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Optimal Targeting Regimes and Instrument Rules in the Basic New Keynesian Model

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OPTIMAL TARGETING REGIMES AND INSTRUMENT RULES IN THE BASIC NEW
KEYNESIAN MODEL

by

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Abstract

OPTIMAL TARGETING REGIMES AND INSTRUMENT RULES IN THE BASIC NEW
KEYNESIAN MODEL

by

Luc Marest

Adviser: Professor Thom Thurston

In the first chapter, the objective is to measure the value of commitment in executing

monetary policies in the context of the New Keynesian model and with value that is represented

by the standard quadratic welfare function containing weighted output gap and inflation

variances. It is found that there is a substantial potential for improvement of welfare under

commitment, but this depends on some key parameters in the model. The range of parameter

calibrations most often found in the literature, however, suggests the improvements will be large.

We then consider specific monetary policies akin to Taylor rules. From the optimality

conditions we derive a Taylor-type rule that meets the optimal path conditions exactly. Further,

we derive a form which is actually guaranteed to produce a determinate outcome the model's

endogenous variables and welfare.

The first chapter outlined the baseline of the New Keynesian model and provided an

analytical solution to measure welfare for discretion, pre-commitment and global policies. The

analysis was done under the presumption that policy makers seek to maximize the standard

social welfare. The second chapter measures the benefit of "delegating" authority to the central

bank to maximize other welfare functions. It is assumed that the public is convinced the central

bank will maximize this particular welfare on the long run. These alternative approaches are

referred as speed limit, price level targeting, and nominal income growth targeting. The chapter shows to what extent some of those approaches may provide higher welfare than when adopting the global target and how it depends on the parameters. The chapter shows also that another alternative would be to including volatility of interest rate in the welfare function. While this strategy would not often provide a higher welfare than the global target, it has the advantage of making the volatility of interest rate low, which could be beneficial when trying to avoid the lower bound issue.

The third chapter investigates on how to implement the policy that will replicate the optimal paths found in previous chapter for each of the approaches. In the New Keynesian model, the most common procedure that central banks use is the Taylor rule which links the nominal interest rate with inflation and output gap. However, this raises questions about determinacy. This chapter demonstrates that when using a money rule, one which uses money as the instrument, determinacy constraints are not very different from the ones encountered with an interest rate rule. In the same vein, the adoption of a Wicksellian rule (the interest rate or money responds to the price level and output gap) does not make it easier to find a stable and unique solution.

Preface

The financial crisis of 2007-2008 and the role of the central bank to deal with the situation has been the source of a vast literature. It re-emphasized the need to examine the benefit of different monetary policies. This thesis is about the gain in welfare of adopting different targeting approaches and the choice of instrument rules to implement monetary policies.

At the origin of the thesis were two questions that Professor Thom Thurston asked me to work on as part of the Monetary Theory class at the CUNY Graduate Center. How do we calculate welfare? Would the use of a money rule help to deal with the problem of indeterminacy encounter when using interest rate rules such as the Taylor rule? It has been the starting point of a very fruitful collaboration. One of the main contributions of the thesis to the literature is that I was able to find an analytical solution for the calculation of welfare. The first chapter is the result of our common effort. Furthermore, the third chapter is the extension of an original idea of Professor Thurston.

I would like to thank Professor Thurston for his patience, availability and constant willingness to listen to my comments and to share his point of view.

I would like to thank also Professor John Devereux and Professor Merih Uctum for their valuable comments.

I want to emphasize that any error is my sole responsibility.

Luc Marest
New York, NY
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CHAPTER 1

The value of central bank commitment to the price level
in the baseline New Keynesian model

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1.0 Introduction

In the "Science of Monetary Policy" Clarida, Gali, and Gertler (1999) derived a "global

optimum" policy commitment and the associated theoretical optimal paths for endogenous

variables (output gap, price level, inflation, among others) in the baseline New Keynesian Model

(NKM). This followed a discussion of paths they had derived for the more-familiar

"discretionary" paths where authorities cannot commit to future policies, and inflation-

commitment (which they called "commitment") paths where the authority can commit to follow

a policy that optimizes over current and expected output gaps and inflation. In this chapter we

will call this level of commitment "inflation-commitment."[1] The key difference between the

"global" and the earlier alternatives is the assumption that authorities commit to a plan to control

the level of future prices rather in a particular way than just inflation rates. We will call this

"price level-commitment" in our paper. As this chapter shows with various parameter

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[1] The use of the word "commitment" to refer to commitment limited to future inflation rather than prices, appears to have been short-lived. In Gali (2008) the term "commitment" (optimum) is used to refer to the "global optimum" policy in Clarida, Gali, and Gertler (1999). A policy which sets policy to a first-order condition that involves expected period-ahead inflation (formerly, "commitment") is not mentioned in his book. We do consider it and call it "inflation-commitment" in this paper.

assumptions, this commitment to price level may have considerable value in terms of welfare, depending on the value of certain key parameters. It also has some important side-effects like reducing the volatility of the interest rate and eliminating the possibility of trend inflation.

It is important to emphasis that in conventional use, as in "Science of Monetary Policy" Clarida, Gali, and Gertler (1999), "discretionary" refers to the time-consistent solution. It is considered a Nash equilibrium. Given a situation with no commitment, the central bank will use at each period a first order condition that is developed using the two current values of x and π. The public recognizes that condition and forms expectations based on the point where the marginal cost of choosing a particular inflation rate just equals its benefit. However, if commitment power is added, then the first order condition and the target change.

The sections are organized as follows. With the baseline NKM model we (Section 1) compare the paths of the endogenous variables in this set-up with the more familiar "discretionary" and "inflation-commitment" regimes. We then (Section 2) show analytically how price-level commitment improves welfare and how much this improvement is sensitive to parameter values. Using calibrations we (Section 3) measure the magnitudes by which welfare is improved by commitment and how these measurements vary with parameter calibration. We also (Section 4) consider some side-effects of price level commitment, in particular on trend rates of inflation (they are eliminated) and in the volatility of interest rates (nominal volatility is reduced; what will happen to real interest rate volatility depends on the parameter structure). We (Section 5) derive a modified Taylor rule which will bring about the price-level commitment paths under this theory. Finally (Section 6) we analyze the determinacy conditions for solutions to the global model, finding the there exists a Taylor rule which satisfies the optimality conditions as well as determinacy.

1.1 The NKM baseline model and paths of endogenous variables

The baseline NKM consists of a Calvo-type Phillips Curve (1.a) and a NKM IS curve

(1.b). Variables are expressed as deviations from steady-state.

$$(1.a) \quad \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

$$(1.b) \quad x_t = E_t x_{t+1} - \frac{i_t - E_t \pi_{t+1}}{\sigma} + g_t$$

where π_t is t-1 to t inflation, x_t is the "output gap," u_t is a "markup shock" or supply-side shock,

and g_t is a demand-side shock arising from shifts in perceived productivity of capital. The shocks

u_t and g_t are assumed to be fully observed and to follow first-order processes, $u_t = \rho u_{t-1} + \eta_t$ and g_t

$= \lambda g_{t-1} + \varepsilon_t$ where ρ and λ are fractions and η_t and, ε_t are white noise. The standard welfare

function, which we will use throughout this paper, can be written

$$(2) \quad W = -\frac{1}{2} \sum_{i=0}^{\infty} \beta^i E_t \left(\pi_{t+i}^2 + \Gamma x_{t+i}^2 \right)$$

where β is a discount factor and Γ is society's - and the authority's - weight on the output gap.

This specification is essentially that presented by Clarida, Gali, and Gertler (1999) and is a

simplification of the more complete model developed by Woodford (2003, chapter 6) and in

standard usage in more recent years.[2] Given the set of u and g shocks, policy makers implement

a monetary policy that affects the interest rate to take levels that will meet the f.o.c.'s.[3]

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[2] The IS curve often is written with a real interest rate term to represent the demand-side check. There is no particular advantage of this as compared with adding the g_t term at the end of (1.b). Also, more recent presentations and derivations present the u-shock above (1.a) in the form $\kappa\xi$, where ξ is the actual markup shock. This does not affect our results, so we stick to the u-shock approach. There are some additional parameters that imply some interrelationships among the parameters above. We will introduce and discuss them below.

[3] The authorities maximize (2) subject to (1). The first order condition (f.o.c.) for the discretionary and the inflation commitment regimes are i=0….∞, $x_{t+i} = (-\kappa/\Gamma) /\pi_{t+i}$ for discretion and $x_{t+i} = (-\kappa/\Gamma(1-\beta\rho)\pi_{t+i}$, i=0…..∞, for inflation commitment. The f.o.c. for the price level commitment regime is $x_{t+i} = (-\kappa/\Gamma) /(p_{t+i} - p_{-1})$, i=0….∞ , where p_{t+i} is the level of price at t+i, and p_{-1} is the lagged price level at the period before the start of the policy and fixed thereafter.

The more precise nature of this policy is unnecessary for this section but will be addressed later. The variables π_t and x_t in (1.a) and (1.b) have been solved for in the literature subject to maximization of (2) under the following three alternative time-consistency assumptions: (1) agents assume the monetary authority will maximize (2) each period without reference or consideration of the policy choice on future inflation (*discretionary* policy); (2) the authorities are able to commit convincingly to a policy that incorporates the model's forecast of inflation in future periods (*inflation-commitment* policy); and (3) authorities commit convincingly to a policy that involves not only commitment to influence future inflation rate but also the level of future prices (*price level commitment*). As we shall show, price level commitment has great value in terms of improved welfare and some other helpful side effects in the context of this model. Table 1 illustrates the path solutions in these three cases. Recall that $u_t = \rho u_{t-1} + \eta_t$ and $g_t = \lambda g_{t-1} + \varepsilon_t$. It should be noted that no IS shocks (g_t) enter the solutions, since these unambiguously should be offset through movements in interest rates. The optimal policy (whatever particular form it takes) should preclude g_t from entering the paths of x and π.

The parameters involved in (1.a and 1.b)-(2) above are all involved in these paths except σ and λ, which appear in the IS (1.b). Whatever specific monetary policy is employed must move the interest rate in a way to meet the first order conditions while at the same time just offsetting any of the IS shocks (g_t); essentially, the IS side is not involved in determining the optimal paths. The following Chart 1 illustrates with one specification of the parameters ($\beta=0.99$, $\kappa=0.024$, $\Gamma=0.0048$, $\rho=0.35$) the impulse response functions implicit in these paths. These are responses to one standard deviation of η_t (assumed to equal 1.0).

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Table 1a: First order conditions for the three regimes

Regime	First order condition
Discretionary case:	$x_t = -\dfrac{\kappa}{\Gamma}\pi_t$
Inflation-commitment case:	$x_t = -\dfrac{\kappa}{\Gamma(1-\beta\rho)}\pi_t$
Price- level commitment case:	$\pi_t = -\dfrac{\Gamma}{\kappa}(x_t - x_{t-1})$

Table 1b: Optimal paths for the three time-consistent cases

Policy regime	Inflation or prices	Output gap
Discretionary case: inflation and output gap paths	$\pi_t = \dfrac{\Gamma u_t}{\kappa^2 + \Gamma\left(1 - \beta\rho\right)}$	$x_t = \dfrac{\kappa u_t}{\kappa^2 + \Gamma\left(1 - \beta\rho\right)}$
Inflation-commitment case: inflation and output gap paths	$\pi_t = \dfrac{\Gamma u_t}{\kappa^2 + \Gamma\left(1 - \beta\rho\right)^2}$	$x_t = \dfrac{\kappa u_t}{\kappa^2 + \Gamma\left(1 - \beta\rho\right)^2}$
Price- level commitment case: price level and output gap paths	$\pi_t = \delta\pi_{t-1} + \dfrac{\delta}{1-\delta\beta\rho}(u_t - u_{t-1})$ $x_t = \delta x_{t-1} - \dfrac{\kappa\delta}{(1-\beta\delta\rho)\Gamma}u_t$ with $\delta = \dfrac{1 - \sqrt{1 - 4\beta a^2}}{2a\beta}$ ($0 < \delta < 1$) and $a = \dfrac{\Gamma}{\kappa^2 + \Gamma(1+\beta)}$	

Chart 1.a: Impulse responses of inflation, output gap, and interest rate to a unit shock in η_t
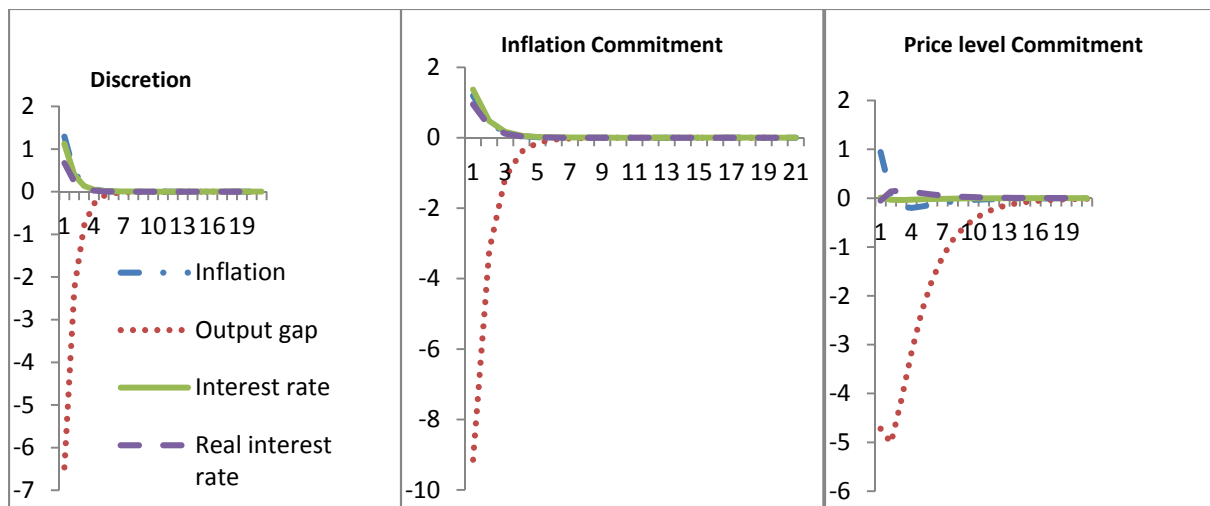
Chart 1.b: Impulse responses of inflation, the price level, and money to a unit shock in η_t – case where interest elasticity of real money is equal to 0.05
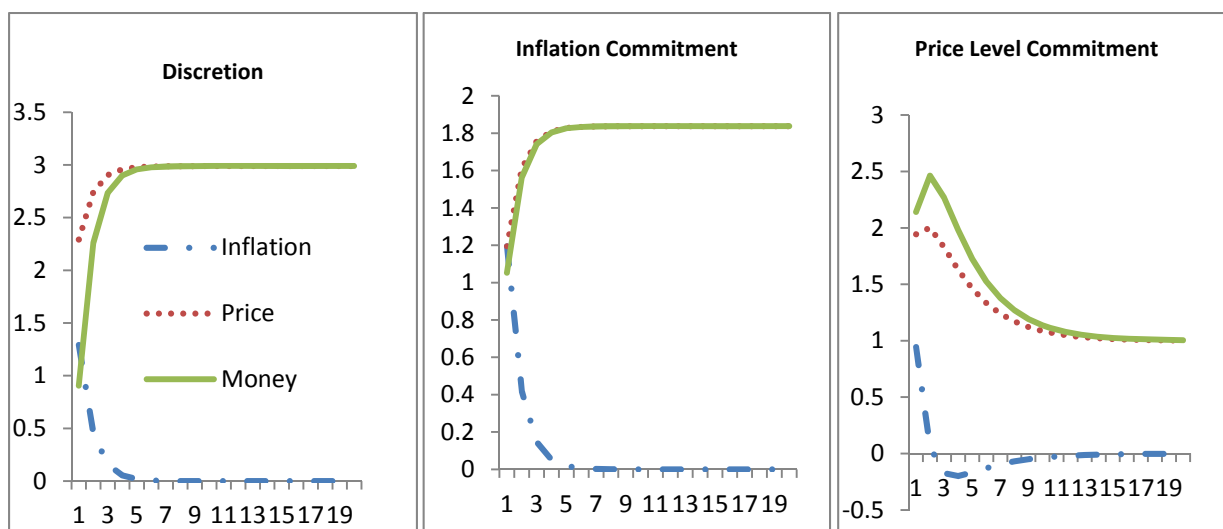
<u>Chart 1.c:</u> Impulse responses of inflation, the price level, and money – interest elasticity of real money demand equal to 1 for discretion and inflation-commitment and equal to 5 for price-level commitment
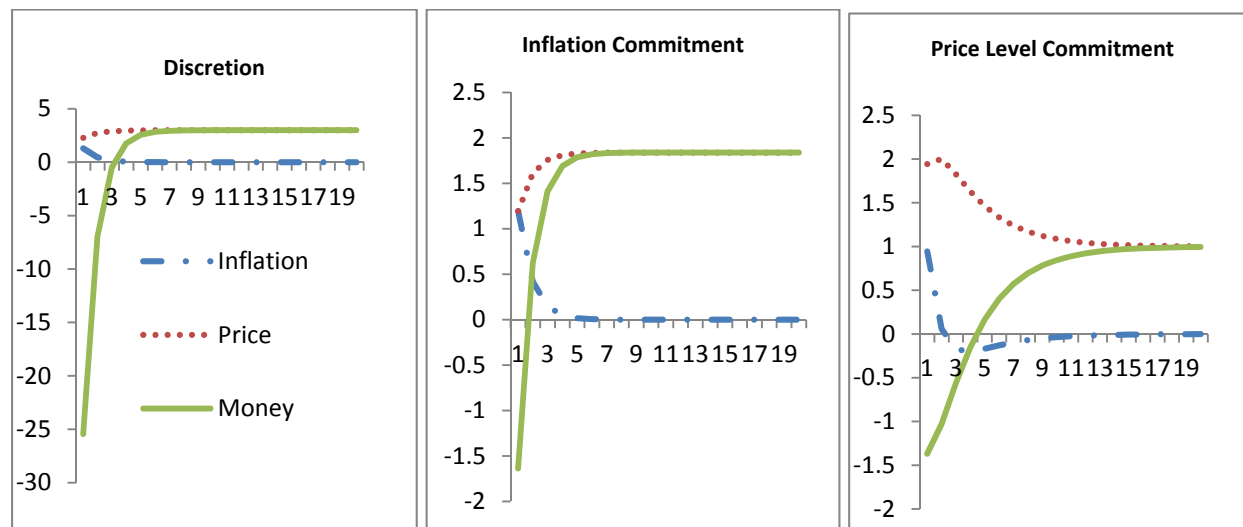
A number of things stand out from these paths. First, the volatility of inflation appears to increasingly diminish as the level of commitment is increased. This is the key difference among regimes. The commitment to the price level, making it stationary (in this case at an arbitrary value of 1.0), requires that future inflation eventually be negative and in an amount large enough to cancel out the positive effects on prices in the earlier periods. Since inflation is linked to expected future inflation (1.a) this dampens the response to shocks of current inflation.

Second, the volatility of the output gap is actually increased from discretion to inflation-commitment, but this volatility for price-level commitment is the least in all three regimes. Third, the volatility of interest rates appears diminished as commitment increases (we will show in Section 4 that this is parameter-specific for the discretionary vs. inflation-commitment regime, although interest rate volatility will always be lowest for the price-level commitment regime). Fourth, the money supply paths are well-defined and generally reflect the price level paths,

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although for high enough interest elasticity of money demand money initially moves in the opposite direction of prices and inflation (Chart 1.c).[4]

Finally, the price-level commitment path unlike the other two involves a lagged price and lagged output gap. The presence of lagged terms is referred to as "inertial" in contrast to the purely "forward-looking" role of expectations in the other two paths. The important role of these lagged effects arises not from the impact of the actual t-1 (or earlier) values of these variables; rather, it is to the pattern of future movements they imply – in effect, "forward inertia." A literature (Woodford (2003a), Giannoni (2010, 2012) and others) has referred to "inertial policy" in which interest rate rules like the Taylor rules include lagged variables (including lagged interest rates) inhibit movement or "smooth" interest rates over time. We return to this issue in Section 5, where we introduce a Taylor rule. At this point we emphasize that the interest volatility reductions of price-level commitment we have illustrated are independent of any particular rule that may be employed to arrive at the optimal path solutions. It would be inappropriate to infer from these paths, therefore, that a motive of interest rate smoothing has anything to do with them.

1.2 Welfare and improvements from commitment

In order to evaluate improvements in welfare, we express the different regimes' solution for (2) in Section 1. The expressions turn out to be:

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[4] This assumes that money is present, which is not required in a 'cashless" version of this model. The money demand-supply side are typically ignored under the argument that that any money-based optimal policy would be equivalent in this model where shocks are fully observed. A money supply-oriented policy would be more complicated, however, on account of the likely presence of money demand and money supply shocks which are automatically accommodated (and have no effect) when an interest rate approach is used.

Table 2:	Solutions for W (equation 2) for three regimes

W under discretion

$$-\frac{1}{2}\frac{\Gamma^2\left(1+\frac{\kappa^2}{\Gamma}\right)\sigma_\eta^2}{\left(\kappa^2+(1-\beta\rho)\,\Gamma\right)^2\left(1-\beta\rho^2\right)(1-\beta)}$$

W under inflation-commitment

$$-\frac{1}{2}\frac{\Gamma^2\left((1-\beta\rho)^2+\frac{\kappa^2}{\Gamma}\right)\sigma_\eta^2}{\left(\kappa^2+(1-\beta\rho)^2\,\Gamma\right)^2\left(1-\beta\rho^2\right)(1-\beta)}$$

W under price-level commitment

$$-\frac{1}{2}\Big(\delta^2\left(\left(1+\beta\rho^2\right)\left(\beta\delta\rho+\delta\rho\beta^2-2\beta\delta+1+\beta\right.\right.$$
$$-2\beta\rho)\,(\beta\delta\rho+1)\,\Gamma-\kappa^2\beta\delta\rho\left(\beta\rho\left((2\beta-2)\,\delta^3\right.\right.$$
$$+4\rho\,(-1+\beta)\,\delta^2+\left(\rho^2\,(-2+\beta)-1\right)\delta-2\rho)-2)$$
$$+\kappa^2+\kappa^2\beta\rho^2\big)\big)\Big/\Big(\left(-1+\beta\delta^2\right)\left(-1+\beta^2\delta^2\rho^2\right)(-1$$
$$\left.+\beta\delta\rho\right)^2\Gamma\,(-1+\beta)\left(-1+\beta^2\rho^4\right)\Big)\,\sigma_\eta^2$$

1.3 Welfare "levels" and improvements for different parameter settings

In this section we consider the impacts that different parameters (ρ, β, Γ, κ) will have on measured welfare (W), later in the section how these parameter values will determine how much "improvement" in welfare would appear across regimes. Our objective is to find parameter ranges in which prospective improvements would be major. We do not calibrate or estimate these parameters ourselves, but we do compare them with the ranges typically found in the literature (Table 3).

It should be noted that the parameters are interrelated. The parameter Γ in the Woodford (2003, chapter 6) derivation is $\Gamma=\kappa/\theta$ where $\theta>1$ is the price elasticity of demand for the goods produced by monopolistically competitive firms. The parameter κ, from the Phillips curve (1.a) follows:

$$\kappa = \frac{(1-\omega)(1-\beta\omega)(\sigma+\tau)}{\omega(1+\tau\Theta)}$$

where $\tau > 0$ is the elasticity the representative firm's real marginal costs with respect to its own price level. Although κ and Γ are not independent, we have enough free parameters independently to select hypothetical combinations of κ and Γ that would imply major impact of price commitment on W. The restriction that $\theta > 1$ does suggest a restriction that $\Gamma < \kappa$ which according to Table 3 is only sometimes observed. Table 3 illustrates the ranges used in some recent literature. The ranges employed in the literature are [0.024, 0.33] for κ, [0.1, 0.8] for ρ, [0.002, 0.5] for Γ, [0.157, 1] for σ, and it seems that there is a large consensus for β=0.99.

Table 3: Calibrations in recent literature

Authors	Year	σ	b	β	κ	ρ	Γ	ω	θ	τ
Hoffman, Rasche, Tieslau	1995		2							
Roberts	1995				0.075					
Rotemberg, Woodford	1997	0.16		0.99	0.024					0.47
Holman	1998		0.1							
McCallum, Nelson	2000			0.99	[0.01,0.05]		0.25			
Chari, Kehoe, McGrattan	2000		2.6							
Jensen	2001			0.99	0.1		0.25			
Walsh	2001			0.99	0.05		0.25			
Ireland	2001		5.26 and 8.33							
Woodford	2002	0.157		0.99	0.024	0.35				
Aoki, Nikolov	2003	0.157		0.99	0.0238	0.35				
Woodford	2003	0.16		0.99	0.024		0.05			0.47
Giannoni, Woodford	2005	0.16		0.99	0.024				7.69	
Bils, Klenow	2004							0.5		
Schabert	2005	2		0.99				0.8		
Gaspar,Smets, Vestin	2006			0.99	0.019		0.002		10	
Billi	2008	0.16		0.9926	0.024	0.1	0.003	0.66	7.66	
Walsh	2010	1		0.99	0.05		0.25	0.8		1
Giannoni	2010					0.35				
Bauducco, Caputo	2010			0.99	0.33	0.8	0.5			
Walsh	2011			0.99	0.25		0.25			
Giannoni	2012	0.157		0.99	0.0238	0.35				

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Impact of ρ and β

Chart 2 illustrates the effects of different values of $0 \le \rho < 1$, then of $0 < \beta < 1$, holding other parameters at fixed values. As the chart shows, higher values of both parameters have negative impacts on W and the deterioration is most pronounced where commitment is least. The sensitivity of this effect is most pronounced at the high end of these parameters.

Chart 2: Impact of ρ and β
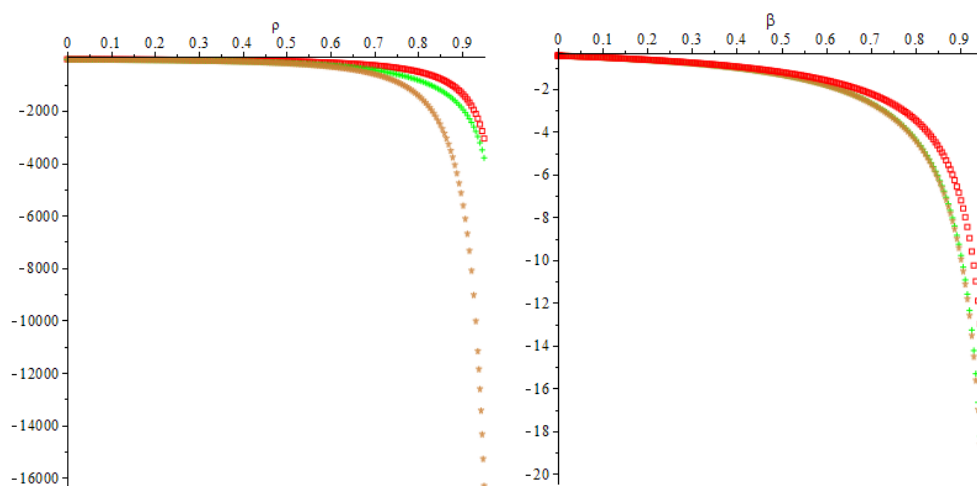
Note: κ held at 0.024, Γ held at 0.0048. In the left chart β held at 0.99. In the right chart ρ held at 0.35.

Higher ρ implies persistence in u-shocks spread over time and whose impacts are reflected in higher variance of current inflation. Higher values of β have similar effects and relate the impact of future expectations price on current prices in the Phillips curve (1.a). In the literature (see also Table 3) the calibration of ρ is usually in the range of 0.10 to 0.8, where the variation across regimes is not very great. The parameter β is almost invariably chosen to be 0.99 which gives a major "edge" to the price-level commitment regime but leaves the others close to each other. In what follows we select ρ=0.35 and β = 0.99 and concentrate on the effects of κ and Γ but the results of course are sensitive to the choice of ρ. A different selection of ρ may have major impacts on the variances and surfaces.

Impact of κ and Γ

  Variation in κ and Γ can have pronounced effects on W and the differences in W across regimes over some ranges (Chart 3). Higher κ, which increases with price flexibility, improves W; indeed, large enough κ eventually will make W=0 in all three cases – that is, as κ →∞, the model becomes essentially classical. Larger Γ will reduce W in all three cases continuously. The higher weight on the output gap variance will result in less inflation-"hawkishness" now and in the future, leading to greater impacts of u-shocks on inflation.

Chart 3: Welfare under three regimes
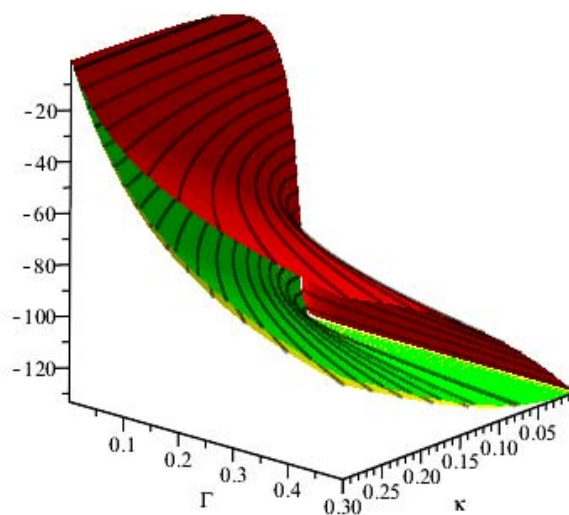
Assumed: β=0.99, σ=0.16

Measuring improvement from commitment

  If we ask the question how much improvement in welfare we see with increased commitment, we can ask what happens to the ratio of welfare under different parameter settings. In particular, measures of proportional improvement are the ratios of W_π to W_P or W_d to W_P. As already suggested by Chart 3, the major jump in welfare occurs in the jump from W_π to W_P.

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Chart 4 focuses on the ratio of W_d to W_P.[5] It shows that the improvement from commitment lie

in a "corridor" for κ and Γ. Improvement is indicated by the "improvement ratio" W_d/W_P or

W_d/W_π. Most of this improvement is concentrated in the jump from inflation-commitment to

price level commitment. For example, W_d/W_P is 1.503 at $\kappa=2.5$, and $\Gamma=6$. The ratio of W_d/W_π

(not shown) is only 1.1067.

Chart 4: Improvement in welfare
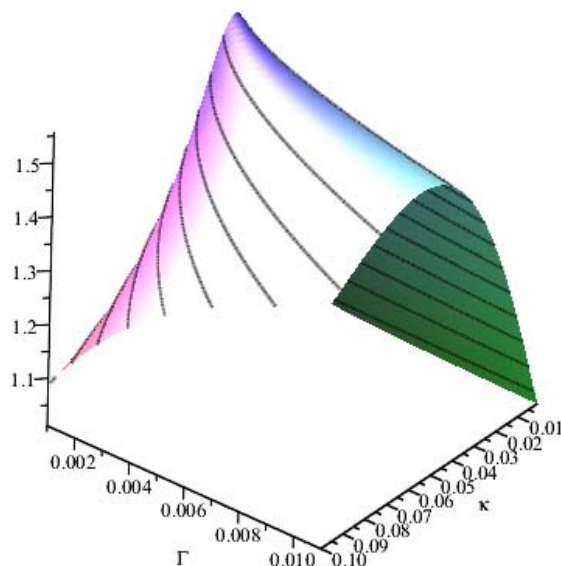
Along a kind of "corridor" the "improvement ratio" is high and stable and along a

particular contour is constant. This corridor is illustrated in contour lines Chart 1.5 (for W_d/W_P

only). It might be noted that the "maximum" ratio (1.74) involves a nonlinear relation between

in κ and Γ in which the ratio Γ/κ falls as the level of both fall. The restriction implied by

Woodford's relation noted above that $\Gamma=\kappa/\theta$ with $\theta>1$ thus suggests that the values which meet

this restriction must be on the low side as shown in the right panel of the chart. It would appear

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[5] These welfare measures are negative with higher absolute values implying lower welfare. The ratios of lower-commitment to higher-commitment can be viewed as a proportional measure of improvement

from Table 3 that the parameters are lower than those we have been using in our examples (κ=0.024, Γ=0.0048) but that it is no uncommon to violate the condition that $\Gamma < \kappa$ (as we have). These particular issues aside, it seems safe to conclude that the parameters chosen in the literature are in a range that would imply substantial improvements in W from commitment.

<u>Chart 5:</u> Contours of welfare improvement with different values of κ and Γ
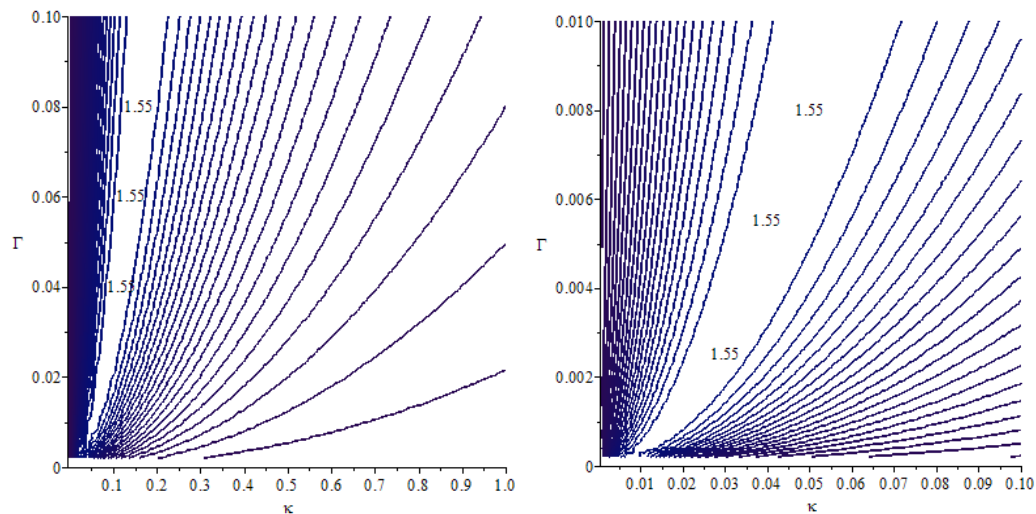
<u>1.4 Two useful side-effects of commitment</u>

There are two side-effects of the price level commitment that deserve attention. First and perhaps obvious is that a policy that achieves the optimal paths is guaranteed to eliminate trend inflation. A number of authors (viz., Ascari and Ropele (2007), Kiley (2007)) have shown that the presence of trend inflation considerably complicates the derivation of the Phillips curve (it is no longer of the tractable form as in 1.a). Further, this revised Phillips curve can severely limit the range of policy choices that will have determinate solutions. Price-level commitment appears to avoid this problem.

A second beneficial effect is that commitment to control price level optimally can substantially reduce the volatility of interest rates (cf. Woodford (2003a), Giannoni (2010, 2012).

The model in this chapter does not specify a benefit of reduced interest rate variance.[6] A particular benefit in the context of this model of reduced interest rate variance of any optimal policy is reduced chance of hitting the lower bound interest rate. There may be other virtues to reducing inflation and its variance, but they are generally not addressed in models of this type.[7]

Chart 6.a: Nominal interest rate volatilities: effects of Γ at three different values of κ
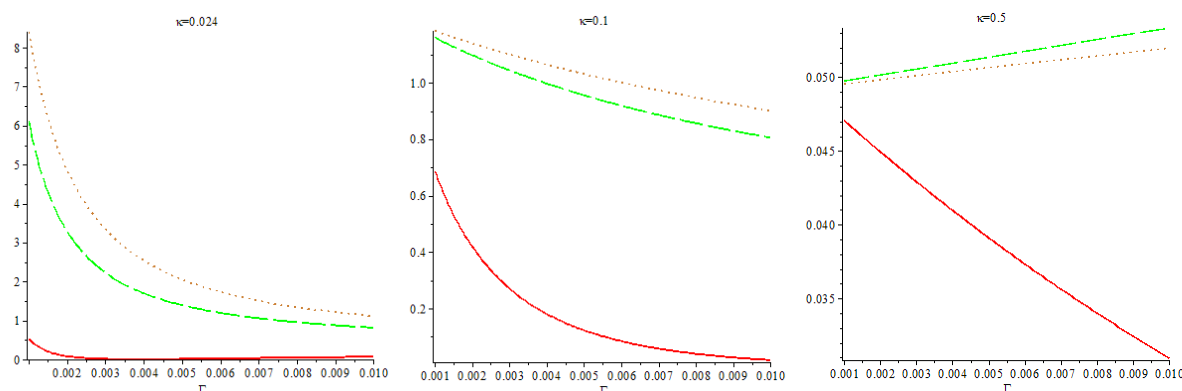
Chart 6.b: Real interest rate volailities: effects of Γ at three different values of κ

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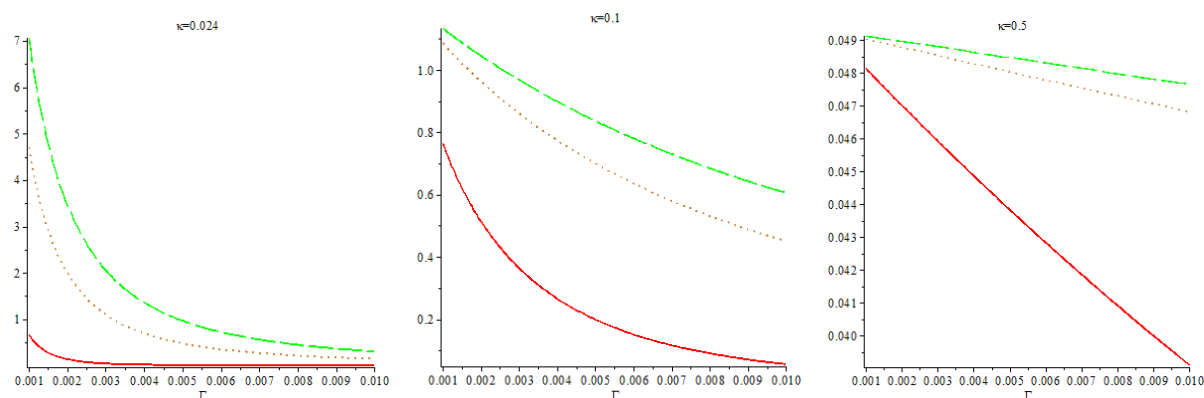
[6] Woodford (2003a) and Giannoni (2010, 2012) however do. Welfare (more precisely, social loss) includes the variance of the interest rate in their work.
[7] Woodford (2003a), Giannoni (2010, 2012) are the exceptions as they put the variance of interests into their social cost function.

Chart 6.a shows that nominal interest rate volatility (σ_i^2) is in all cases lowest (and possibly considerably lower) under price-level commitment. (These volatility measures include only the component of volaility that arises from the u-shocks. Volatilty arising from the independent g-shocks are systematically offset by means of interest rate "jumps" in the amount of σg_t, implying an independent and constant component of interest rate volaility equal to $\sigma^2 \sigma_g^2$.) Whether inflation-commitment produces lower interest volitility than discretion depends on the range of Γ. Real interest rate volatility across regimes (Chart 6.b), including even the price-level commitment one, appears to depend on the relative magnitude of κ vs. Γ.

More perspective on the effects of parametization on interest rate volatility across regimes is available in Chart 7,which shows the effect of different sets of κ and Γ on both the ratio $\sigma_{iP}^2 / \sigma_{id}^2$ and the ratio $\sigma_{i\pi}^2 / \sigma_{id}^2$. The much lower ratios $\sigma_{iP}^2 / \sigma_{id}^2$ in some ranges indicates considerable power of price-level commitment to lower interest rate volaility. Inflation commitment appears to offer modest reductions in interest rate volatility at best. At low values of κ and Γ a movement from discretion to inflation-commitment may greatly increase interest rate volatility.

Chart 7: Improvement of interest rate volatility
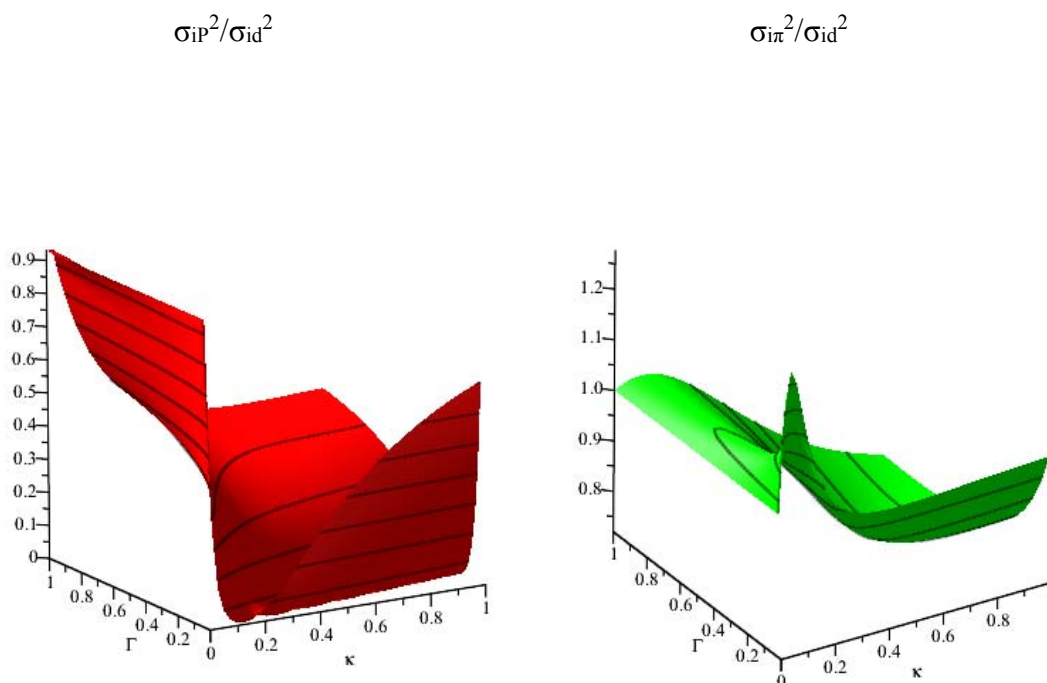
$\sigma_{iP}^2/\sigma_{id}^2$ $\qquad\qquad\qquad\qquad\qquad\qquad$ $\sigma_{i\pi}^2/\sigma_{id}^2$

The contours for $\sigma_{iP}^2/\sigma_{id}^2$ is illustrated in Chart 8. Interestingly, the locus of minimum 0.0 points is linear, reflecting a condition that $\kappa\sigma/\Gamma=1$. When this is true, σ_{iP}^2 (or at least that component arising from u-shocks) is 0.[8] Again, the range of κ and Γ that would imply large suppression of interest volatility appears to be large, and includes the range suggested by the recent literature (Table 3).

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[8] The slope of unity in Chart 8 is an artifact of the fact we used the assumption $\sigma=0.16$ in order to construct the graph.

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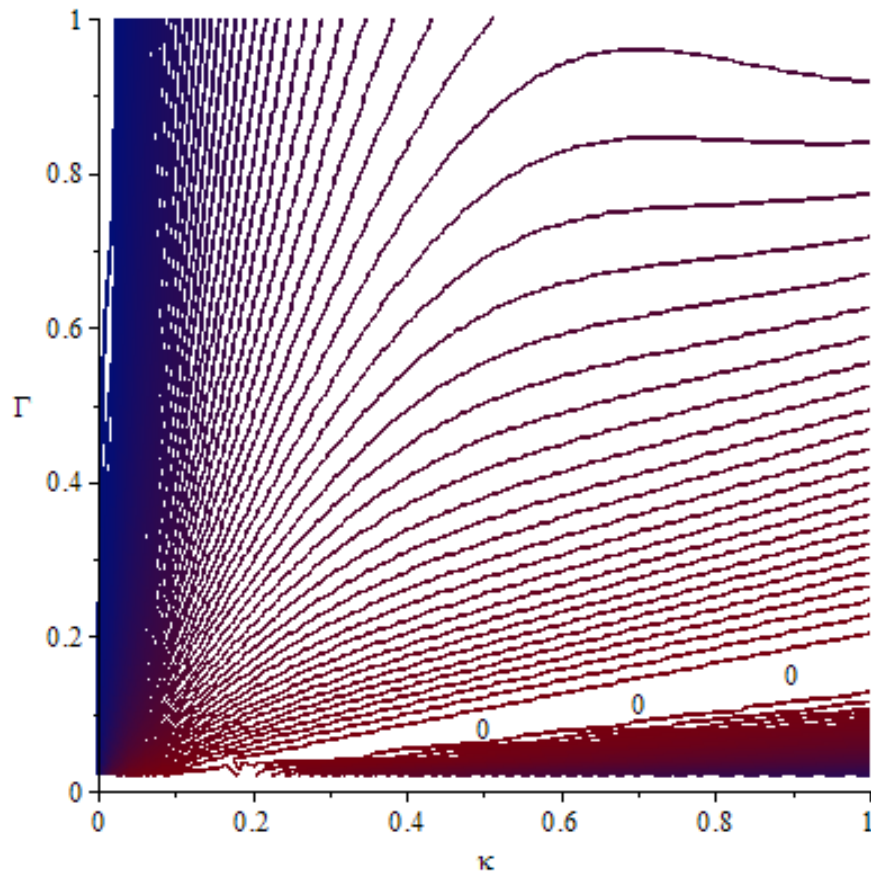
1.5 A modified Taylor rule

To this point we have discussed only the paths of the endogenous variables including the interest rate, and no specific policy procedure or rule has been specified. The main classes of monetary policy rules include money stock-driven rules (in which money responds to endogenous variables) or interest rate policies. The standard approach is interest rate rules, of which Taylor rules have been the most studied. Any interest rate rule must, for determinacy, involve reactions to endogenous variables in the model (determinacy to be discussed in Section 6).

The Taylor rule that applies most directly to the version of the model we have been considering is of the form

$$(3) \quad i_t = \phi_\pi \pi_t + \phi_x x_t + \phi_g g_t$$

The shock term g_t represents a "demand shift" (more precisely, a shift in the expected marginal efficiency of capital) that shifts the demand for spending on the IS schedule (1.b). As mentioned earlier and emphasized by Woodford (2001) and Clarida, Gali and Gertler (1999), an optimal monetary policy would shift the interest rate just enough to offset this shock perfectly. Thus the optimal value of ϕ_g should equal to σ in (1.b). In the price level commitment regime at hand, it is not possible to find combinations of parameters of form (3) that would produce the optimal path, but a modification is consistent with the optimal path, namely:

$$(4) \quad i_t = \phi_p \hat{P}_t + \phi_{p-1} \hat{P}_{t-1} + \phi_g g_t,$$

where $\hat{P}_t = p_t - p_{-1}$ and $\hat{P}_{t-1} = p_{t-1} - p_{-1}$, and provided that the parameters of the model including these ϕ's satisfy the eigenvalue conditions for determinacy. These values will not necessarily satisfy the conditions, but the rule can be rewritten as

$$(5) \quad i_t = \phi_p \hat{P}_t + \phi_{p-1} \hat{P}_{t-1} + \phi_x x_t + \phi_g g_t,$$

which will exactly match the optimal path provided the ϕ's follow:

$$\varphi_{p-1} = (\frac{\kappa\sigma}{\Gamma} - 1)\rho\delta, \quad \varphi_p = \frac{\kappa}{\Gamma}\varphi_x + (1 - \frac{\kappa\sigma}{\Gamma})\rho + (1-\delta)(\frac{\sigma\kappa}{\Gamma} - 1),$$ and $\phi_g = \sigma$.[9]

---

[9] Place the interest rate rule (5) in IS curve (1.b), and using the paths $x_t = \delta x_{t-1} - \frac{\kappa\delta}{\Gamma(1-\delta\beta\rho)}u_t$ and $\pi_t = (\delta-1)\hat{p}_{t-1} + \frac{\delta}{1-\beta\delta\rho}u_t$ with the first order condition $x_t = (-\kappa/\Gamma)\hat{P}_t$, and we have: $(\varphi_p - \delta + 1 - ((1-\delta)\sigma + \varphi_x)\frac{\kappa}{\Gamma})\frac{p_t}{\sigma} = -\frac{\varphi_{p-1}}{\sigma}\hat{p}_{t-1} + (\frac{1}{\sigma} - \frac{\kappa}{\Gamma})\frac{\delta}{1-\beta\delta\rho}\rho u_t + (-\frac{\varphi_g}{\sigma} + 1)g_t$. To

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The $\hat{P}$ variables are deviations of the price from a fixed starting value at t-1 (p-1): for example, $\hat{P}_t$ = p_t - p_{-1}. The obvious and key difference from the Taylor rule (4) is the separation of the coefficients on $\hat{P}_t$ and $\hat{P}_{t-1}$. This permits the rule to bring the long-run level back to the fixed level of p-1.

There are two more interesting remarks that can be made about the Taylor rule and its coefficients. First, as discussed in the previous section, provided there is commitment it is not necessary that the Γ weight be treated as the "true" (Γ*) value. We might wish to vary Γ in order to affect the variance of interest rates or perhaps even the preferences of the central bank different from the society's. Or, there may simply be different or varying societal Γ*. All of these affect the Taylor rule in a way which makes "inflation hawkishness" inversely and monotonically a measure of the "inflation hawkishness" of monetary policy. This is clear from the expressions for W in Table 2. For the price-level commitment case, the relation is more complicated but inflation volatility also varies directly with Γ. For example, Chart 9 illustrates the relationship for various three κ's.

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conform with the path of the price level we need to have $\hat{p}_t = \delta \hat{p}_{t-1} + \frac{\delta}{1-\beta\delta\rho} u_t$. So we have the system of equations:

$$\delta = -\frac{\varphi_{p-1}}{\varphi_p - \delta + 1 - ((1-\delta)\sigma + \varphi_x)\frac{\kappa}{\Gamma}}$$

and

$$\frac{\delta}{1-\beta\delta\rho} = \frac{(1-\frac{\kappa\sigma}{\Gamma})\rho\delta}{(1-\beta\delta\rho)(\varphi_p - \delta + 1 - ((1-\delta)\sigma + \varphi_x)\frac{\kappa}{\Gamma})}$$

Then we obtain the formulas above.

<u>Chart 9</u>: Inflation volatility for different choices of Γ

red: κ=0.024; blue: κ=0.1; green: κ=0.5
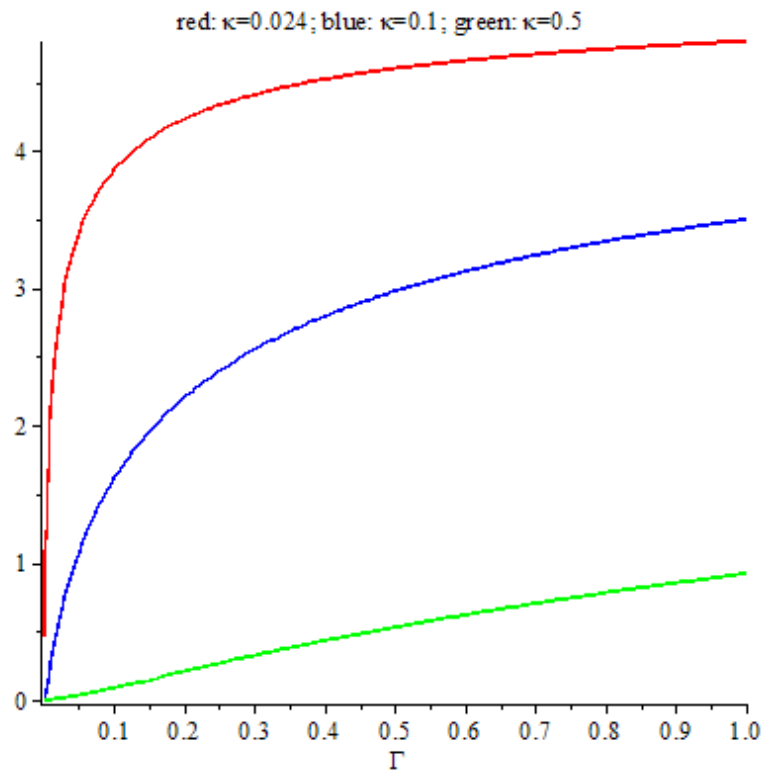
In this context, the choice of Γ can be regarded as an inverse measure of "hawkishness" (toward inflation) in all the three regimes.

The second remark has to do with the lagged dependence of the Taylor rule interest rate on the price level. Since a lagged endogenous variable appears in the rule, it is not improper to describe this as an "inertial" interest rate policy. But some researchers have gone further to impute a desire to suppress interest rate volatility as a reason for policy "inertia," and suggested that monetary policy adjusts partially to desired interest rate levels in order to achieve this "interest rate smoothing." In empirical estimates of the Taylor rule, lagged interest rate coefficients appear to be substantial and hard to reject statistically. Further, in simulation experiments allowing for a lagged interest rate or other endogenous variables ("Wicksellian"

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rules) has improved simulated outcomes for inflation and output. The optimal W (2) in this paper

however reveals no motive for suppressing interest rate volatility; rather the reduced interest rate

volatility seems to be a natural by-product of commitment to a stationary price level. We would

attribute the improved theoretical performance with lagged interest terms and Wicksellian rules

as a result of price-level commitment (which of course appears to improve W). Strong estimated

estimates of lagged interest rates in Taylor rules may of course have multiple potential causes of

which might be included a desire to dampen the trend in the price level (implicitly a commitment

to future price levels).[10]

1.6 Determinacy and the Taylor rule

Considerable attention has been paid to the determinacy conditions in models of this type.
The standard analysis was developed in Blanchard and Kahn (1980) and Bullard and Mitra
(2002).

The model with explicit treatment of the price level can be written:

$$(6) \qquad \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

The IS curve with the Taylor rule in the previous section is

$$(7) \quad x_t = E_t x_{t+1} - \frac{\varphi_p}{\sigma} \hat{p}_t - \frac{\phi_{p-1}}{\sigma} \hat{p}_{t-1} - \frac{\varphi_x}{\sigma} x_t - \frac{\varphi_g}{\sigma} g_t + \frac{1}{\sigma} E_t \pi_{t+1} + g_t$$

and the price level must evolve as

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[10] We have experimented with many different ways of rewriting (5) with the optimal restrictions on the φ's. One
of these is

$$i_t = \delta i_{t-1} + (2\delta\rho + \delta^2 - 2\delta - \rho - \delta^2\rho + 1)(1 - \frac{\kappa\sigma}{\Gamma})\frac{\Gamma}{\kappa} x_t + \delta(\rho + \delta - 1 - \delta\rho)(1 - \frac{\kappa\sigma}{\Gamma})\pi_t$$

$$+ \frac{1 - \frac{\kappa\sigma}{\Gamma}}{1 - \beta\delta\rho} \delta^2 \rho (u_t - u_{t-1}) + \sigma(g_t - \delta g_{t-1})$$

. The two moving average errors would seem to present
estimation difficulties, but we believe it would not be surprising for an estimated coefficient on i_{t-1} to appear to be
significant (although not necessarily consistent).

$$(8)\ \hat{P}_t = \pi_t + \hat{P}_{t-1}.$$

The system can therefore be written in the form

$$(9)\ \begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ \hat{p}_t \end{bmatrix} = M \begin{bmatrix} x_t \\ \pi_t \\ \hat{p}_{t-1} \end{bmatrix} + N \begin{bmatrix} g_t \\ u_t \end{bmatrix} =$$

where $M = \begin{bmatrix} 1 + \dfrac{\varphi_x}{\sigma} + \dfrac{\kappa}{\sigma\beta} & \dfrac{\varphi_p}{\sigma} - \dfrac{1}{\sigma\beta} & \dfrac{\varphi_p + \varphi_{p-1}}{\sigma} \\ -\dfrac{\kappa}{\beta} & \dfrac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{bmatrix}$ and $N = \begin{bmatrix} 1 - \dfrac{\varphi_g}{\sigma} & \dfrac{1}{\sigma\beta} \\ 0 & -\dfrac{1}{\beta} \\ 0 & 0 \end{bmatrix}$.

The Blanchard-Khan conditions for determinacy require that of the three eigenvalues in M, two must be outside the unit circle and one inside. Deriving an analytical expression of these conditions in terms proved difficult, so we studied cases with a variety of assumed parameter values for the model parameters σ, β, ρ, κ, and Γ. As an example for $\sigma=0.16$, $\beta=0.99$, $\rho=0.35$, $\kappa=0.024$, and $\Gamma=0.0048$, we find the range of determinant combinations of φ_p and φ_x as in Chart 10.

The Blanchard-Khan condition will be met only if the combinations of φ_p and φ_x lie to the right of the boundary in the left panel of Chart 10 and, on the right panel, above the horizontal line when $\varphi_x>0$, not restricted when $\varphi_x <0$. The optimal combinations are pictured as the roughly diagonal line in the right panel. The same basic pattern appears with other parameter assumptions.

Chart 10: Regions of Determinacy for φ_p and φ_x
The indeterminate region is in shadow

$$(1-\frac{\kappa\sigma}{\Gamma})\rho+(1-\delta)(\frac{\kappa\sigma}{\Gamma}-1)=0.12$$

$$\varphi_p=\frac{\kappa}{\Gamma}\varphi_x+(1-\frac{\kappa}{\Gamma})(\rho+\delta-1)$$

$$\varphi_p=(1-\frac{\kappa\sigma}{\Gamma})\rho\delta$$

$$-((1-\frac{\kappa\sigma}{\Gamma})\rho+(1-\delta)(1+\frac{\kappa\sigma}{\Gamma}))\frac{\Gamma}{\kappa}=-0.11$$
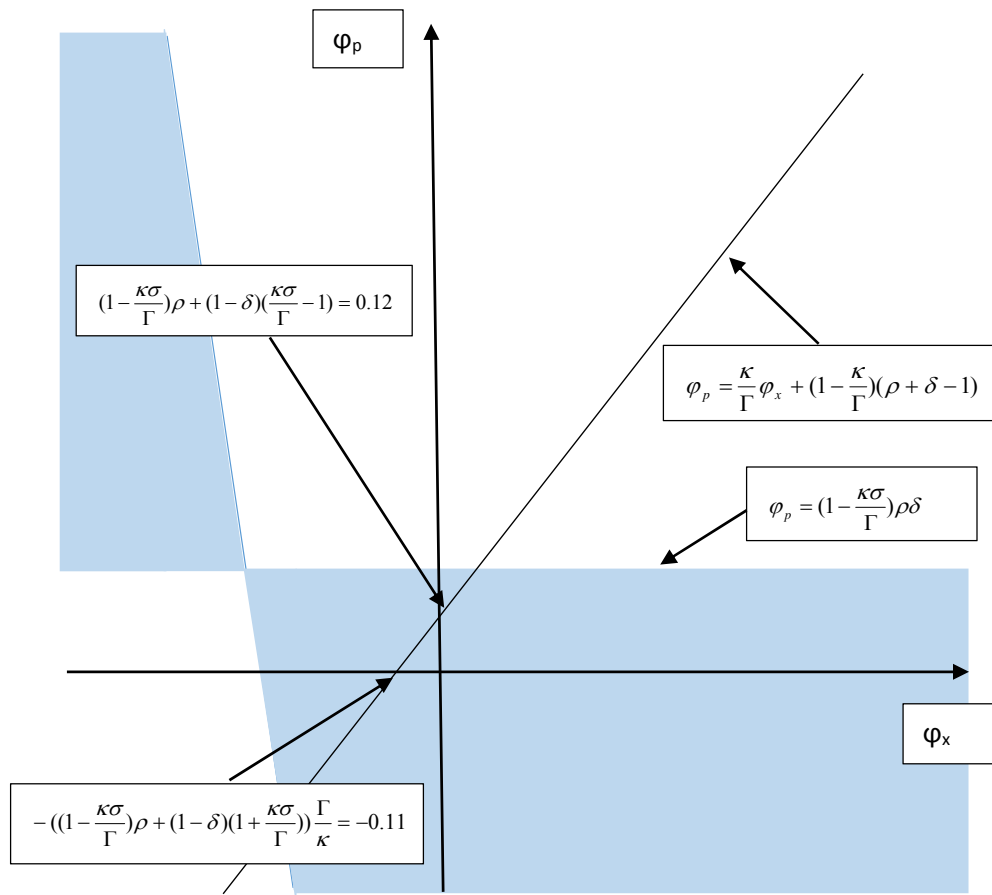
Evident in Chart 10 is the fact that there are an infinity of combinations of φ_p and φ_x that will lead to determinate solutions both on the NE and SW portions of the optimal-combination line. With the downward boundary on the left panel and the horizontal restrictions on the $\varphi_x>0$ side of the right panel, it is clear geometrically that the optimal path can always be obtained with the appropriate combination of φ_p and φ_x.

1.7 Concluding remark

The chapter (1) shows how different degrees of commitment to future monetary policies can be improve welfare in the NKM model. It (2) measures the improvement in welfare that can result from a commitment to period-ahead inflation to commitment to bring the future price level back to a starting level, and (3) shows how the improvement can be sensitive to the basic parameters in the NKM formulation. The parameter ranges used in the recent literature happen to be the ranges in which the potential improvement in the welfare would be large. The chapter also (4) points to important side-effects of increased commitment, including elimination of expected trend inflation and a reduction in interest rate volatility. The chapter (5) shows a modified Taylor rule that can actually bring about the paths necessary to the improvements in welfare, and demonstrates that (6) such a rule which maximizes welfare can be set to avoid determinacy problems.

CHAPTER 2

Gain from Commitment to Different Monetary Targets

2.0 Introduction

The previous chapter outlined the baseline of the New Keynesian model and provided an analytical solution to measure welfare for discretion, pre-commitment and global targets. That analysis was done under the presumption that policy makers seek to maximize the standard social welfare. It revealed that the gain under commitment is important but varies depending on the parameters. A number of authors, however, have acknowledged that the time-inconsistency problem associated with commitment may be difficult to overcome. To deal with the problem, one solution would be to assign a welfare function and assume delegation. Through this institutional device, the central bank would have to implement that target.

The concept is to "delegate" authority to the central bank to maximize other welfare functions in order to get a better result for the standard social welfare. It assumes that the public is convinced the central bank will maximize that alternative welfare function on the long run. The rule will be to follow a first order condition obtained by optimization of the welfare selected. The central will be expected to meet that condition in the present and also later indefinitely. Some authors called this approach as discretionary targeting in the sense that the issue of time-inconsistency is assumed away under delegation. If delegation is implementable, the natural tendency would be to select the global target as it was the optimal solution when deriving the

social welfare. The fact that an institutional mechanism constrains the central bank to follow a specific first order condition overtime is a way to ensure that commitment is implemented indefinitely.

In the previous chapter, it was remained that the notion of "discretion" refers to the time-consistent solution of a maximization of a welfare function. In "Science of Monetary Policy" Clarida, Gali, and Gertler (1999), it is considered as a Nash equilibrium and at each period the central bank will use a first order condition that is derived by considering the two current values of x and π. It is important to note that the first order condition and the target change when any form of commitment is added. The alternative approaches considered in this chapter involve commitment in order for them to be time-consistent.

There are some reasons to select a welfare function different than the social welfare. First, another welfare function may be easier to understand from the public. Second, there may be some difficulties in observing the values of some variables such as the output gap. So alternative targets may deal with that problem by making observations more accurate; for example the speed limit approach targets the difference of output gap instead of the level of output gap and it is supposed to be more precise. To illustrate the problem, Orphanides (2000) mentioned that output gap was not measured correctly during the 1970's. The potential output was not estimated properly. It was attributed not only to the optimistic assumption about the level of unemployment compatible with full employment but also largely to the optimistic view of how an improvement of the rate of labor productivity would translate into the growth of output. Policy makers during that period became more activists to make sure that output reaches the level of potential output and as a consequence they created excess inflation. Walsh (2001) argues that using the growth of potential output would be measured more accurately than its

level. Thus, the use of the difference of output gap rather than the level of output gap in the loss function would prevent in part some of the mismeasurements.

Finally, by choosing a different target, it could be possible to get a higher social welfare than the one from global targeting. The previous chapter showed that policy makers can get a higher social welfare under commitment than under discretion. An alternative approach would be for the central bank to maximize another welfare function, so having another targeting regime and choosing the weight between the targets, but still try to maximize the social welfare as a final objective. This chapter provides evidence that directly adopting some alternative targeting regimes could produce a higher social welfare than the one obtained when adopting global targeting. Thus there is a possibility that maximizing other welfare can generate optimal paths that would improve the standard social welfare even when optimized using global targeting.

This chapter evaluates the performance of different targeting approaches suggested by various authors. Four targets are considered. First, Walsh (2003) suggested that choosing the growth in output relative to the growth in potential output rather than the output gap itself in the loss function may reproduce the same kind of result than with the global target. It is the speed limit target and it was introduced to diminish the problem of error in output gap calculation. Second, one loss function that a central bank can adopt is one that incorporates the volatility of price level rather than the volatility of inflation, and it is referred as the price level targeting. It is compatible with the idea of bringing back price to its original level after a shock as in commitment. Third, Jensen (2002) considered to add a nominal income growth objective into the loss function of the central bank. Nominal income growth targeting is motivated by its ability to a nominal target so it is in monetary terms. Finally, Woodford (2003) introduced the idea of an interest-smoothing target. It is an approach that incorporates the volatility of interest rate into the

welfare function. The inertia that would then appear in the evolution of the interest rate would create paths in inflation and output gap that would be similar to ones found in the global target.

Originally, the objective of using those alternative targeting objectives was to reproduce the results of the global target but adopting a discretionary approach. As mentioned above, discretion is related to delegation to central bank. One of the most surprising results is that the central bank could actually get a higher social welfare while implementing price level or nominal income growth targeting than when using the global strategy. It may be counterintuitive but in fact, due to the specific constraints associated with the optimization with the global strategy, the result for welfare is not automatically the absolute optimal result that can be obtained. The chapter shows that for any values of the parameters, the central bank can choose a weight on output gap in the welfare function that makes price level targeting the strategy that provides the higher welfare.

Furthermore, the chapter shows that the interest-smoothing targeting objective would allow to dealing better with the lower bound issue because with that approach the volatility of interest rate is much lower than when using other targets. In addition, while it may trigger more inflation, the volatility of output gap would be diminished a lot so that strategy may be easier to implement and prevent the temptation from the central bank to switch. The interest-smoothing targeting objective however does not improve welfare much and then provide then lower results than the previous approaches.

2.1 Model

The basic New Keynesian model was introduced by Yun (1996), Rotemberg and Woodford (1997), Goodfriend and King (1997) and others. The version established by Clarida,

Gali, and Gertler in "The Science of Monetary Policy" (1999) and further described by

Woodford in "Interest and Prices" (2003) is widely used in recent work in policy design. The

paper follows the version of the model presented by Walsh (2003). Households provide labor,

purchase goods for consumption, hold money and have bonds. Firms produce different goods in

a monopolistically competitive goods market, designed by Dixit and Stiglitz (1977). Capital

stock is ignored. Price stickiness is specified by the Calvo's model in which it is assumed that

prices adjust infrequently. Each period, there is a constant probability 1-ω that the firm can

adjust its price. So there is a probability ω that the firm must keep its price unchanged.

Furthermore, it is forward looking because expectations of future variables are in structural

equation describing behavior of consumers and firms. We have the traditional log linearized

equations:

$$x_t = E_t x_{t+1} - \frac{\hat{i}_t - E_t \pi_{t+1}}{\sigma} + g_t \qquad (1) \qquad \text{with } g_t = \lambda g_{t-1} + \varepsilon_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \qquad (2) \qquad \text{with } u_t = \rho u_{t-1} + \eta_t$$

$$\hat{m}_t - \hat{p}_t = \frac{1}{b}(\sigma \hat{y}_t - \hat{i}_t) + \omega_t \qquad (3) \qquad \text{with } x_t = \hat{y}_t - \hat{y}_t^{\,f} \text{ and } \hat{y}_t^{\,f} = \frac{1+\eta}{\sigma+\eta}\hat{z}_t$$

$$\text{with } \omega_t = \delta \omega_{t-1} + \psi_t$$

Equation (1) is the "IS curve". Equation (2) corresponds to the supply curve; it is the

Phillips curve, or inflation curve. Equation (3) determines the quantity of money. It is the

demand for money. The following variables, ε_t, η_t, and ψ_t are white noises.

The other part of the model is the welfare function. Using targeting rules, policy makers

try to find the solution of a stochastic dynamic optimal control problem. There is a general

consensus among academics (Svensson 1999) to adopt a loss function that depends on the

variability of inflation and output gap. By adopting an inflation target rule, central banks have for

objective to stabilizing inflation around an inflation target. It is flexible in the sense that it takes

also into consideration the stability of the real economy. It is recognized that central banks are

concerned also about output gap when implementing their policies. So the loss function takes the

form of $L = (\pi_t - \pi^*)^2 + \Gamma x_t^2$ where π^* is the inflation target. The inflation target π_t^* here is zero.

Woodford (2003) demonstrated that it can be derived by optimizing household utility. In the

function loss, π_t is the inflation at time t, x_t the output gap and Γ the relative weight on stabilizing

the output gap. Γ is the relative importance of output gap volatility in the preference of central

bank. The goal for the central bank is then to maximize the welfare $-\dfrac{1}{2} E_t (\sum_{i=0}^{\infty} \beta^i (\Gamma x_{t+i}^2 + \pi_{t+i}^2))$

2.2 Gain from alternative regimes

2.2.1 Speed limit

As mentioned by Woodford (1999) and McCallum (1999), the optimal results are

obtained with monetary policies that are history-dependent. Walsh (2002) proposed a speed limit

targeting rule implemented in a discretion mode to provide the kind of inertia that would

improve social welfare. The central bank reacts to the change of output gap rather than its level.

Using a hybrid Phillips curve, they conclude that this rule follows closely the global strategy.

Stracca (2006) reaches also the same conclusion using data from the euro area. It was also

confirmed by Yetman (2006). Using neo-classical model, Hatcher (2008) reaches similar

conclusion. In this paper, using a forward Phillips curve, it is argued that the central bank can

adopt a weight on output that makes that approach superior to the pure discretion strategy. However, it cannot do better than the global strategy.

We should be then able to extract the optimal solution for the relationship between x_t and π_t that is established by the central bank. In the discretionary case, as it has already been well established that the central bank should minimize $\pi_t^2 + \Gamma_x x_t^2$ subject to not only the Phillips curve but also the IS curve because it involves interest rate.

In the speed limit case, the loss function takes the form:

$$L = \pi_t^2 + \Gamma_{sl}(x_t - x_{t-1})$$

The objective is to minimize

$$L = E_t \sum_{j=0}^{+\infty} \beta^j [-\frac{1}{2}(\pi_{t+j}^2 + \Gamma(x_t - x_{t-1})^2) + \psi_{t+j}(\pi_{t+j} - \beta\pi_{t+j+1} - \kappa x_{t+j} - u_{t+j})$$

Optimizing using discretion, we get the following first order conditions:

By deriving by π_{t+j}: $-\pi_{t+j} + \psi_{t+j} = 0$

By deriving by x_{t+j}: $-\Gamma(x_{t+j} - x_{t+j-1}) - \kappa\psi_{t+j} = 0$

So we get the following first order condition: $\pi_t = -\frac{\Gamma}{\kappa}(x_t - x_{t-1})$

The first order condition for speed limit under discretion is the same as the one from the global case. The paths are then identical to the ones from global target. Incorporating the paths of inflation and output gap found by optimizing the speed limit welfare with a weight Γ_{sl} into the social welfare that has a weight $\Gamma = \frac{\kappa}{\theta}$, the welfare from the speed limit target is always superior

to the welfare of pure discretion when Γ_{sl} is low enough (see Chart 11 b/ in which Γ_{sl} is below

0.01), but it is always lower than the welfare from the global approach (See Chart 11 c/). The

inertia created by using the change of output gap instead of its level improve welfare of

discretion but the relationship Woodford (2003) found between Γ and κ provides the optimal

weight on output.

Chart 11: Comparison of welfare for speed limit with welfare for discretion and global. Salmon
is the color used to represent the speed limit case

a/ Welfare for discretion, global   b/ Difference of welfare between   c/ Percentage difference between
and speed limit   speed limit and discretion   speed limit and global
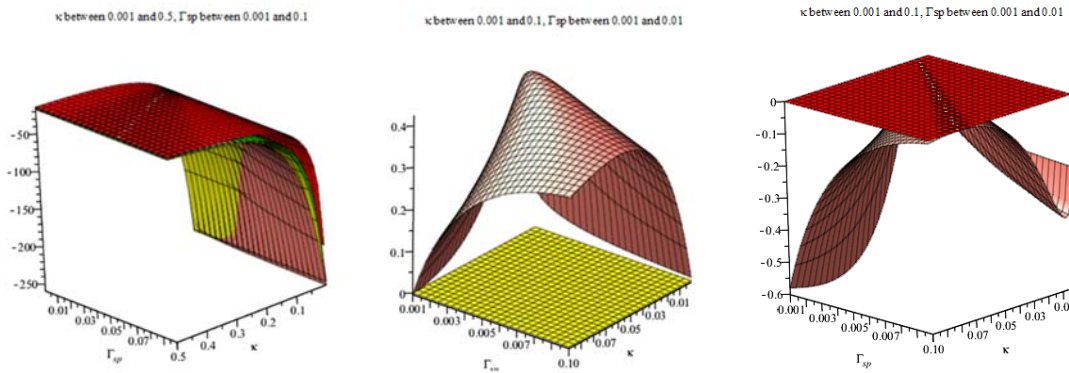
2.2.2 Price level targeting

Some authors (Fisher (1994), Haldane and Salmon (1995) and Kiley (1998)) use the idea

of an increase of volatility of the output gap to oppose a price level targeting regime. In contrast,

Dittmar, Gavin and Kydland (1999) and Svensson (1999) argue that this regime shows a

reduction of the volatility of not only inflation but also output gap. However, those results are

obtained using a neo-classical Phillips curve. On the other hand, Dittmar and Gavin (2000) and

Vestin (2003) found that this result holds with the New Keynesian Phillips curve, which is

forward looking. The policy provides a more favorable combination of volatility of inflation and output gap.

The stickiness of prices may be a factor that plays a role in the decision to target price level. According to the New Keynesian Phillips curve, it is the difference between current inflation and expected inflation that makes output gap fluctuating (Kiley 1998). When future inflation is expected to increase then output gap decreases. Due to the fact that price are sticky, firms increase their price to response to anticipated increase of future demand. As a consequence, the aggregate supply shifts negatively, and it as for effect to decrease current output. Price level targeting implies deflation after an initial inflationary shock, so it provokes more volatility in output gap. In contrast, Svensson (1999) weighted on the debate by claiming that policy makers may get a "Free Lunch" by adopting a price level targeting. However, he was using "Neoclassical" Phillips curve with incorporates lag inflation.

We should be then able to extract the optimal solution for the relationship between x_t and π_t that is established by the central bank. In the price level targeting case, as it has already been well established that the central bank should minimize $p_t^2 + \Gamma x_t^2$ subject to the Phillips curve, or inflation equation. Under discretion, it should thus minimize

$$L = \frac{1}{2}(p_t^2 + \Gamma_p x_t^2) + \psi_t(p_t - p_{t-1} + \beta p_t - \kappa x_t - u_t)$$

The first order condition is: $p_t = -\frac{(1+\beta)\Gamma_p}{\kappa} x_t$

So we obtain

$$p_t = \delta p_{t-1} + \frac{\delta}{1-\beta\delta\rho} u_t \quad \text{with } \delta = \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \quad \text{and } a = \frac{(1+\beta)\Gamma_p}{\kappa^2 + \Gamma(1+\beta)^2}$$

As a consequence, we obtain the following paths

$$x_t = \delta x_{t-1} - \frac{\kappa\delta}{\Gamma_p(1-\beta\delta\rho)(1+\beta)}u_t$$

$$\pi_t = \delta\pi_{t-1} + \frac{\delta}{1-\beta\delta\rho}(u_t - u_{t-1})$$

$$i_t = \delta i_{t-1} + \frac{\delta}{1-\beta\delta\rho}(1-\frac{\kappa\sigma}{\Gamma_p(1+\beta)})(\delta-1+\rho)u_t - \frac{\delta^2\rho}{1-\beta\delta\rho}(1-\frac{\kappa\sigma}{\Gamma_p(1+\beta)})u_{t-1} + \sigma(g_t - \delta g_{t-1})$$

We can find also the path for the real interest rate and money:

$$r_t = \delta r_{t-1} - \frac{\delta}{1-\beta\delta\rho}\frac{\kappa\sigma}{\Gamma_p(1+\beta)}(\delta-1+\rho)u_t + \frac{\delta^2\rho}{1-\beta\delta\rho}\frac{\kappa\sigma}{\Gamma_p(1+\beta)}u_{t-1} + \sigma(g_t - \delta g_{t-1})$$

$$m_t = \delta m_{t-1} + (1-\frac{\kappa\sigma}{b\Gamma_p(1+\beta)} + \frac{1}{b}(1-\delta-\rho)(1-\frac{\kappa\sigma}{\Gamma_p(1+\beta)}))\frac{\delta}{1-\beta\delta\rho}u_t + \frac{\delta^2\rho}{b(1-\beta\delta\rho)}(1-\frac{\kappa\sigma}{\Gamma_p(1+\beta)})u_{t-1}$$

$$+ (1-\delta)p_{-1} + \frac{\sigma}{b}(y_f^{t} - \delta y_f^{t-1}) - \frac{\sigma}{b}(g_t - \delta g_{t-1}) + \omega_t - \delta w_{t-1}$$

The paths shown in Chart 12 have the same characteristics than the ones from the global regime. Inertia seems even more pronounced that with the global or speed limit approaches.

Chart 12: Optimal paths for price level targeting
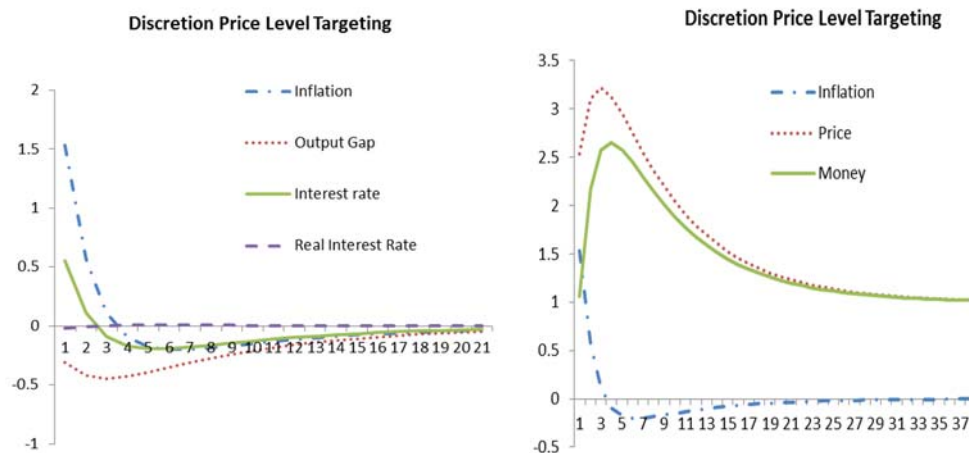
In chart 13, welfare with price level targeting is in blue and it has the same kind of
pattern than the welfare from other policies. However, price level targeting seems to be a
superior target. For any price stickiness κ, the central bank can select a low weight Γ_p on output
gap that would provide a welfare that is higher than welfare with not only discretion but also
global cases. Again, Γp is the weight that a central bank put on the volatility of output gap when
optimizing the welfare for price level targeting. However, the ultimate objective is still to
optimize the social welfare. Blake (2001) demonstrated that the global first order condition found
when maximizing social welfare is not the optimal solution. It is possible that the targeting of
other welfare function could provide better outcome. Chart 13 demonstrates that conclusion. In
chart 13 a, it can be seen that for some combination of Γp and κ, the social welfare obtained
when optimizing the welfare for price level targeting is greater than the social welfare resulting
from the global regime. The percentage difference between the two is seen in chart 13 b.

<u>Chart 13</u>: Comparison of welfare for price level targeting with welfare for discretion, pre-commitment and global. Blue is the color used to represent price level targeting

a/ Welfare for discretion, pre-commitment, global and price level

b/ Percentage difference between price level targeting welfare and global welfare

κ between 0.001 and 0.1, Γp between 0.0001 and 0.1

κ between 0.001 and 0.1, Γp between 0.0001 and 0.1
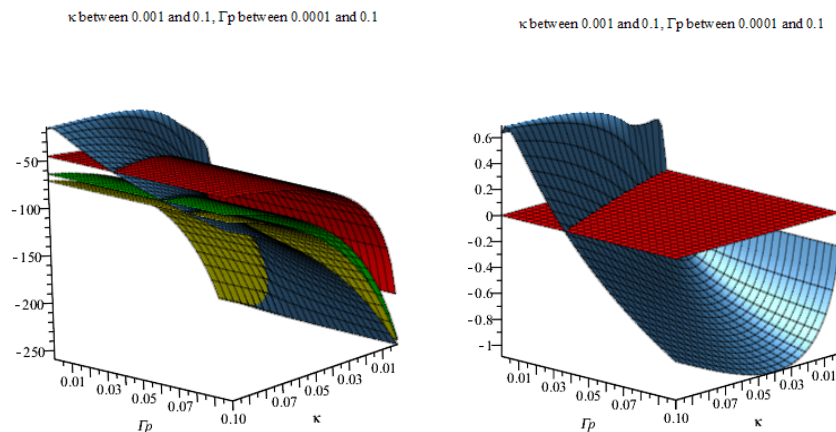
2.2.3 Nominal income growth

Stability of the nominal income targeting regime has been studied for a while. However, the result depends mainly on the model adopted, and particularly the form of the Phillips curve. Ball (1999) used a backward looking Phillips curve and concluded that not only nominal income growth targeting is not efficient but also it creates instability in the sense that the variances of inflation and output gap are infinite. In contrast, McCallum (1997) argue that stability depends on the specifications of the Phillips curve. With a forward looking model, he finds that nominal income growth targeting is not instable. Dennis (2001) reaches similar conclusion. Malik (2005) found that nominal income growth targeting performs better than the price level targeting rule, using a continuous New Keynesian model. Incorporating endogenous persistence, Hanson and Kapinos (2006) showed that the alternative rules such as nominal income growth targeting do not perform as well. If there is a forward looking component in the model, it is enough to make the system stable. It depends then on the form of the Phillips curve.

The loss function for the nominal income growth targeting is:

$$L = \pi_t^2 + \Gamma_{gdp}(\pi_t + x_t - x_{t-1})$$

Under discretion, the first order condition is: $\pi_t = -\dfrac{\Gamma_{gdp}(1+\kappa)}{\kappa + \Gamma_{gdp}(1+\kappa)}(x_t - x_{t-1})$

It is found by the following derivations:

By deriving by π_{t+j}: $\quad -\pi_{t+j} - \Gamma_{gdp}(\pi_{t+j} + x_{t+j} - x_{t+j-1}) + \psi_{t+j} = 0$

By deriving by x_{t+j}: $\quad -\Gamma_{gdp}(\pi_{t+j} + x_{t+j} - x_{t+j-1}) - \kappa\psi_{t+j} = 0$

We can transform this relationship to obtain

$$x_t = -\frac{\kappa + \Gamma_{gdp}(1+\kappa)}{\Gamma_{gdp}(1+\kappa)}\hat{p}_t \quad \text{with } \hat{p}_t = p_t - p_{-1} \text{ and p-1 being the price level just before the}$$

targeting objective took place.

As a consequence, the central bank is implementing an objective using also price level as a target.

We then have the path for $\hat{p}_t$ and it is

$$\hat{p}_t = \delta\hat{p}_{t-1} + \frac{\delta}{1-\beta\delta\rho}u_t \quad \text{with } \delta = \frac{1-\sqrt{1-4\beta a^2}}{2a\beta} \text{ and } a = \frac{\Gamma_{gdp}(1+\kappa)}{\kappa^2 + \Gamma_{gdp}(1+\kappa)(1+\beta+\kappa)}$$

We can then deduct the paths for the other variables.

For the output gap we get: $\quad x_t = \delta x_{t-1} - \dfrac{\kappa\delta}{(1-\beta\delta\rho)\Gamma_{gdp}}u_t$

And for inflation
$$\pi_t = \delta\pi_{t-1} + \frac{\delta}{1-\delta\beta\rho}(u_t - u_{t-1})$$

For the interest rate, we would obtain:

$$i_t = -(1-\delta)(1-\frac{\kappa\sigma}{\Gamma_{gdp}})\hat{p}_t + \frac{\delta}{1-\beta\delta\rho}\rho(1-\frac{\kappa\sigma}{\Gamma_{gdp}})u_t + \sigma g_t$$

Or

$$i_t = \delta\tilde{i}_{t-1} + \frac{\delta}{1-\beta\delta\rho}(\rho+\delta-1)(1-\frac{\kappa\sigma}{\Gamma_{gdp}})u_t - \frac{\delta^2}{1-\beta\delta\rho}\rho(1-\frac{\kappa\sigma}{\Gamma_{gdp}})u_{t-1} + \sigma(g_t - \delta g_{t-1})$$

We can get also the real interest rate: $r_t = i_t - E_t\pi_{t+1}$

$$r_t = \delta r_t - \frac{\kappa\sigma\delta(\delta+\rho-1)}{\Gamma_{gdp}(1-\beta\delta\rho)}u_t + \frac{\kappa\sigma\delta^2\rho}{\Gamma_{gdp}(1-\beta\delta\rho)}u_{t-1} + \sigma(g_t - \delta g_{t-1})$$

For the money, we use $m_t = p_t + \frac{\sigma}{b}y_t - \frac{1}{b}i_t$ to get:

$$m_t = \delta m_{t-1} + (1 - \frac{\kappa\sigma}{\Gamma_{gdp}b} + \frac{1}{b}(1-\delta-\rho)(1-\frac{\kappa\sigma}{\Gamma_{gdp}}))\frac{\delta}{1-\beta\delta\rho}u_t + \frac{\delta^2}{1-\beta\delta\rho}\frac{\rho}{b}(1-\frac{\kappa\sigma}{\Gamma_{gdp}})u_{t-1}$$

$$+ (1-\delta)p_{-1} + \frac{\sigma}{b}(y_t^f - \delta y_{t-1}^f) - \frac{\sigma}{b}(g_t - \delta g_{t-1}) + \omega_t - \delta\omega_{t-1}$$

The paths for nominal income targeting offer also the kind of inertia that is similar to global, price level targeting and speed limit cases (see chart 14).

<u>Chart 14</u>: Optimal paths for the nominal income growth case
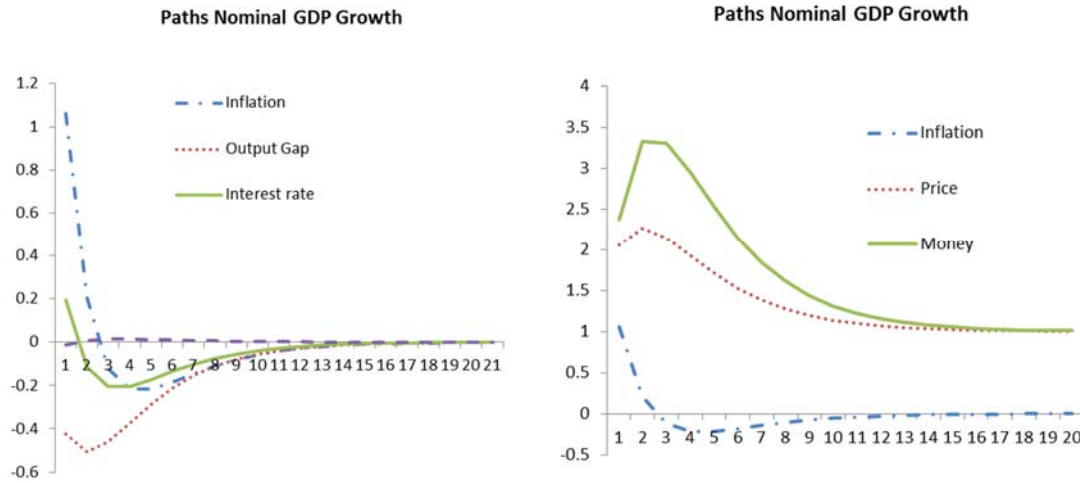
In a similar analysis used in the previous section, chart 15 b shows that generally nominal income growth targeting is superior to pure discretion, particularly when price stickiness κ is lower and the weight on output is larger. Furthermore, chart 15 c indicates that the nominal income growth regime is not providing a better welfare than when implementing a price level targeting. It should be noticed that when the two welfares in chart 15 c are compared, the weights Γ_p and Γ_{gdp} are moving the same amount.

<u>Chart 15</u>: Comparison of welfare for nominal income growth with welfare for discretion, pre-commitment and global. Violet is the color used to show nominal growth targeting

a/ Welfare for discretion, pre-commitment, global and nominal income growth difference	b/ Percentage difference between welfare for nominal income growth and welfare for discretion	c/ Percentage difference between welfare for nominal income growth and welfare for global

κ between 0.001 and 0.5, Γgx1 between 0.001 and 0.1

κ between 0.001 and 0.5, Γgx1 between 0.001 and 0.1

κ between 0.001 and 0.5, Γgx1 between 0.001 and 0.1
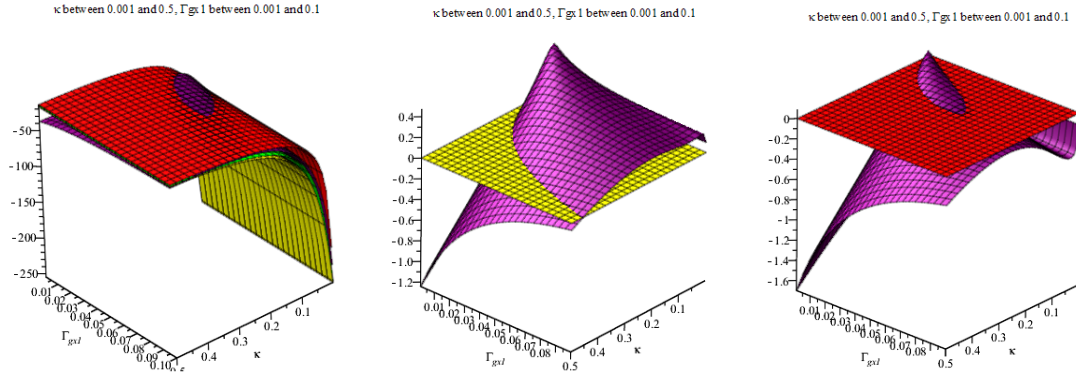
<u>2.2.4 Welfare gain with different regimes</u>

Chart 16 shows how price level targeting is a superior regime. The central bank can choose a low enough weight Γ_p on output in price level targeting to get a higher welfare than with any other regime. It should be reminded that the objective is still to maximize the social welfare $-\frac{1}{2}E_t(\sum_{i=0}^{\infty}\beta^i(\Gamma x_{t+i}^2 + \pi_{t+i}^2))$. We compare thus different regimes to obtain the maximum social welfare. As described above, those regimes include strategies that optimize a different welfare with their own weight Γ_p (Γ_p is used here as a symbol for all the weights for the different regimes) that reflects the importance that the central bank put on variables such as inflation, price level and output gap. In chart 16 a, the level of welfare is represented for the different regimes depending on the price stickiness κ and the policy weight Γ_p. We observe that for any level of stickiness κ, the central bank would select a weight Γ_p low enough to get the highest welfare and

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every time the price level targeting offers the best solution. Chart 16 b/ indicates how price level targeting is proportionally superior to nominal income growth targeting. It is noticeable that the more sticky prices are, the lower Γ_p needs to be to select the price level targeting over the other regimes.

Chart 16: Comparison of welfare for discretion, pre-commitment, global, price level targeting and nominal income growth targeting

a/ Welfare for discretion, pre-commitment, global, speed limit, nominal income growth and price level targeting

b/ percentage difference between welfare for price level targeting and welfare for nominal income growth
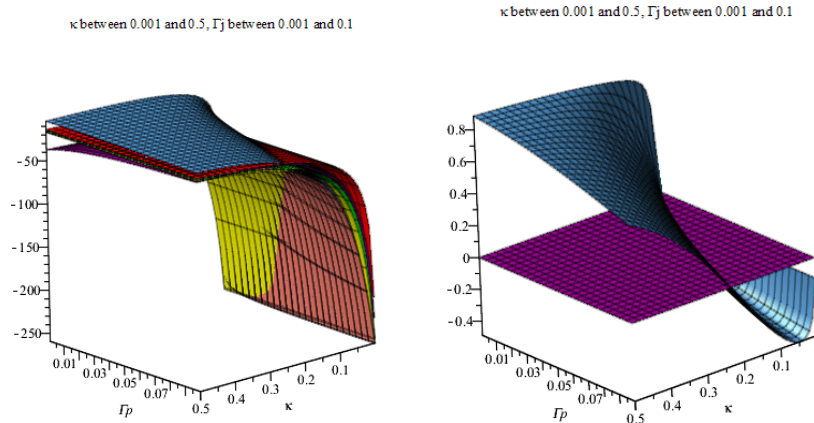
2.2.5 Interest-smoothing targeting

In the interest-smoothing regime, it is assumed that the central bank is also concerned by interest rate volatility. Initially, Woodford and Rotemberg (1997) raised the concern that interest rate could hit the zero lower bound and suggested that incorporating the interest rate volatility into welfare would minimize that risk. Woodford (1999) also argued that the addition of an interest rate smoothing objective in the loss function would introduce inertia and improve welfare.

The goal for the central bank is then to maximize the welfare

$$-\frac{1}{2}E_t(\sum_{i=0}^{\infty}\beta^i(\pi_{t+i}^2+\Gamma_x x_{t+i}^2+\Gamma_i i_{t+j}^2))$$

Again, it is optimized under discretion. As mentioned before, it is defined as a regime that is implemented at the beginning of each period by policy makers after they examined data from the economy and they optimize their decision. The central bank does not consider future dates, so it cannot affect the expectation of private agents. Individual agents know that the central bank proceeds this way. The expectation from the private sector is that policy makers will continue to adopt that strategy. The policy makers have no incentive to modify their behavior, and the expectations of the private sector are rational; here it is implemented through delegation.

We should be then able to extract the optimal solution for the relationship between x_t and π_t that is established by the central bank. Under discretion, as it has already been well established that the central bank should minimize $\pi_t^2 + \Gamma_x x_t^2 + \Gamma_i i_t^2$ subject to not only the Phillips curve but also the IS curve because it involves interest rate. It should thus minimize

$$L=E_t\sum_{j=0}^{+\Leftrightarrow}\beta^j[-\frac{1}{2}(\pi_{t+j}^2+\Gamma_x x_{t+j}^2+\Gamma_i i_{t+j}^2)+v_{t+j}(x_{t+j}-x_{t+j+1}+\frac{1}{\sigma}(i_{t+j}-\pi_{t+j+1})-g_{t+j})$$

$$+\psi_{t+j}(\pi_{t+j}-\beta\pi_{t+j+1}-\kappa x_{t+j}-u_{t+j})]$$

For the discretion case, we get the following first order conditions:

By deriving by i_{t+j}: $-\Gamma_i i_{t+j}+v_{t+j}\frac{1}{\sigma}=0$

By deriving by π_{t+j}: $-\pi_{t+j}+\psi_{t+j}=0$

By deriving by x_{t+j}: $-\Gamma_x x_{t+j} + V_{t+j} - \kappa\psi_{t+j} = 0$

So we get the following first order condition: $-\Gamma_x x_t + \sigma\Gamma_i i_t - \kappa\pi_t = 0$

Or $x_t = -\dfrac{\kappa}{\Gamma_x}\pi_t + \sigma\dfrac{\Gamma_i}{\Gamma_x}i_t$

Here, ψ_t is the Lagrangian multiplier associated with the inflation equation and γ_t is the Lagrangian multiplier associated with the IS curve. It should be noticed that $E_t\pi_{t+1}$ disappeared in this equation because the central bank does not consider future dates.

We get the following optimal paths:

$$\pi_t = D_{d1}u_t + D_{d2}g_t$$

With $\quad D_{d1} = \dfrac{\sigma^2\Gamma_i(\rho-1)-\Gamma_x}{A\kappa}$

$$A = -\kappa + \left(-\frac{\sigma\beta\rho^2}{\kappa} + \left(1+\frac{\sigma}{\kappa}+\frac{\sigma\beta}{\kappa}\right)\rho - \frac{\sigma}{\kappa}\right)\sigma\Gamma_i + (\rho\beta-1)\frac{\Gamma_x}{\kappa}$$

$$D_{d2} = -\frac{\sigma^2\Gamma_i}{B}$$

$$B = -\kappa + \left(-\frac{\sigma\beta\lambda^2}{\kappa} + \left(1+\frac{\sigma}{\kappa}+\frac{\sigma\beta}{\kappa}\right)\lambda - \frac{\sigma}{\kappa}\right)\sigma\Gamma_i + (\lambda\beta-1)\frac{\Gamma_x}{\kappa}$$

$$x_t = D_{d3}u_t + D_{d4}g_t$$

With $\quad D_{d3} = \dfrac{\sigma^2\Gamma_i(\rho-1)-\Gamma_x}{A\kappa^2}(1-\beta\rho)-\dfrac{1}{\kappa}$

$$D_{d4} = (\beta\lambda - 1)\frac{\sigma^2\Gamma_i}{\kappa B}$$

$$i_t = D_{d5}u_t + D_{d6}g_t$$

With $D_{d5} = \dfrac{\sigma^2\Gamma_i(\rho-1)-\Gamma_x}{A\kappa}(\rho+\dfrac{\sigma(1-\beta\rho)}{\kappa}(\rho-1))+\dfrac{\sigma}{\kappa}(1-\rho)$

$$D_{d6} = (\frac{\sigma(\beta\lambda-1)}{\kappa}(\lambda-1)-\lambda)\frac{\sigma^2\Gamma_i}{B}+\sigma$$

with $u_t = \rho u_{t-1}+\eta_t$ and η_t is a white noise process with constant variance σ_u^2.

We can find also the real interest rate.

We get:

$$r_t = (\frac{\sigma^2\Gamma_i(\rho-1)-\Gamma_x}{A\kappa}\frac{\sigma(1-\beta\rho)}{\kappa}(\rho-1)+\frac{\sigma}{\kappa}(1-\rho))u_t + (\frac{\sigma(\beta\lambda-1)}{\kappa}(\lambda-1)\frac{\sigma^2\Gamma_i}{B}+\sigma)g_t$$

For the money path, we incorporated σg_t into the path for interest rate to neutralize the shock from the demand curve. These paths are the optimal paths after a shock u_t from the Phillips curve. This shock u_t represents usually a difference between the marginal cost and the output gap. In this process, deviations of inflation and output gap from steady state in the past are neglected. What counts is to bring back inflation and output back to target.

If the economy is subject to an impulse in the Phillips curve, contrary to inflation and output gap, price level is not forced to go back to previous levels, and goes up without limits. We can see that by deducting the path for the price level. We have $p_t = \pi_t + p_{t-1}$ so we obtain:

$$p_t = p_{t-1} + \frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_x}{A\kappa} u_t - \frac{\sigma^2 \Gamma_i}{B} g_t$$

Even if monetary policies use interest rate rules, money is not completely absent in the model. Then we can use the money demand equation to find the optimal path for money:

$$m_t - p_t = \frac{\sigma}{b} y_t - \frac{1}{b} i_t + \omega_t$$

With $\qquad p_t = \pi_t + p_{t-1}$

$$y_t = x_t + y_t^f$$

we get:

$$m_t = m_{t-1} + [\frac{\sigma}{b}(\frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_x}{A\kappa^2}(1 - \beta\rho) - \frac{1}{\kappa}) - \frac{1}{b}(\frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_x}{A\kappa}(\rho + \frac{\sigma(1 - \beta\rho)(\rho - 1)}{\kappa} + \frac{\sigma(1 - \rho)}{\kappa})) + \frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_x}{A\kappa}]u_t$$

$$-[\frac{\sigma}{b}(\frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_x}{A\kappa^2}(1 - \beta\rho) - \frac{1}{\kappa}) - \frac{1}{b}(\frac{\sigma^2 \Gamma_i (\rho - 1) - \Gamma_x}{A\kappa}(\rho + \frac{\sigma(1 - \beta\rho)(\rho - 1)}{\kappa} + \frac{\sigma(1 - \rho)}{\kappa}))]u_{t-1}$$

$$+[\frac{\sigma}{b}(\beta\lambda - 1)\frac{\sigma^2 \Gamma_i}{\kappa B} - \frac{1}{b}((\sigma\frac{(\beta\lambda - 1)(\lambda - 1)}{\kappa} - \lambda)\frac{\sigma^2 \Gamma_i}{B} + \sigma) - \frac{\sigma^2 \Gamma_i}{B}]g_t$$

$$+[\frac{\sigma}{b}(\beta\lambda - 1)\frac{\sigma^2 \Gamma_i}{\kappa B} - \frac{1}{b}((\sigma\frac{(\beta\lambda - 1)(\lambda - 1)}{\kappa} - \lambda)\frac{\sigma^2 \Gamma_i}{B} + \sigma)]g_{t-1}$$

$$+\frac{\sigma}{b}(y_t^f - y_{t-1}^f) + \omega_t - \omega_{t-1}$$

According to Woodford (2003), Γ_i and Γ_x have a specific relationship. When calculating the welfare obtained when using this regime, only Γ_x will change so we can compare with other

regimes. Chart 17 shows the path for the different variables when choosing the interest-smoothing regime. It is noticeable that these paths have the same kind of shape than the paths derived under pure discretion. No inertia is created and it will have as a consequence a lower welfare than the other regimes.

Chart 17: Optimal paths for discretion for interest rate smoothing approach
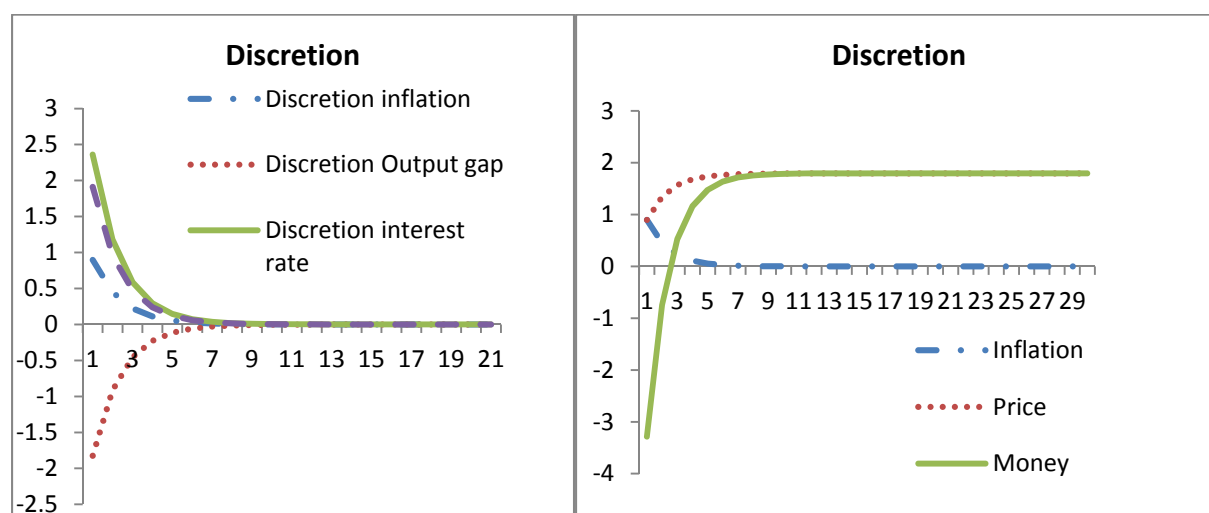
2.3 Comparison of different regimes

The idea is the same as in the previous section. The paths from the different alternative regimes are used to calculate this new welfare. Analyzing the gain from the alternative regimes, the result of this paper shows that the price level targeting rule still dominates the other approaches. Even considering the volatility of interest rate, the benefit of emphasizing on bringing back price to its original level is predominant. However, it is the case when the central bank may choose the weight Γ_p (again it symbolizes here the weight for each of the different regimes) that appears on the loss function. The highest is price stickiness, the lowest this weight has to be to reach the highest welfare through price level targeting. If Γ_p is not low enough, then

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nominal income growth targeting would be superior in case of high stickiness. It is possible now to see the results in chart 18.

Chart 18: Welfare for discretion, speed limit, nominal income growth, price level targeting and interest rate smoothing approach. Aqua is the color used for interest rate smoothing approach. Both charts are the same but the range for κ and Γ_p are different
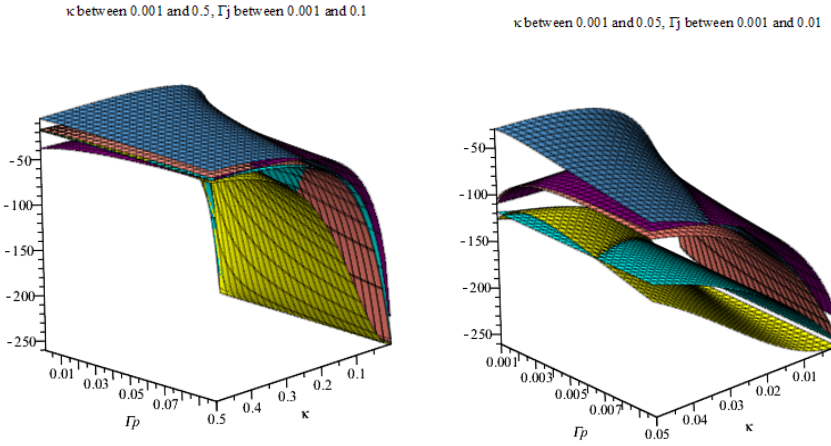
Chart 19 shows the percentage gain of the different policies against selected others. In chart 19 a, it can be seen that the interest-smoothing regime provides a higher welfare than pure discretion when using a large enough Γ_p (which is Γ_x for the interest-smoothing regime). In the same vein, chart 19 b shows how the speed limit approach has a higher welfare than in the interest-smoothing case when Γ_p is low enough (Γ_{sl} for speed limit and Γ_x for interest-smoothing). Finally chart 19 c demonstrates that price level targeting provides a higher welfare than speed limit. All of these conclusions are found for a specific range of Γ_p and κ that are appropriate considering the standard calibration from the literature. For a different range of selection, the shape of the charts would be different.

<u>Chart 19</u>: Percentage gain of different regimes in terms of welfare

a/ Percentage gain of discretion against interest rate smoothing

b/ Percentage gain of speed limit against interest rate smoothing

c/ Percentage gain of price level against speed limit
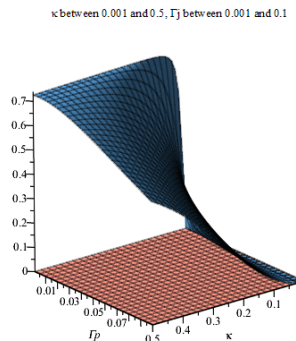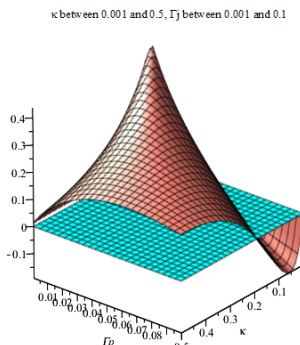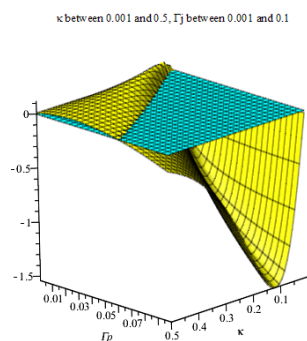
The key to obtain the highest welfare is for the central bank to use the weight on the output gap (or other variable in certain regimes) inside the loss function. From the graphs, it is clear that policy makers would get a higher welfare with either price level targeting or nominal income growth targeting depending on the weight on output gap. Chart 20 shows the percent difference between the two regimes. At any level of stickiness, the highest welfare that a central bank could produce when choosing among the regimes described above is the one by selecting price level targeting. It can do so when choosing a weight Γ_p low enough.

Chart 20: Percentage gain of nominal income growth against price level targeting in terms of welfare

κ between 0.001 and 0.5, Γj between 0.001 and 0.1
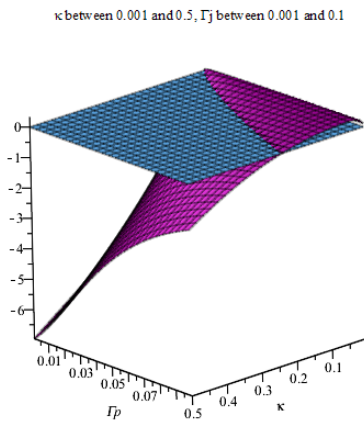
The calculation of welfare involves the trade-off between the volatility of inflation and the volatility of output gap. Traditionally, the idea is that inertia created through the different regimes described in this chapter would lower the volatility of inflation and increase the volatility of output gap. It is not as simple as that however. In any case, it is the change of the trade-off between the two that can potentially increase welfare. The ultimate goal would be then to search for the best combination. The volatility of inflation and output gap for the different regimes are shown in chart 21. In Appendix C, we can see the graphs showing the differences between each policy.

Chart 21: Volatility of inflation and output gap for discretion, global, price level targeting, nominal income growth targeting and interest rate smoothing approach

a/ Volatility of Inflation   b/ Volatility of Output Gap
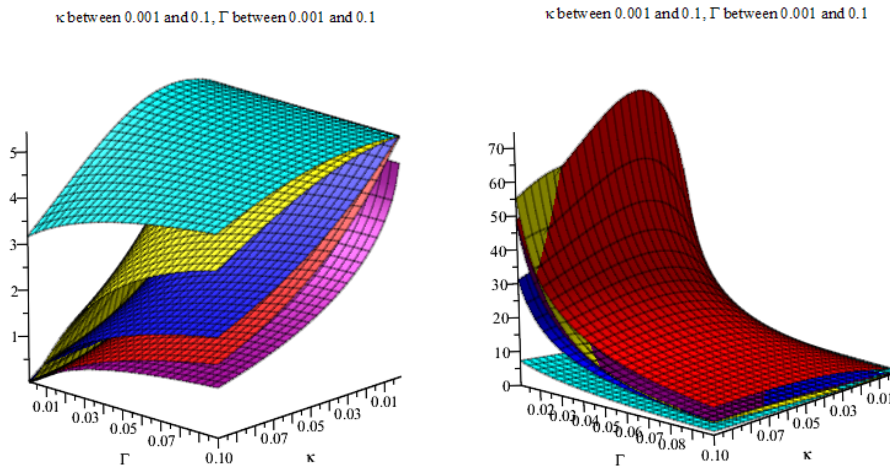
It can be seen that the approach including the volatility of interest rate into the loss function provides the lowest volatility of output gap but also the highest volatility of inflation. Thus that approach would be easier to implement in terms of consistency but the welfare is not improving a lot compared to the traditional discretion case.

What can be observed also is that, contrary to earlier conclusions from authors, price level targeting decreases output gap volatility. It is actually the key point that makes that strategy superior to others when trying to reach the highest welfare. Even if that policy increases volatility of inflation, policy makers can find the highest welfare with it when decreasing the weight on output gap. Furthermore, output gap volatility is lower with price level targeting than with nominal income targeting.

2.4 Volatility of interest rate and lower bound issue

In a low inflation environment and when nominal interest rates are close to zero monetary policy makers would have a limited ability to ease in response to adverse shocks. The focus of the literature is to propose strategies when traditional monetary policies become apparently inefficient. An increasing number of economists revive the idea of stabilizing nominal income growth targeting as a strategy. This paper has another perspective. Having found analytical solution to calculate welfare for different regimes, it is also possible to develop formulas to assess the volatility of interest rate. As a consequence it is possible to evaluate to what extend the different approaches can reduce this volatility to prevent the situation in which the nominal interest rate is immobilized at zero.

Chart 22 shows, for a specific level of price stickiness, how volatility evolve when central bank manipulate the weight Γ_p on output that appears on different monetary approaches. Using the calibration from Rotemberg and Woodford (1997), Γ is considered to be around 0.0048 in the social welfare. This chapter showed that the central bank can obtain the highest welfare when adopting a price level targeting rule with a very low Γ_p. According to the results shown in chart 19, it would require to increase that weight Γ_p to reduce the volatility of interest rate. However, it can be seen that that the central bank could be significantly more effective at lower that volatility by choosing discretion with the social welfare incorporating transaction frictions, or the interest-smoothing regime. It implies that policy makers would switch from price level targeting to inflation targeting policy. As shown in chart 18 and 19, it would have for consequence to reduce welfare. It is the cost of avoiding the problems created by the lower bound for interest rate.

Chart 22: Volatility of interest rate for the different regimes considered in this chapter
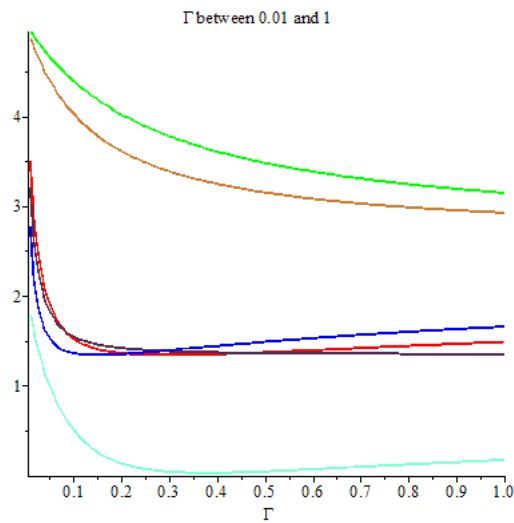
2.5 Concluding remark

A commitment rule should in general produce higher social welfare compared to discretion. However, several authors suggested other different targets to reproduce the inertial behavior observed in commitment. In the speed limit case, the central bank would focus on the growth of output gap instead of its level. For the price level targeting strategy, the emphasis would be on price level rather than inflation. Finally, the nominal income growth targeting would replace output gap by the growth of output gap plus inflation.

This chapter used an analytical solution that shows the gains from these different scenarios compared to the traditional commitment of social welfare. First, the relative gain of the different targeting regimes depends on the values of the parameters. Second, the interest-smoothing regime would be the easiest to implement because of low volatility of output gap but the resulting welfare is not higher than the global approach. Third, this interest-smoothing regime has the advantage of having a much lower volatility of interest rate which is an advantage in case policy makers encounter the lower bound issue.

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CHAPTER 3

Optimal Instrument Rules and Determinacy in Targeting Regimes

<u>3.0 Introduction</u>

In the previous chapters, the welfare gain of different targeting approaches were derived and calculated using an analytical solution. Those results provided optimal paths but there was no mention about specific monetary policies. However, when a central bank selects a target then it has to choose among different instrument rules to guide the economy to the corresponding optimal paths. In the New Keynesian model, the Taylor rule is traditionally used as the instrument but some authors have argued that the determinacy issue may be difficult to solve, determinacy being the existence of a stable solution of a single rational expectation dynamic system. However, it is widely recognized how difficult it is to get uniqueness and stability in interest rate rules and it has been at the origin of a large literature. As a consequence some authors have proposed alternative rules.

First, following the methodology from Thurston (2012), this chapter finds the optimal Taylor rules for not only the discretion and global policies but also for the price level and nominal income targeting policies. It can be seen that policy makers can always find a determinate solution by choosing certain parameters in the Taylor rule.

Second, some authors have proposed to use a money rule instead of an interest rate rule. It is recognized that money demand and the stock of money play a minor role in current monetary theories and is not taken in consideration in the decision making processes at most of

central banks. As Woodford (2008) emphasized, however, it is true that money supply is not totally absent in the New Keynesian model, and can be integrated through a money demand curve. One of the results of this paper is that it can be proved that conditions for determinacy are automatically satisfied when utilizing a particular exogenous money rule. However, those types of rules cannot allow policy maker to implement an optimal rule. As a consequence, it is necessary to adopt a kind of 'Taylor money rule' that controls for inflation and output gap and in this case determinacy constraints are not very different from the one encountered with an interest rate rule.

Third, as observed in the previous chapters, it has been demonstrated that inertia and then price level stability would enhance welfare[11]. Even if inflation can be controlled under some conditions in the New Keynesian model, there is less obvious reasons to believe that the price level is somewhat controlled using a Taylor rule. In his seminal book, Woodford (2003) proposed the idea of using a Wicksellian rule. This rule links the nominal interest rate with the price level rather than the rate of inflation. In this chapter, it is shown that even if a monetary rule would allow central banks to anchor price level, policy makers may also adopt an interest rate rule to obtain the same result and it does not materially affect the determinacy issue. A central bank would still have to face the same kind of problem and select the parameters in a way that will ensure to be in the determinate region.

3.1 Model

As in the previous chapter, the model used is the basic New Keynesian model. The model that is followed is the one presented by Walsh (2003). Households provide labor, purchase goods

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[11] See Woodford (2003). For detailed analytical solutions see Marest and Thurston (2013) and Marest (2015)

for consumption, hold money and have bonds. Firms produce different goods in a monopolistically competitive goods market, designed by Dixit and Stiglitz (1977). Capital stock is ignored. We have the traditional log linearized equations:

$$x_t = E_t x_{t+1} - \frac{\hat{i}_t - E_t \pi_{t+1}}{\sigma} + g_t \qquad (1) \qquad \text{with } g_t = \lambda g_{t-1} + \varepsilon_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \qquad (2) \qquad \text{with } u_t = \rho u_{t-1} + \eta_t$$

$$\hat{m}_t - \hat{p}_t = \frac{1}{b}(\sigma \hat{y}_t - \hat{i}_t) + \omega_t \qquad (3) \qquad \text{with } x_t = \hat{y}_t - \hat{y}_t^f \text{ and } \hat{y}_t^f = \frac{1+\eta}{\sigma+\eta}\hat{z}_t$$

$$\text{with } \omega_t = \delta \omega_{t-1} + \psi_t$$

Equation (1) is the "IS curve". Equation (2) corresponds to the supply curve; it is the Phillips curve, or inflation curve. Equation (3) determines the quantity of money. It is the demand for money. The following variables, ε_t, η_t, and ψ_t are white noises.

In the literature, authors look for interest rate rules that would provide the welfare closest to the optimal one. McCallum and Nelson (2004) argue for example that policymakers have difficulties to come up with a true model in terms of theory but also due to imperfect knowledge of the value of the parameters. It becomes then difficult operationally to implement those models. Not only is it difficult for the central bank to find with accuracy the current value of output but also the natural-rate level of that output, the one that the economy would generate if wages and prices were flexible. However, this chapter argues that we can get the path of interest rate that corresponds to the optimal solution and find a corresponding interest rule that matches

this path. It is a consensus into the literature that the central bank minimizes the following

welfare: $E_t(\sum_{i=0}^{\infty} \beta^i (\Gamma x_{t+i}^2 + \pi_{t+i}^2))$

3.2 Determinacy and Optimal Rules

For each of the instrument rules proposed in the chapter, the analysis contains three steps. The first one is to find the determinacy conditions of the model when adopting an instrument rule: it involves to obtaining the eigenvalues of the auto-regressive matrix of the systems of equations of the model and to see at what conditions they are inside or outside the unit circle. In the second step, policy makers have to choose the parameters of the instrument rule to reproduce the optimal paths established in the previous chapters. Finally, by combining the two later results, the objective is to determine the range of values of the parameters of the optimal instrument rule that makes the solution unique and stable.

3.3 Taylor rules or inflating targeting rules

First, the question of determinacy is raised when applying an interest rule. The New Keynesian Taylor rule method is widely used in monetary economics for the determination of inflation. However, with a Taylor rule, some conditions between parameters of the rule are necessary to obtain a unique and stable solution.

3.3.1 Interest rate Taylor rule

The Taylor rule controls for inflation and output gap: $i_t = \phi_\pi \pi_t + \phi_x x_t + v_t$. As a well-known result, in order to get unique and stable equilibrium in this model with a Taylor Rule, the φ's must satisfy the following condition: $0 < \kappa(\phi_\pi - 1) + \phi_x(1-\beta)$. To find the optimal instrument rule for the discretionary policy, the solution for ϕ_π and ϕ_x from the Taylor principal can be

found with the method of unknown coefficient[12]. There is a simpler way to arrive at a solution.

First, following "The Science of Monetary policy" from Clarida and al. (1999), we have from

equation (7) representing the optimal path of inflation: $\pi_t = \dfrac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)} u_t$

So, we have also

$$\pi_{t+1} = \frac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)} u_{t+1}$$

Then

$$\pi_{t+1} = \frac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)}(\rho u_t + \eta_{t+1})$$

So

$$E_t \pi_{t+1} = \frac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)} \rho u_t = \rho \pi_t$$

And the same way, we have:

$$E_t x_{t+1} = \rho x_t$$

As a consequence, the IS curve $x_t = E_t x_{t+1} - \dfrac{\hat{i}_t - E_t \pi_{t+1}}{\sigma} + g_t$ can be transformed the following

way.

First we replace: i_t with $i_t = \phi_\pi \pi_t + \phi_x x_t + \varphi_g g_t$

$E_t x_{t+1}$ with $E_t x_{t+1} = \rho x_t$

$E_t \pi_{t+1}$ with $E_t \pi_{t+1} = \rho \pi_t$

---

[12] See Lecture notes for Thom Thurston's Macroeconomics I (Econ 711, CUNY Graduate Center) contain probably the only analytical solution for optimal parameters of the Taylor Rule in the context of the basic New Keynesian model.

So we get: $x_t = \rho x_t - \dfrac{\varphi_\pi \pi_t + \varphi_x x_t + \varphi_g g_t - \rho \pi_t}{\sigma} + g_t$

Or, after transformations and using $\varphi_g = \sigma$ to cancel g_t, we get:

$$0 = (-1 + \rho - \frac{\varphi_x}{\sigma}) x_t + (-\frac{\varphi_\pi}{\sigma} + \frac{\rho}{\sigma}) \pi_t$$

But, if we incorporate the fact that $x_t = -\dfrac{\kappa}{\Gamma} \pi_t$ then:

$$\varphi_\pi = \rho + \kappa \sigma \frac{1 - \rho + \dfrac{\varphi_x}{\sigma}}{\Gamma}$$ and it is what professor Thurston found through the method of

unknown coefficients.

It can be transformed, and we get $\varphi_\pi = \rho + \kappa \sigma \dfrac{1 - \rho}{\Gamma} + \dfrac{\kappa}{\Gamma} \varphi_x$ so the slope of the curve

representing this relationship is $\dfrac{\kappa}{\Gamma}$. It is the optimal relationship between φ_π and φ_x.

Chart 23 shows the stability and uniqueness condition and the optimal Taylor rule with the

discretion policy. It is the graph found in Thurston (2012).

Chart 23: Determinacy zone and optimal relationship between the φ_π and φ_x in the Taylor rule for discretion
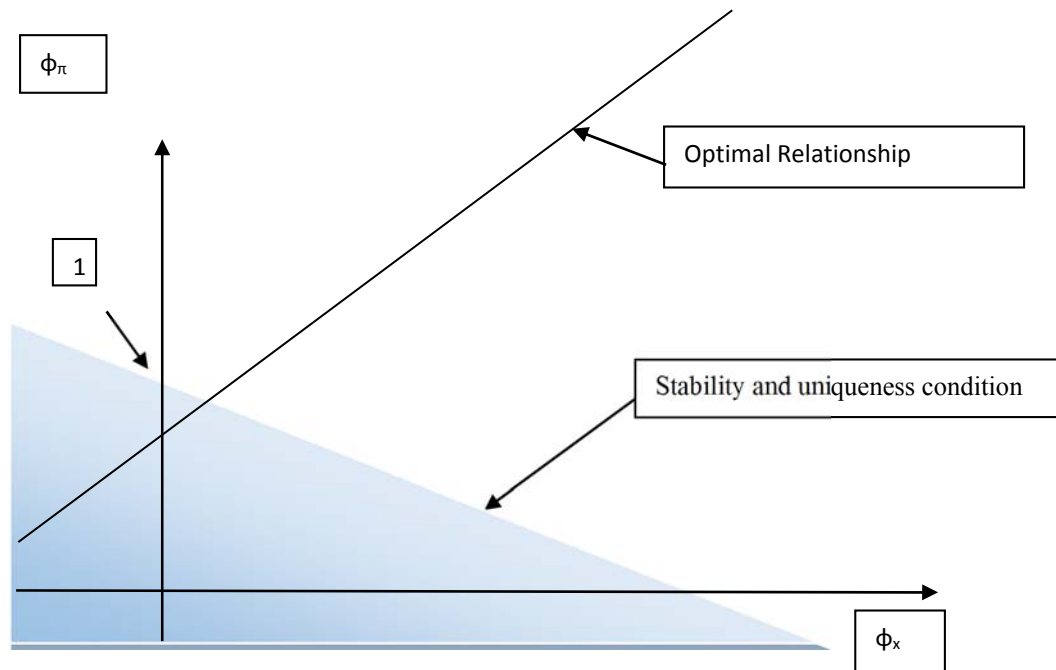
The region of indeterminacy is the shadow part. We can notice that we have always stable and unique solution with $\varphi_\pi > 0$ and $\varphi_x > 0$ if we are on the optimal line and if:

$$\rho + \kappa\sigma\,\frac{1-\rho}{\Gamma} > 1$$

$$\Leftrightarrow \frac{\kappa\sigma}{\Gamma}(1-\rho) > 1 - \rho$$

$$\Leftrightarrow \kappa\sigma > \Gamma$$

As mentioned in the previous chapter, the central bank may adopt other targets that require a certain level of commitment. Those approaches are "global" target, "speed limit", "price level" targeting, and "nominal income" targeting. "Global" is inflation targeting using the social welfare but assuming commitment, so the price level goes back to its original level. In "speed limit", the central bank reacts to the change of output gap rather than its level. When

following "price level" targeting, the welfare that the central bank tries to optimize has the volatility of the price level instead of the volatility of inflation. Finally, for the "nominal income" targeting, the volatility of output gap in welfare is replaced by the volatility of nominal income[13].
It is possible to find the optimal Taylor rules for each of these policies using the same methodology that was used for discretion. Some of them require to compensating for the supply shock u_t. The list of optimal rules is shown below.

Those rules are Taylor rules and the relationship between φ_π and φ_x that allows the central bank to replicate optimal paths for each targeting approach are found in the same way as described above for discretion. It is important to notice that it is necessary to compensate for the cost-push shock u_t in order to find a solution. Indeed, when linking forward variables with current ones, this cost-push shock is present and is not eliminated automatically thus the interest rate has to compensate for it.

Discretion: $i_t = \varphi_\pi \pi_t + \varphi_x x_t + \sigma g_t$ with $\varphi_\pi = \dfrac{\kappa}{\Gamma} \varphi_x + \rho + \dfrac{\kappa\sigma}{\Gamma}(1-\rho)$

Global: $i_t = \varphi_\pi \pi_t + \varphi_x x_t + \varphi_u u_t + \sigma g_t$ with $\varphi_\pi = -\dfrac{\kappa}{\Gamma}\dfrac{\delta}{1-\delta}\varphi_x + (1-\dfrac{\kappa\sigma}{\Gamma})\delta$

And $\varphi_u = -\dfrac{1}{1-\beta\delta\rho}\varphi_\pi - \dfrac{\kappa\delta\rho\sigma}{(1-\beta\delta\rho)\Gamma} + \dfrac{\delta\rho}{1-\beta\delta\rho}$

Price level targeting: $i_t = \varphi_\pi \pi_t + \varphi_x x_t + \varphi_u u_t + \sigma g_t$

With $\varphi_\pi = -\dfrac{\kappa}{\Gamma(1+\beta)}\dfrac{\delta}{1-\delta}\varphi_x + (1-\dfrac{\kappa\sigma}{\Gamma(1+\beta)})\delta$

And $\varphi_u = -\dfrac{1}{1-\beta\delta\rho}\varphi_\pi - \dfrac{\kappa\delta\rho\sigma}{(1-\beta\delta\rho)\Gamma(1+\beta)} + \dfrac{\delta\rho}{1-\beta\delta\rho}$

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[13] See the previous chapter: Marest (2015)

Nominal GDP growth: $i_t = \varphi_\pi \pi_t + \varphi_x x_t + \varphi_u u_t + \sigma g_t$

$$\text{With} \quad \varphi_\pi = -\frac{\kappa + \Gamma(1+\kappa)}{\Gamma(1+\kappa)}\frac{\delta}{1-\delta}\varphi_x + (1 - \frac{(\kappa + \Gamma(1+\kappa)\sigma}{\Gamma(1+\kappa)})\delta$$

$$\text{And} \quad \varphi_u = -\frac{\kappa(1+\kappa)\Gamma}{1-\beta\delta\rho}\varphi_\pi - \frac{\kappa^2}{\kappa+\Gamma(1+\kappa)} + \frac{\delta\rho}{(1-\beta\delta\rho)\Gamma}$$

Chart 24 shows the optimal rules for the different policies, presented the same way has in chart 23. The calibration used is Γ=0.0048, ρ=0.35, b=1, σ=0.16, κ=0.024 and β=0.99. It is conformed to what Woodford (2003) uses as calibration. In the first chapter, table 3 shows the calibration used by different authors[14]. With different values for the calibration, the slopes and intersections with the axis change slightly but the concept and the shape stay the same.

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[14] See Marest and Thurston 2015. "The Value of Central Bank Commitment to the Price Level in the Baseline New Keynesian Model"

Chart 24: Determinacy condition and optimal relationship between the φ_π and φ_x in the Taylor rule for discretion (yellow), global (also speed limit in red), price level targeting (blue) and nominal income targeting (violet)
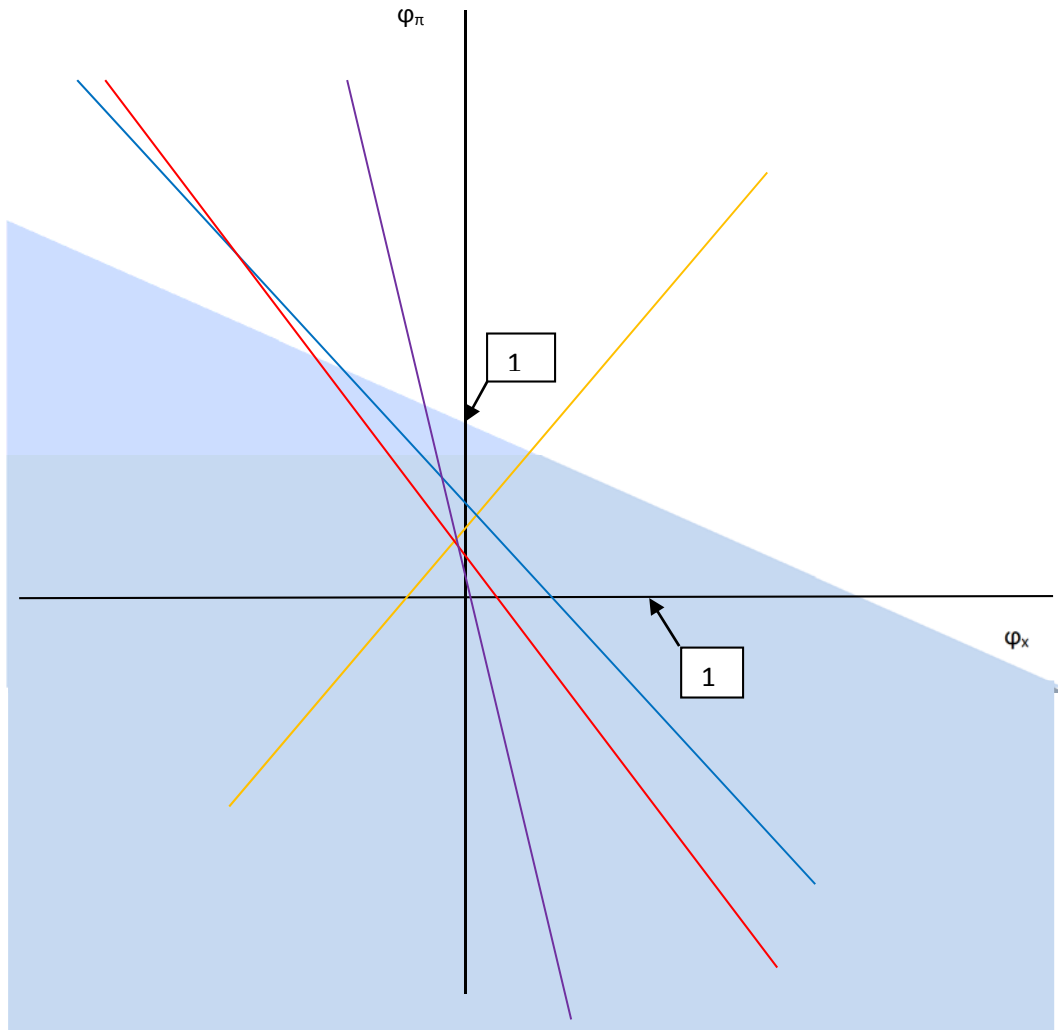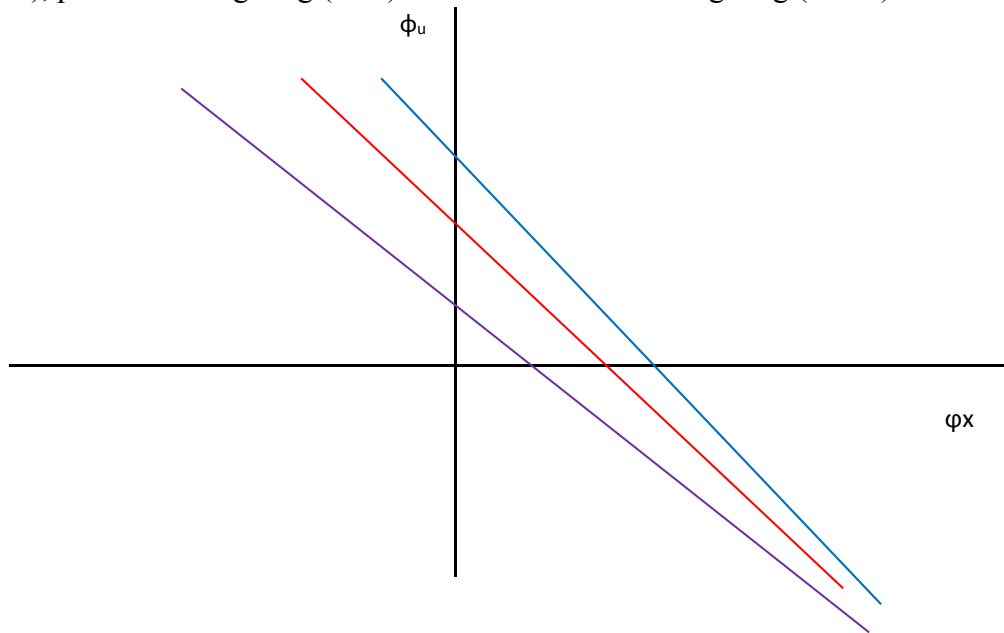
Chart 25: Optimal relationship between the φu and φx in the Taylor rule for global (also speed limit in red), price level targeting (blue) and nominal income targeting (violet)

Charts 24 and 25 show how it is always possible to find a combination of φ_π and φ_x (but also φ_u) of a Taylor rule to replicate optimal paths in a way that gives a unique and stable solution. The two parameters of the Taylor rule, φ_π and φ_x, have to be conform to the equations developed above and large enough and positive to be in the determinacy zone for the pure discretion case. However, for the other regimes, φ_x has to be negative enough and φ_π positive enough to be in the determinacy zone.

3.3.2 Taylor type money rules

Some authors suggested to using a monetary rule instead of an interest rate rule in order to deal with the determinacy issue mentioned above. This paper is looking at not only exogenous monetary rules but also rules similar to the Taylor rule with money as an instrument.

Exogenous money: $m_t = \rho_m m_{t-1} + v_t$

We use the following equations:

$$x_t = E_t x_{t+1} - \frac{i_t - E_t \pi_{t+1}}{\sigma} + g_t \qquad (4) \qquad \text{with } g_t = \lambda g_{t-1} + \varepsilon_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \qquad (5) \qquad \text{with } u_t = \rho u_{t-1} + \eta_t$$

$$m_t - p_t = \frac{1}{b}(\sigma y_t - i_t) + \omega_t \qquad (6) \qquad \text{with } x_t = y_t - y_t^f \text{ and } y_t^f = \frac{1+\eta}{\sigma+\eta} z_t$$

$$\text{with } \omega_t = \delta \omega_{t-1} + \psi_t$$

Then putting (6) into (4) eliminating i_t, we get:

$$x_t = E_t x_{t+1} - \frac{-b(m_t - p_t) + \sigma y_t^f + \sigma x_t + b\omega_t - E_t \pi_{t+1}}{\sigma} + g_t$$

So

$$E_t x_{t+1} + \frac{b}{\sigma} m_t + \frac{1}{\sigma} E_t p_{t+1} = 2x_t + \frac{1+b}{\sigma} p_t + y_t^f + \frac{b}{\sigma} \omega_t - g_t \quad (7)$$

Considering that $\pi_t = p_t - p_{t-1}$, equation (5) can be transformed as:

$$p_t - p_{t-1} = \beta(E_t p_{t+1} - p_t) + \kappa x_t + u_t \quad (8)$$

Then, we consider m_t as exogenous, so

$$m_t = \rho_m m_{t-1} + v_t \qquad (9)$$

and we get, following the same kind of reasoning as Walsh when he considered it as exogenous (p245-246), using (7), (8), and (9):

Then the system of equations should be

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & \dfrac{1}{\sigma} & 0 \\ 0 & 0 & \beta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} m_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \\ p_t \end{bmatrix} = \begin{bmatrix} \rho_m & 0 & 0 & 0 \\ -\dfrac{b}{\sigma}\rho_m & 2 & \dfrac{b}{\sigma} & \dfrac{b}{\sigma} \\ 0 & -\kappa & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} m_{t-1} \\ x_t \\ \pi_t \\ p_{t-1} \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ -\dfrac{b}{\sigma} & 1 & -1 & 0 & \dfrac{b}{\sigma} \\ 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} v_t \\ y_t^{\,f} \\ g_t \\ u_t \\ w_t \end{bmatrix}$$

So

$$\begin{bmatrix} m_t \\ E_t x_{t+1} \\ E_t \pi_{t+1} \\ p_t \end{bmatrix} = \begin{bmatrix} \rho_m & 0 & 0 & 0 \\ -\dfrac{b}{\sigma}\rho_m & 2+\dfrac{\kappa}{\sigma\beta} & \dfrac{b}{\sigma}-\dfrac{1}{\sigma\beta} & \dfrac{b}{\sigma} \\ 0 & -\dfrac{\kappa}{\beta} & \dfrac{1}{\beta} & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}\begin{bmatrix} m_{t-1} \\ x_t \\ \pi_t \\ p_{t-1} \end{bmatrix}$$

The eigenvalues are complicated formulas. Following Blanchart and Kahn, we look to see if we have two eigenvalues outside the unit circle and two forward undetermined variables. We would then get a unique and stable solution. Here the eigenvalues may be complex, meaning that the variables may vary in an oscillating way. In these cases, we need to consider the absolute values of the eigenvalues to evaluate the uniqueness and stability. The first eigenvalue is ρ_m which we assume to be positive and below 1. So among the three other eigenvalues, we need to have one below 1 and two above one when considering absolute values. According to simulations, the system is always determinate.

<u>Money rule that reacts to the shock u_t</u> $m_t = \varphi u_t$

From previous paragraphs, we have:

$$E_t x_{t+1} + \frac{1}{\sigma} E_t p_{t+1} = -\frac{b}{\sigma} m_t + (\frac{b}{\sigma} + \frac{1}{\sigma}) p_t + 2x_t + y_t^f + \frac{b}{\sigma} \omega_t - g_t$$

So $E_t x_{t+1} + \frac{1}{\sigma} E_t p_{t+1} = -\frac{b}{\sigma} \varphi u_t + (\frac{b}{\sigma} + \frac{1}{\sigma}) p_t + 2x_t + y_t^f + \frac{b}{\sigma} \omega_t - g_t$

The Phillips Curve provides as in previous paragraphs: $p_t - p_{t-1} = \beta(E_t p_{t+1} - p_t) + \kappa x_t + u_t$

We have the IS curve $x_t = E_t x_{t+1} - \dfrac{-b(m_t - p_t) + \sigma y_t^f + \sigma x_t + b\omega_t - E_t \pi_{t+1}}{\sigma} + gt$

So we get by re-arranging:

$$E_t x_{t+1} + \frac{1}{\sigma} E_t \pi_{t+1} = 2x_t + \frac{b}{\sigma} \pi_t + \frac{b}{\sigma} p_{t-1} - \frac{b}{\sigma} \varphi u_t + y_t^f + \frac{b}{\sigma} \omega_t - g_t$$

So considering the Phillips curve and the $\pi_t = p_t - p_{t-1}$, we get the following system of equations:

$$
\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ p_t \end{bmatrix} =
\begin{bmatrix} 2 + \dfrac{\kappa}{\sigma\beta} & \dfrac{b}{\sigma} - \dfrac{1}{\sigma\beta} & \dfrac{b}{\sigma} \\ -\dfrac{\kappa}{\beta} & \dfrac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{bmatrix}
\begin{bmatrix} x_t \\ \pi_t \\ p_{t-1} \end{bmatrix} +
\begin{bmatrix} -\dfrac{b}{\sigma}\varphi + \dfrac{1}{\sigma\beta} & \dfrac{b}{\sigma} & -1 & 1 \\ -\dfrac{1}{\beta} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} u_t \\ \omega_t \\ g_t \\ y_t^f \end{bmatrix}
$$

The eigenvalues are also very complicated. However, the three eigenvalues are the three eigenvalues other than ρ_m for the case in which $m_t = \rho_m m_{t-1} + v_t$ so we have the same conditions.

The exogenous money rule and the money rule that reacts to the cost push shock are always determinate but policy makers cannot replicate the optimal policy with those. As a consequence, the instrument that the central has to use is a version of the Taylor rule but using money as the policy tool.

Taylor money rule $m_t = \varphi_\pi \pi_t + \varphi_x x_t + \varphi_{p-1} p_{t-1} + \varphi_y y_t^f + \varphi_g g_t + \varphi_\omega \omega_t$

Stability and uniqueness:

$$x_t = E_t x_{t+1} + \frac{b}{\sigma} m_t - \frac{b}{\sigma} p_t - y_t^f - x_t - \frac{b}{\sigma} \omega_t + \frac{1}{\sigma} E_t \pi_{t+1} + g_t$$

with $p_t = \pi_t + p_{t-1}$

So we get

$$E_t x_{t+1} + \frac{1}{\sigma} E_t \pi_{t+1} = -\frac{b}{\sigma} m_t + \frac{b}{\sigma} \pi_t + \frac{b}{\sigma} p_{t-1} + 2x_t + y_t^f + \frac{b}{\sigma} \omega_t - g_t$$

$$\Leftrightarrow E_t x_{t+1} + \frac{1}{\sigma} E_t \pi_{t+1} = -\frac{b}{\sigma} (\varphi_x x_t + \varphi_\pi \pi_t + \varphi_{p-1} p_{t-1} + \varphi_y y_t^f + \varphi_g g_t + \varphi_\omega \omega_t)$$

$$+ \frac{b}{\sigma} \pi_t + \frac{b}{\sigma} p_{t-1} + 2x_t + y_t^f + \frac{b}{\sigma} \omega_t - g_t$$

$$\Leftrightarrow E_t x_{t+1} + \frac{1}{\sigma} E_t \pi_{t+1} = (2 - \frac{b}{\sigma} \varphi_x) x_t + (\frac{b}{\sigma} - \frac{b}{\sigma} \varphi_\pi) \pi_t + (\frac{b}{\sigma} - \frac{b}{\sigma} \varphi_{p-1}) p_{t-1}$$

$$+ (1 - \frac{b}{\sigma} \varphi_y) y_t^f + (\frac{b}{\sigma} - \frac{b}{\sigma} \varphi_\omega) \omega_t + (-\frac{b}{\sigma} \varphi_g - 1) g_t$$

We have also the Philips Curve:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t$$

Let's consider the special case where $\varphi_{p-1}=1$. It means that we consider the money rule

$$m_t = \varphi_\pi \pi_t + \varphi_x x_t + p_{t-1} + \varphi_y y_t^f + \varphi_g g_t + \varphi_\omega \omega_t$$

So we get:

$$\begin{pmatrix} 1 & \frac{1}{\sigma} \\ 0 & \beta \end{pmatrix} \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} = \begin{pmatrix} 2 - \frac{b}{\sigma} \varphi_x & \frac{b}{\sigma} - \frac{b}{\sigma} \varphi_\pi \\ -\kappa & 1 \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}$$

$$\Leftrightarrow \begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \end{pmatrix} = \begin{pmatrix} 2 - \dfrac{b}{\sigma}\varphi_x + \dfrac{\kappa}{\sigma\beta} & \dfrac{b}{\sigma} - \dfrac{b}{\sigma}\varphi_\pi - \dfrac{1}{\sigma\beta} \\ -\dfrac{\kappa}{\beta} & \dfrac{1}{\beta} \end{pmatrix} \begin{pmatrix} x_t \\ \pi_t \end{pmatrix}$$

The eigenvalues are:

$$\lambda_1 = \frac{1}{2}\frac{1}{\sigma\beta}(\sigma + 2\sigma\beta - b\beta\varphi_x + \kappa +$$

$$\sqrt{\sigma^2 - 4\sigma^2\beta + 2\sigma b\beta\varphi_x + 2\sigma\kappa + 4\sigma^2\beta^2 - 4\sigma\beta^2 b\varphi_x + 4\sigma\beta\kappa + b^2\beta^2\varphi_x^2 - 2b\beta\varphi_x\kappa + \kappa^2 - 4\sigma\kappa b\beta + 4\sigma\kappa b\beta\varphi_\pi})$$

$$\lambda_2 = \frac{1}{2}\frac{1}{\sigma\beta}(\sigma + 2\sigma\beta - b\beta\varphi_x + \kappa -$$

$$\sqrt{\sigma^2 - 4\sigma^2\beta + 2\sigma b\beta\varphi_x + 2\sigma\kappa + 4\sigma^2\beta^2 - 4\sigma\beta^2 b\varphi_x + 4\sigma\beta\kappa + b^2\beta^2\varphi_x^2 - 2b\beta\varphi_x\kappa + \kappa^2 - 4\sigma\kappa b\beta + 4\sigma\kappa b\beta\varphi_\pi})$$
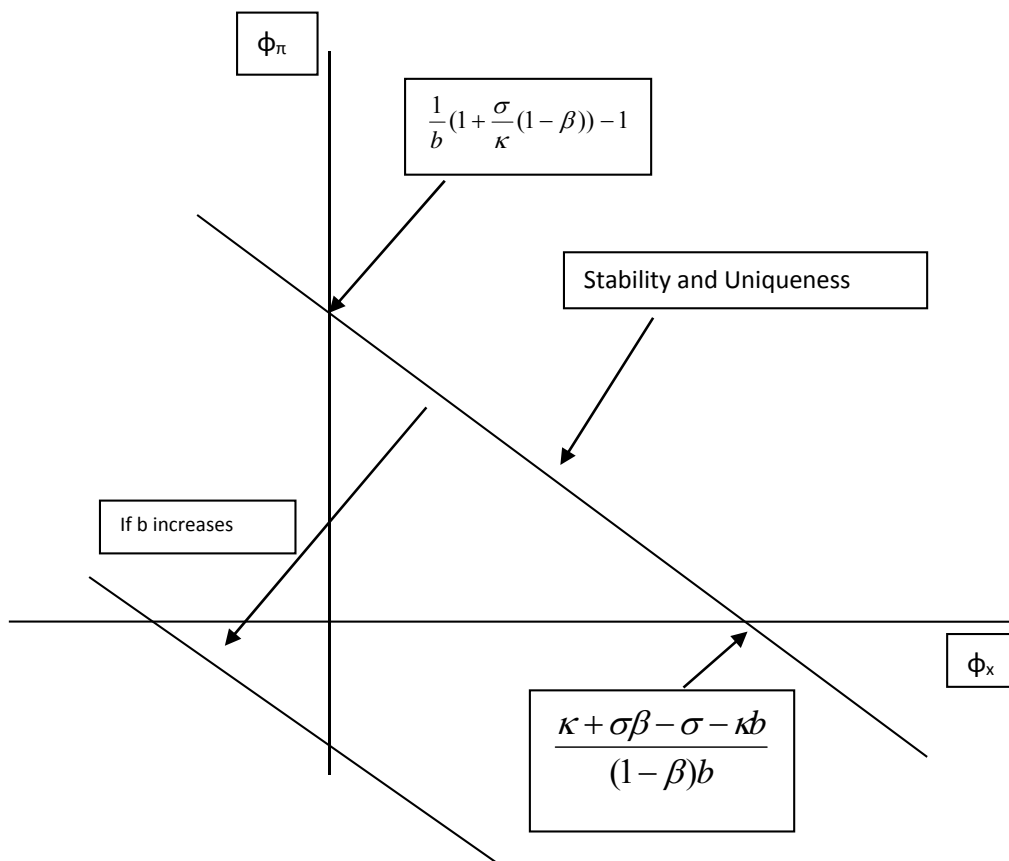
Both of the eigenvalues will be outside the unit circle if $\lambda_2 > 1$

$$\Leftrightarrow \frac{1}{2}\frac{1}{\sigma\beta}(\sigma + 2\sigma\beta - b\beta\varphi_x + \kappa -$$

$$\sqrt{\sigma^2 - 4\sigma^2\beta + 2\sigma b\beta\varphi_x + 2\sigma\kappa + 4\sigma^2\beta^2 - 4\sigma\beta^2 b\varphi_x + 4\sigma\beta\kappa + b^2\beta^2\varphi_x^2 - 2b\beta\varphi_x\kappa + \kappa^2 - 4\sigma\kappa b\beta + 4\sigma\kappa b\beta\varphi_\pi}) > 1$$

$$\Leftrightarrow \sigma - b\beta\varphi_x + \kappa >$$

$$\sqrt{\sigma^2 - 4\sigma^2\beta + 2\sigma b\beta\varphi_x + 2\sigma\kappa + 4\sigma^2\beta^2 - 4\sigma\beta^2 b\varphi_x + 4\sigma\beta\kappa + b^2\beta^2\varphi_x^2 - 2b\beta\varphi_x\kappa + \kappa^2 - 4\sigma\kappa b\beta + 4\sigma\kappa b\beta\varphi_\pi}$$

$$(1-\beta)b\varphi_x + (\sigma - \sigma\beta - \kappa + \kappa b) + \kappa b\varphi_\pi < 0$$

Contrary to the condition under the interest rate rule, this condition creates more limits in the sense that the φ's must be below the stability and uniqueness line. The other element that plays a role in the shape of determinacy zone is b. The expression $\dfrac{1}{b}$ is the interest elasticity of money demand. When b is increasing, the stability and uniqueness line that delimits the determinacy zone is moving down as shown is chart 26.

Chart 26: Limit for the determinacy zone in case of the sort of money Taylor rule

$$m_t = \varphi_\pi \pi_t + \varphi_x x_t + \varphi_{p-1} p_{t-1} + \varphi_y y_t^f + \varphi_g g_t + \varphi_\omega \omega_t$$

ϕ_π

$$\frac{1}{b}(1 + \frac{\sigma}{\kappa}(1 - \beta)) - 1$$

Stability and Uniqueness

If b increases

$$\frac{\kappa + \sigma\beta - \sigma - \kappa b}{(1 - \beta)b}$$

ϕ_x

We can see that the intercepts with the axis in the above chart are positive if

$b < 1 - \dfrac{\sigma}{\kappa}(1 - \beta)$ or using the standard calibration, b<0.966. In this case, for any choice of φ_π

and φ_x negative the system is stable and unique. However, if $b > 1 - \dfrac{\sigma}{\kappa}(1 - \beta)$ then we may have

some constraints, but it depends again on b. We can see that the system will be always

determinate for any choice in the optimal line as long as φ_π and φ_x are negative and

$$b > \frac{\kappa - (1-\beta)(\frac{\rho\Gamma}{\kappa} + \sigma(3-\rho))}{\kappa - (1-\beta)\frac{\Gamma}{\kappa}}$$ or using the standard calibration, b>1.0358.

The optimal rule for the discretion case can be found using the same methodology that in the previous section. To incorporate money in the IS curve, we use equation (6) which corresponds to money demand: $m_t - p_t = \frac{1}{b}(\sigma y_t - i_t) + \omega_t$

Isolating it and replacing in the IS curve, we get

$$x_t = E_t x_{t+1} - \frac{-b(m_t - p_t) + \sigma y_t^f + \sigma x_t + b\omega_t - E_t \pi_{t+1}}{\sigma} + g_t$$

So

$$x_t = E_t x_{t+1} + \frac{b}{\sigma}m_t - \frac{b}{\sigma}p_t - y_t^f - x_t - \frac{b}{\sigma}\omega_t + \frac{1}{\sigma}E_t \pi_{t+1} + g_t$$

First we replace $\quad$ m$_t$ with $m_t = \varphi_\pi \pi_t + \varphi_x x_t + \varphi_{p-1} p_{t-1} + \varphi_y y_t^f + \varphi_g g_t + \varphi_\omega \omega_t$

E$_t$x$_{t+1}$ with $E_t x_{t+1} = \rho x_t$

E$_t$π$_{t+1}$ with $E_t \pi_{t+1} = \rho \pi_t$

p$_t$ with $p_t = \pi_t + p_{t-1}$

And we get

$$x_t = \rho x_t + \frac{b}{\sigma}(\varphi_\pi \pi_t + \varphi_x x_t + \varphi_{p-1} p_{t-1} + \varphi_y y_t^f + \varphi_g g_t + \varphi_\omega \omega_t)$$

$$-\frac{b}{\sigma}\pi_t - \frac{b}{\sigma}p_{t-1} - y_t^f - x_t - \frac{b}{\sigma}\omega_t + \frac{1}{\sigma}\rho\pi_t + g_t$$

Using $\varphi_{p-1}=1$ to cancel out p_{t-1}

$$\varphi_y = \frac{\sigma}{b}$$ to cancel out y_t^f

$$\varphi_g = -\frac{\sigma}{b}$$ to cancel out g_t

$\varphi_\omega=1$ to cancel out ω_t

We get then:

$$0 = (-1+\rho+\frac{b}{\sigma}\varphi_x - 1)x_t + (\frac{b}{\sigma}\varphi_\pi - \frac{b}{\sigma} + \frac{\rho}{\sigma})\pi_t$$

Then using $x_t = -\frac{\kappa}{\Gamma}\pi_t$ we get

$$\varphi_\pi = 1 - \frac{\rho}{b} - \frac{\kappa\sigma(2-\rho)}{b\Gamma} + \frac{\kappa}{\Gamma}\varphi_x$$

We can notice in chart 27 that the slope of that line is the same as the one for the relationship for the interest rate rule. However, the intercept is different. Again, the influence of b is significant. If b is decreasing, the optimal line will move downward.

Chart 27: The optimal relationship between φ_π and φ_x for discretion when using the sort of money Taylor rule $m_t = \varphi_\pi \pi_t + \varphi_x x_t + \varphi_{p-1} p_{t-1} + \varphi_y y_t^f + \varphi_g g_t + \varphi_\omega \omega_t$
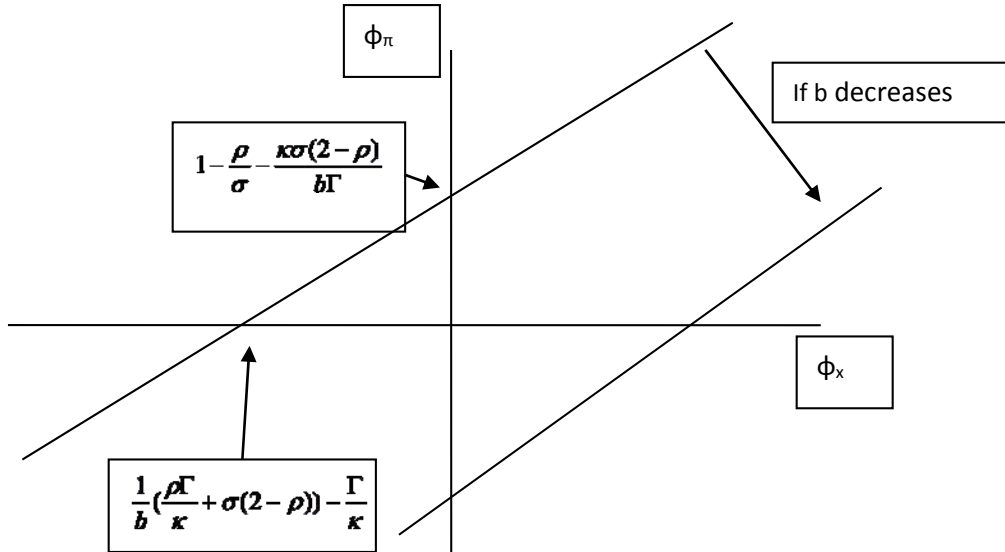
We can see that the intercept of the optimal line with the φ_π axis is above zero if

$b > \rho + \dfrac{\kappa\sigma(2-\rho)}{\Gamma}$ or if we use the standard calibration, b>0.725. The combination of the

conditions for determinacy and the optimal rule for the discretion policy is shown in chart 28.

<u>Chart 28</u>: Condition for determinacy and optimal relationship between φ_π and φ_x for discretion when using the sort of money Taylor rule $m_t = \varphi_\pi \pi_t + \varphi_x x_t + \varphi_{p-1} p_{t-1} + \varphi_y y_t^f + \varphi_g g_t + \varphi_\omega \omega_t$
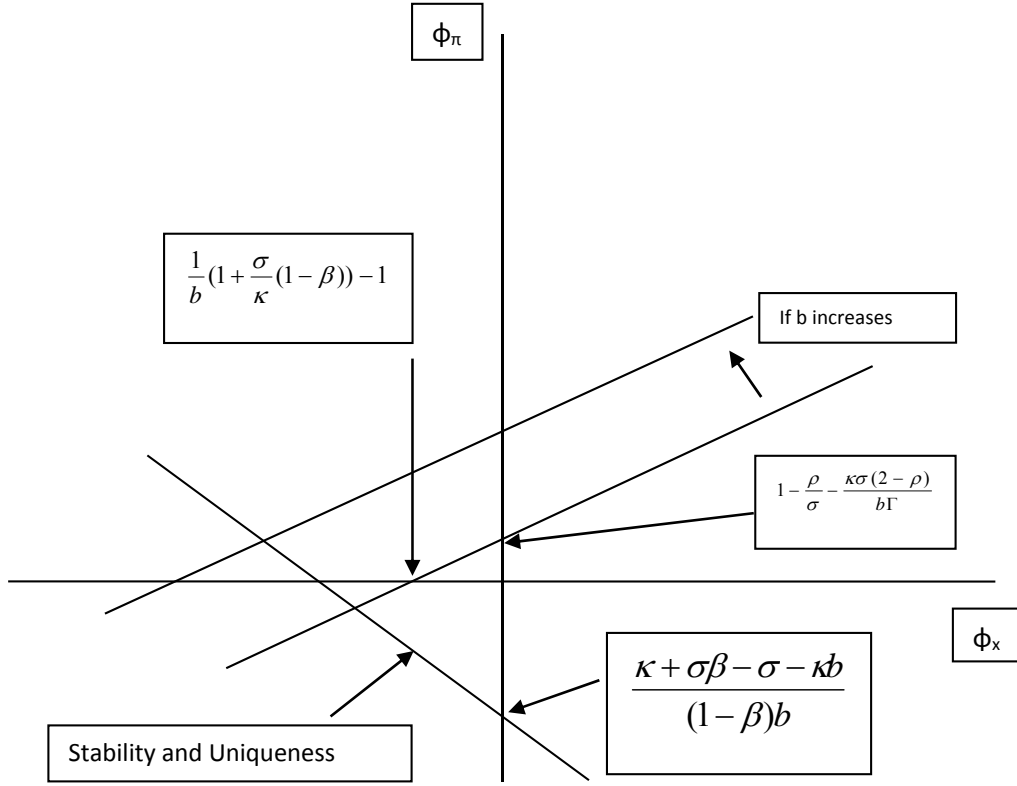
The same way, we can find the optimal rules for the global policy, price level targeting and nominal income targeting.

For global we have: $\quad \varphi_\pi = -\dfrac{\kappa}{\Gamma}\dfrac{\delta}{1-\delta}\varphi_x + \dfrac{(2-\delta)\sigma\kappa\delta}{b\Gamma(1-\delta)} + 1 - \dfrac{\delta}{b}$

And $\quad \varphi_u = -\dfrac{1}{(1-\beta\delta\rho)}\varphi_\pi + \dfrac{1}{b(1-\beta\delta\rho)}(\dfrac{\kappa\delta\sigma\rho}{\Gamma} + b - \delta\rho)$

with $\delta = \dfrac{1-\sqrt{1-4\beta a^2}}{2a\beta}$ and $a = \dfrac{\Gamma}{\kappa^2 + \Gamma(1+\beta)}$

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For price level we have:
$$\varphi_\pi = -\frac{\kappa}{\Gamma(1+\beta)}\frac{\delta}{1-\delta}\varphi_x + \frac{(2-\delta)\kappa\sigma}{b(1+\beta)\Gamma}\frac{\delta}{1-\delta} + 1 - \frac{\delta}{b}$$

And
$$\varphi_u = -\frac{1}{1-\beta\delta\rho}\varphi_\pi + \frac{\kappa}{b\Gamma(1-\beta\delta\rho)(1+\beta)}\left(\frac{\delta\sigma}{\rho}+b\right) - \frac{\delta\rho}{(1-\beta\delta\rho)b}$$

with $\delta = \dfrac{1-\sqrt{1-4\beta a^2}}{2a\beta}$ and $a = \dfrac{(1+\beta)\Gamma_p}{\kappa^2 + \Gamma(1+\beta)^2}$

For nominal income growth targeting we have:
$$\varphi_\pi = -\frac{\kappa+\Gamma(1+\kappa)}{\Gamma(1+\kappa)}\frac{\delta}{1-\delta}\varphi_x + \frac{\delta(2-\delta)\sigma}{1-\delta}\frac{\kappa+\Gamma(1+\kappa)}{\Gamma(1+\kappa)} - 1 - \frac{\delta}{b}$$

And
$$\varphi_u = -\frac{\Gamma(1+\kappa)\kappa}{(1-\beta\delta\rho)\Gamma(\kappa+\Gamma(1+\kappa))}\varphi_x + \frac{1}{b(1-\beta\delta\rho)}\left(\frac{\kappa\delta\sigma}{\Gamma}+\frac{(1+\kappa)(b-\delta)\kappa}{\kappa+\Gamma(1+\kappa)}-\delta(\rho-1)\right)$$

with $\delta = \dfrac{1-\sqrt{1-4\beta a^2}}{2a\beta}$ and $a = \dfrac{\Gamma(1+\kappa)}{\kappa^2+\Gamma(1+\kappa)(1+\beta+\kappa)}$

We can see in Chart 29, using the same calibration used in the previous section, that the conditions for determinacy are relatively similar to the ones when using the traditional Taylor rule. The different optimal lines have different slopes than with the interest rate Taylor rule. In addition, the determinacy region is different. This time the system will be always determinate for the discretion case if φ_π and φ_x are negative enough. For the other cases, it will be so if φ_π is negative enough and φ_x positive enough. Using money as an instrument does not provide any advantage in that regard.

Chart 29: Determinacy condition and optimal relationship between the φ_π and φ_x in the sort of money Taylor rule for discretion (yellow), global (also speed limit in red), price level targeting (blue) and nominal income targeting (violet)
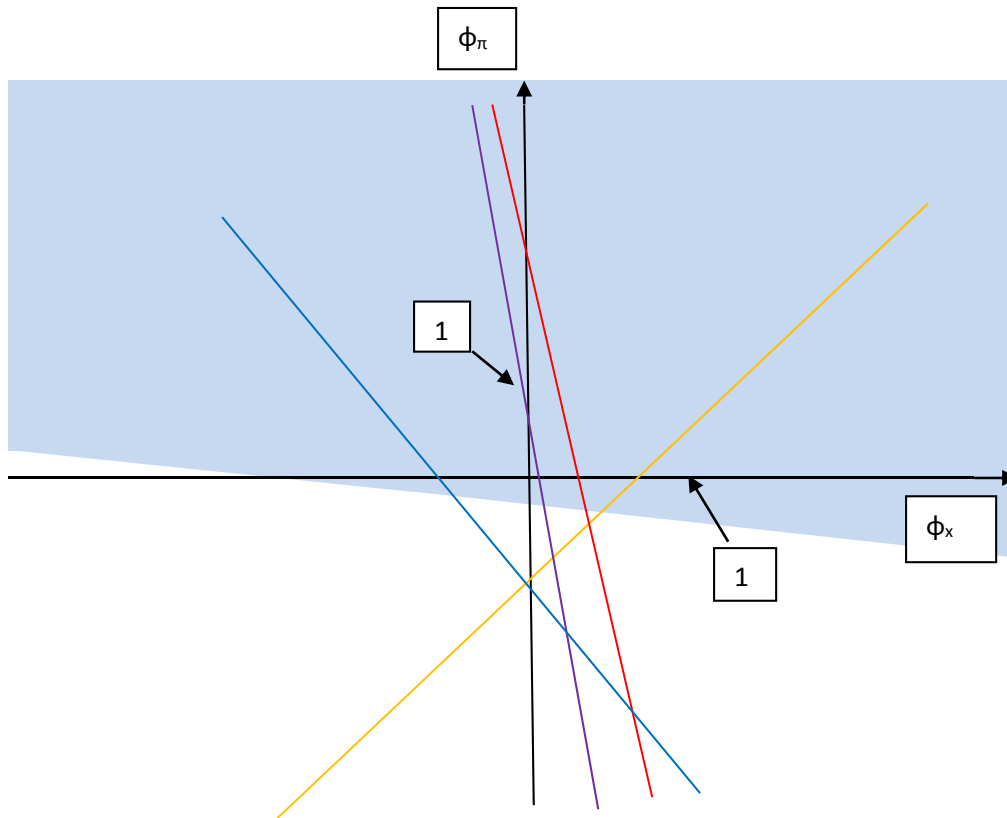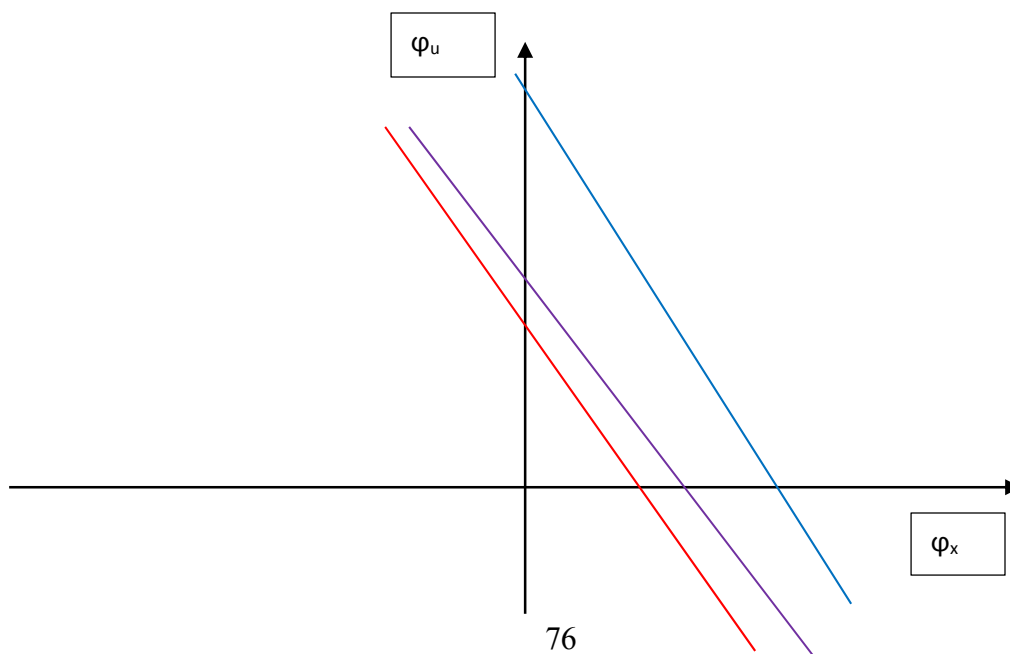

Chart 30: Optimal relationship between the φ_u and φ_x in the sort of money Taylor rule for global (also speed limit in red), price level targeting (blue) and nominal income targeting (violet)

3.4 Wicksellian rules or price level targeting rules

It has been demonstrated that inertia and then price level stability would enhance welfare[15]. Actually, Woodford (2003) based his theory on the idea that instability in price level is the source of disturbance and distortions that prevent sectors of an economy to coordinate in an efficient manner. The idea was already promoted by the Swedish economist Knut Wicksell (1898). He was one of the first economists to articulate comprehensively a theory of fiat money. That kind of price level regime was actually implemented in Sweden in the 1930's. Wicksell was recommending in his theory to adopt a rule that would link the nominal interest rate to the price level: if prices rise then the interest rate should be raised and if prices decrease then the interest rate should be lowered (Wicksell 1898).

3.4.1 Wicksellian interest rate rule

In appendix C, a list of different Wicksellian rules is shown with details on determinacy. Some of them however don't allow to finding optimal rules. The interest rate rule that central bank should focus on to get optimal instruments has the form $i_t = \varphi_p \, \hat{p}_t + \varphi_{p-1} \, \hat{p}_{t-1} + \varphi_x x_t + \varphi_g g_t$ with $\hat{p}_t = p_t - p_{-1}$.

The original price level when the policy starts is p-1. In term of stability and uniqueness we have the following equations:

$$x_t = E_t x_{t+1} - \frac{i_t - E_t \pi_{t+1}}{\sigma} + g_t \qquad (10) \qquad \text{with } g_t = \lambda g_{t-1} + \varepsilon_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \qquad (11) \qquad \text{with } u_t = \rho u_{t-1} + \eta_t$$

---

[15] See Woodford (2003). For detailed analytical solutions see Marest and Thurston (2013) and Marest (2015)

and using $E_t \pi_{t+1} = E_t \, p_{t+1} - p_t$ and $\pi_t = \hat{p}_t - \hat{p}_{t-1}$ replacing i_t with the interest rate rule discussed, we get:

$$\begin{bmatrix} 1 & \dfrac{1}{\sigma} & -\dfrac{\varphi_p}{\sigma} \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ \hat{p}_t \end{bmatrix} = \begin{bmatrix} 1+\dfrac{\varphi_x}{\sigma} & 0 & \dfrac{\varphi_{p-1}}{\sigma} \\ -\kappa & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ \hat{p}_{t-1} \end{bmatrix} + \begin{bmatrix} -1+\dfrac{\varphi_g}{\sigma} & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} g_t \\ u_t \end{bmatrix}$$

Or

$$\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ p_t \end{bmatrix} = \begin{bmatrix} 1+\dfrac{\varphi_x}{\sigma}+\dfrac{\kappa}{\sigma\beta} & \dfrac{-1+\beta\varphi_p}{\sigma\beta} & \dfrac{\varphi_{p-1}+\varphi_p}{\sigma} \\ -\dfrac{\kappa}{\beta} & \dfrac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ \hat{p}_{t-1} \end{bmatrix} + A \begin{bmatrix} g_t \\ u_t \end{bmatrix}$$

To find the determinacy conditions, it may be useful to establish a value for $\varphi_{p\text{-}1}$. When looking for the optimal global rule, the methodology of the previous section is used. We would obtain an optimal path if we have:

$$(\varphi_p - \delta + 1 - ((1-\delta)\sigma + \varphi_x)\frac{\kappa}{\Gamma})\frac{\hat{p}_t}{\sigma} = -\frac{\varphi_{p-1}}{\sigma}\hat{p}_{t-1} + (\frac{1}{\sigma} - \frac{\kappa}{\Gamma})\frac{\delta}{1-\beta\delta\rho}\rho u_t + (-\frac{\varphi_g}{\sigma} + 1)g_t$$

So we need the system of equations, with $\varphi_g = \sigma$:

$$\delta = -\frac{\varphi_{p-1}}{\varphi_p - \delta + 1 - ((1-\delta)\sigma + \varphi_x)\dfrac{\kappa}{\Gamma}}$$

And

$$\frac{\delta}{1-\beta\delta\rho} = \frac{(1-\dfrac{\kappa\sigma}{\Gamma})\rho\delta}{(1-\beta\delta\rho)(\varphi_p - \delta + 1 - ((1-\delta)\sigma + \varphi_x)\dfrac{\kappa}{\Gamma})}$$

Then we obtain: $\varphi_{p-1} = (\dfrac{\kappa\sigma}{\Gamma} - 1)\rho\delta$

And a relationship between φ_p and φ_x: $\varphi_p = \dfrac{\kappa}{\Gamma}\varphi_x + (1 - \dfrac{\kappa\sigma}{\Gamma})\rho + (1-\delta)(\dfrac{\sigma\kappa}{\Gamma} - 1)$

If we put the φ_{p-1} that is just above in the interest rate rule we can find more easily the conditions for uniqueness and stability by simulation. Chart 31 shows the constraints for determinacy and the optimal instrument rules for the different policies considered: discretion, global, price level targeting and nominal income targeting.

For discretion: $\qquad \varphi_p = \dfrac{\kappa}{\Gamma}\varphi_x + \dfrac{\kappa}{\Gamma} + (1 - \dfrac{\kappa\sigma}{\Gamma})\rho$ and $\varphi_{p-1} = -\varphi_p$

For price level targeting, because the first order condition links directly the price level and the output gap, the Wicksellian interest rate rule becomes $i_t = \varphi_p\, p_t + \varphi_{p-1}\, p_{t-1} + \varphi_x x_t + \varphi_g g_t$ and we get:

$$\varphi_p = \dfrac{\kappa}{\Gamma(1+\beta)}\varphi_x + (\delta - 1 + \rho)(1 - \dfrac{\kappa\sigma}{\Gamma(1+\beta)})$$

$$\text{And } \varphi_{p-1} = (\dfrac{\kappa\sigma}{\Gamma(1+\beta)} - 1)\delta\rho$$

$$\text{with } \delta = \dfrac{1 - \sqrt{1 - 4\beta a^2}}{2a\beta} \text{ and } a = \dfrac{(1+\beta)\Gamma_p}{\kappa^2 + \Gamma(1+\beta)^2}$$
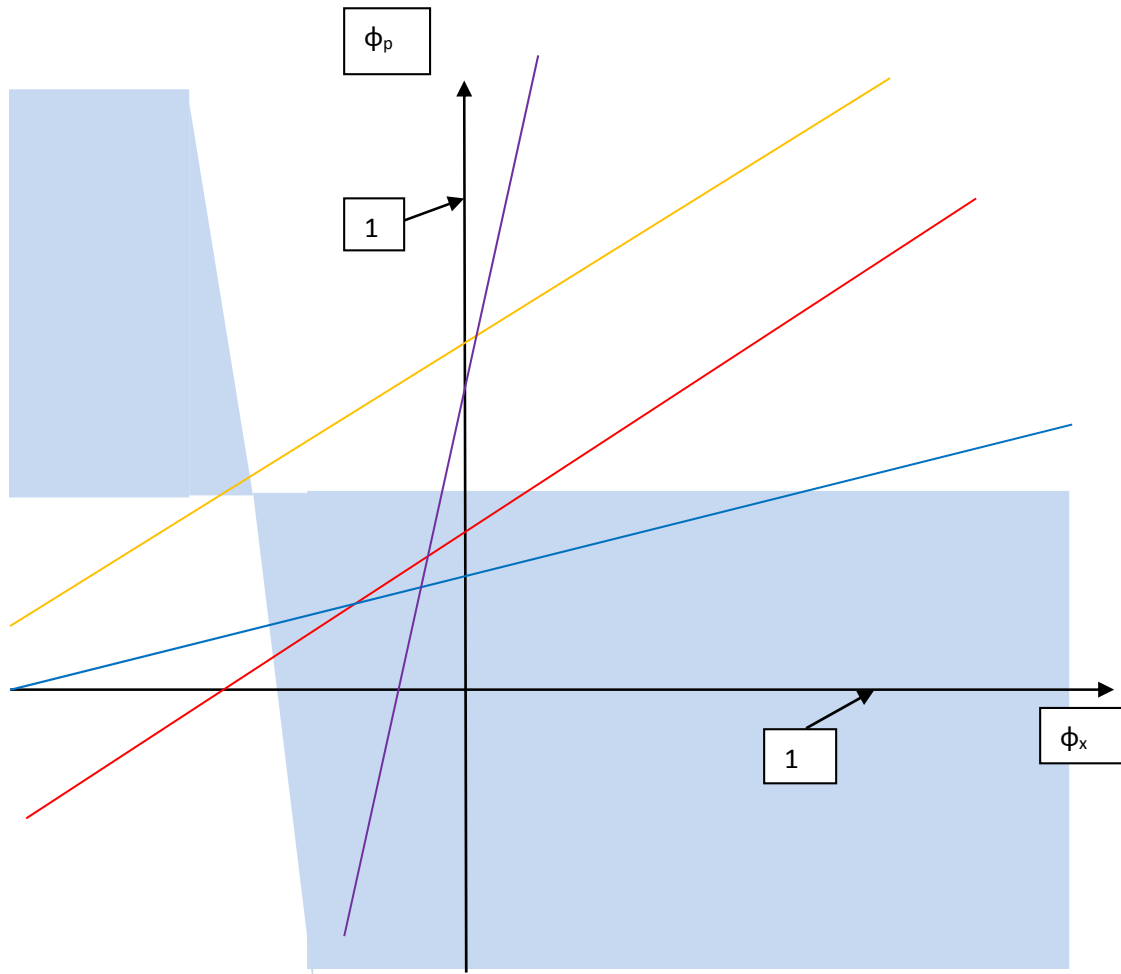
For nominal income growth targeting:

$$\varphi_p = \dfrac{\kappa + \Gamma(1+\kappa)}{\Gamma(1+\kappa)}\varphi_x + (1 - \dfrac{\kappa + \Gamma(1+\kappa)}{\Gamma(1+\kappa)}\sigma)(\delta - 1) + (1 - \dfrac{\kappa\sigma}{\Gamma})\rho$$

$$\text{And } \varphi_{p-1} = (\dfrac{\kappa\sigma}{\Gamma} - 1)\delta\rho$$

$$\text{with } \delta = \dfrac{1 - \sqrt{1 - 4\beta a^2}}{2a\beta} \text{ and } a = \dfrac{\Gamma(1+\kappa)}{\kappa^2 + \Gamma(1+\kappa)(1+\beta+\kappa)}$$

Chart 31: Determinacy condition and optimal relationship between the φ_π and φ_x in the Wicksellian rule for discretion (yellow), global (also speed limit in red), price level targeting (blue) and nominal income targeting (violet)

When $\varphi_x<0$, and more exactly when $\varphi_x<$around 2.18 using standard calibration, then the determinacy conditions change. As a consequence, a Wicksellian rule would not solve the determinacy issue. It is possible also in this case to find an optimal rule for each of the policies used in previous chapters, but it requires to selecting some range of the value of the parameters of the rules to be determinate. In appendix D, it is shown how the determinacy region may shift

and the optimal combination of parameters of the rules may change when the value of a factor is changing.

3.4.2 Wicksellian Money rule

Another idea to deal with the determinacy issue would be to adopt a Wicksellian money rule: the instrument is money but it controls the price level and the output gap.

Money rule $m_t = \varphi_p \hat{p}_t + \varphi_{p-1} \hat{p}_{t-1} + \varphi_x x_t + \varphi_y y_t^f + \varphi_g g_t + \varphi_\omega \omega_t + \varphi_{-1} p_{-1}$

In terms of stability and uniqueness, we get the following system of equations:

$$
\begin{pmatrix} 1 & \dfrac{1}{\sigma} & \dfrac{b}{\sigma}\varphi_p - \dfrac{b}{\sigma} \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{pmatrix}
\begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ \hat{p}_t \end{pmatrix}
=
\begin{pmatrix} 2 - \dfrac{b}{\sigma}\varphi_x & 0 & \dfrac{b}{\sigma}\varphi_{p-1} \\ -\kappa & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}
\begin{pmatrix} x_t \\ \pi_t \\ \hat{p}_{t-1} \end{pmatrix}
+ B
\begin{bmatrix} y_t^f \\ g_t \\ \omega_t \\ \varphi_{-1} \end{bmatrix}
$$

Or

$$
\begin{pmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ \hat{p}_t \end{pmatrix}
=
\begin{pmatrix} 2 - \dfrac{b}{\sigma}\varphi_x + \dfrac{\kappa}{\sigma\beta} & -\dfrac{1}{\sigma\beta} - \dfrac{b(\varphi_p-1)}{\sigma} & \dfrac{b}{\sigma}\varphi_{p-1} - \dfrac{b(\varphi_p-1)}{\sigma} \\ -\kappa & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix}
\begin{pmatrix} x_t \\ \pi_t \\ \hat{p}_{t-1} \end{pmatrix}
+ C
\begin{bmatrix} y_t^f \\ g_t \\ \omega_t \\ \varphi_{-1} \end{bmatrix}
$$

Following the same procedure than in the previous section, it is useful to find a value for $\varphi_{\text{p-1}}$.

For the global case, if we put this money rule in the IS curve with money, then we get

$$
(-\frac{b}{\sigma}\varphi_p + \frac{b}{\sigma} - \frac{\delta-1}{\sigma} - (2 - \delta - \frac{b}{\sigma}\varphi_x)\frac{\kappa}{\Gamma})\hat{p}_t = \frac{b}{\sigma}\varphi_{p-1}\hat{p}_{t-1} + (\frac{1}{\sigma} - \frac{\kappa}{\Gamma})\frac{\delta}{1-\beta\delta\rho}u_t
$$

With $\quad \varphi_g = -\dfrac{\sigma}{b}$

$$\varphi_y = \dfrac{\sigma}{b}$$

$$\varphi_\omega = 1$$

$$\varphi_{-1} = 1$$

Then, to have a solution in accordance to the optimal path, we should have

$$\delta = \dfrac{b\varphi_{p-1}}{\sigma\left(-\dfrac{b}{\sigma}\varphi_p + \dfrac{b}{\sigma} - \dfrac{\delta-1}{\sigma} - (2 - \delta - \dfrac{b}{\sigma}\varphi_x)\dfrac{\kappa}{\Gamma}\right)}$$

And

$$\dfrac{\delta}{1-\beta\delta\rho} = \dfrac{\dfrac{1}{\sigma} - \dfrac{\kappa}{\Gamma}}{\sigma\left(-\dfrac{b}{\sigma}\varphi_p + \dfrac{b}{\sigma} - \dfrac{\delta-1}{\sigma} - (2 - \delta - \dfrac{b}{\sigma}\varphi_x)\dfrac{\kappa}{\Gamma}\right)} \dfrac{\delta}{1-\beta\delta\rho}$$

So we obtain from these two equations:

$$\varphi_{p-1} = \dfrac{\delta}{b}(1 - \dfrac{\kappa\sigma}{\Gamma})$$

And

$$\varphi_p = \dfrac{\kappa}{\Gamma}\varphi_x + 1 - \dfrac{\delta}{b} - (1-\delta)\dfrac{\kappa\sigma}{\Gamma b}$$

In the case $\varphi_{p-1} = \dfrac{\delta}{b}(1 - \dfrac{\kappa\sigma}{\Gamma})$ we examine what are the condition for stability and

uniqueness. Due to the complexity of the eigenvalues, we have to use the standard numerical

values adopted already previously. The results are in chart 32. The determinacy area is found

through simulation.

Discretion: $\varphi_p = \dfrac{\kappa}{\Gamma}\varphi_x + 1 - \dfrac{\rho}{b} + (\rho - 2)\dfrac{\kappa\sigma}{b\Gamma}$ and $\varphi_p + \varphi_{p-1} = 1$

For price level targeting, the Wicksellian money rule takes the form

$$m_t = \varphi_p p_t + \varphi_{p-1} p_{t-1} + \varphi_x x_t + \varphi_y y_t^f + \varphi_g g_t + \varphi_\omega \omega_t + \varphi_{-1} p_{-1}$$

with $\varphi_p = \dfrac{\kappa}{(1+\beta)\Gamma}\varphi_x + \dfrac{(\delta-2)\kappa\sigma}{b(1+\beta)\Gamma} + 1 - \dfrac{\delta}{b} - \dfrac{1}{b}(\rho(1-\dfrac{\kappa\sigma}{(1+\beta)\Gamma})-1)$

And $\varphi_{p-1} = \dfrac{\delta\rho}{b}(1 - \dfrac{\kappa\sigma}{(1+\beta)\Gamma})$

with $\delta = \dfrac{1-\sqrt{1-4\beta a^2}}{2a\beta}$ and $a = \dfrac{(1+\beta)\Gamma_p}{\kappa^2 + \Gamma(1+\beta)^2}$

For the nominal income growth targeting, we get

$$\varphi_p = \dfrac{\kappa+\Gamma(1+\kappa)}{\kappa(1+\kappa)}\varphi_x + \dfrac{\sigma(\delta-2)}{b}\dfrac{\kappa+\Gamma(1+\kappa)}{\Gamma(1+\kappa)} + 1 - \dfrac{\delta}{b} - \dfrac{\rho}{b}(1-\dfrac{\kappa\sigma}{\Gamma}) + \dfrac{1}{b}$$

And $\varphi_{p-1} = \dfrac{\delta\rho}{b}(1 - \dfrac{\kappa\sigma}{(1+\beta)\Gamma})$

with $\delta = \dfrac{1-\sqrt{1-4\beta a^2}}{2a\beta}$ and $a = \dfrac{\Gamma(1+\kappa)}{\kappa^2 + \Gamma(1+\kappa)(1+\beta+\kappa)}$

Chart 32: Determinacy condition and optimal relationship between the φ_π and φ_x in the sort of money Wicksellian rule for discretion (yellow), global (also speed limit in red), price level targeting (blue) and nominal income targeting (violet)
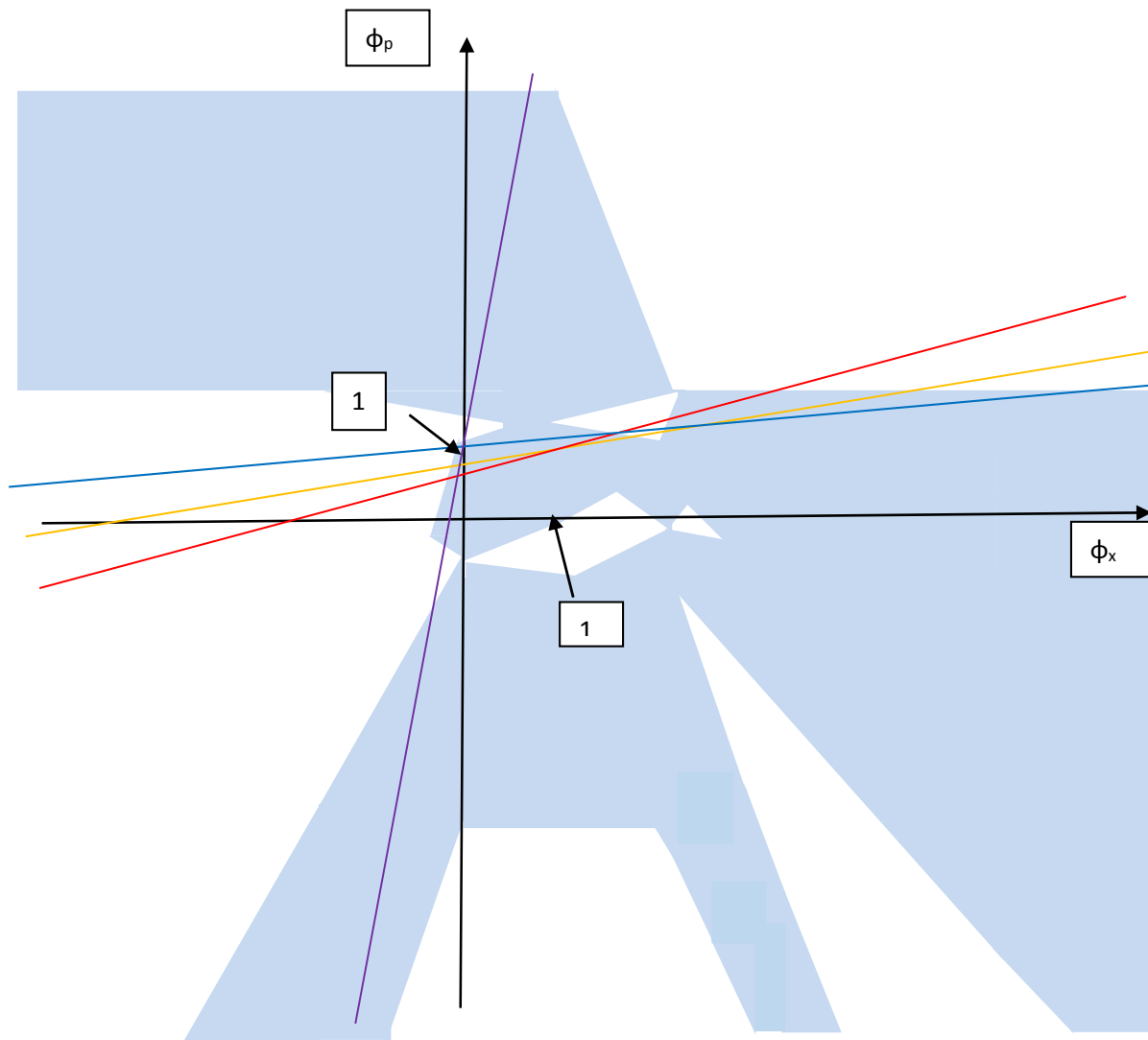
3.5 Concluding remark

The concept of determinacy, which is a unique and stable solution, has become a major issue in monetary policy analysis. The action of a central bank may be decomposed in two parts. First, as shown in the previous chapter, the central bank may try to optimize a different welfare than the social welfare and it could obtain a better result, assuming that the government can delegate effectively. Second, the central bank can choose among different policies. In the New

Keynesian model, the instrument used by central banks is an interest rate rule, and principally the Taylor rule that controls for inflation and output gap. To deal with the determinacy issue associated with this Taylor rule, many authors have proposed different rules. One would be to adopt a money rule instead of an interest rate rule. Another would be to choose a Wicksellian rule that would control for price level and output gap. This paper showed that in any case, when looking for the optimal rule, the constraints we can observe related to determinacy with the Taylor rule are still present in a similar fashion when implementing a money rule or a Wicksellian rule.

As a result, this chapter demonstrated that a central bank can use indifferently a traditional interest rate Taylor rule, a "money Taylor rule", an interest rate Wicksellian rule or a "money Wicksellian rule" and still guide with certainty the economy to the optimal paths for the different regimes. Policy makers can do this provided they are free to select the φ's of the respective rules according to the equations established above and in conformity with the conditions for determinacy. There are infinite sets of combinations of that sort that have identical consequences.

Appendix

Appendix A:

Appendix A shows the basic calculation of welfare for discretion, pre-commitment and global. Those results were used and formulas were transformed to calculate welfare for the alternative regimes.

Welfare for Discretion

If inflation and output gap follow the paths developed above in the discretion case, then the resulting loss function is:

$$L = \pi_t^{\;2} + \Gamma x_t^{\;2}$$

$$\Leftrightarrow L = (\frac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)})^2 u_t^{\;2} + \Gamma(\frac{\kappa}{\kappa^2 + \Gamma(1-\beta\rho)})^2 u_t^{\;2}$$

$$\Leftrightarrow L = \frac{\Gamma(\Gamma + \kappa^2)}{(\kappa^2 + \Gamma(1-\beta\rho))^2} u_t^{\;2}$$

In the "Science for Monetary Policy" (Clarida, Gali, and Gertler, 1999), the objective function has the form:

$$\max - \frac{1}{2} E_t (\sum_{i=0}^{\infty} \beta^i (\Gamma x_{t+i}^2 + \pi_{t+i}^2))$$

Welfare is then:

$$W_{dis} = -\frac{1}{2}(\frac{\Gamma^2}{(\kappa^2 + (1-\beta\rho)\Gamma)^2})(\frac{1}{1-\beta\rho^2})(\frac{1}{1-\beta})(1 + \frac{\kappa^2}{\Gamma})\sigma_\eta^{\ 2}$$

Welfare for pre-commitment:

So we can see that the calculation for the welfare will be similar to the calculation of the welfare for the discretion case, except a few changes.

The factor $D_d = \dfrac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)}$

Becomes $D_{presmp} = \dfrac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)^2}$

Also we have to multiply the result for inflation by (1-βρ). Thus, the formula for the welfare of the pre-commitment case according to the paths extracted from the "Science of Monetary Policy" paper is:

$$W_{presmp} = -\frac{1}{2}(\frac{\Gamma^2}{(\kappa^2 + (1-\beta\rho)^2\Gamma)^2})(\frac{1}{1-\beta\rho^2})(\frac{1}{1-\beta})((1-\beta\rho)^2 + \frac{\kappa^2}{\Gamma})\sigma_\eta^{\ 2}$$

And we can obviously see that this welfare is above the welfare from discretion.

It is true if we have:

$$\frac{(1-\beta\rho)^2 + \dfrac{\kappa^2}{\Gamma}}{\kappa^2 + (1-\beta\rho)^2\Gamma} < \frac{1 + \dfrac{\kappa^2}{\Gamma}}{\kappa^2 + (1-\beta\rho)\Gamma}$$

Or

$0 < \beta\rho\kappa^2 + (1-\beta\rho)^2\Gamma\beta\rho$ which is always true.

So we have

$$W_{discretion} < W_{precommitm\,entsmp}$$

Welfare for global approach:

The welfare due to inflation is:

$$W_{global\pi} = -\frac{1}{2}(\frac{\delta}{1-\beta\delta\rho})^2(\frac{1}{1-\beta})[1+\frac{(\delta+\rho-1)^2\beta}{1-\beta\delta^2}+\frac{2\delta(\delta+\rho-1)\rho(\rho-1)\beta^2}{1-\beta^2\delta^2\rho^2}+\frac{\rho^2(\rho-1)^2\beta^2}{1-\beta^2\rho^4}$$

$$+\frac{2\delta^2(\delta+\rho-1)(\delta+\rho)\rho(\rho-1)\beta^3}{(1-\beta\delta^2)(1-\beta^2\delta^2\rho^2)}+\frac{(\delta+\rho)^2\rho^2(\rho-1)^2\beta^3}{(1-\beta\delta^2)(1-\beta^2\rho^4)}$$

$$+\frac{2\delta\rho^4(\rho-1)^2(\delta+\rho)\beta^4}{(1-\beta^2\delta^2\rho^2)(1-\beta^2\rho^4)}+\frac{2\delta^2\rho^4(\rho-1)^2(\delta+\rho)^2\beta^5}{(1-\beta\delta^2)(1-\beta^2\rho^4)(1-\beta^2\delta^2\rho^2)}]\sigma_\eta^2$$

The welfare due to output gap is:

$$W_{globalx} = -\frac{1}{2}(\frac{\delta}{1-\beta\delta\rho})^2\frac{\kappa^2}{\Gamma}(\frac{1}{1-\beta})(\frac{1}{1-\beta^2\rho^4})[1+\frac{(\delta+\rho)^2\beta}{1-\beta\delta^2}+\frac{2\delta(\delta+\rho)\rho^2\beta^2}{1-\beta^2\delta^2\rho^2}+\frac{2\delta^2(\delta+\rho)^2\rho^2\beta^2}{(1-\beta\delta^2)(1-\beta^2\delta^2\rho^2)}]\sigma_\eta^2$$

So the total welfare is

$$W_{global} = W_{globalx} + W_{global\pi}$$

We can observe with all different values for the parameters that the welfare with the global approach is above the welfare with the pre-commitment from the "Science of Monetary Policy" Clarida, Gali, and Gertler (1999) and then above the welfare with discretion. So at the end we obtain:

$$W_{discretion} < W_{precommitm\,entsmp} < W_{global}$$

Appendix B shows that the results are robust regardless the value of other parameters. It is robust in the sense that the superiority of certain regimes against others is not changed when the value of those parameters change, but of course the difference in welfare will be more or less important. The other parameters such as ρ, β, or θ don't play a major role in how to select the different targeting regimes.

Chart 33: Influence of ρ and β on welfare for discretion, pre-commitment and global
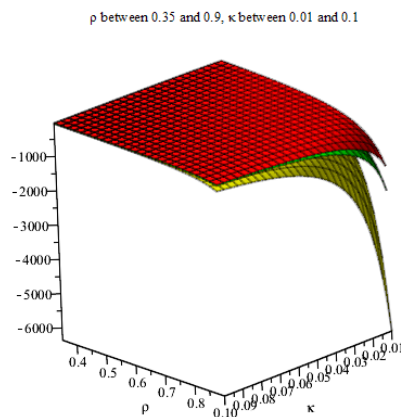
a/ Influence of ρ on welfare   b/ Influence of β on welfare
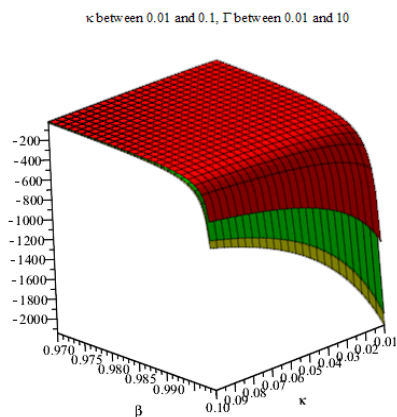
Chart 33 a/ shows that the highest the persistence, the lowest welfare is. Again, similar to the reason behind the influence of price stickiness, inflation and output gap adjust much more slowly with a higher persistence. As a consequence, welfare is lower because the difference

between the evolution of the economy and inflation and the path when prices are flexible is larger for a longer period of time. Furthermore, the difference between global and discretion is accentuated when persistence is higher. The selection of regime makes more difference when persistence is higher. Chart 33 b/ indicates also that the higher β, the lower welfare. Indeed, welfare is negative of the discounted value of the future volatilities of inflation and output gap. As a consequence, the higher the discount factor, the lower the welfare. It corresponds to the situation in which the society in general is discounting less the future losses, putting more value on the losses.

Using the relationship between κ and Γ according to Woodford, there we can observe the influence of θ on welfare on chart 34. It seems that it does not have any strong influence. However, κ has also θ in its term, so it may be better if we use a figure that shows how welfare changes when θ and ω change.

Chart 34: Influence of θ on welfare for discretion, pre-commitment and global

a/ Welfare for discretion, pre-commitment and global

b/ percentage difference between welfare for global and welfare for discretion
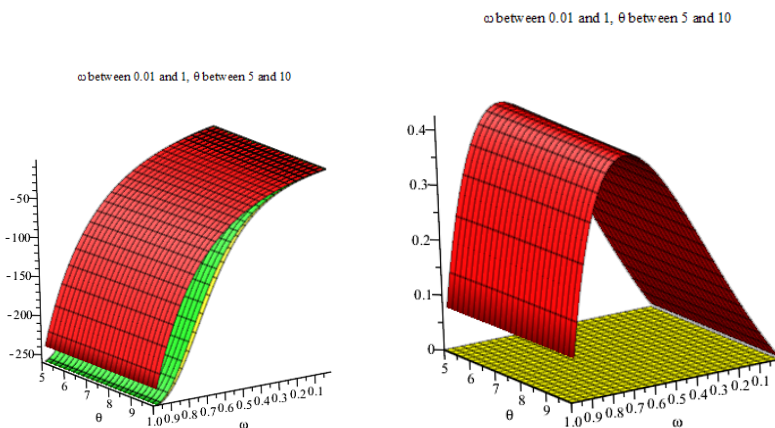
Volatility of Inflation

We can calculate the volatility of inflation with the same methodology than Appendix A. The gain in welfare from commitment has been attributed to a better trade-off between the volatility of inflation and volatility of output gap. The volatility of inflation is decreasing and the volatility of output gap is increasing but in a more favorable way for the calculation of welfare. Appendix A shows the calculation of the volatilities and the trade-offs described above for the calculation of welfare.

For discretion, we get:

$$E_t(\pi_{t+n}{}^2) = \frac{D_d{}^2}{1-\rho^2}\sigma_\eta{}^2 \text{ with } D_d = \frac{\Gamma}{\kappa^2 + \Gamma(1-\beta\rho)}$$

For the pre-commitment case, we just have to transform D_d as:

$$D_{pre} = \frac{\Gamma(1-\beta\rho)}{\kappa^2 + \Gamma(1-\beta\rho)^2}$$

So we would get:

$$E_t(\pi_{t+n}{}^2) = \frac{D_{pre}{}^2}{1-\rho^2}\sigma_\eta{}^2 \text{ with } D_{pre} = \frac{\Gamma(1-\beta\rho)}{\kappa^2 + \Gamma(1-\beta\rho)^2}$$

For the global case, we get:

$$D_g = \frac{\delta}{1-\beta\delta\rho}$$

$$E_t(\pi_{t+n}^2) = D_g(1 + \frac{(\delta + \rho - 1)^2}{1 - \delta^2} + \frac{2\delta(\delta + \rho - 1)\rho(\rho - 1)}{(1 - \delta^2)(1 - \delta\rho)} + \rho^2(\rho - 1)^2 + \frac{(\delta + \rho)^2 \rho^2(\rho - 1)^2}{1 - \delta^2}$$

$$+ \frac{2\delta\rho^5(\rho - 1)^2}{(1 - \delta^2)(1 - \delta\rho)(1 - \rho^2)} + \frac{\rho^6(1 - \rho)}{(1 - \delta^2)(1 + \rho)} + \frac{2\delta^2\rho^4(1 - \rho)^2}{(1 - \delta^2)(1 - \delta\rho)})\sigma_\eta^2$$

Volatility of output gap

In contrast, volatility of output gap should decrease when adopting a commitment approach. Svensson (1997) argued that welfare could improve with commitment having both volatility of inflation and volatility of output gap increase. It is what he called a "free lunch". However, he obtained this result with a backward looking Phillips curve.

In the discretion case, we have also $x_t = -\frac{\kappa}{\Gamma}\pi_t$

So $x_t = -\frac{\kappa}{\kappa^2 + \Gamma(1 - \beta\rho)}u_t$

So we can use the results we had for inflation. We just have to change D_d by:

$$D_x = -\frac{\kappa}{\Gamma}D_d$$

$$E_t(x_{t+n}^2) = (\frac{\kappa^2}{\Gamma^2})\frac{D_d^2}{1 - \rho^2}\sigma_\eta^2 \text{ with } D_d = \frac{\Gamma}{\kappa^2 + \Gamma(1 - \beta\rho)}$$

For the pre-commitment case, we have a similar situation in the sense that we get the volatility of x_t using the volatility of πt but we just have to transform D_{pre} by:

$$D_{prex} = -\frac{\kappa}{\kappa^2 + \Gamma(1 - \beta\rho)^2}$$

we get $E_t(x_{t+n}{}^2) = \dfrac{D_{prex}{}^2}{1-\rho^2}\sigma_\eta{}^2$ with $D_{prex} = -\dfrac{\kappa}{\kappa^2 + \Gamma(1-\beta\rho)^2}$

For the global case,

With

$$D_{gx} = -\frac{\kappa}{\Gamma}D_g$$

With

$$D_g = \frac{\delta}{1-\beta\delta\rho}$$

We get

$$E_t(x_{t+n}{}^2) = D_g\left(1 + \frac{(\delta+\rho)^2}{1-\delta^2} + \frac{2\delta(\delta+\rho)\rho^2}{(1-\delta^2)(1-\delta\rho)} + \rho^4 + \frac{(\delta+\rho)^2\rho^4}{1-\delta^2}\right.$$

$$\left. + \frac{2\delta\rho^7}{(1-\delta^2)(1-\delta\rho)(1-\rho^2)} + \frac{\rho^8}{(1-\delta^2)(1-\rho^2)} + \frac{2\delta^2\rho^6}{(1-\delta^2)(1-\delta\rho)}\right)\sigma_\eta{}^2$$

The chart 35 provides a perspective of the difference between the discretion, pre-commitment and global regimes. Chart 35 confirms that the volatility of inflation is decreasing when adopting commitment. It is particularly true for lower κ and Γ. It is noticeable however, that when prices are more flexible volatility of inflation is higher for global than for pre-commitment.

Chart 35: Volatilities of inflation and output gap for discretion, pre-commitment and global

a/ Volatility of inflation
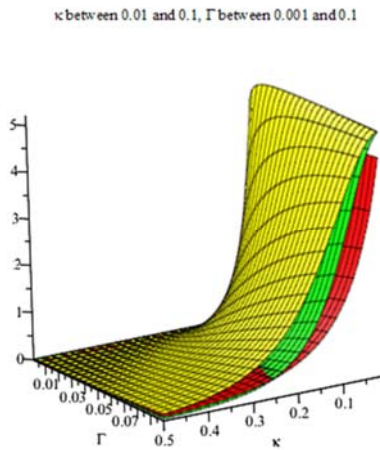
b/ Volatility of output gap
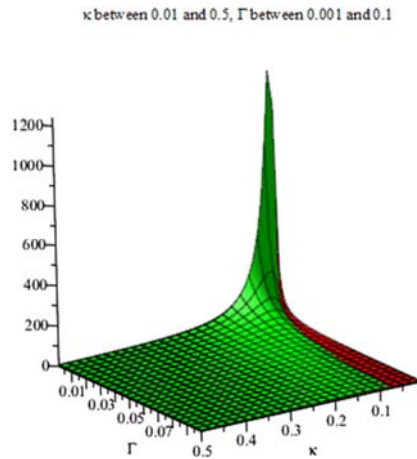
We can obtain the same way the volatilities for other policies, and they are shown in the graph

36. Charts 37 to 42 show the comparison between the different regimes.

Chart 36: Volatility of inflation and output gap for discretion, global, price level targeting, nominal income growth and interest rate smoothing

a/ Volatility of Inflation
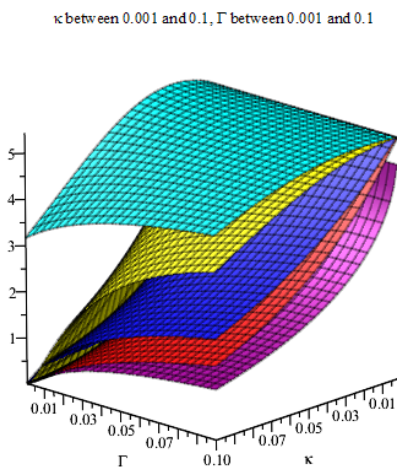
b/ Volatility of Output Gap
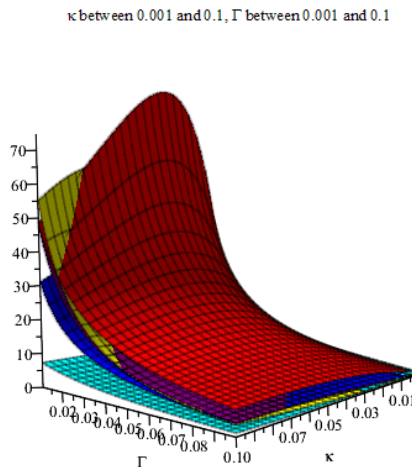
Chart 37: Comparison between price level targeting and global target

a/ Volatility Inflation
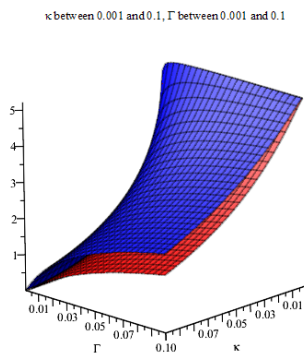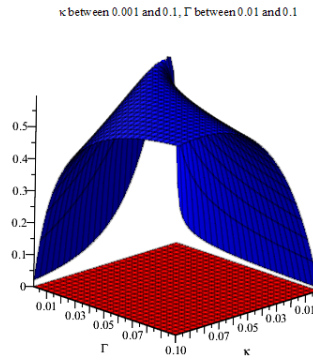
b/ Difference

c/ Percentage Difference
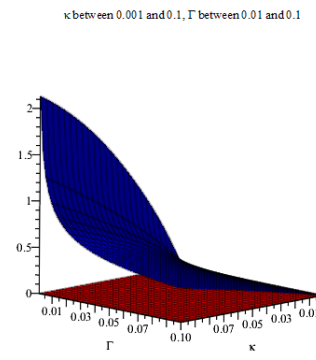
d/ Volatility Output Gap
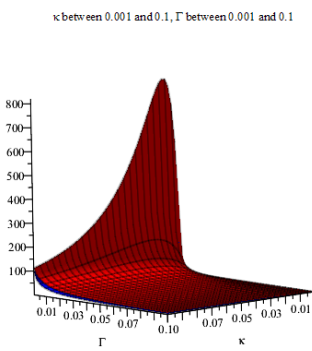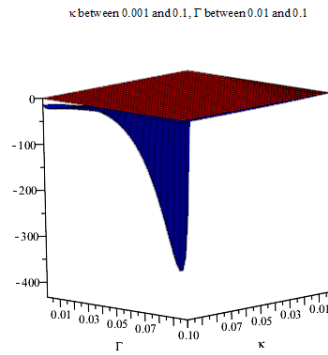
e/ Difference

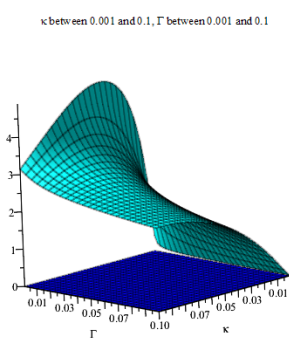f/ Percent Difference
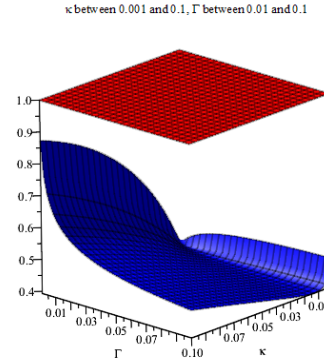
Chart 38: Comparison between price level targeting and interest rate smoothing approach
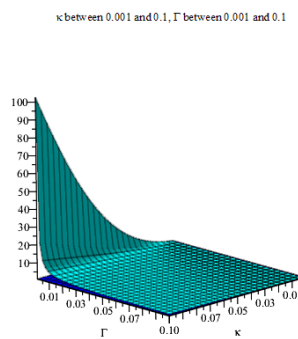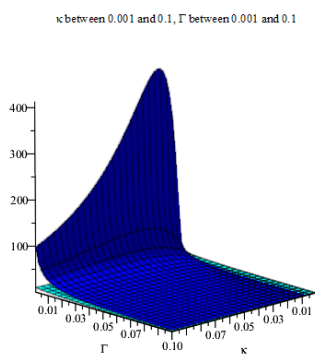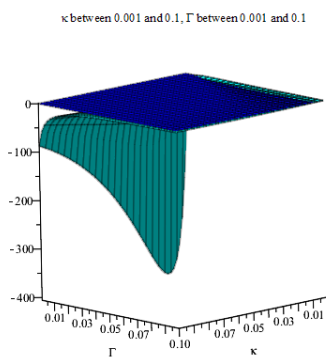
a/ Volatility Inflation   b/ Difference   c/ Percent Difference
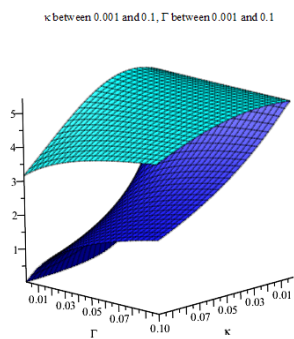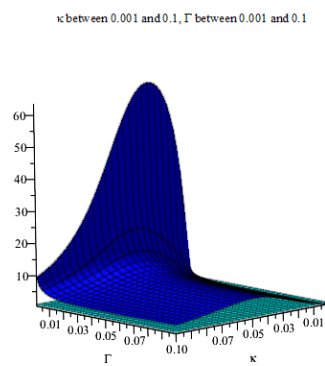

d/ Volatility Output Gap   e/ Difference   f/ Percent Difference

Chart 39: Comparison between price level targeting and nominal income targeting

a/ Volatility Inflation

b/ Difference

c/ Percent Difference
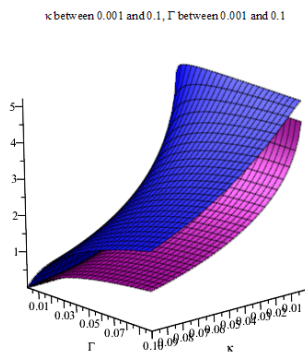

d/ Volatility Output Gap
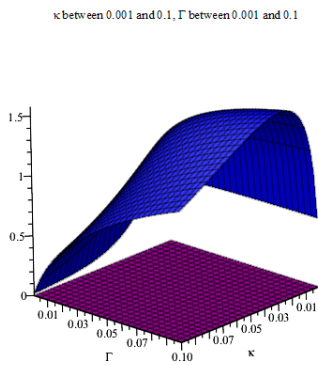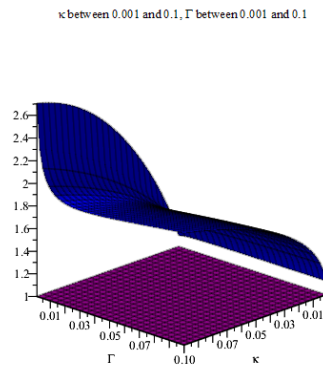
e/ Difference

f/ Percent Difference

Chart 40: Comparison between interest rate smoothing approach and global

a/ Volatility inflation

b/ Difference

c/ Percent Difference

κ between 0.001 and 0.1, Γ between 0.001 and 0.1

κ between 0.001 and 0.1, Γ between 0.001 and 0.1

κ between 0.001 and 0.1, Γ between 0.001 and 0.1

d/ Volatility Output Gap

e/ Difference

f/ Percent Difference

κ between 0.001 and 0.1, Γ between 0.001 and 0.1

κ between 0.001 and 0.1, Γ between 0.001 and 0.1

κ between 0.001 and 0.1, Γ between 0.001 and 0.1

Chart 41: Comparison between interest rate smoothing approach and nominal income growth targeting

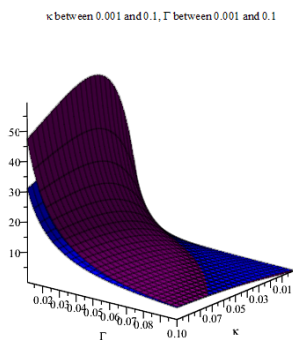a/ Volatility Inflation   b/ Difference   c/ Percent Difference
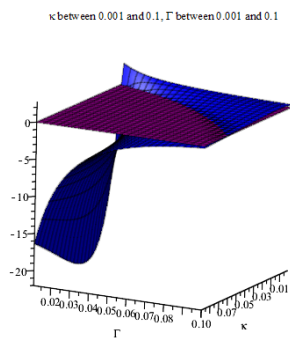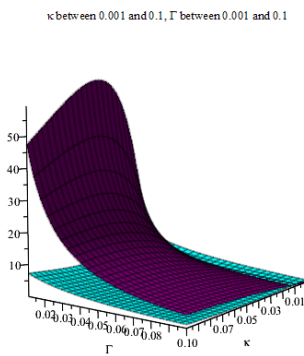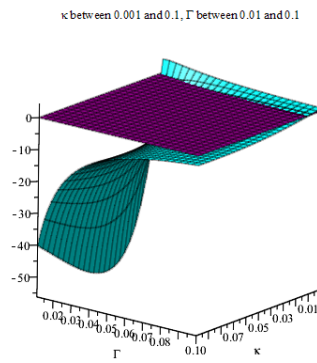

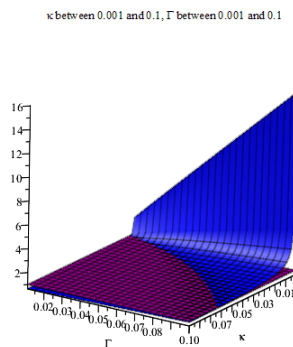d/ Volatility Output Gap   e/ Difference   f/ Percent Difference
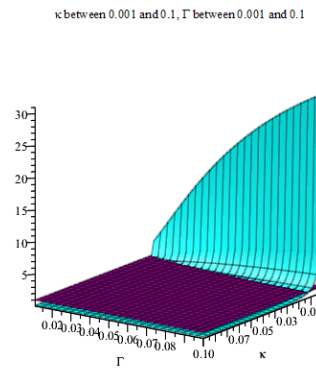
Chart 42: Comparison between nominal income growth targeting and global

a/ Volatility inflation
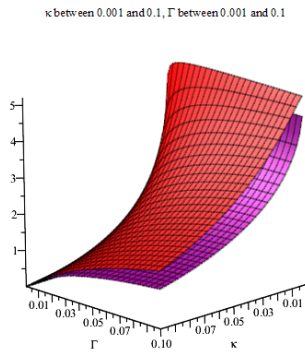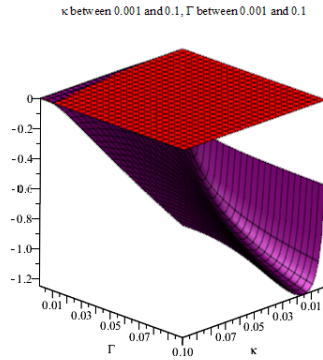
b/ Difference

c/ Percent Difference
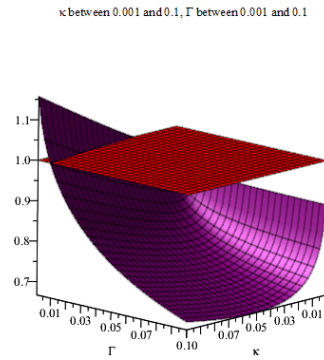
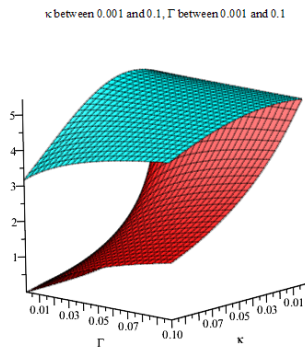d/ Volatility Output Gap
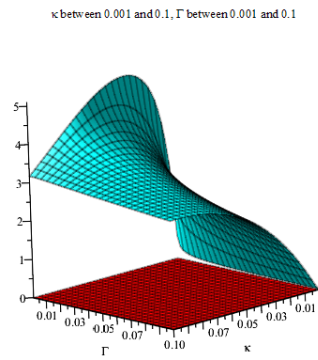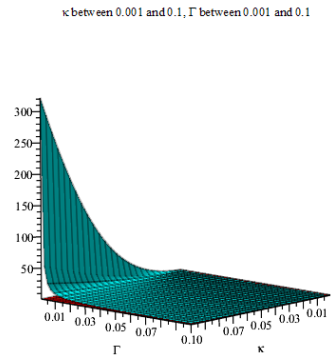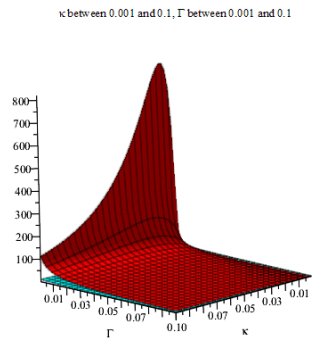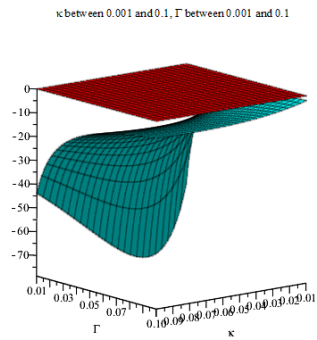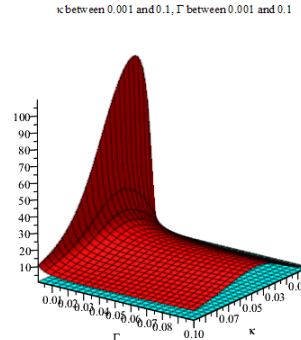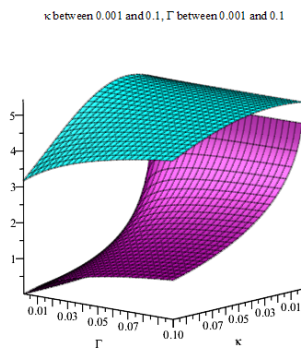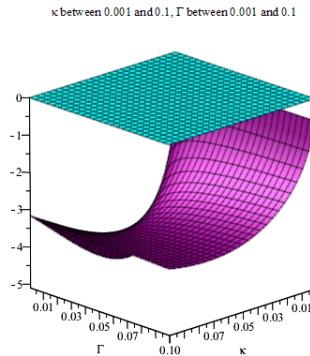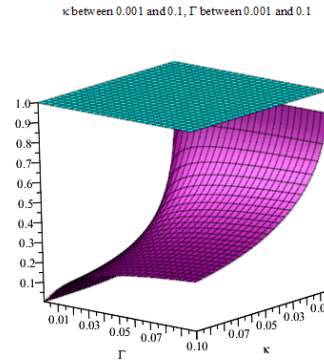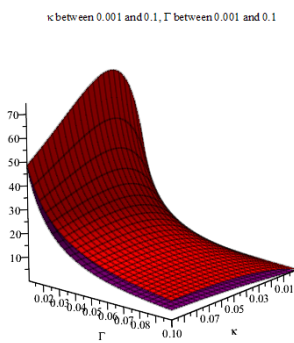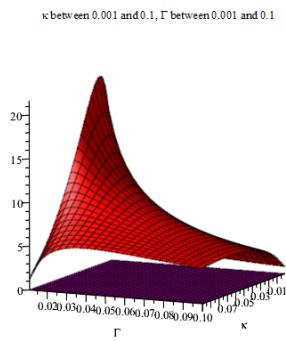
e/ Difference

f/ Percent Difference
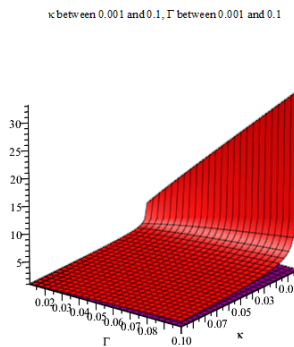
<u>Volatility of interest rate with shock g_t:</u>

If we want to incorporate the IS shock, then we just add σg_t to it for the discretion, pre-commitment, and global cases.

For the discretion case, we then get:

$$i_t = \frac{\rho\Gamma(1+(1-\rho)\frac{\kappa\sigma}{\rho\Gamma})}{\kappa^2 + \Gamma(1-\beta\rho)} u_t + \sigma g_t \text{ with } u_t = \rho u_{t-1} + \eta_t \text{ and } g_t = \lambda g_{t-1} + \varepsilon_t$$

so the total variance is:

$$E_t(i_{t+n}{}^2) = \frac{D_i{}^2}{1-\rho^2}\sigma_\eta{}^2 + \frac{\sigma^2}{1-\lambda^2}\sigma_\varepsilon{}^2 \text{ with } D_i = \frac{\rho\Gamma(1+(1-\rho)\frac{\kappa\sigma}{\rho\Gamma})}{\kappa^2 + \Gamma(1-\beta\rho)}$$

For the pre-commitment case, we proceed the same way and we get:

$$E_t(i_{t+n}{}^2) = \frac{D_{ipre}{}^2}{1-\rho^2}\sigma_\eta{}^2 + \frac{\sigma^2}{1-\lambda^2}\sigma_\varepsilon{}^2$$

For the global case, we have:

$$i_t = \delta i_{t-1} + \frac{\delta}{1-\delta\beta\rho}(\rho+\delta-1)(1-\frac{\kappa\sigma}{\Gamma})u_t - \frac{\delta^2}{1-\beta\delta\rho}\rho(1-\frac{\kappa\sigma}{\Gamma})u_{t-1} + \sigma(g_t - \delta g_{t-1})$$

So at each step, the g_{t-1} due to i_{t-1} disappears because of δg_{t-1} at the end of the expression and we are left with σg_t as with the other cases, so the variance is:

$$E_t(i_{t+n}{}^2) = (D_{gi1}{}^2 + \frac{Y^2}{1-\delta^2} + \frac{2\delta YZ}{(1-\delta^2)(1-\delta\rho)} + Z^2 + \frac{(\delta+\rho)^2 Z^2}{1-\delta^2}$$

$$+ \frac{2\delta\rho^3 Z^2}{(1-\delta^2)(1-\delta\rho)(1-\rho^2)} + \frac{\rho^4 Z^2}{(1-\delta^2)(1-\rho^2)} + \frac{2\delta^2\rho^2 Z^2}{(1-\delta^2)(1-\delta\rho)})\sigma_\eta^2$$

$$+ \frac{\sigma^2}{1-\lambda^2}\sigma_\varepsilon^2$$

Volatility of real interest rate without g_t:

In the discretion case,

with:

$$D_r = \frac{(1-\rho)\kappa\sigma}{\kappa^2 + \Gamma(1-\beta\rho)}$$

we get $E_t(r_{t+n}^2) = \dfrac{D_r^2}{1-\rho^2}\sigma_\eta^2$ with $D_r = \dfrac{(1-\rho)\kappa\sigma}{\kappa^2 + \Gamma(1-\beta\rho)}$

For the pre-commitment case, we just have to transform D_r as:

$$D_{rpre} = \frac{\kappa\sigma(1-\rho)}{\kappa^2 + \Gamma(1-\beta\rho)^2}$$

So we would get:

$$E_t(r_{t+n}^2) = \frac{D_{rpre}^2}{1-\rho^2}\sigma_\eta^2 \text{ with } D_{ipre} = \frac{\kappa\sigma(1-\rho)}{\kappa^2 + \Gamma(1-\beta\rho)^2}$$

For the global case,

with

$$D_{gr1} = -\frac{\kappa\sigma\delta}{\Gamma(1-\delta\beta\rho)}(\rho+\delta-1)$$

$$Y = \delta D_{gr1} + \rho D_{gr1} - D_{gr2} \text{ with } D_{gr2} = -\frac{\kappa\sigma\delta^2}{\Gamma(1-\beta\delta\rho)}\rho$$

$$Z = \rho(\rho D_{gr1} - D_{gr2})$$

We get

$$E_t(r_{t+n}^2) = (D_{gi1}^2 + \frac{Y^2}{1-\delta^2} + \frac{2\delta YZ}{(1-\delta^2)(1-\delta\rho)} + Z^2 + \frac{(\delta+\rho)^2 Z^2}{1-\delta^2}$$

$$+ \frac{2\delta\rho^3 Z^2}{(1-\delta^2)(1-\delta\rho)(1-\rho^2)} + \frac{\rho^4 Z^2}{(1-\delta^2)(1-\rho^2)} + \frac{2\delta^2\rho^2 Z^2}{(1-\delta^2)(1-\delta\rho)})\sigma_\eta^2$$

If we want to incorporate the IS shock, then we just add σg_t to it for the discretion, pre-commitment, and global cases. As for the volatility of interest rate, we just have to add

$\frac{\sigma^2}{1-\lambda^2}\sigma_\varepsilon^2$ to the volatilities calculated above.

Chart 43 shows how the interest rate volatility for the global case is lower than the one for discretion. The difference is significant and it can be used as an argument to implement the global regime when a central bank has to face the issue of lower bound.

Chart 43: Volatility of interest rate with shock g_t for discretion, pre-commitment and global

a/ Volatility of interest rate
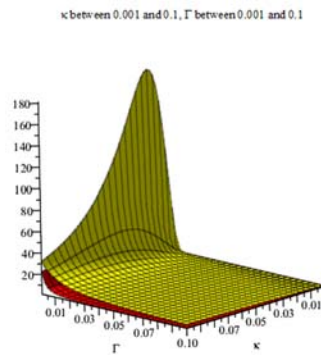
b/ Percentage difference between volatility of interest rate for global and the one for discretion
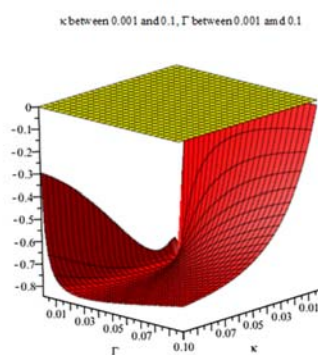
c/ Volatility of interest rate depending on κ only
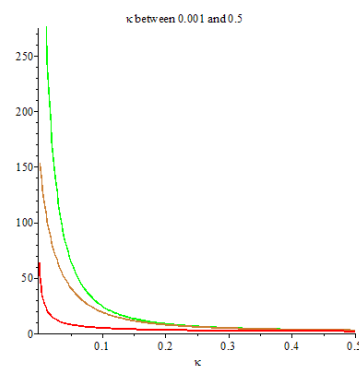
Appendix C

In appendix C, calculations are shown with different interest rate rules. However, those rules are not useful in the sense that they don't allow central banks to replicate optimal paths.

Interest rate rule $i_t = \varphi_p \hat{p}_t + \varphi_g g_t$: we could start with that interest rate, and it is the one Gali (2008) used with in addition the terms with gt to get rid of demand shock.

$$x_t = -\frac{\kappa}{\Gamma} \hat{p}_t \text{ where } \hat{p}_t = p_t - p_{-1}$$

p-1 is the price level before t=0.

Gali (2008) explains how he obtains the optimal path for $\hat{p}_t$ and x_t:

$$\hat{p}_t = \delta \hat{p}_{t-1} + \frac{\delta}{(1-\delta\beta\rho)} u_t$$

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With $\delta = \dfrac{1-\sqrt{1-4\beta a^2}}{2a\beta}$ and $a = \dfrac{\Gamma}{\Gamma(1+\beta)+\kappa^2}$

Then the optimal path for x_t is:

$$x_t = \delta x_{t-1} - \frac{\kappa\delta}{\Gamma(1-\delta\beta\rho)}u_t$$

with $x_0 = -\dfrac{\kappa\delta}{\Gamma(1-\delta\beta\rho)}u_0$

Using his approach, we can also find the path for inflation: $\pi_t = (\delta-1)\hat{p}_{t-1}+\dfrac{\delta}{1-\beta\delta\rho}u_t$

And we can get also the optimal path for interest rate:

$$i_t = \delta(\delta-1)(1-\frac{\sigma\kappa}{\Gamma})\hat{p}_{t-1}+\frac{\delta(\delta-1+\rho)\Gamma-\kappa\delta(\delta(\delta-1)+\sigma\rho)}{(1-\beta\delta\rho)\Gamma}u_t+\sigma g_t$$

We can then look at different interest rules.

Replacing it in the IS curve $x_t = E_t x_{t+1} - \dfrac{\hat{i}_t - E_t\pi_{t+1}}{\sigma}+g_t$ and using $x_t = \delta x_{t-1} - \dfrac{\kappa\delta}{\Gamma(1-\delta\beta\rho)}u_t$ and

$$\pi_t = (\delta-1)\hat{p}_{t-1}+\frac{\delta}{1-\beta\delta\rho}u_t$$

We obtain:

$$x_t = \delta x_t - \frac{\kappa\delta}{(1-\beta\delta\rho)}\rho u_t - \frac{\varphi_p\hat{p}_t+\varphi_g g_t-(\delta\hat{p}_t+\dfrac{\delta}{1-\beta\delta\rho}\rho u_t-\hat{p}_t)}{\sigma}+g_t$$

And having $x_t = -\dfrac{\kappa}{\Gamma}\hat{p}_t$, we get then:

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$$(\varphi_p - \delta + 1 - (1-\delta)\frac{\sigma\kappa}{\Gamma})\frac{\hat{p}_t}{\sigma} = (\frac{1}{\sigma} - \frac{\kappa}{\Gamma})\frac{\delta}{1-\beta\delta\rho}\rho u_t + (-\frac{\varphi_g}{\sigma} + 1)g_t$$

With φg=σ, we still cannot see a relationship in the form of $\hat{p}_t = \delta \hat{p}_{t-1} + \frac{\delta}{(1-\delta\beta\rho)}u_t$

because we don't have $\hat{p}_{t-1}$ in the expression. We cannot get then an optimal path using this

interest rate rule; $\hat{p}_{t-1}$ needs to be added.

In term of stability and uniqueness for the interest rate rule $i_t = \varphi_p \hat{p}_t + \varphi_g g_t$, we have the

following equations:

$$x_t = E_t x_{t+1} - \frac{\hat{i}_t - E_t \pi_{t+1}}{\sigma} + g_t \qquad (1) \qquad \text{with } g_t = \lambda g_{t-1} + \varepsilon_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \qquad (2) \qquad \text{with } u_t = \rho u_{t-1} + \eta_t$$

and replacing it with the interest rate rule discussed, we get:

$$\begin{bmatrix} 1 & \frac{1}{\sigma} & -\frac{\varphi_p}{\sigma} \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ \hat{p}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ \hat{p}_{t-1} \end{bmatrix} + \begin{bmatrix} -1+\frac{\varphi_g}{\sigma} & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} g_t \\ u_t \end{bmatrix}$$

Or

$$
\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ \hat{p}_t \end{bmatrix} = \begin{bmatrix} 1+\dfrac{\kappa}{\sigma\beta} & \dfrac{-1+\beta\varphi_p}{\sigma\beta} & \dfrac{\varphi_p}{\sigma} \\ -\dfrac{\kappa}{\beta} & \dfrac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ \hat{p}_{t-1} \end{bmatrix} + A \begin{bmatrix} g_t \\ u_t \end{bmatrix}
$$

The eigenvalues of the matrix are complicated and difficult to transform, so it is better to use calibration, using σ=1, κ=0.3 and β=0.99, which seem to be the standard calibration.

Gali finds that it is always stable and unique.

<u>Interest rate rule</u> $i_t = \phi_p p_t + \phi_x x_t + v_t$

This time, we associate the New Keynesian model with an interest rule that depends on p$_t$, not π$_t$.

$$i_t = \phi_p p_t + \phi_x x_t + v_t$$

We look for the solutions for φ$_\pi$ and φ$_x$. As a consequence, the IS curve

$$x_t = E_t x_{t+1} - \frac{i_t - E_t \pi_{t+1}}{\sigma} + g_t$$

can be transformed the following way.

First we replace   i$_t$ with $i_t = \phi_p p_t + \phi_x x_t + \varphi_g g_t$

      E$_t$x$_{t+1}$ with $E_t x_{t+1} = \rho x_t$

      E$_t$π$_{t+1}$ with $E_t \pi_{t+1} = \rho \pi_t$

So we get:

$$x_t = \rho x_t - \frac{\varphi_p p_t + \varphi_x x_t + \varphi_g g_t - \rho \pi_t}{\sigma} + g_t$$

Or, after transformations and using $\varphi_g = \sigma$ to cancel g_t

$$0 = (-1 + \rho - \frac{\varphi_x}{\sigma}) x_t + (-\frac{\varphi_p}{\sigma} + \frac{\rho}{\sigma}) p_t - \frac{\rho}{\sigma} p_{t-1}$$

But, we have to incorporate the fact that $x_t = -\frac{\kappa}{\Gamma} \pi_t$ to get a relationship between φ_x and φ_p then

we use $p_t = \pi_t + p_{t-1}$ to get:

$$0 = (-1 + \rho - \frac{\varphi_x}{\sigma}) x_t + (-\frac{\varphi_p}{\sigma} + \frac{\rho}{\sigma})(\pi_t + p_{t-1}) - \frac{\rho}{\sigma} p_{t-1}$$

$$\Leftrightarrow 0 = (-1 + \rho - \frac{\varphi_x}{\sigma}) x_t + (-\frac{\varphi_p}{\sigma} + \frac{\rho}{\sigma}) \pi_t - \frac{\varphi_p}{\sigma} p_{t-1}$$

Then we need to have $\varphi_p = 0$ to use the optimal relationship between x_t and π_t.

As a consequence we get:

$$0 = (-1 + \rho - \frac{\varphi_x}{\sigma}) x_t + \frac{\rho}{\sigma} \pi_t$$

$$\Leftrightarrow 0 = (-1 + \rho - \frac{\varphi_x}{\sigma}) x_t - \frac{\rho}{\sigma} \frac{\Gamma}{\kappa} x_t$$

$$\Leftrightarrow -1 + \rho - \frac{\varphi_x}{\sigma} = \frac{\rho}{\sigma} \frac{\Gamma}{\kappa}$$

$$\Leftrightarrow \varphi_x = (-1 + \rho)\sigma - \rho \frac{\Gamma}{\kappa}$$

It doesn't seem realistic because φ_x is then negative using the standard calibration. At this point, because of the strange result, we could have tried to do two things. One thing we could have done is to try to add also $\varphi_{p-1}p_{t-1}$ in the interest rate rule. However, to get an optimal relationship between x_t and π_t, we would have needed the relationship $\varphi_{p-1}=-\varphi_p$, so we would have gotten again the Taylor rule.

Another thing we could have done is to try to get a relationship between x_t and p_t so we could have obtained the optimal relationship between φ_p and φ_x. We could for example find the first order condition of the Lagrange with the loss function $\max-\dfrac{1}{2}E_t(\sum_{i=0}^{\infty}\beta^i(\Gamma x_{t+i}^2+\pi_{t+i}^2))$ and the Phillips curve relative to p_t for the discretion case, but it does not make sense because we have to find the first order conditions with π_t and x_t. Anyway, we would have found the following relationship:

$$\Gamma x_t = -\kappa\frac{p_t-p_{t-1}}{1+\beta}$$

$$\pi_t = -\frac{\Gamma(1+\beta)}{\kappa}x_t \text{ which is different from what we got previously.}$$

So it seems that it is not really possible to follow an interest rule with p_t and x_t when the loss function depends on π_t and x_t.

For the stability and uniqueness conditions, we have:

$$x_t = E_t x_{t+1} - \frac{i_t - E_t\pi_{t+1}}{\sigma} + g_t$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t + u_t$$

So the IS curve becomes

$$x_t = E_t x_{t+1} - \frac{\varphi_p p_t + \varphi_x x_t + \varphi_g g_t - E_t \pi_{t+1}}{\sigma} + g_t$$

$$x_t = E_t x_{t+1} - \frac{\varphi_p}{\sigma} \pi_t - \frac{\phi_p}{\sigma} p_{t-1} - \frac{\varphi_x}{\sigma} x_t - \frac{\varphi_g}{\sigma} g_t + \frac{1}{\sigma} E_t \pi_{t+1} + g_t$$

So we have

$$
\begin{bmatrix} 1 & \frac{1}{\sigma} & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}
\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ p_t \end{bmatrix}
=
\begin{bmatrix} 1+\frac{\varphi_x}{\sigma} & \frac{\varphi_p}{\sigma} & \frac{\varphi_p}{\sigma} \\ -\kappa & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix}
\begin{bmatrix} x_t \\ \pi_t \\ p_{t-1} \end{bmatrix}
+
\begin{bmatrix} 1-\frac{\varphi_g}{\sigma} & 0 \\ 0 & -1 \\ 0 & 0 \end{bmatrix}
\begin{bmatrix} g_t \\ u_t \end{bmatrix}
$$

$$
\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ p_t \end{bmatrix}
=
\begin{bmatrix} 1+\frac{\varphi_x}{\sigma}+\frac{\kappa}{\sigma\beta} & \frac{\varphi_p}{\sigma}-\frac{1}{\sigma\beta} & \frac{\varphi_p}{\sigma} \\ -\frac{\kappa}{\beta} & \frac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{bmatrix}
\begin{bmatrix} x_t \\ \pi_t \\ p_{t-1} \end{bmatrix}
+
\begin{bmatrix} 1-\frac{\varphi_g}{\sigma} & -\frac{1}{\sigma\beta} \\ 0 & \frac{1}{\beta} \\ 0 & 0 \end{bmatrix}
\begin{bmatrix} g_t \\ u_t \end{bmatrix}
$$

And the eigenvalues are complicated formulas and not easy to transform. As a consequence, we can use calibration to try to figure out if the system is stable and unique. We use σ=1, κ=0.3, and β=0.99. Following Blanchart and Kahn, we look to see if we have two eigenvalues outside the unit circle and two forward undetermined variables. We would then get a unique and stable solution. Here the eigenvalues may be complex, meaning that the variables may vary in an oscillating way. In these cases, we need to consider the absolute values of the eigenvalues to evaluate the uniqueness and stability. Using a wide range of values for φp and φx we have some restrictions.

Interest rate rule $i_t = \phi_p \, p_t + \phi_x x_t + \varphi_{p-1} p_{t-1} + v_t$

Because we cannot get the optimal path using the previous interest rate rule, an idea would be to include $\varphi_{p-1} p_{t-1}$ into the rule. Using the same system of equations as above but with $\varphi_{p-1} p_{t-1}$ into the rule, then we would need to have $\varphi_{p-1} = -\varphi_p$, so we would have gotten again the traditional Taylor rule $i_t = \phi_\pi \pi_t + \phi_x x_t + v_t$. In term of determinacy, we would have the following system of equations:

$$
\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ \hat{p}_t \end{bmatrix} =
\begin{bmatrix} 1 + \dfrac{\varphi_x}{\sigma} + \dfrac{\kappa}{\sigma\beta} & \dfrac{-1+\beta\varphi_p}{\sigma\beta} & \dfrac{\varphi_{p-1}+\varphi_p}{\sigma} \\ -\dfrac{\kappa}{\beta} & \dfrac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{bmatrix}
\begin{bmatrix} x_t \\ \pi_t \\ \hat{p}_{t-1} \end{bmatrix} + ?\begin{bmatrix} g_t \\ u_t \end{bmatrix}
$$

It is similar to the system previously found but with φ_{p-1}.

Interest rate rule $i_t = \varphi_p \, \hat{p}_t + \varphi_{p-1} \, \hat{p}_{t-1} + \varphi_g g_t$

As described above, it is necessary to include φ_{p-1} into the interest rate rule in order to obtain an optimal path. In this case, we then obtain the following relationship when looking for the optimal path:

$$
(\varphi_p - \delta + 1 - (1-\delta)\frac{\sigma\kappa}{\Gamma})\frac{\hat{p}_t}{\sigma} = -\frac{\varphi_{p-1}}{\sigma}\hat{p}_{t-1} + (\frac{1}{\sigma} - \frac{\kappa}{\Gamma})\frac{\delta}{1-\beta\delta\rho}\rho u_t + (-\frac{\varphi_g}{\sigma}+1)g_t
$$

And this time we can relate to the equation $\hat{p}_t = \delta \, \hat{p}_{t-1} + \dfrac{\delta}{(1-\delta\beta\rho)}u_t$

And the system of equations we get is:

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$$\delta = -\frac{\varphi_{p-1}}{\varphi_p - \delta + 1 - (1-\delta)\sigma\frac{\kappa}{\Gamma}}$$

And

$$\frac{\delta}{1-\beta\delta\rho} = \frac{(1-\frac{\kappa\sigma}{\Gamma})\rho\delta}{(1-\beta\delta\rho)(\varphi_p - \delta + 1 - (1-\delta)\sigma\frac{\kappa}{\Gamma})}$$

So we get:

$$\varphi_{p-1} = (\frac{\kappa\sigma}{\Gamma} - 1)\rho\delta \text{ and } \varphi_p = (1-\frac{\kappa\sigma}{\Gamma})\rho + (1-\delta)(\frac{\sigma\kappa}{\Gamma} - 1) = (1-\frac{\kappa\sigma}{\Gamma})(\rho + \delta - 1)$$

In term of stability and uniqueness for the interest rate rule $i_t = \varphi_p \hat{p}_t + \varphi_{p-1} \hat{p}_{t-1} + \varphi_g g_t$, we have the following equations:

$$x_t = E_t x_{t+1} - \frac{i_t - E_t \pi_{t+1}}{\sigma} + g_t \qquad\qquad \text{with } g_t = \lambda g_{t-1} + \varepsilon_t$$

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \qquad\qquad \text{with } u_t = \rho u_{t-1} + \eta_t$$

and using $E_t \pi_{t+1} = E_t \hat{p}_{t+1} - \hat{p}_t$ and $\pi_t = \hat{p}_t - \hat{p}_{t-1}$ replacing i_t with the interest rate rule discussed, we get:

$$\begin{bmatrix} 1 & \frac{1}{\sigma} & -\frac{\varphi_p}{\sigma} \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ \hat{p}_t \end{bmatrix} = \begin{bmatrix} 1 & 0 & \frac{\varphi_{p-1}}{\sigma} \\ -\kappa & 1 & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \pi_t \\ \hat{p}_{t-1} \end{bmatrix} + ? \begin{bmatrix} g_t \\ u_t \end{bmatrix}$$

Or

$$
\begin{bmatrix} E_t x_{t+1} \\ E_t \pi_{t+1} \\ \hat{p}_t \end{bmatrix} = \begin{bmatrix} 1+\dfrac{\kappa}{\sigma\beta} & \dfrac{-1+\beta\varphi_p}{\sigma\beta} & \dfrac{\varphi_{p-1}+\varphi_p}{\sigma} \\ -\dfrac{\kappa}{\beta} & \dfrac{1}{\beta} & 0 \\ 0 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_t \\ \hat{\pi}_t \\ \hat{p}_{t-1} \end{bmatrix} + ? \begin{bmatrix} g_t \\ u_t \end{bmatrix}
$$

See below when $\varphi_x=0$: according to the determinacy conditions below when we consider x_t, then it is enough to be determinate if $\varphi_p > \varphi_{p-1}$. Using the calibration $\rho=0.5$, $\beta=0.99$, $\Gamma=2$, $\kappa=0.3$, $\sigma=1$, we can notice that for the optimal case the system is not determinate. It would be determinate only if $\Gamma<0.3$

Appendix D

In this appendix, through an example, it is shown how the determinacy condition and the line representing the optimal relationship between the parameters of a rule may change when a parameter changes. In the following example, the central bank adopt a Wicksellian rule for a global targeting, and the weight on output gap in the welfare function changes from $\Gamma=2$ to $\Gamma=1$. In chart 44, we can see that the slope of the optimal combination of φ_p and φ_x is changing. In addition, the entire determinacy region is shifting to the lower part of the graph. The calibration is the one above in the paper.

Chart 44: Shift of determinacy zone and change of slope of optimal combination of φ_p and φ_x when adopting a Wicksellian rule and a global target
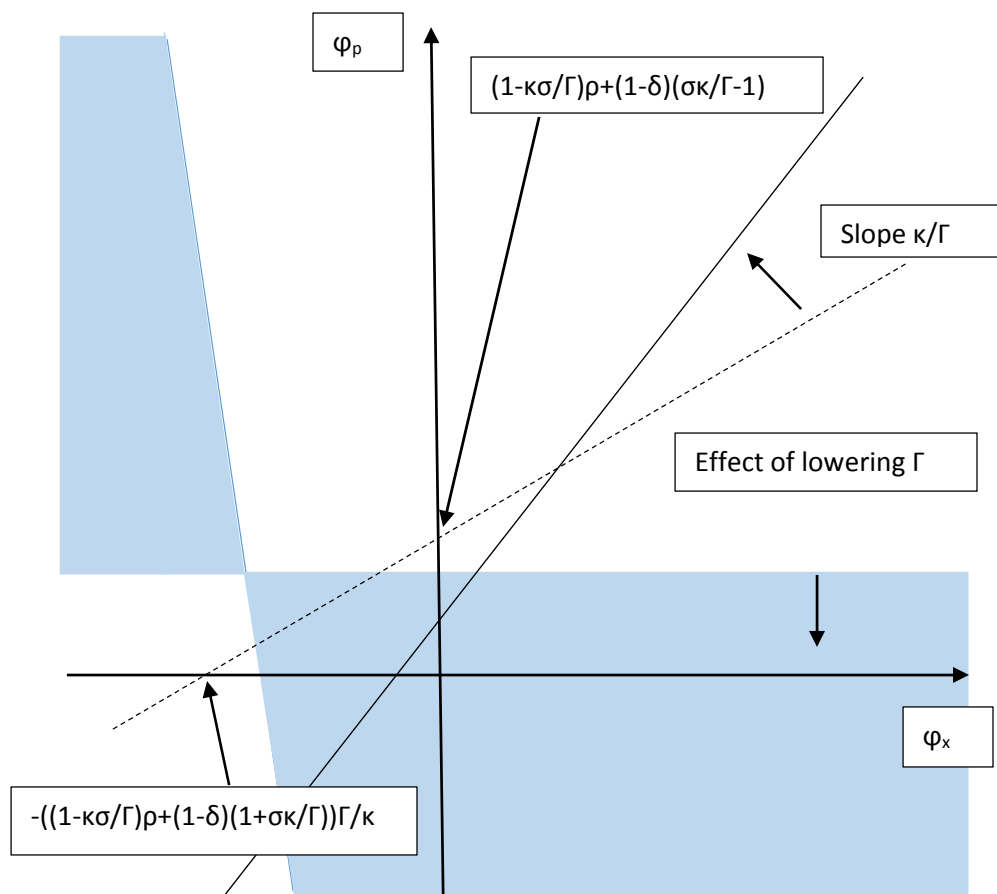
We get $\varphi_p=-\varphi_{p-1}$ when $\varphi_x=0$ at $\Gamma=0.3$. Then $\varphi_p=\varphi_{p-1}=\varphi_x=0$.

As illustrated in chart 45, for $\Gamma<0.3$ then we have determinacy for $\varphi_x=0$ because $\varphi_p>\varphi_{p-1}$
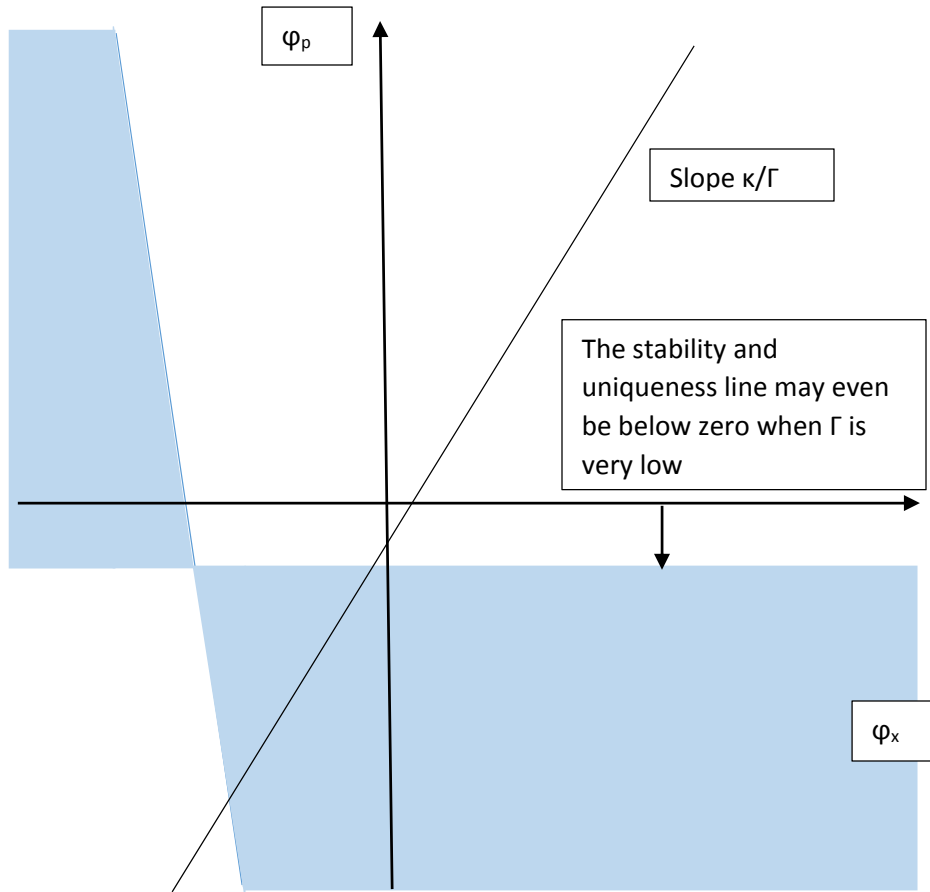
Chart 45: Determinacy zone and optimal combination of φ_p and φ_x when adopting a Wicksellian rule and a global target with $\Gamma<0.3$, here $\Gamma=0.2$

We get $\varphi_p=-\varphi_{p-1}$ when $\varphi_x=0$ at $\Gamma=0.3$. Then $\varphi_p=\varphi_{p-1}=\varphi_x=0$.

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