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Machine Learning in Finances

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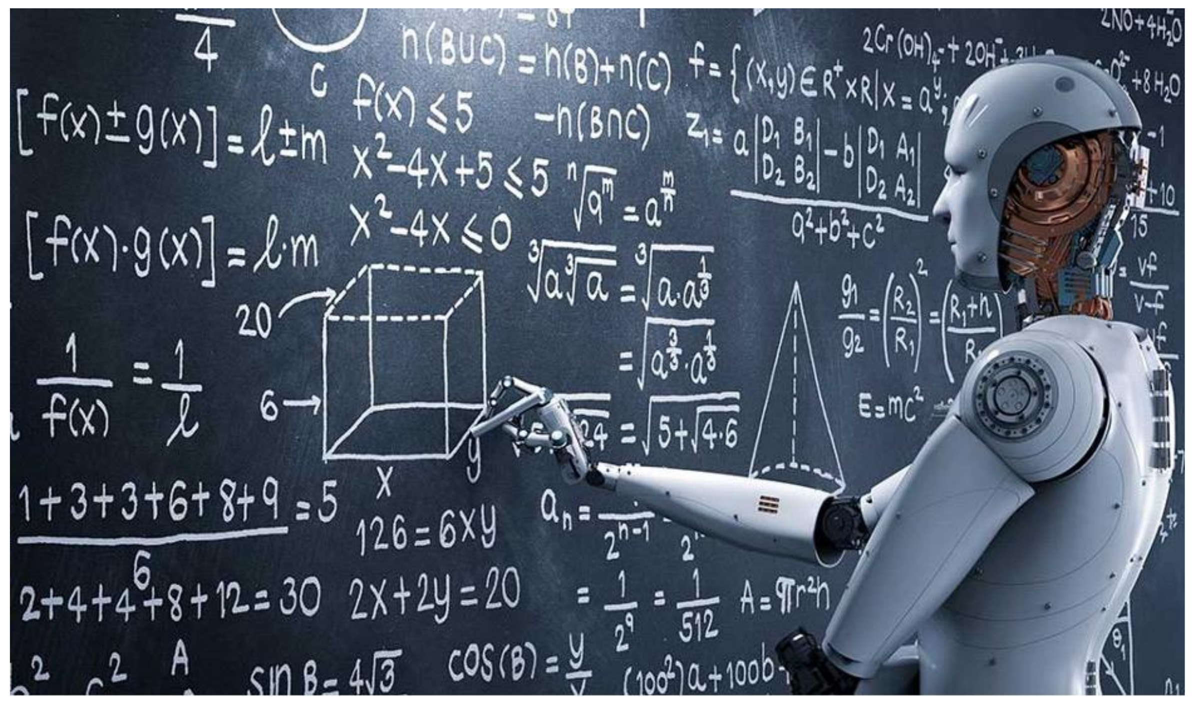
ABSTRACT

In our study we work on an optimization of an appropriate stock portfolio based on available information. Our work takes into consideration the average return and any associated risk. We produce an investment strategy that predictively allows a portfolio to grow with high yields.



INTRODUCTION

"Purchase low, sell high" may seem like wise guidance, however it is difficult to follow. Assuming we open the everyday paper to the stock citations and see that right now a portion of IBM stock sells for 103¾, down from 105½ the other day, is the stock presently "low" and prepared for a development, or is it "high" and on the way down? Even if a stock has been relatively stable over a period, its daily volatility may cause us anxiety. We need a fair profit from our speculations, however not at the expense of having an uncertain outlook on their dangers.

There are numerous types of data that can be used to predict stock performance: general economic conditions, the health of the industry that the stock represents, the company's productivity as shown in its annual report, and so forth. Our goal here isn't to show you how to win at the stock market, but rather to show how, with stock quotes and some statistics, you can figure out the average rate of return on investment and the risk of investing. We will use Lagrange multipliers to construct an optimal investment portfolio based on these estimates.

METHODS

There are several methods we went through in this project.

1. **Average Rate of Return and Risk**
2. **Risk and Risk Aversion**
3. **Solving the Optimization Problem**
4. **The Portfolio Separation Theorem**

We prefer high rates of return versus low rates of return. However, we don't know a stock's rate of return on any given day since we have to wait for its price to be monitored the next day. However, we have missed our investing chance to sell or acquire more stock. To deal with this, assume that, at least over a short time period, there is a constant but unknown "average" or expected rate of return on the stock, denoted , about which swings daily. A decent technique to determine this theoretical anticipated rate of return is to track the stock for as many days as possible and compute the average total $\bar{R} = \frac{1}{n}\sum_{t=1} R_t$ its daily rates of return. The variance of the stocks daily rates of return is:

$$S^2 = \frac{1}{n-1}\sum_{t=1}^{n}\left(R_t - \bar{R}\right)^2$$

Maximizing the risk averse expected return time $\mu - a\sigma^2$ the portfolio problem gets reduced to the constrained optimization problem:

Maximize :$(w_1, w_2, w_3, w_4) = w_1\mu_1 + w_2\mu_2 + w_3\mu_3 + w_4\mu_4 - a(w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + w_4^2\sigma_4^2)$

PROBLEM 3.1

Use the partial derivatives. $\frac{\partial L}{\partial w_i}$ $(i = 1,2,3,4)$ and $\frac{\partial L}{\partial \lambda}$ of the Lagrangian

$$L(w_1, w_2, w_3, w_4) = f(w_1, w_2, w_3, w_4) - \lambda g(w_1, w_2, w_3, w_4)$$

to obtain system of five equations in the unknowns $\lambda, w_1, w_2, w_3, w_4$. Show that the solution to the system's simulations equations is:

$$w_2 = \frac{\sigma_1^2}{\sigma_2^2}w_1 + \frac{\mu_2 - \mu_1}{2a\sigma_2^2}, w_3 = \frac{\sigma_1^2}{\sigma_3^2}w_1 + \frac{\mu_3 - \mu_1}{2a\sigma_3^2}, w_4 = \frac{\sigma_1^2}{\sigma_4^2}w_1 + \frac{\mu_4 - \mu_1}{2a\sigma_4^2}$$

Where,

$$w_1 = \frac{1 - \frac{\mu_2 - \mu_1}{2a\sigma_2^2} - \frac{\mu_3 - \mu_1}{2a\sigma_3^2} - \frac{\mu_4 - \mu_1}{2a\sigma_4^2}}{1 + \frac{\sigma_1^2}{\sigma_2^2} + \frac{\sigma_1^2}{\sigma_3^2} + \frac{\sigma_1^2}{\sigma_4^2}}$$

SOLUTION

We find partial derivatives for *L*, with respect to $\boldsymbol{w_1, w_2, w_3, w_4, \lambda}$, we get that each of the variables $\boldsymbol{w_1, w_2, w_3, w_4, \lambda}$, have solutions:

$$\boldsymbol{w_2 = \frac{\sigma_1^2}{\sigma_2^2}w_1 + \frac{\mu_2 - \mu_1}{2a\sigma_2^2}, w_3 = \frac{\sigma_1^2}{\sigma_3^2}w_1 + \frac{\mu_3 - \mu_1}{2a\sigma_3^2}, w_4 = \frac{\sigma_1^2}{\sigma_4^2}w_1 + \frac{\mu_4 - \mu_1}{2a\sigma_4^2}}$$

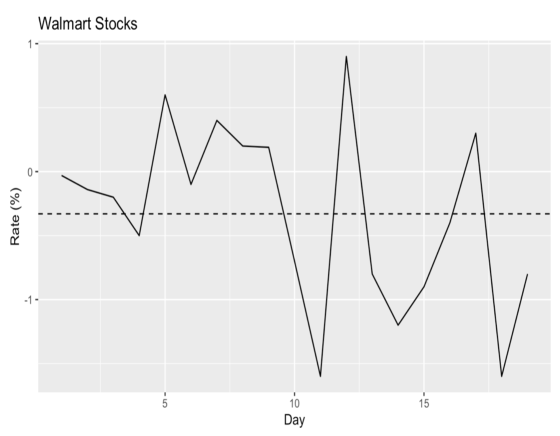
And

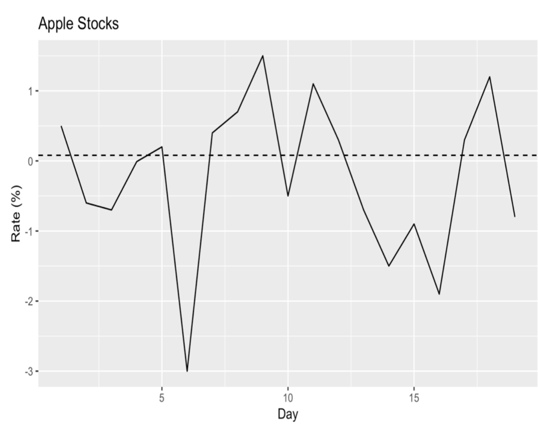
$$\boldsymbol{w_1 = \frac{1 - \frac{\mu_2 - \mu_1}{2a\sigma_2^2} - \frac{\mu_3 - \mu_1}{2a\sigma_3^2} - \frac{\mu_4 - \mu_1}{2a\sigma_4^2}}{1 + \frac{\sigma_1^2}{\sigma_2^2} + \frac{\sigma_1^2}{\sigma_3^2} + \frac{\sigma_1^2}{\sigma_4^2}}}$$

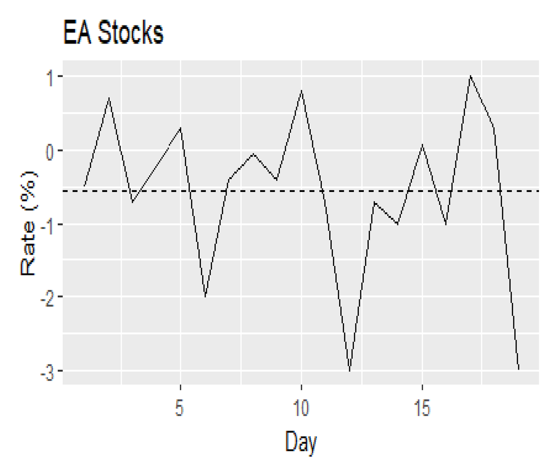
RESULTS

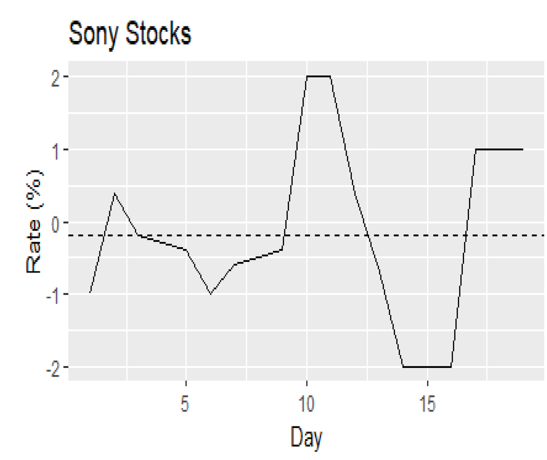
Based on the application of the average rate of return method, the following results are gotten for Walmart, Apple, EA and Sony Stocks:

	A	B	C	D	E	F	G	H	I	J	K
1	Walmart	R1		Apple	R2		EA	R3		Sony	R4
2	150.06			166.69			129.25			91.78	
3	150.01	-0.03%		167.63	0.50%		128.18	-0.50%		90.97	-1%
4	149.85	-0.14%		166.47	-0.60%		129.04	0.70%		91.31	0.40%
5	149.52	-0.20%		165.23	-0.70%		128.08	-0.70%		91.15	-0.20%
6	148.48	-0.50%		165.21	-0.01%		127.87	0.20%		90.86	-0.30%
7	149.49	0.60%		165.56	0.20%		128.28	0.30%		90.49	-0.40%
8	149.34	-0.10%		160.1	-3%		126.13	-2%		89.44	-1%
9	150.07	0.40%		160.8	0.40%		125.58	-0.40%		88.91	-0.60%
10	150.51	0.20%		162.03	0.70%		125.62	-0.05%		88.51	-0.50%
11	150.8	0.19%		164.66	1.50%		125.16	-0.40%		88.17	-0.40%
12	149.67	-0.70%		163.76	-0.50%		126.15	0.80%		89.65	2%
13	147.23	-1.60%		165.63	1.10%		125.24	-0.70%		91.01	2%
14	148.69	0.90%		167.17	0.30%		121.35	-3%		91.33	0.40%
15	147.45	-0.80%		164.9	-0.70%		120.45	-0.70%		90.65	-0.70%
16	145.67	-1.20%		162.36	-1.50%		119.1	-1%		89.31	-2%
17	144.23	-0.90%		160.77	-0.90%		119.19	0.07%		87.87	-2%
18	143.61	-0.40%		157.65	-1.90%		118.02	-1%		85.82	-2%
19	144.17	0.30%		158.28	0.30%		118.64	1%		86.64	1%
20	141.8	-1.60%		160.25	1.20%		119.03	0.30%		87.53	1%
21	140.65	-0.80%		158.93	-0.80%		116.04	-3%		88.27	1%
22	R1 Mean=-0.335%			R2 Mean = -0.232%			R3 Mean = -0.552%			R4 Mean = -0.174%	
23	σ = 0.706%			σ = 1.127%			σ = 1.256%			σ = 1.461%	









PROBLEM 3.2

Let us see what happens if one of the assets has no risk (i.e., $\boldsymbol{\sigma = 0}$):

(a) Suppose that there are three mutually independent assets, with $\boldsymbol{\mu_1 = 5\%, \sigma_1 = 0\%, \mu_2 = 8\%, \sigma_2 = \sqrt{2}\%(i.e.\, \sigma_2^2 = 2), \mu_3 = 12\%, \sigma_3 = 2\%}$. What is the risk-averse optimal portfolio for an investor whose risk aversion is *a=2*? And for *a=3*?

(b) How do these expected returns compare with the maximum risk-free expected return?

SOLUTION

Using the formulas for $\boldsymbol{w_1, w_2, w_3, w_4, \lambda}$ from **Problem 3.1** and substituting values of $\mu_1, \sigma_1, \mu_2, \sigma_2, \mu_3, \sigma_3$, and ***a=2,*** we get that $\boldsymbol{w_1 = \frac{3}{16}, w_2 = \frac{3}{8}, w_3 = \frac{7}{16}}$.

To consider the portfolio's actual daily rate of return we use formula:

$$\boldsymbol{R_p = w_1R_1 + w_2R_2 + w_3R_3}$$

Therefore, then $\boldsymbol{R_{a=2} = 0.091875}$

For ***a=3***:

We get that $\boldsymbol{w_1 = \frac{11}{24}, w_2 = \frac{1}{4}, w_3 = \frac{7}{24}}$.

CONCLUSION

In this section, we see that if, in addition to risky investments such as stocks ($\sigma^2 > 0$), we consider a risk-free investment option such as a bond or a savings account (σ^2=0), then under reasonable conditions the weights Risky investments are always positive. In other words, in this more realistic context, the problem of negative equity weights does not arise. We find that as risk aversion increases, more wealth moves from the more speculative stocks, Walmart and Apple to the EA and Sony. To continue, we use our results to prove important theorem in investment economics.

Portfolio Separation Theorem. If one possible investment in the portfolio is risk-free, the proportions of the optimal weights of the remaining investments do not depend on the investor's risk capacity.

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