongly wavelength-dependent ($\propto \frac{1}{\lambda^4}$) and it scatters light in all directions. Rayleigh scattering is dominant in scattering of the sunlight by the air, and it is the reason for the colors of the sky. The long wavelength limit of interstellar dust scattering is Rayleigh scattering.

1.4.2. Mie Scattering. Mie scattering is the scattering of light by particles with sizes similar to or larger than the wavelength of light such as dust or fog in the atmospheres. It is not strongly wavelength-dependent and it has strong forward scattering and weak back scattering. Mie scattering causes the white-gray colors of clouds, mist, and fog as well as the white glare seen around the Sun when the air is dense. The short wavelength limit of interstellar dust scattering is Mie scattering. Mie scattering is also used to describe scattering of light by dust particles in substellar/planetary atmospheres.
Figure 1.4.1. Comparison of Rayleigh scattering and Mie scattering. Rayleigh scattering is strongly wavelength-dependent and scatters light in all directions. Mie scattering is not strong wavelength-dependent and has strong forward scattering.
CHAPTER 2

Demonstrating the Existence of Sub-micron size dust grains in the Atmospheres of Red L Dwarfs

This chapter is focused on our effort to explain the observed red $J-K$ colors of some L dwarfs by introducing a high-altitude layer of sub-micron size dust grains in model atmospheres. Sub-micron size dust grains are not included in current atmospheric models of brown dwarfs, which have yet to give a reasonable explanation for the observed reddening. Our model that combines sub-micron size dust grains and the Hansen particle size distribution reproduce the observed reddening, which motivates the inclusion of small dust grains in atmosphere models.

This chapter is a reprinting of a paper, of which I am the primary author, submitted to the Astrophysical Journal with co-authors Kelle L. Cruz, Mark S. Marley, Stephanie T. Douglas, and Vivienne F. Baldassare. The manuscript has been reviewed by a referee and is currently being edited for primarily grammatical and aesthetic reasons.

2.1. Abstract

Here, we examine a hypothesis that the red NIR colors of some L dwarfs could be explained by a “dust haze” of small particles in their upper atmospheres in addition to the regular clouds that consist of larger particles. We developed a model which uses Mie theory and Hansen particle size distributions to reproduce the extinction due to the proposed dust haze. We apply our method to 20 young L dwarfs and 36 red field L dwarfs. We constrain the properties of the dust haze including particle size distribution and column density using Markov-Chain Monte Carlo methods. We find that sub-micron range silicate grains reproduce the observed reddening. We also find that Hansen particle size distributions reproduce the shape of the observed reddening better than power law particle size distributions. Current brown dwarf atmosphere models include large grain (1–100 $\mu$m) dust clouds but not ISM-size small dust grains. Our results provide a strong proof of concept and motivate a combination of large and small dust grains in brown dwarf atmosphere models.

2.2. Introduction and Problem

Brown dwarfs are substellar objects with intermediate masses between stars and planets. Brown dwarfs do not have high enough mass to sustain hydrogen fusion in their cores, so they keep cooling with time.
brown dwarfs age, they also contract and their surface gravities increase. The cool temperatures of brown
dwarfs allow condensates to form in their atmospheres, which shape their emergent spectra.

There is a wide spread in $J - K$ colors of L dwarfs. So-called ‘red’ L dwarfs have redder-than-average
NIR colors, and have notably redder spectral slopes through the NIR than normal L dwarfs (20). Low-
gravity objects tend to be systematically redder but some field-aged L dwarfs also have redder colors. Red,
low-gravity L dwarfs are considered ‘exoplanet analogs’ (14) and have been of particular interest (30; 18).
Faherty et al. (20) observationally showed that low-gravity L dwarfs and giant exoplanets have similar dusty
atmospheres.

While the range of NIR colors of L dwarfs has not been fully explained, it is commonly attributed to
variation in metallicity, gravity, and/or cloud properties (47; 39). Brown dwarfs have clouds of refractory
particles typically 1–100 $\mu$m in their atmospheres and these clouds cause extinction and shape their emergent
spectra (1; 16). Current brown dwarf atmosphere models include discrete cloud decks that consist of large
dust grains typically 1–100 $\mu$m. These atmosphere models reproduce the general trend of L dwarf spectra
but fail to give a reasonable explanation for the observed reddening in the NIR. Model SED fitting sometimes
gives unrealistically low gravities and small $f_{sed}$ (cloud sedimentation efficiency parameter) values (16) and
radii too small compared to evolutionary models (33) for red L dwarfs. We need a better dust treatment
that can account for the observed reddening in L dwarf spectra.

In efforts to understand the red objects empirically, it has been found that dust described by the inter-
stellar reddening law can de-redden red L dwarf spectra to look like standard objects (35; 43). Looper et al.
(35) de-redden the optical spectra of TWA 30 (young M5 star) using the interstellar reddening law described
by Cardelli, Clayton, and Mathis (13). Marocco et al. (43) de-redden the spectra of ULAS J222711-004547
(L7pec) using two extinction curves (13; 23). In both cases, the spectra have been successfully de-reddened
by the interstellar reddening law even though small grains are not in the atmosphere models of brown dwarfs.
This result implies that brown dwarfs with red spectral energy distributions may have small grains like ISM
in their atmospheres that scatter and absorb the emergent light.

Interstellar reddening is the extinction of starlight caused by the interstellar dust. The interstellar grains,
which have radii less than 1 $\mu$m, block blue light more effectively than red light. As a result, distant stars look
redder than they actually are. The interstellar reddening law (extinction law) is an empirical relationship
between the absolute extinction and the visible extinction (13). We do not expect significant interstellar
reddening in brown dwarfs since they are so close to the Sun on a galactic scale, so the use of the interstellar
reddening law to de-redden brown dwarf spectra is not physically motivated. However, ISM-like grains in
the atmospheres of red L dwarfs might reproduce the results of the interstellar reddening law and explain
the observed reddening.
In this work, we propose a prescription for a dust haze in L dwarfs (Figure 2.8.1). We aim to understand the physical cause of the reddening in brown dwarfs with the dust haze prescription. The sample of L dwarfs studied in this analysis is presented in §2.3. In §2.4, we present our method of isolating the observed reddening. We explain our dust haze model in detail in §2.5, and model fitting method in §2.6. Finally, we present our results in §2.7 and conclusion in §2.8.

Both Marocco et al. (43) and this work aim to explain the observed reddening in L dwarfs by introducing a layer of small dust grains in their upper atmospheres. This is motivated by the success of the interstellar reddening law in de-reddening unusually red L dwarf spectra. Despite the similar ideas and concepts, the method we describe here is unique and different from that of Marocco et al. (43). As described in §2.5, we use different grain size distributions and grain species to model the dust haze. Also, we carry out Bayesian analysis to constrain the properties of the dust haze as shown in §2.6, while Marocco et al. (43) perform $\chi^2$ minimization. In §2.7.3, we further discuss the similarities and differences between the two studies.
2.3. Sample and Spectral Observations of the Reddening

In order to study the observed reddening, we compiled a sample of low-resolution NIR spectra of 20 L dwarfs with low-gravity features in the optical (14) and 36 red field L dwarfs (31).

Our red field objects have spectral features indicative of field gravity with \( J - K \) colors redder than the spectral standards used in this analysis (31). This definition of ‘red’ is specific to the purpose of this analysis. For example, Faherty et al. (20) defined ‘red’ as having a redder \( J - K \) color than the mean \( J - K \) of normal objects in the spectral type as opposed to comparing to the spectral standard. Therefore, red objects in our sample may be different from objects that are defined red by Faherty et al. (20) or other papers. All objects in our sample are described in Table 1.

All objects were observed with IRTF/SpeX in clear and dry conditions. We used the 0.5" slit and prism-dispersed mode to obtain \( \lambda / \Delta \lambda \approx 120 \) spectra covering 0.7-2.5 \( \mu \)m. Nearby A0 V stars were observed for flux calibration and telluric correction. Internal flat field and Ar arc lamp exposures were obtained for pixel response and wavelength calibration. We reduced data using SpeXtool (52; 15).

2.4. Isolating the Observed Reddening

In order to isolate the observed reddening, we treat the overall shapes of the emergent spectra of both red L dwarfs and field L dwarf spectral standards to be identical, except for the reddening. Therefore, our analysis is independent of physical properties of L dwarfs such as metallicity, radius, and the properties of non-haze clouds. We compared a sample of red L dwarfs (including young and field) to the field spectral standards to isolate and characterize the observed reddening. Table 1 summarizes the objects we studied in this analysis.

Top panels of Figure 2.8.2 show spectra of a red L dwarf (in red) and the spectral standard (in black), illustrating the redder spectral slope of the red object. Bottom panels of Figure 2.8.2 visualize the observed reddening. The observed reddening was obtained by dividing the spectrum of the field standard by the spectrum of the red L dwarf. The small-scale spectral features seen in the observed reddening are due to gravity-sensitive spectral features such as peaky H band. These gravity-sensitive spectral features appear different in low-gravity and field-gravity L dwarfs. We assume these features are not caused by reddening, so we are treating the overall shape of the observed reddening as a smooth curve.

2.5. Dust Haze Prescription

In this paper, we explore a prescription for the hypothesized dust haze of small particles in the atmospheres of red L dwarfs which could be responsible for the observed reddening as illustrated in Figure 2.8.1. Our model prescribes that the dust haze must be high in the atmospheres so that it is too cool (since the
temperature decreases as the altitude increases) to radiate in the NIR, only scatter. The dust haze should also be above the regular clouds to affect the emergent spectra. Our prescription does not have thickness or height, therefore we are not constraining the position or dimensions of the dust haze any further than lying above the main cloud deck.

We hypothesize that the upper atmospheres of red L dwarfs have a “dust haze” of small particles (comparable to the sub-micron size of the ISM) to explain the reddening observed in the SEDs of L dwarfs. Our proposed dust haze could explain the observed reddening in young L dwarfs and so-called ‘red’ field L dwarfs. In Figure 2.8.1, we show an illustration of the proposed dust haze in the upper atmospheres of red L dwarfs.

We used Mie scattering to model the effect of the proposed dust haze on the emergent spectra of red L dwarfs. Mie scattering occurs when the scattering particle is spherical and its size is similar to the wavelength of the scattered light. We used Mie scattering because L dwarf spectra peak in the NIR and we are interested in studying the effects of sub-micron size dust grains in the atmospheres of red L dwarfs. For larger particles, Mie scattering is independent of wavelength while for smaller particles, it is wavelength dependent. Mie scattering reduces to strongly wavelength dependent Rayleigh scattering when particle sizes are much smaller than wavelength.

We utilized Mie extinction curves to model the observed reddening curves. Extinction is the sum of absorption and scattering, and is the fraction of incoming light that gets affected by interactions with particles. Reddening is a type of extinction, where extinction is more effective at shorter (bluer) wavelengths than at longer (redder) wavelengths, making the spectral slope redder.

The dust haze grains were modeled by forsterite grains. As shown by Lodders and Fegley (34), forsterite (Mg$_2$SiO$_4$) is thought to exist in L dwarf atmospheres among other dust species such as corundum (Al$_2$O$_3$), enstatite (MgSiO$_3$), and iron. Corundum condenses at higher temperatures in the atmospheres of late M dwarfs. Liquid iron and silicates condense in early L dwarfs between 1600 – 1840 K. Iron grains form deeper in the atmosphere, so the dust haze is most likely silicate. The silicate grains in the L dwarf atmospheres are thought to be a mixture of forsterite and enstatite. Extinction curves of both forsterite and enstatite have similar shapes which could explain the observed reddening. For simplification, forsterite was used in our analysis.

We computed forsterite extinction coefficients for various radii between 0.01 and 10 $\mu$m at wavelengths for which forsterite refractive indices were available. We used a fortran subroutine that computes Mie scattering by a stratified sphere (a particle consisting of a spherical core and a spherical shell) described in Toon and Ackerman (50) to compute forsterite extinction coefficients for each radius as a function of wavelength. In our analysis, we treated the properties of the core and the shell of a stratified sphere to be the same.
In order to find the relationship between the observed reddening and modeled extinction, we assume that the observed flux, $I$, from a red L dwarf can be modeled as

$$I(\lambda) = fI_0(\lambda)e^{-\tau(\lambda)} \quad (2.5.1)$$

where $I_0(\lambda)$ is the flux of the field standard L dwarf, $f$ is a scaling factor, and $\tau(\lambda)$ is the optical depth of the dust haze in the red L dwarf atmosphere assuming the red L dwarf can be described by the field L dwarf surrounded by the dust haze.

The scaling factor $f$ is determined by the distances and sizes of the objects.

$$f = \frac{d_0^2 R^2}{d^2 R_0^2} \quad (2.5.2)$$

where $d_0$ and $d$ are the distances to the field and the red L dwarfs, and $R$ and $R_0$ are the radii of the red and the field L dwarfs, respectively.

Solving Equation 2.5.1 for the optical depth we get

$$\tau(\lambda) = \ln f + \ln \frac{I_0(\lambda)}{I(\lambda)} \quad (2.5.3)$$

The optical depth $\tau(\lambda)$ is related to the Mie extinction coefficient $Q_{\text{ext}}(\lambda)$ as

$$\tau(\lambda) = N\pi a^2 Q_{\text{ext}}(\lambda) \quad (2.5.4)$$

where $N$ is the column density of the dust haze and $a$ is the effective radius of the scattering grains which we assume are composed of forsterite. $Q_{\text{ext}}(\lambda)$ is the averaged forsterite extinction coefficient which we calculated using the Mie code by Toon and Ackerman (50). We averaged the forsterite coefficients over a particle size distribution $n(r)$ (detail in subsection 2.5.1) and interpolated onto the observed wavelengths so that we can compare with the data.

We assume we can break $\tau(\lambda)$ down to two parts: A wavelength-independent gray component due to big dust particles ($\gg \lambda$) and a wavelength-dependent component that arises from a population of small dust grains ($\lesssim \lambda$).

$$\tau(\lambda) = \tau_{\text{big}} + \tau_{\text{small}}(\lambda) \quad (2.5.5)$$

where $\tau_{\text{big}}$ and $\tau_{\text{small}}(\lambda)$ are the optical depths due to bigger and smaller particles, respectively. $\tau_{\text{big}}$ and $\tau_{\text{small}}(\lambda)$ can be expressed as follows.

$$\tau_{\text{big}} = N_{\text{big}} \pi a_{\text{big}}^2 Q_{\text{big}} \quad (2.5.6)$$

$$\tau_{\text{small}}(\lambda) = N_{\text{small}} \pi a_{\text{small}}^2 Q_{\text{small}}(\lambda) \quad (2.5.7)$$
2.5. DUST HAZE PRESCRIPTION

where \( N_{\text{big}} \) and \( N_{\text{small}} \), \( a_{\text{big}} \) and \( a_{\text{small}} \), \( Q_{\text{big}} \) and \( Q_{\text{small}}(\lambda) \) are column densities, effective radii, and forsterite extinction coefficients of big and small particles, respectively. \( \tau_{\text{big}} \) and \( \tau_{\text{big}} \) are independent of wavelength because \( Q_{\text{ext}}(\lambda) \) approaches a constant for \( a >> \lambda \). These bigger particles would correspond to the regular clouds in L dwarfs. Whereas \( \tau_{\text{small}}(\lambda) \) and \( Q_{\text{small}}(\lambda) \) depend on wavelength and correspond to the hypothesized dust haze.

Substituting Equations 2.5.6 and 2.5.7 into Equation 2.5.5 we get

\[
\tau(\lambda) = N_{\text{big}} \pi a_{\text{big}}^2 Q_{\text{big}} + N_{\text{small}} \pi a_{\text{small}}^2 Q_{\text{small}}(\lambda)
\]

(2.5.8)

Combining with Equation 2.5.3, we get

\[
\ln \frac{I_0(\lambda)}{I(\lambda)} = N_{\text{small}} \pi a_{\text{small}}^2 Q_{\text{small}}(\lambda) + N_{\text{big}} \pi a_{\text{big}}^2 Q_{\text{big}} - \ln f
\]

(2.5.9)

The last two terms are constant with wavelength. So it can be written as

\[
\ln \frac{I_0(\lambda)}{I(\lambda)} = N_{\text{small}} \pi a_{\text{small}}^2 Q_{\text{small}}(\lambda) + C
\]

(2.5.10)

As explained later in §2.6, we used MCMC methods to constrain the parameters on the RHS so that they will reproduce the observed reddening.

2.5.1. Particle Size Distribution of the Dust Haze. In order to smooth over the small-scale interference patterns in forsterite extinction coefficients, we computed ‘effective’ extinction coefficients by averaging the extinction coefficients over a particle size distribution.

Effective extinction coefficients were defined as

\[
Q_{\text{ext}}(\lambda) = \frac{\int_{r_{\text{min}}}^{r_{\text{max}}} \pi r^2 Q_{\text{ext}}(r, \lambda) n(r) dr}{\int_{r_{\text{min}}}^{r_{\text{max}}} \pi r^2 n(r) dr}
\]

(2.5.11)

where \( n(r) dr \) is the number of particles per unit volume with radius between \( r \) and \( r + dr \). The integration limits we employed are 0.01–10 \( \mu m \). Particles smaller than this range would be too small (a few atoms) to scatter light and particles that exceed 10 \( \mu m \) tend to be grey at all wavelengths (i.e. the extinction they cause will be independent of wavelength). Since the reddening we observe is wavelength dependent, we are not interested in a grey component.

We tested two different distributions: a power law distribution and Hansen distribution. In Figure 2.8.4, we compare Hansen, power law, and Gaussian distributions. In the following subsubsections, we describe the motivation to use these particle size distributions and results.
2.5.1.1. Hansen Distribution. The Hansen particle size distribution is used to describe the particle size distribution of water droplets in Earth’s clouds (27). The Hansen particle size distribution successfully reproduces the observed particle size distributions of different types of clouds (fair weather cumulus, altostratus, and stratus clouds) as shown in Figure 1 in Hansen (27). The microphysics and structure of Earth’s clouds may also apply to clouds in brown dwarf atmospheres, so we adopted the Hansen particle size distribution for our analysis.

The Hansen particle size distribution is a variation of the gamma distribution and is expressed as follows.

\[ n(r) = r^{1-3b} e^{-\frac{r}{a}} \]  

where \( a \) is the mean effective radius and \( b \) is the effective variance. According to Hansen (27), \( a \) and \( b \) are defined as

\[ a = \frac{\int_0^\infty \pi r^2 n(r) dr}{\int_0^\infty \pi r^2 n(r) dr} \]  

\[ b = \frac{\int_0^\infty (r - a)^2 \pi r^2 n(r) dr}{a^2 \int_0^\infty \pi r^2 n(r) dr} \]

respectively.

We first computed effective forsterite extinction coefficients on a rough grid of \( a \) and \( b \) using extinction coefficients calculated directly from the code by Toon and Ackerman (50). We then linearly interpolated the coefficients onto a finer grid. In Figure 2.8.3, we show our model grid of forsterite extinction coefficients for various Hansen particle size distributions.

By eye comparison, we found that forsterite curves for small \( a \) resemble the observed reddening. For small particle sizes, extinction coefficients are wavelength dependent and resemble power law shapes of the observed reddening (Figure 2.8.3). For particles greater than 0.4 \( \mu m \), extinction coefficients are less wavelength dependent and the shapes are flat (‘grey’), which do not fit the observed reddening. Thus, we decided to use Hansen particle distribution with a parameter grid of \( a \) between 0.05 and 0.4 \( \mu m \), \( b \) between 0.1 and 1.0 for the rest of our analysis. Detailed model fitting is described below in §2.6.

2.5.1.2. Power Law Distribution. We also considered power law particle size distributions \( n(r) \propto r^{-p} \) to model theoretical extinction due to the proposed small dust grains. Power law particle size distributions with \( p \approx -3.5 \) are typically used to characterize interstellar dust and grains in the circumstellar disks around young brown dwarfs. (42; 17; 36; 10) We considered two different power law particle size distributions with \( p = -3 \) and -3.5 shown in Figure 2.8.4.
In the bottom left panel of Figure 2.8.3, we show forsterite extinction coefficients averaged over the two power law particle size distributions. Compared to the top panel of Figure 2.8.3, these curves are much flatter and do not reproduce the shapes of the observed reddening.

Marocco et al. (43) used a power law grain size distribution $n(r) = r^{-2.5}$ for iron to de-redden ULAS J222711−004547 and found the maximum grain size to be 0.3 $\mu$m.

2.5.1.3. **Gaussian Distribution.** Marocco et al. (43) adopted a Gaussian distribution to de-redden red L dwarfs using corundum and enstatite with a width $\sqrt{2\sigma} = 0.1 \times \mu$ where $\mu$ is the characteristic grain radius. They de-reddened ULAS J222711−004547 using corundum and enstatite, and other red L dwarfs using corundum. They found $\mu \sim 0.5\mu$m.

In Figure 2.8.3, we show forsterite extinction coefficients averaged over the two Gaussian particle size distributions. These extinction curves do not reproduce the shapes of the observed reddening as well as the extinction curves averaged over the Hansen particle size distributions. As shown in Figure 2.8.4, the Gaussian particle size distribution is concentrated at $\mu$.

### 2.6. Methods: Fitting the Model to the Data

We use Markov-chain Monte Carlo (MCMC) fitting to estimate the best-fit parameters and their uncertainties. MCMC is a Bayesian inference method provides a sampling approximation of the posterior probability distribution function (PDF). An MCMC run produces a chain of positions in parameter space, and a histogram of these positions provides the approximation of the posterior PDF. MCMC allows for more in-depth probabilistic data analysis than, e.g. $\chi^2$ minimization, because it efficiently approximates the full posterior PDF, which in turn provides uncertainties on and illustrates covariances between model parameters.

One of the most straightforward MCMC algorithms is the Metropolis-Hastings (M-H) algorithm, which generates a single chain of parameter sets that approximate the posterior PDF. At each step in the fit, a trial position in the parameter space is randomly generated based on the last position in the chain. A different step size is assigned for each parameter to determine how far away the trial position is from the last position. The probabilities at the trial and last positions are compared, and if the trial position is more probable, then the corresponding parameters are added to the chain and the process repeats. If the trial position is less probable, there is some probability that the corresponding parameters are added to the chain, and the rest of the time the last position is duplicated again. This builds up the chain of parameter positions which approximates the posterior PDF. (see, e.g., Ford (24) for a step-by-step description of the M-H algorithm.) Although the M-H algorithm is powerful, the step sizes must be hand-tuned for efficiency; this may be impossible in cases with many covariant parameters or an otherwise complex probability space.
The Goodman-Weare (G-W) algorithm improves upon the M-H algorithm by changing the method for choosing trial positions (26). The G-W algorithm deploys an ensemble of chains, known as “walkers”, instead of a single chain. The trial position for each walker is chosen from the ensemble’s location in parameter space, with some probability for choosing a position outside the occupied region. This method does not require hand-tuning the step size for each parameter, and the selection of trial positions can be parallelized. The G-W algorithm more efficient than the M-H algorithm in both human working hours and computation time. (26; 25)

We use the open-source python implementation of the G-W algorithm, *emcee* (25), to fit the observed reddening with a set of forsterite extinction curves. The extinction curves are parameterized by the mean particle size $a$ and effective variance $b$ for the Hansen distribution and the column density $N$ of forsterite grains. The extinction at each wavelength point is calculated as Equation 2.5.10.

We also model the vertical offset between the observed reddening and the extinction curve with a constant $C$, and include a tolerance parameter $s$. The tolerance $s$ estimates the uncertainty in the model as a single value across the extinction curve; it accounts for the fact that the photon-noise uncertainties are smaller than the typical difference between each observed reddening point and the corresponding point on the extinction curve. If we denote observed reddening points as $r = \{r_i\}$ and the corresponding uncertainties as $\sigma_0 = \{\sigma_{0,i}\}$, we compute the natural logarithm of the likelihood function as

$$
\mathcal{L}(\{r_i\}|a, b, N, C, s, \sigma_0) = -\frac{1}{2} \sum_i \left( \frac{(r_i - Q_i)^2}{\sigma_{0,i}^2 + s^2} + \ln(2\pi(\sigma_{0,i}^2 + s^2)) \right)
$$

The natural logarithm of the posterior PDF is given by

$$
\ln(\text{PDF})(a, b, N, C, s|\{r_i\}, \sigma_0) = \mathcal{L}(\{r_i\}|a, b, N, C, s, \sigma_0) + P(a, b, N, C, s, \sigma_0)
$$

We assume a flat prior on each parameter, so $P(a, b, N, C, s, \sigma_0) = 0$.

We pass a function for $\ln(\text{PDF})$ to *emcee*, which uses that function to determine acceptance of each step in parameter space. We typically use 100 walkers. After we iterate for 200 steps to generate a new set of initial positions for the walkers, we reset the walkers and restart from the new initial positions. We iterate for 2000 steps after a burn-in period of 200 steps. At the end of the run, we plot the 1D and 2D marginalized posterior PDFs for all parameters; these plots are shown in Figure 2.8.5. Models corresponding to 100 randomly-drawn parameter sets from posterior PDF are shown with the data in Figure 2.8.61.

2.7. Results and Discussion
2.7. RESULTS AND DISCUSSION

2.7.1. Fitting Dust Haze Parameters. We used the MCMC method described in §2.6 to fit dust haze extinction models to the observed reddening and constrain the physical properties of the proposed dust haze. The constrained properties of the dust haze include mean effective radius, effective variance of the Hansen distribution (§2.5.1), and column density of the dust haze. Figure 2.8.5 and 2.8.61 show the posterior distribution for each parameter and model fits for each of the 61 objects in our sample.

In Figure 2.8.5, we show the posterior distributions for each parameter. Each figure has 1-D distributions for the parameters and 2-D contours for each combination of parameters. Gaussian-like 1-D distributions and round 2-D contours indicate no covariances. Quantiles (16, 50, 84 %) are shown with dashed lines and are used to calculate the uncertainties on the parameter fits. In many objects, the PDFs for the mean effective radius $a$, column density $N$, tolerance parameter $log(s)$, and vertical offset constant $C$ have clear peaks and therefore are well constrained. The variance $b$, on the other hand, does not have clear peaks and is not well constrained in most objects. There is a correlation between parameters $a$ and $N$ as seen in the 2-D contours. The relationship between $a$ and $N$ is shown in Equation 2.5.10. In order to compute the optical depth, we multiply the column density $N$ by the scattering cross section $\pi a^2$. The extinction curves are almost the same over a small range of grain radii, so $a^2$ and $N$ are inversely proportional and this relation appears in the posterior distributions.

In Figure 2.8.61, we show the resulting model fits to the observed reddening. The black line is the observed reddening (Figure 2.8.2) and the green lines are 100 models randomly drawn from the posterior distributions. The models reproduce the overall shape of the observed reddening.

In Figure 2.8.117, we compare a de-reddened spectrum of a red L dwarf (green), the spectrum of the field standard L dwarf (black), and the original red L dwarf spectrum (red). The de-reddened spectrum is the spectrum of a red L dwarf corrected by the best-fit forsterite extinction curve determined by the MCMC analysis. The de-reddened spectrum looks much closer to the standard spectrum than the original red L dwarf spectrum. The submicron-size dust haze prescription successfully corrects the red $J - K$ colors of L dwarfs.

In Figure 2.8.173, we show improvement in $\chi^2$ due to the proposed dust haze prescription. The ratio of $\chi^2$ before de-reddening to $\chi^2$ after de-reddening is plotted against $\Delta(J - K)$ color, which is the difference in $J - K$ color between the red L dwarf and the field standard L dwarf. We use $\Delta(J - K)$ because we compare the spectra of red L dwarfs to the standards to isolate the observed reddening. The value of $\chi^2$ is improved for all objects, which shows that the de-reddened spectra fit the field standards much better than the original red L dwarf spectra.
2.7.2. Lack of Correlation with Gravity. It has been widely noted that low-gravity L dwarfs have redder NIR SEDs compared to the field-gravity spectral standards. One might expect to see a correlation between dust haze properties and low-gravity spectral features. We hypothesized that the proposed dust haze might dissipate over time due to grain growth by condensation. Large condensed particles are expected to fall out of the dust haze as a result of the sedimentation rate exceeding the remixing rate by eddy turbulence (38). Increasing surface gravity with age could also contribute to dust settling. Thus, young, low-gravity L dwarfs might have optically thicker dust hazes which may explain their red NIR colors. The proposed dust haze could also explain the reddening within field L dwarfs and the properties of the dust haze might be correlated with age.

Even though a correlation between dust haze properties and gravity is expected, there is no apparent difference in $a$ and $N$ between low-gravity and red field L dwarfs. In Figures 2.8.174 and 2.8.175, we show scatter plots of mean effective radius $a$ and column density $N$ versus $\Delta(J - K)$ color, respectively. Green symbols denote low-gravity objects and magenta denote field-gravity. Circles denote objects with PDFs with clear peaks, while squares and diamonds denote objects with unclear peaks for some of the parameters.

There are several possible explanations for this inconsistency between our hypothesis and results.

First, the lack of correlation between the dust haze properties and gravity might be due to a model grid not spanning a wide enough range of effective variance $b$. We consider $0.1 < b < 1.0$ because Hansen particle size distributions for large $b$ look like a power law size distribution (Figure 2.8.4; Equation 2.5.12). The effective variance was not well constrained for many of the objects. This makes the results for these objects somewhat unreliable. In most of those objects, $b$ tends to hit the upper limit and therefore is not well constrained.

Second, our initial assumptions about low-gravity and field L dwarfs may be unrealistic. We assumed that low-gravity and field L dwarfs with the same base spectral type (e.g. L2 and L2γ) have the same effective temperatures and the low-gravity L dwarf has the hypothetical dust haze of small grains in the upper atmosphere. However, recent evidence suggests that low-gravity L dwarfs do not necessarily share the same physical properties with field L dwarfs just because they share the same base spectral type (37; 22). Young, low-gravity L dwarfs might have cooler effective temperatures than field L dwarfs in the same spectral classification (20). This suggests that an earlier type standard would be a better comparison. We could be comparing objects with different effective temperatures and thus, not setting accurate estimates of the dust haze properties. Since the overall shape of a spectrum is very sensitive to the effective temperature, comparing objects with the same effective temperatures might be more useful for our analysis. Our hypothesis and method would still be robust and reliable, except that it might be more appropriate to compare red
objects to field objects with the same effective temperatures instead of field spectral standards. We will explore this topic in future work.

Finally, the overlapping distributions of $a$ and $N$ may indicate that the dust hazes of low-gravity and red field L dwarfs are not different from one another. It might be the case that gravity does not play a major role in determining the properties of the dust haze and the same distribution of dust haze properties exist in both low-gravity and red field L dwarfs. Some red field L dwarfs might have a dust haze for reasons other than gravity.

### 2.7.3. Comparison to Marocco et al. (43)

Independently, Marocco et al. (43) (hereafter, M14) present results from a similar analysis. Like us, they are motivated by the utility of the interstellar extinction law in de-reddening red L dwarf spectra, use Mie theory to characterize a high-altitude population of sub-micron dust grains, isolate the observed reddening by comparing red L dwarf spectra to spectral standards. However, M14 consider corundum ($\text{Al}_2\text{O}_3$), enstatite ($\text{MgSiO}_3$), and iron while we use forsterite ($\text{Mg}_2\text{SiO}_4$). We use forsterite as a test dust particle because extinction curves for forsterite, enstatite –both of which are silicates– and corundum all behave similarly in the near infrared. We use silicate instead of iron because silicate grains form higher in the atmosphere than iron. Actual dust grains in the brown dwarfs atmospheres are most likely a mixture of these species so we will consider enstatite and corundum in future work.

As discussed in §2.5.1, M14 used Gaussian grain size distributions for enstatite and corundum to model the extinction due to the dust haze. For iron, they used a power law grain size distribution because the fit was poor with a Gaussian grain size distribution. As described in §2.5.1.2, we found that power law grain size distributions do not reproduce the observed reddening. Instead, we found that Hansen grain size distributions do provide good results. As shown in Figure 2.8.4, Hansen distributions include greater amounts of small particles compared to Gaussian distributions, but they resemble Gaussian for small variances.

Our model includes more parameters than M14. They have two parameters: characteristic grain radius $r$ and normalization of the extinction curve at 2.20 $\mu$m. Our model includes mean effective grain radius $a$, effective grain size variance $b$, column density $N$, vertical offset $C$, and tolerance factor $s$ as described in §2.5. $N$ and $C$ account for normalization as shown in Equation 2.5.10. M14 perform $\chi^2$ minimization to select the best fit parameters and therefore do not have uncertainties, while we use MCMC to determine the best fit parameters and their uncertainties.

The range of the characteristic grain radius M14 found for corundum and enstatite are slightly larger than what we found. They obtained $r = 0.4 - 0.6 \mu m$ for corundum and enstatite, while our results for the mean effective radius for forsterite are generally smaller ($0.15 - 0.35 \mu m$). Their results for the maximum radius of iron are $0.15 - 0.3 \mu m$, closer to the values we found for forsterite grains.
M14 applied their method to 5 red L dwarfs, and we applied our method to 58 red L dwarfs including low-gravity and field-gravity. They used objects with later spectral types (L5 – L7), so there is only one common object between their sample and ours. 2MASS 0355+1133 is used in both studies but compared to different spectral standards. M14 used SDSS J0835+1953 as the L5 standard while we used 2M1507. They found the characteristic grain radius for 2MASS 0355 to be 0.4 \( \mu m \) and we found the mean effective radius to be \( 0.3^{+0.03}_{-0.02} \mu m \).

Regardless of the different methods, these two studies show similar results. The reproducibility of the results demonstrates the viability of sub-micron size dust grains, and warrants further study and inclusion of small dust grains in future atmosphere models.

2.8. Conclusion and Summary

Motivated by the success of the interstellar reddening law, we proposed and tested the validity of a prescription of a dust haze of small particles in the upper atmospheres of red L dwarfs that can possibly explain their red NIR colors. The success of the interstellar reddening law in de-reddening L dwarf spectra led us to investigate the possibility of a population of small ISM-size grains in L dwarf atmospheres.

In order to isolate and characterize the reddening, we compared spectra of red L dwarfs to field spectral standards. The observed reddening was treated as smooth power-law shaped curves.

We used Mie theory to model the dust haze of small particles with theoretical extinction curves. We also found that Hansen grain size distributions reproduce the shape of the reddening curve better than power law grain size distributions.

We used a Markov-Chain Monte Carlo algorithm to fit the theoretical extinction curves to the observed reddening in order to find the best-fit values and uncertainties for the properties of the dust haze. We applied this method to 20 L dwarfs with low-gravity spectral features and 36 field L dwarfs with redder \( J - K \) colors than the spectral standards. We found that small forsterite grains (\(< 0.5 \mu m\)) can reproduce the observed reddening. There is no apparent difference in grain properties between low-gravity and field L dwarfs. The calculated column densities of forsterite grains are reasonable compared to typical brown dwarf atmospheres (\(\sim 10^8 \text{ cm}^{-2}\)). These results suggest that a dust haze of small spherical particles with a Hansen particle size distribution can explain the observed red NIR spectral energy distributions of brown dwarfs.

In order to further study the role of small grains in brown dwarf atmospheres, future work will include other grain species such as corundum and enstatite. This work is a proof of concept and it provides a strong motivation for including small dust grains in future atmosphere models of brown dwarfs.

The dust haze analysis can be applied to other studies including variability in brown dwarfs, exoplanet atmospheres, and the interstellar/intergalactic medium.
Figure 2.8.1. A conceptual representation of our dust haze model. The regular clouds of large particles (∼10 µm, grey cloud symbol) exist in both normal and red L dwarfs. An additional haze of small particles (green layer) is present in the red L dwarf atmosphere, which causes the observed reddening. We do not know yet the specific location of the dust haze but it has to be at an altitude where the temperature is low enough so that the dust grains do not emit in the near IR.
Figure 2.8.2. Top panel shows spectra of a red L0 dwarf spex prism 0141-4633 (red) and field standard L0 dwarf 2M0345 (black). Both spectra are normalized by the mean flux. The young object has excess flux longward of 1.5 \( \mu m \). Bottom panel shows the observed reddening for the same object. The observed reddening was derived by dividing the spectrum of the standard object by the spectrum of the red object and taking the log of the flux ratio. The overall shape of the observed reddening resembles a power law curve.
Figure 2.8.3. Top panel shows forsterite extinction coefficients according to Mie theory averaged over Hansen particle size distributions with various combinations of mean effective radius \( a \) and effective variance \( b \). Different colors correspond to different effective radii. Different line style correspond to different effective variance. The shapes of the extinction curves for smaller particles \((0.1 - 0.4 \, \mu m)\) resemble the observed reddening but larger particles \((1.0 \, \mu m)\) do not. Bottom right panel shows forsterite extinction coefficients averaged over power law particle size distributions with indices of -3 and -3.5. The extinction curves are too flat and do not reproduce the observed reddening. Bottom right panel shows forsterite extinction coefficients averaged over Gaussian particle size distributions mean radius of 0.3 and 0.5 \( \mu m \).
2.8. CONCLUSION AND SUMMARY

Figure 2.8.4. Comparison of different particle size distributions. The gray lines are the power law distributions with power indices of -3 and -3.5. The magenta is a Gaussian distribution with characteristic grain size of 0.5 μm. The green lines are Hansen distributions for $a = 0.2 \mu m$ and $b = 0.1, 0.5$. The Hansen distributions are a variation of the gamma distribution and they are wider than the Gaussian, but narrower than the power law distributions. The Hansen distributions can reproduce the shapes of the observed reddening better than the other distributions.
Figure 2.8.5. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.6. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.7. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.8. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.9. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.10. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.11. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.12. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.13. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.14. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.15. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.16. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.17. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.18. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.19. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.20. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.21. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.22. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.23. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.24. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1σ uncertainties.
Figure 2.8.25. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.26. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.27. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.28. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.29. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.30. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.31. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.32. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.33. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and $1 \sigma$ uncertainties.
Figure 2.8.34. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.35. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1σ uncertainties.
Figure 2.8.36. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.37. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.38. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.39. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.40. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1σ uncertainties.
Figure 2.8.41. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.42. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.43. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.44. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.45. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and $1\sigma$ uncertainties.
Figure 2.8.46. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.47. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1σ uncertainties.
Figure 2.8.48. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.49. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.50. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.51. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and $1 \sigma$ uncertainties.
Figure 2.8.52. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.53. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and $1\sigma$ uncertainties.
Figure 2.8.54. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1σ uncertainties.
Figure 2.8.55. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.56. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 $\sigma$ uncertainties.
Figure 2.8.57. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.58. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 \( \sigma \) uncertainties.
Figure 2.8.59. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.60. Posterior distributions of mean effective radius, effective variance, tolerance, column density, and offset showing 1-D distributions for each parameter and 2-D distributions for each combination of parameters. Dashed lines in the 1-D distributions represent 16, 50, 84 percent quantiles, corresponding to the median and 1 σ uncertainties.
Figure 2.8.61. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.63. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.64. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
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Figure 2.8.67. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.68. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.69. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.70. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.71. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.72. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.73. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.74. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.75. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.76. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.78. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.79. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.80. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.81. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.82. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.83. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.84. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.85. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.86. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.87. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.89. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.91. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.92. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.93. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.94. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.95. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.96. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.97. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.98. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.99. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.100. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.101. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.102. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.103. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.104. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.105. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.106. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.107. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.108. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.109. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.110. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.111. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
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Figure 2.8.116. Green lines are 100 models randomly drawn from the posterior distribution overplotted on the observed reddening in black. The observed reddening was derived by dividing the spectrum of the spectral standard object by the spectrum of the red object and taking the log of the flux ratio. The models reproduce the overall shape of the observed reddening well.
Figure 2.8.117. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.118. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. \( \chi^2 \) values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller \( \chi^2 \) value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
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Figure 2.8.120. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. \( \chi^2 \) values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller \( \chi^2 \) value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.121. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
2.8. CONCLUSION AND SUMMARY

Figure 2.8.122. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
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Figure 2.8.130. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.131. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. \( \chi^2 \) values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller \( \chi^2 \) value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.132. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
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Figure 2.8.140. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
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Figure 2.8.142. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.143. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.144. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
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Figure 2.8.161. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.162. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. \( \chi^2 \) values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller \( \chi^2 \) value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
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Figure 2.8.164. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.165. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.166. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.167. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.168. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. \( \chi^2 \) values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller \( \chi^2 \) value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
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Figure 2.8.170. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
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Figure 2.8.172. The spectrum of a red L dwarf de-reddened by the dust haze prescription is compared to the spectra of the field standard object. Black is the standard, red is the red object, and green is the spectrum of the red object de-reddened by the forsterite extinction curve for the best fit parameters from MCMC fits. $\chi^2$ values before and after the de-reddening are also shown. The de-reddened spectrum fits the standard spectrum much better and has a smaller $\chi^2$ value than the original red spectrum. This shows that the proposed dust haze prescription successfully corrects the red NIR slopes of red L dwarfs to look like the standard object.
Figure 2.8.173. Improvement on $\chi^2$ due to the dust haze prescription. The ratio of $\chi^2_{\text{before}}$ to $\chi^2_{\text{after}}$ is plotted against $\Delta(J - K)$ color. $\chi^2_{\text{before}}$ is $\chi^2$ between standard and red L dwarf spectra, and $\chi^2_{\text{after}}$ is $\chi^2$ between standard and corrected red L dwarf spectra. Green markers denote low-gravity L dwarfs and magenta markers denote field-aged red L dwarfs. In all objects, $\chi^2$ value is reduced after the correction.
Figure 2.8.174. A scatter plot of mean effective radius $a$ [µm] against $\Delta(J - K)$ color. Green markers denote low-gravity L dwarfs and magenta markers denote field-aged red L dwarfs both ranging between L0 – L5. Circles denote objects with PDFs with clear peaks. Diamonds denote objects with PDFs for $b$ hitting the limit. Thin diamonds denote objects with PDFs for $a$ hitting the limit. Squares denote objects with PDFs for both $a$ and $b$ hitting the limits. Our sample includes objects with $\Delta(J - K) < 0.1$. There is no noticeable trend in radius in relation to color. There is also no visible difference between young and red field L dwarfs.
Figure 2.8.175. A scatter plot of column density $N \, [10^8 \text{cm}^{-2}]$ against $\Delta(J - K)$ color.
Table 1: L dwarfs used in this paper

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Red field L dwarfs
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CHAPTER 3

Future Work and Summary

3.1. Incorporating the Dust Haze into Atmospheric Models

The success of the sub-micron size dust grains described in Chapter 2 motivates an incorporation of such small dust grains into current atmospheric models of brown dwarfs. As described in Chapter 1, atmospheric models compute structures and spectra with a set of assumptions. The incorporation of sub-micron size dust grains would involve changing the assumptions made for clouds in the atmospheric model to the properties of the dust haze, specifically the particle size distribution and column density. The free parameters for the dust haze would include the mean effective radius $a$ and the effective variance $b$ of the Hansen particle size distribution, the column density $N$, and the haze height. Spatially non-uniform dust hazes may be described by additional geometrical parameters.

Current atmospheric models of brown dwarfs reproduce the general trend of red L dwarf spectra, but model SED fitting sometimes results in unreasonable physical parameters like too low surface gravities and too small radii (16; 33). If the sub-micron size dust grains properly reflect the dust properties of dusty brown dwarfs, the modification will not affect the temperature-pressure profile and the resulting spectra will better reproduce the spectra of dusty brown dwarfs. Specifically, the modified spectra should explain the red spectral slopes of these brown dwarfs with realistic physical parameters.

3.2. Variability in L/T Transition Brown Dwarfs

The spatially non-uniform dust haze of small grains could possibly explain the photometric variability observed in some brown dwarfs.

Artigau et al. (5) and Radigan et al. (45) observed an early T dwarf with large-amplitude variability in the infrared. This observed variability cannot be reproduced by atmosphere models with a single set of parameters. They have found that combinations of models are able to reproduce the variability. Figure 3.2.1 shows the $J$-band variability of a T1.5 dwarf 2M2139. Figure 3.2.2 shows model fits to the NIR spectrum of 2M2139 and photometric variability of for linear combinations of 1-D cloud models. The good fits of these combinations of models indicate that the variability of L/T transition objects is possibly due to a combination of thin and thick clouds, or ‘patchy’ clouds.
Recent observations of a T2 dwarf SIMP1629+03 (an unpublished discovery from Artigau et al. (6)) revealed its unique variability response. Unlike other L/T transition brown dwarfs, the water absorption band ($\sim 1.35 - 1.5 \mu m$) is absent, which could be indicative of a high altitude patchy haze layer. According to the dust haze model described in Chapter 2, this ‘masking’ of the water band could be explained by a dust haze with large grain sizes. Further observations and analyses of SIMP1629+03 could be used to constrain the properties of the dust haze in this L/T transition object and possibly provide evidence of grain growth in the dust haze.

3.3. Exoplanet Atmospheres

Planets, like L dwarfs, have clouds and hazes in their atmospheres. The closest thing to exoplanetary atmospheres that have been observed is brown dwarf atmospheres. The similarities between brown dwarfs and exoplanets make brown dwarfs great laboratories for studying exoplanetary atmospheres. Figure 3.3.1 illustrates clouds in the atmospheres of different types of objects. Condensates form in planetary atmospheres the same way as in brown dwarf atmospheres, except different kinds of grain species are present in planetary atmospheres due to the cooler temperatures (46). In Jupiter-like atmospheres, refractory grains such as silicates, iron, and corundum (also present in brown dwarf atmospheres) form deeper in the atmosphere, and volatile species (that do not exist in brown dwarf atmospheres) form higher in the atmosphere.
Figure 3.2.2. Figure from Radigan et al. (45). In each panel, simultaneous model fits to both the NIR spectrum of 2M2139 (black line) and photometric variability (black filled circles) are shown. The top and bottom panels show one of the two best-fitting model combinations.

Figure 3.3.2 is a color-magnitude diagram of brown dwarfs and planetary mass companions. Planets (triangles), like low-gravity brown dwarfs (circles with black bordering), have redder $J-K$ colors than field-gravity brown dwarfs (small filled circles). The red colors of exoplanets may also be attributed to sub-micron size dust grains similarly to red L dwarfs.
Figure 3.3.1. Figure from Marley (41). Comparison of clouds in the atmospheres of young giant planets like HR 8799c, T dwarfs, and Jupiter. Clouds in the atmospheres of young gas giants may be similar to clouds in young brown dwarf atmospheres. At cooler temperatures mature giant planets like Jupiter, a wider variety of grain species may be present.

Young gas giants like HR 8799c may have dust properties similar to those of dusty brown dwarfs which may be modeled with the same sub-micron size silicate dust grains used to describe dust in brown dwarf atmospheres in Chapter 2. Dust in the atmospheres of cooler gas giants may also be modeled similarly with volatile grain species. Nonuniform dust hazes can be used to characterize patchy clouds seen in Jupiter and Earth, and possibly shared among exoplanets.

Furthermore, the dust haze analysis may also be applied to habitability studies. Our dust haze model uses the Hansen particle size distribution, which is used to describe the size distribution of water droplets in Earth’s clouds, so it is likely applicable to clouds in other terrestrial planetary atmospheres. Clouds in terrestrial planetary atmospheres are important for habitability. In particular, detection of water clouds implies the existence of water, which is the primary condition for habitability. The atmospheric opacity of a planet affects its albedo, which is the ratio of reflected light from the surface of a planet to incident light. The albedo determines the equilibrium temperature of the planet as well as the greenhouse effect. The greenhouse effect from clouds affects the habitability by absorbing and re-emitting, or scattering thermal radiation back to the surface.

The dust in the atmospheres of terrestrial planets may be characterized by a dust haze that consists of water droplets and aerosol. By studying the atmospheres of both habitable and non-habitable planets, we may reveal atmospheric conditions for habitability.
3.4. Summary

In this dissertation, we have described our efforts to test the validity of a hypothesized dust haze of sub-micron size grains in the upper atmospheres of red L dwarfs as a possible explanation for their red spectral slopes. This work was motivated by the success of the interstellar reddening law in de-reddening red L dwarf spectra.

We used low-resolution NIR spectra of L dwarfs to isolate the reddening, which was modeled by theoretical extinction curves calculated using Mie theory and Hansen grain size distributions. In order to fit the theoretical extinction curves to the observed reddening, we used a Markov-Chain Monte Carlo method. We applied our method to 56 red L dwarfs and found that a dust haze of small (< 0.5 µm) can reproduce the observed reddening.

Figure 3.3.2. This is a color-magnitude diagram of brown dwarfs and planetary mass companions from Faherty et al. (21). Small filled circles are field-gravity brown dwarfs, big circles with black bordering are low-gravity brown dwarfs, and triangles are planets. Colors separate spectral classes. The low-gravity brown dwarfs and planets have redder \( J - K \) colors than the field-gravity objects.
This is a proof-of-concept work which motivates an incorporation of small grains into atmosphere models of brown dwarfs. A proscription of a dust haze may also be applied to exoplanet atmospheres and variability in brown dwarfs.
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