Increasing Accessibility for Map Readers with Acquired and Inherited Color Vision Deficiencies: A Re-Coloring Algorithm for Maps

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Increasing Accessibility for Map Readers with Acquired and Inherited Color Vision Deficiencies: A Re-Coloring Algorithm for Maps

by

Gretchen Maria Culp

A dissertation submitted to the Graduate Faculty in Earth and Environmental Sciences in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

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THE CITY UNIVERSITY OF NEW YORK
Abstract

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Gretchen Maria Culp

Advisor: Dr. Juliana Maantay

Approximately 8% of the male population suffer from an inherited form of color vision deficiency (CVD). Age, diabetes, macular degeneration, cataracts and glaucoma result in eye defects including an acquired form of CVD. Inherited CVD is marked by a difficulty in discerning red from green, while acquired CVD is marked by a difficulty in discerning blue from green. A recent review of the cartographic literature revealed a deficit in studies on accessible maps for readers with the acquired form of CVD. In addition, research on accessible maps for readers with the inherited form of CVD was restricted to the design or pre-publication stage. An approach is needed to render maps already in circulation accessible to an audience with CVD. The purpose of this research is to improve the accessibility of maps post-publication. Image re-coloring is a method of altering an image’s color composition in such a way as to make it accessible to a color vision deficient audience. An innovative algorithm is presented that produces a re-colored map that can be perceived by individuals with red-green (inherited) CVD, blue-green CVD (acquired) and normal color vision alike.
The algorithm was tested on a control group of participants with normal color vision and a case group of participants with impaired color vision through a series of matching, content and personal preference questions about six pairs of maps. Each map pair represented one of the following color schemes: balance, diverging, qualitative area, qualitative dot, sequential polychrome, and two variable. Each map pair is composed of two renditions: a map using a color palette that is potentially confusing to viewers with impaired color vision (original rendition) and a map where the original color palette has been re-colored by the algorithm (re-colored rendition). According to the results of a Wilcoxon signed-rank test, the performance of the case group improved when using the re-colored renditions compared to when using the original renditions while the performance of the control group was the same for both renditions. A Mann-Whitney rank sum test revealed that while the scores of the case group were lower than the control group when using the original renditions, they were the same when using the re-colored renditions. A binomial test revealed that subjects in the case group displayed a preference towards all the re-colored renditions while subjects in the control group displayed a preference to two of the six original renditions.
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Preface

All of the work presented henceforth was approved by the Herbert H. Lehman College (CUNY) HRPP Office [586203-2].

This dissertation is original, independent work by the author, G.M. Culp. All images included in this dissertation were produced by the author unless otherwise specified.

A previous version of the re-coloring algorithm described in Chapter 3 has been published (Culp, 2012). I was the lead investigator, responsible for all major areas of concept formation, data collection and analysis, as well as manuscript composition.
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Chapter 1. Introduction

Arthur Robinson, notable American cartographer and creator of the Robinson projection, wrote that “the eye-brain mechanism is constantly processing whatever appears on the retina...What makes visual sense to our brains is not necessarily what makes geographical sense. It is up to the cartographer to try to bring the two together (Robinson, 1978, p. 6).” But what if a map does not make visual sense? How, then, could said map ever expect to make geographic sense? Approximately 4% of the population (8% of the male population) suffer from inherited color vision deficiency (CVD) (Judd, 1943; Alpern et al., 1983a,b; Brettel et al., 1997). Age, diabetes, macular degeneration, cataracts and glaucoma result in acquired CVD (Krastel and Moreland, 1991; Chioran et al., 1985; Roy et al., 1986; Bresnick et al., 1985; Hardy et al., 1992; Verriest, 1964; Rockett et al., 1987; Pacheco-Cutillas et al., 1999). A study on Korean website usability for disabled users found that low vision and senior citizen participants frequently made errors based on lack of color recognition (Choi et al., 2008).

Data from the 2014 National Diabetes Statistics Report show 9.3% (29.1 million) of the United States population have diabetes. Approximately 11.2 million of these cases are among persons age 65 years or older. On average, 1.7 million new cases of diabetes are diagnosed in people aged 20 years and older annually (CDC, 2014). Age-related macular degeneration, cataracts, diabetic retinopathy and glaucoma affect around 2.1 million, 24.4 million, 7.7 million and 2.7 million individuals aged 40 years and older in the United States, respectively (NEI, 2015). While damage from cataracts is typically repairable, glaucoma and macular
degeneration cause irreversible vision loss. Results of the 2010 Census show that the age group 60 to 64 years experienced the largest percent increase (55.6%) with the second largest increase found in the age group 55 to 59 years (46.0%). In addition, the median age reached a new high of 37.2 years (Howden and Meyer, 2011). In order to better serve this large and rapidly increasing population of visually impaired individuals, information graphics such as maps must be reevaluated.
1.1 Research Question and Hypotheses

This study presents the following research question: *Is the proposed re-coloring algorithm a valid approach to improving map accessibility for viewers with impaired color vision while not having a detrimental impact on viewers with normal color vision?* To answer this question, an experiment was conducted on two groups of human subjects: a normal color vision group (control group) and a color vision deficient (CVD) group (case group). The groups evaluated maps with potentially confusing color schemes (original renditions) and their re-colored counterparts (re-colored renditions). Using the results of this experiment, three hypotheses were tested.

1. The re-colored renditions have a positive impact on CVD individuals. Specifically, the case group performed better when using the re-colored maps ($CVD_{RC}$) than when using the original maps ($CVD_{OR}$):

$$H_1 : CVD_{OR} < CVD_{RC}$$

2. The re-colored renditions have no impact on individuals with normal color vision. Specifically, the control group’s performance when using the original maps ($NORMAL_{OR}$) was the same as when using the re-colored maps ($NORMAL_{RC}$):

$$H_1 : NORMAL_{OR} = NORMAL_{RC}$$

3. The positive impact of the re-colored renditions on CVD individuals is such that their performance is equal to that of individuals with normal color vision. Specifically, when using the re-colored maps, case group performance is equal to that of the control group:

$$H_1 : CVD_{RC} = NORMAL_{RC}$$
1.2 Background

1.2.1 The Eye

A brief description of the eye (see Figure 1.1) is helpful in further understanding how maps are perceived, particularly by map readers that have sustained ocular injury. Light initially passes through the cornea, the transparent anterior part of the eye. The iris, located behind the cornea, consists of pigmented tissue and serves as an aperture which regulates light intake. Following the iris, the lens converges light onto a membrane of nerve cells referred to as the retina. Age and cataracts result in yellowing and clouding of the lens. Macular degeneration is the destruction of the cells of the central portion of the retina referred to as the macula. Diabetes retinopathy is a common side effect of diabetes caused by hemorrhages in the retinal arteries resulting in damage to, and in severe cases detachment of, the retina (Garg, 2014). The retina is composed of two types of photoreceptor cells which receive light and convert it into signals. The rod cells are highly sensitive, responsible for vision at low light levels (night), and absent in the fovea, a small pit near the center of the macula. The cone cells are less sensitive, control vision at higher light levels (day), and are present in higher concentrations in the fovea (see Figure 1.2). All humans are temporarily color blind when there is insufficient light as rods are the only photoreceptors that can function in lowlight conditions.

According to the Young-Helmholtz three-component theory (also referred to as the trichromatic theory or tristimulus model), normal color vision is achieved through three spectral classes of cone photoreceptors with different spectral sensitivities: long wave ($\lambda_{\text{max}} \approx 560$ nm), medium wave ($\lambda_{\text{max}} \approx 530$ nm) and short wave ($\lambda_{\text{max}} \approx 420$ nm) (Baylor et al., 1987; Schnapf et al., 1987) (see Figure 1.4a). In other words, every color can be represented using...
just three primaries. The cone receptors’ response is proportional to the logarithm of the intensity of the stimulus. There is great variability among the spectra of individual long and medium wave cones. In comparison to long and medium wave cones, short wave cones are present in much lower numbers, constituting approximately 10% of all cones, and are absent from the center of the fovea (Ruddock, 1991) (see Figure 1.3). Retinal ganglia cells then transmit these signals to the optic nerve, a cable consisting of axons or nerve fibers located in the rear of the eyeball. The optic nerve then disseminates these signals to the brain. Glaucoma causes damage to the optic nerve resulting in loss of retinal nerve cells (Garg, 2014).

The lateral geniculate nucleus (LGN) is a section of the brain which processes input from the cones. There are three classes of spectrally opponent cells within the LGN: one achromatic (white-black) and two chromatic (red-green and yellow-blue). The red/green class may be better described as a color between red and magenta and a color between cyan and green (Conway, 2009). Opponent cells were first theorized by Hering (1964) and explain mutually exclusive colors. We can perceive reddish yellow (orange) and blueish red (purple) but not yellowish blue or reddish green. The combination of the trichromatic theory at the photoreceptor level and the opponent-color theory at the signal process level is known as the two-stage model (von Kries, 1905). Ingling and Tsou (1977) developed a transformation (see Figure 1.4b) for mapping cone responses (see Figure 1.4a) to an opponent-color space (see Figure 1.4c). The variable $V_\lambda$ represents the luminance channel $WS$, and the variables $r - g$ and $y - b$ represent the red/magenta – cyan/green and yellow – blue opponent chromatic channels, respectively.
Figure 1.1. Diagram of the human eye.

Figure 1.2. Distribution of rod and cone photoreceptors across the horizontal meridian of the human retina (Curcio et al., 1990).

Figure 1.3. Illustration of a foveal cone mosaic for an individual with normal color vision.
Figure 1.4. Two-stage model of human color vision.

(a) The trichromatic stage occurs in the retina and is composed of the three types of cone photoreceptors ($L$, $M$ and $S$). This graph depicts human cone spectral sensitivities for an individual with normal color vision (Stiles and Burch, 1959; Stockman et al., 1999; Stockman and Sharpe, 2000).

![Graph showing human cone spectral sensitivities]

(b) The transformation from trichromatic stage to opponent stage occurs between the ganglion cells in the retina and the cerebral cortex (Ingling and Tsou, 1977).

$$\begin{bmatrix} V_{\lambda} \\ y - b \\ r - g \end{bmatrix} = \begin{bmatrix} 0.600 & 0.400 & 0.000 \\ 0.240 & 0.105 & -0.700 \\ 1.200 & -1.600 & 0.400 \end{bmatrix} \begin{bmatrix} L \\ M \\ S \end{bmatrix}$$

(c) The opponent stage is composed of an achromatic channel ($V_{\lambda}$) which represents lightness and the red/magenta – cyan/green ($r - g$) and yellow – blue ($y - b$) chromatic channels which represent color. The chromatic channels are mutually exclusive. We cannot perceive yellowish blue or reddish green, however, we can perceive reddish yellow (orange) and blueish red (purple).
1.2.2 Color Vision Deficiencies

I found that persons in general distinguish six kinds of color in the solar image, namely, red, orange, yellow, green, blue, and purple. I see only two, or at most three, distinctions. These I should call yellow and blue, or yellow, blue, and purple. My yellow comprehends the red, orange, yellow, and green of others; and my blue and purple coincide with theirs.

- John Dalton (1798)

The term 'color blind' is frequently misused. True color blindness is to view the world in black, white and gray. For a compelling account of this extremely rare condition known as monochromatism or achromatism, refer to Oliver Sacks (1996) bestseller *The Island of the Colorblind*. Incomplete achromatism results in difficulties distinguishing desaturated colors (Tränkner, 2008). More commonly, the classification 'color blind' is mistakenly used to describe an individual with color vision deficiency (CVD) or Daltonism, named in honor of the English atomic theorist John Dalton who was afflicted with this vision disorder. CVD is a reduced form of color vision. Recall that color vision is initiated in the retina through three types of cone photoreceptors. The severity of CVD is related to the magnitude in shift of a malfunctioning cone type. Individuals with normal color vision are referred to as trichromats and possess three fully functioning cones types. Anomalous trichomats have full use of two cone types and partial use of one cone type. Dichromats have full use of two cone types and lack use of one cone type. In their study on CVD as an occupational handicap, Seward and Cole (1989) found that 25% of the anomalous trichomat participants had no prior knowledge of their impaired color vision compared to only 5% of the dichromat participants. The malfunctioning cone type determines the CVD type: protans or 'red blind' individuals have impaired long wave (L) cones, deutans or 'green blind' individuals have impaired middle
wave \((M)\) cones, and tritans or 'blue blind' individuals have impaired short wave \((S)\) cones. Referring to figure 1.5, a partial displacement of \(L\) cones, for example, indicates protanomaly while full overlap with \(M\) cones indicates protanopia (Ruddock, 1991).

Under conditions such as indirect vision, insufficient size, insufficient brightness, insufficient time, and chromatic fatigue, anomalous trichomats make the same errors as dichromats (Judd, 1948). In fact, as stimulus size is reduced, color perception in individuals with normal color vision is also reduced. As blue/green discrimination deteriorates more rapidly than
red/green, this effect is referred to as small-field tritanopia. For a comparison of CVD types, refer to Table 1.1.

What does the world look like through a dichromats eyes? Although there are several other approaches which simulate dichromatic vision, Brettel et al. (1997); Viénot et al. (1999); Viénot and Brettel (2001) color appearance model (BVM) is the method most often used in color vision research and is therefore utilized in all CVD simulations in this dissertation (see Figure 1.6 and Appendix A). The model makes the following assumptions:  a) because neutral colors are perceived identically by both normal and dichromatic viewers, the simulation must leave neutral colors unchanged; b) based on research on unilateral dichromats, a stimulus of 575 nm is perceived as yellow and a stimulus of 475 nm as blue by both normal and red/green type dichromatic viewers; and c) based on research on unilateral acquired tritanopia, the corresponding two hues for a tritanope are red (λ = 660 nm) and cyan (λ = 485 nm). LMS color space is used as it decorrelates colors into cone responses. The BVM algorithm simulates only dichromatic vision. Machado et al. (2009) developed an algorithm that simulates anomalous trichomacy at various levels of severity. The simulation process is not reciprocal, it is impossible to simulate a normal color experience for color vision deficient individuals.

In sum, dichromats lack functionality in one class of cone photopigments. Essentially, normal vision is three dimensional while CVD vision is two dimensional (see Figure 1.7a). Individuals with red–green type CVD (i.e., protanopes and deutanopes) perceive the world using the yellow/blue chromatic channel and the achromatic channel (see Figure 1.7b). Individuals with blue–green type CVD (i.e., tritanopes) perceive the world using the red/cyan chromatic channel and the achromatic channel (see Figure 1.7c).
Figure 1.6. Simulation of red-green types (protan and deutan) and blue-green type (tritan) dichromatic vision.

(a) Normal vision.
(b) Protan vision.
(c) Deutan vision.
(d) Tritan vision.
Figure 1.7. The colors an individual can perceive are referred to as a gamut. The normal color vision gamut’s three dimensions are represented by the achromatic channel and both chromatic channels. The dichromatic vision gamut’s two dimensions are represented by the achromatic channel and a single chromatic channel, the yellow/blue channel in the case of red-green CVD and the red/cyan channel in the case of blue-green CVD.

(a) Normal color vision gamut.

(b) Red/green type (protan and deutan) CVD vision gamut.

(c) Blue/green type (tritan) CVD vision gamut.
1.2.2.1 Inherited Color Vision Deficiencies

Congenital color vision deficiency is caused by an inherited trait. The bulk of this group displays a reduced ability in varying degrees to discriminate between reds and greens. Approximately 8% of the Caucasian male population falls within this category. These deficiencies are inherited through a sex-linked recessive gene carried on the X chromosome. The probability of a color vision deficiency in the male population is 0.01 for protanomaly, protanopia and deuteranopia, and 0.05 for deuteranomaly. Roughly 15% of women are carriers of X-linked color vision deficiencies. There is substantial evidence that color vision in heterozygous women is characterized by abnormal spectral sensitivity although color matching is unaffected (Ruddock, 1991). Subtle differences have been detected in their color perception including a slight reduction of red sensitivity in protan carriers (Schmidt, 1934) and impairment of chromatic discrimination along a red-green axis in deutan carriers (Hood et al., 2006). A carrier of two protan deficiencies (protanomaly and protanopia) will likely exhibit the milder form (i.e. protanomaly). The same is true for deutan color vision deficiencies (Piantanida, 1991).

Tritanopia is thought to be inherited as an autosomal (non sex-linked) dominant condition with complete penetrance (all individuals carrying the mutation possess symptoms of the disorder). In other words, the gene for tritanopia is equally likely to be transmitted either gender of offspring, and only one copy of the gene is necessary for emergence of the condition. Due to its fluctuating manifestation, the same gene can produce variable degrees of tritanopia which may account for the accounts of incomplete tritanopia (Piantanida, 1991). The presence of a reduced number of normal blue sensitive cones in the retina of the majority of tritanopes is indicative that the defect is likely due to the absence of the blue-
sensitive response mechanism. Rather conservative figures of congenital tritanopia incidence are estimated at 1 in 15,000 to 1 in 50,000 (Ruddock, 1991).

Optical literature recognizes up to seven varieties of congenital achromatic (monochromatic) vision phenotypes. Typical and atypical complete achromatopsia are thought to be inherited as autosomal recessive conditions. The sex-ratios are similar, often affecting multiple siblings yet lacking prior familial history of achromatopsia.

Color vision is advantageous. The high incidence of dichromatism, however, has led to speculation that there could be a compensating advantage to this vision abnormality. Research has shown that deutanopes and protanopes are capable of detecting objects camouflaged with red-green variation between elements. Color is an important means of dividing an image into regions and can also interfere with segregation through texture. Dichromats lack of color discrimination makes them less susceptible to such interference (Morgan et al., 1992).

1.2.2.2 Acquired Color Vision Deficiencies

There are several notable differences between acquired and congenital CVD (summarized in Table 1.2). Changes in color vision competency over time are indicative of an acquired impairment (Krastel and Moreland, 1991). While inherited CVD is bilateral in nature, acquired CVD is often found in just one eye (Ruddock, 1991). Color vision in both eyes is often tested simultaneously therefore it is likely that most unilateral defects are overlooked (Judd, 1948). A genetic history of inherited color deficiency does not exclude the possibility of an acquired one. Congenital color vision deficiency in females is infrequent compared to occurrence in males (0.4% to 8%, respectively) thus the detection of a color deficiency in a female patient is suggestive of a potential underlying disease. In other words, the frequency
of an acquired color deficiency is not related to gender but to the incidence of the causative disorder (Krastel and Moreland, 1991).

Acquired tritanopia, often referred to as classical tritanopia, was once thought to be the only form. This assumption is quite reasonable as inherited tritanopia is extremely rare, affecting only 1 in 13,000 to 65,000 people. The rarity of inherited tritan deficiencies (affecting less than 0.01% the population) (Birch, 1993) is in marked contrast to the blue-green CVD typically associated with ophthalmic diseases, age and diabetes. Alpern et al. (1983a) studied a subject with a dichromatic left eye and normal trichromatic right eye who had acquired unilateral tritanopia as a result of central serous chorio-retinopathy. Measurements of the left-eye distimulus color-matching functions, spectral luminosity, and wave-length discrimination functions were indistinguishable from those of congenital tritanopes. Thus, all tritanopes, both classical and congenital, are physiologically the same (Alpern et al., 1983a). Certain acquired deficiencies are more difficult to classify as they may possess characteristics of multiple types of congenital deficiency and are therefore often described as red-green CVD rather than distinctly protan or deutan. While acquired red-green color vision confusion resulting from disease or exposure to toxic chemicals can be distinguished from deuteranopia mainly by its poorer light-dark discrimination, acquired tritanopia can be identified by a subject's memory of former yellow and blue sensations (Judd, 1943).

Short-wavelength absorbance markedly increases with age. The formation of cataracts escalates this process, particularly in nuclear yellowish and brownish cataracts (Krastel and Moreland, 1991). Yellow-blue thresholds are more affected by age than red-green thresholds (Chioran et al., 1985).

Color discrimination is abnormal in diabetic patients with either background retinopathy
or proliferative retinopathy and maculopathy (Roy et al., 1986; Bresnick et al., 1985). Overt vascular damage (diabetic retinopathy) was assumed to be the cause of color discrimination loss. Hardy et al. (1992) assessed color vision function in 38 non complicated type 1 diabetic patients between the age of between 16 and 40 years and 36 age-matched, non diabetic controls. All subjects were healthy and taking no medication except insulin. Despite no evidence of either macro or micro vascular disease, color discrimination was abnormal in 57% of the diabetic subjects. These findings imply that color discrimination may be abnormal in uncomplicated type 1 diabetic patients before the onset of retinopathy and may be of a non-vascular aetiology. The cause of color discrimination loss, whether the result of changes in blood glucose derangement or other metabolic pathways, is as yet unknown (Hardy et al., 1992). Blue-green color matching tests aid in the detection of diabetes. Individuals with normal color vision that are suffering from diabetes for more than a few years are exceptional (Verriest, 1964). Rockett et al. (1987) found that diabetic patients with and without demonstrable retinopathy have an heightened incidence of acquired blue-green type CVD of which patients that received laser therapy encounter more frequent severe forms. The authors note that the color-reagent test strips for home glucose monitoring require differentiation of blues and yellows and suggest home blood glucose monitoring machines that do not require accurate color perception for glucose level determination as an alternative. Bresnick et al. (1984) observed poor performance in urine sugar testing which employ color-coded strips. Some of the subjects were ignorant of their color vision difficulties, a discovery which underscores the necessity for tests that do not rely solely upon color vision.

Diseases of the optical nerve, mainly glaucoma, are primarily associated with blue/green defects. A type III (tritan type) defect is the most frequent chromatic anomaly associated
with primary open angle glaucoma (POAG). Acquired red/green CVD accompanied by loss of visual acuity is usually associated with advanced stages of glaucoma. Investigation of color mechanism may allow for detection of POAG prior to the occurrence of extensive and irreversible neuronal damage. Standard clinical color vision tests, including pseudo-isochromatic plates and arrangement tests, can identify patients with advanced glaucoma. Unfortunately, these tests lack the sensitivity and specificity to screen for earlier stages of the disease (Pacheco-Cutillas et al., 1999). Most individuals in the early stages of macular diseases, where visual acuity is well preserved, will have type III (blue-green or tritan-like) defects (Pacheco-Cutillas et al., 1999).

Tritan-like deficiencies can be caused by various pharmaceuticals including anticonvulsant drugs (Bayer et al., 1990) and oral contraceptives (Marre et al., 1974). Prolonged direct ocular exposure to bright sun has been shown to result in acquire tritanopia (Abramov and Gordon, personal communication, Dec. 7, 2011).
Table 1.1. Comparison of CVD by type. Adapted from Judd (1943); Alpern et al. (1983a,b); Brettel et al. (1997).

<table>
<thead>
<tr>
<th>Attribute</th>
<th>CVD Type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Protan</td>
</tr>
<tr>
<td>Visible Hues</td>
<td>yellow blue</td>
</tr>
<tr>
<td>Discrimination Losses</td>
<td>545-650 nm</td>
</tr>
<tr>
<td>Convergence Point</td>
<td>$x = 0.7465$</td>
</tr>
<tr>
<td>Neutral Point</td>
<td>492 nm</td>
</tr>
<tr>
<td>Monochromatic Stimuli</td>
<td>475 nm</td>
</tr>
<tr>
<td>Gender</td>
<td>mostly males</td>
</tr>
<tr>
<td>Percent of Population</td>
<td>2.4%</td>
</tr>
</tbody>
</table>

Table 1.2. Comparison of CVD by source. Adapted from King–Smith (1991).

<table>
<thead>
<tr>
<th>Diagnostic</th>
<th>CVD Source</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Acquired</td>
</tr>
<tr>
<td>Common Deficiency Type</td>
<td>blue–green</td>
</tr>
<tr>
<td>Eyes Affected</td>
<td>unilateral (one eye)</td>
</tr>
<tr>
<td>Onset</td>
<td>occurs after birth</td>
</tr>
<tr>
<td>Classification</td>
<td>may be difficult</td>
</tr>
<tr>
<td>Color Vision Test Results</td>
<td>changes over time</td>
</tr>
<tr>
<td>Deficiency Type and Severity</td>
<td>fluctuates</td>
</tr>
<tr>
<td>Gender</td>
<td>equal among genders</td>
</tr>
<tr>
<td>Cause</td>
<td>abnormality in visual pathway (from retina to brain)</td>
</tr>
<tr>
<td>Location</td>
<td>peripheral region</td>
</tr>
<tr>
<td>Visual Acuity</td>
<td>often reduced</td>
</tr>
</tbody>
</table>
1.2.3 Color Vision Tests

The majority of acquired deficiencies involve loss of blue perception thus tests which can detect tritan deficiencies must be included (Birch, 1991). Large normal variations in threshold blue perception (arising from individual differences in densities of the yellow pigments in the macula and lens as well as cone pigment spectra) make it difficult to design efficient screening tests to detect slight tritan deficiency. In addition, only certain tests can detect tritan deficiencies.

Printed pseudoisochromatic plates are the most broadly employed screening tests for abnormal color vision. The majority of these tests consist solely of ‘vanishing figures’ as these plates are the easiest to design effectively. The Ishihara pseudoisochromatic plates enable diagnosis of red-green CVD types (i.e., protan versus deutan) as well as source (i.e., congenital versus acquired) but lack the ability to detect tritan defects (see Figure 1.8). The Hardy-Rand-Rittler plates pseudoisochromatic plates test for severity of CVD and are able to detect tritan defects (see Figure 1.9). One major drawback to using these tests is that many individuals, particularly the elderly, are unable to form characters from colored dots. Subjects with normal color vision have been known to fail pseudoisochromatic plate tests due to this cognitive deficit (Abramov and Gordon, personal communication, Dec. 7, 2011).

The Farnsworth panel test was developed as a means of vocational screening. The goal of the test is to divide subjects into two groups: a) those with normal color vision or slight CVD; and b) those with moderate or severe CVD (Birch, 1991). This test employs a series of 15 colored caps which subjects arrange in hue order starting from the violet side of the spectrum. Lanthony (1978) developed a panel test to identify acquired color vision defects. As discussed in the previous section, the effect of acquired CVD are often more
mild than congenital CVD and thus more difficult to detect. The Lanthony test is similar to the Farnsworth test in that it involves the arrangement of 15 color caps. The Lanthony test, however, utilizes desaturated colors making it more difficult than the Farnsworth test. The Farnsworth and Lanthony tests are commonly referred to as the D-15 test and the D-15DS test, respectively (see Figure 1.10). Used in tandem, the D-15 and D-15DS tests are an effective method of identifying a range of color vision deficiencies (Birch, 1991). Subjects first complete the D-15 test. If the subject fails the D-15 test, diagnosis of CVD type and severity is attempted. If the subject passes the D-15 test, the D-15DS test is then administered and the results are assessed (Atchison et al., 1991). Panel tests are appealing in that they can be administered to a wide audience, including children, the elderly and non English speakers, with fairly simple instruction (Abramov and Gordon, personal communication, Dec. 7, 2011).

The original method of scoring the D-15 and D-15DS tests was through visual interpretation of results plotted on a hue circle (see Figure 1.11a). Minor errors involving neighboring caps are quite common, even among subjects with normal color vision, and are referred to as transpositions (see Figure 1.11b). Major errors result in lines that cross the hue circle in a diametric fashion where two or more such crossings constitute a failure (see Figures 1.11c-1.11i). The number of crossings indicates severity, the angle of the crossings indicates type and the intersection of crossings indicates scatter.

Several quantitative methods have been developed to standardize the scoring of panel tests. Bowman (1982) summed the the color differences between adjacent caps to produce a single measure of error, the total color difference score (TCDS). Vingrys and King-Smith (1988) developed the color difference vector (CDV) technique which uses averaging. Foutch
et al. (2011) developed the least squares regression (LSR) technique which uses linear regression. Both the CDV and LSR techniques produce three measures:  

1) the angle which indicates type;  
2) the confusion index (CI) which indicates severity;  
3) the selectivity index (SI) which indicates specificity. The CDV technique is more specific to congenital defects and less specific to random or acquired defects while the opposite holds true for the LSR technique. The TCDS and the CDV CI measures are highly correlated (Atchison et al., 1991) as are the CDV CI and the LSR CI measures (Foutch et al., 2011).
Figure 1.8. Ishihara vanishing digit pseudoisochromatic plate (Ishihara, 1972). Normal color vision observers should see a “6” while deutan observers do not see a number.

(a) Normal vision.  

(b) Deutan vision.

Figure 1.9. Hardy-Rand-Rittler vanishing design pseudoisochromatic plate (Cole et al., 2006). Normal color vision observers should see a triangle and a circle while deutan observers do not see any shapes.

(a) Normal vision.  

(b) Deutan vision.
Figure 1.10. Color vision panel tests.

(a) Farnsworth D-15 test.  
(b) Lanthony D-15DS test.

Figure 1.11. Typical D-15 cap arrangements for various types of color vision (Birch, 1991; Vingrys and King-Smith, 1988).

(a) Normal, perfect score.  
(b) Normal, minor error.  
(c) Protan, mild.  
(d) Protan, severe.  
(e) Deutan, mild.  
(f) Deutan, severe.  
(g) Tritan, mild.  
(h) Tritan, severe.  
(i) Achromat.
1.2.4 The Role of Color in the Visualization of Information

The application of color to visual displays of information is a contentious topic. In the introduction of *Interaction of Color*, the German-born artist and color theorist Josef Albers (2006, p. 1) wrote: “In order to use color effectively it is necessary to recognize that color deceives continually.” Famed French cartographer Jacques Bertin (1981, p. 147) warned against “the false aestheticism sought in color.” He felt that “color is not indispensable” as differences in texture or pattern can effectively represent a component (Bertin, 2011, p. 90). American statistician and data visualization critic Edward Tufte (2001, p. 110), however, refers to cross-hatching as a form of “vibrating chartjunk” and suggests replacing patterns with shades of gray. As an aside, a monochromatic map appears the same to a user with CVD as it does to one with normal color vision. American critical cartographer Denis Wood (1992) finds fault with self-evident color assignment such as depicting water using shades of blue. Wood notes that water is not truly blue and has historically been symbolized in red, black, white, brown, pink and green. American cartographer Cynthia Brewer (2005) advises map makers to take care with literal uses of color by avoiding superficial and exaggerated emphasis on color associations. She suggests using a purposely abstract set of easily distinguished hues, particularly where certain colors may have unintended meanings for the subject being mapped. American engineer Willard Brinton (1939, p. 14), an innovator in the field of information graphics, felt, however, that “the question is not ‘Can one afford to use color?’ but ‘Can one afford to omit color?’” The use of primary colors by British mathematician Oliver Byrne (1847) to symbolize points, lines and angles in his *The Elements of Euclid* was a revolutionary and successful method of information visualization. Geometry students were able to grasp concepts more rapidly with Byrnes vibrant diagrams than with traditional
black and white figures (Tufte, 1990). Byrne (1847, p. xiii) does advise that “care must be taken to show that color has nothing to do with the lines, angles, or magnitudes, except merely to name them.”
Chapter 2. Literature Review

2.1 Previous Cartographic Research on Color Vision

Color is often representative. It can stand for quantitative and qualitative variables. Color is frequently used to guide attention. It can establish order on a graphic and assist the user in recognizing differences. Color can be a gauge of map scale. Color schemes are arrangements of color groups in which hue and lightness logically relate to the data being represented.

Olson and Brewer (1997) feel that although the obvious working hypothesis is that the general findings about color vision will transfer to maps, this assumption may be problematic. Publications dealing with colors found to be confusing to individuals with CVD are based on stimuli other than maps. Map color selection is not standardized. A map has a virtually infinite number of potential color combinations. There is a need to test the potential extremes of difficulty that face map readers with CVD. Olson and Brewer (1997) comment that although there has historically been much interest in the map user and the psychology of map reading, research involving color has been relatively limited. Improved technical capacity on the part of academic cartographers may be one reason for the changing interest in color. Psychological and physiological studies on color vision are fundamental to research on color vision in map reading.

Olson and Brewer (1997) researched the affects of red-green type CVD on map reading performance and proposed color schemes to accommodate individuals with red-green type CVD. Red-green CVD confusion lines were utilized to produce color schemes for their study. Color schemes for the “confusing” renditions were selected to lie along a red-green CVD.
confusion line. Color schemes for the “accommodating” renditions were selected to lie across the red-green confusion lines. This technique is described in greater detail in Section 3.3. Sixty-four subjects, thirty-two with red-green CVD and thirty-two with normal color vision, were asked content and matching questions about an “accommodating” rendition and a “confusing” rendition of each of seven color schemes totaling fourteen maps in all. Subjects were then asked which of the two renditions of each color scheme was easier or better to use. The map survey was administered on a computer in a testing room. An initial practice test familiarized subjects with the format. The last choice for every content and matching question, “not answerable”, rapidly indicates a subject’s inability to distinguish colors while avoiding prolonged and frustrating indecision. In order to avoid map sequence or question rendition biasing map comparison, two test versions were produced which differed by the sequence in which the map renditions and corresponding questions were presented. Each test version was administered to half of the subjects. Finally, an Ishihara test was used to determine the subjects color vision status. This test was in the form of a published paper booklet and was administered in a room illuminated by natural or simulated daylight.

Due to the dichotomous nature of the accuracy results (right or wrong responses), Olson and Brewer (1997) opted for a logit analysis where the dependent variable is binary. The model was fit by varying combinations of categorical independent variables and ranking them using the Bayesian information content (BIC) statistic. Two analyses were run, one for content question responses and one for matching question responses. Potential independent variables included: vision group (i.e., normal, red-green impaired), rendition (i.e., confusing, accommodating), map color scheme (i.e., qualitative area, qualitative dot, sequential monochrome, sequential polychrome, diverging, balance, two variable) and test map order.
The best model to fit both content and matching questions was one in which response accuracy was estimated using color vision group, rendition, color scheme, and interaction between vision and rendition.

Reaction time is a quantitative measure that was analyzed only for correct responses using an analysis of variance (ANOVA). Despite shorter reaction times when using accommodating maps (as opposed to confusing maps), subjects with red-green impairments took longer than subjects with normal color vision. Olson and Brewer (1997) found that subjects with CVD are often so accustomed to difficulty interpreting colors that they ceased to trust their first impressions thus color discrimination tasks take longer for them than for subjects with normal color vision. Normal color vision subjects displayed slightly longer reaction times when working with the accommodating maps. The authors suspect that the restricted color space and confined selection of contrasting colors might be the cause behind this trend. As predicted, a binomial test revealed that subjects with normal vision did not prefer one map rendition over the other. Subjects with red-green impairments selected the accommodating rendition more than 50% of the time. Accommodating renditions need not be used for map readers with normal color vision but should be made available as an alternative for users with CVD. Olson and Brewer (1997) feel that slight improvements over their color selections restore or even improve the performance of the normal color vision group as well as accommodate the color vision impaired thus improving overall population performance.

2.2 Cartographic Color Scheme Design Software

Several computerized tools have been developed to assist in the production of accommodating maps. Color Oracle (Jenny and Kelso, 2007a,b) is a piece of software that uses
the BVM algorithm (Brettel et al., 1997; Viénot et al., 1999; Viénot and Brettel, 2001) to simulate how a users’ computer screen would appear to individuals with protan, deutan, and tritan forms of dichromacy.

ColorBrewer is a web-based application developed to assist in the selection, evaluation, and implementation of effective thematic map color schemes (Brewer, 2003). This tool includes several CVD accessible color schemes that were developed by Olson and Brewer (1997) and were further tested on individuals with red-green CVD by Gardner (2005). ColorBrewer schemes are also available within the D3 javascript library for data visualization and the R language statistical computing environment.

Both ColorBrewer and Color Oracle are not specific to a particular operating system or software package and can thus compliment a variety of desktop mapping applications. Jefferson and Harvey (2007) classify techniques and tools that assist in the design of materials accessible to CVD observers as pre-publication methods.

2.3 Related Image Re-Coloring Research

Image re-coloring is a method of altering an image’s color composition in such a way as to make it accessible to a color vision impaired audience. Individuals with CVD have a reduced color gamut. The objective of re-coloring is to preserve an abundance of visual information within the constraints of this limited color range. Developed by researchers in the field of computer science, these algorithms are typically used to process images ranging from photographs and works of art to computer graphics and even video. Image re-coloring occurs post-publication. Surprisingly, the application of re-coloring algorithms to maps is absent from the cartographic literature. Twenty-five re-coloring algorithms are broken down
into four categories (i.e., color contrast enhancing, gamut re-mapping, daltonization, and conversion to grayscale) and briefly described below. Algorithms in the first three categories produce separate images for red-green and blue-green deficiencies. Algorithms in the fourth category produce a single image.

2.3.1 Color Contrast Enhancing Algorithms

The most popular re-coloring method works by changing the colors of the original image such that viewers with color vision deficiencies perceive color contrast in portions of the image where they originally were indistinguishable. A drawback to this approach is that the color remapping may vary based on the original images color composition. Some algorithms were designed for anomalous trichromats where CVD is mild. These algorithms are ineffective for dichromats where the degree of CVD is severe. Yang et al.’s (2004) digital item adaptation algorithm compensates confusing colors according to the severity of CVD defined by the user. Oka et al.’s (2009) color compensation method intensifies either the $L$ or $M$ cone axis. Ichikawa et al.’s (2003) algorithm works by enhancing lightness and color difference. Chao et al.’s (2008) algorithm uses Riemann geometry to create isometries between normal and CVD gamuts that preserve color differences.

Other algorithms are intended for red-green CVD observers and lack a method for blue-green color deficiency. Michelson’s (2008) color contraster algorithm intensifies pixel hue by making red pixels redder and green pixels greener as well as altering the pixels blue component by increasing it for greens and decreasing it for reds. Michelson and Yun’s (2008) color corrector algorithm shifts pixel luminosity and chromatic channels based on redness or greenness. Iaccarino et al.’s (2006) color blind filter service determines the proportion of red and green pixels in the original image and alters pixel hue, saturation, and lightness based
on this proportion. Jefferson and Harvey’s (2007) algorithm transfers chromatic information of the defective cone across the two functioning cones. For protanopes, variation is transferred to $M$ and $S$ cones while for deutanopes, information is transferred to $L$ and $S$ cones. Nakauchi and Onouchi’s (2008) algorithm evaluates and modifies confusing color clusters. Troiano et al.’s (2008) algorithm selects a set of key colors from an image and uses Euclidean distance between each pair in the set to create a set of CVD accessible colors which is then interpolated across the remaining colors in the image.

There are also algorithms that can generate separate images for red-green and blue-green CVD. Huang et al.’s (2008) algorithm works by extracting a set of representative colors from an image and re-mapping them in such a way as to maintain contrast between each pair of these representative colors. Bao et al.’s (2008) improved adaptive mapping algorithm uses the images color distribution and the type of color vision deficiency to map colors to areas of the color plane that are better perceived by CVD observers (see Figure 2.1). Images re-colored for red-green dichromacy will have blue added to green pixels and blue subtracted from all other pixels. Images re-colored for blue-green dichromacy will have red added to blue pixels and red subtracted from all other pixels. Wakita and Shimamura’s (2005) SmartColor uses simulated annealing to re-color images according to the user’s specifications while maintaining contrast.
**Figure 2.1.** Application of improved adaptive mapping color contrast enhancing algorithm (Bao et al., 2008).

(a) Original map, normal vision.

(b) Re-colored for red-green CVD, normal vision.

(c) Re-colored for blue-green CVD, normal vision.

(d) Re-colored for red-green CVD, protan vision.

(e) Re-colored for red-green CVD, deutan vision.

(f) Re-colored for blue-green CVD, tritan vision.
2.3.2 Gamut Re-Mapping Algorithms

A second means of re-coloring an image involves re-plotting the image within the dichromat’s color subspace or gamut. The re-colored images appear similar to both the dichromat that the image was re-colored for and individuals with normal vision. A drawback to this approach is that the color re-mapping may vary based on the original image’s color composition. Rasche et al.’s (2005a) algorithm uses multi-dimensional scaling to collapse the image’s three-dimensional gamut into a dichromat’s two-dimensional gamut. Ma et al.’s (2009) algorithm uses self-organizing mapping to build a two-dimensional gamut of colors present in the original image and then maps these colors to the corresponding position on the dichromat’s two-dimensional gamut. Machado and Oliveira’s (2010) algorithm projects the original colors onto a plane aligned with the direction that maximizes contrast loss and then rotates the plane to align with the color coordinates of the dichromat gamut. Kuhn et al.’s (2008a) algorithm enhances image contrast through mass spring optimization, allowing users to select a number of quantized colors and opt for an exaggerated color contrast setting for re-coloring non-natural images such as scientific and information visualizations (see Figure 2.2). Milić et al. (2015) uses a content-dependent approach to remap confusing colors perpendicular to their confusion line.
Figure 2.2. Application of mass spring optimization gamut re-mapping algorithm (Kuhn et al., 2008a).

(a) Original map, normal vision.

(b) Re-colored for protan, normal vision.

(c) Re-colored for deutan, normal vision.

(d) Re-colored for tritan, normal vision.

(e) Re-colored for protan, protan vision.

(f) Re-colored for deutan, deutan vision.

(g) Re-colored for tritan, tritan vision.
2.3.3 Daltonization Algorithms

A third technique is daltonization in which the error (the difference between the normal and dichromat perception of a color) is adjusted and added back to the original color. This process is named after John Dalton, a dichromatic chemist who was the first to research color vision deficiencies. By altering only the colors that are confusing to dichromats, daltonization algorithms preserve colors that are discernible. Dougherty and Wade’s (2002) Vischeck and Fidaner et al.’s (2005) algorithm both use daltonization to re-color images for observers with red-green type deficiencies (see Figure 2.3). Doliotis et al.’s (2009) intelligent daltonization method begins with an initial modification of the error followed by subsequent scaled modifications until the image is distinguishable to dichromats.
Figure 2.3. Application of Daltonization algorithm (Fidaner et al., 2005).

(a) Original map, normal vision.

(b) Re-colored for protan, normal vision.

(c) Re-colored for deutan, normal vision.

(d) Re-colored for tritan, normal vision.

(e) Re-colored for protan, protan vision.

(f) Re-colored for deutan, deutan vision.

(g) Re-colored for tritan, tritan vision.
2.3.4 Grayscale Conversion Algorithms

A fourth re-coloring approach is conversion to grayscale. Grayscale conversion is easily accomplished in image editing software where there are two common approaches. The first approach, luma encoding, involves weighting RGB channel values but the resulting gray values may appear darker or lighter than the original colors (see Figure 2.4b). The second approach uses the $L^*$ channel of the CIELAB colorspace which closely matches human perception of lightness (see Figure 2.4c). Some loss of visual information is unavoidable during the grayscale conversion process due to the loss of chromatic dimensions. More sophisticated conversion algorithms enhance contrast by setting the perceived gray difference proportional to the perceived hue difference between color pairs. Gooch et al.’s (2005) Color2Gray algorithm works by converting the color image to a perceptually uniform color space and then calculating luminance and chrominance differences among neighboring pixels. These variations as well as user-defined controls are incorporated into an optimization problem that determines the grayscale setting. Rasche et al.’s (2005b) method uses a system of constraints to maintain luminous consistency and preserve contrast resulting in an image where perceived gray differences are proportional to perceived color differences between any pair of colors in the original image. Two other algorithms produce grayscale images with globally consistent color reassignment while retaining the original images gray values. Grundland and Dodgson’s (2007) Decolorize algorithm assigns gray values by sampling color differences by Gaussian pairing and analyzing these differences through predominant component analysis (see Figure 2.4d). Kuhn et al.’s (2008) algorithm uses a mass-spring system to perform a constrained optimization on the luminance values of a set of quantized colors obtained from the original image to produce a set of gray values that are then interpolated across all pixels.
Figure 2.4. Application of Grayscale Algorithms.

(a) Original image, normal vision.

(b) Luma encoding, all vision types.

(c) CIE 1976 $L^*$ lightness channel, all vision types.

(d) Decolorize grayscale algorithm (Grundland and Dodgson, 2007), all vision types.
Chapter 3. Methodology

3.1 OGA Re-Coloring Algorithm Development

The re-coloring methodology described here produces a single color image that is accessible to red-green and blue-green CVD observers (see Figure 3.1). The algorithm works by re-plotting the image within a gamut that is perceptible to protanopes, deutanopes and tritanopes. This gamut was constructed by combining the region of the red-green CVD gamut that is perceptible to blue-green CVD observers (blue region) and the region of the blue-green CVD gamut that is perceptible to red-green CVD observers (red region). Protanopes perceive reds (hue: 0°) as desaturated and blues (hue: 240°) as saturated. Conversely, tritanopes perceive blues (hue: 240°) as desaturated and reds (hue: 0°) as saturated. This disparity in saturation was addressed by rotating the red and blue regions until a more uniform saturation was achieved. The resulting reddish-orange (hue: 25°), azure blue (hue: 205°) and monochromatic gamut is shown in Figure 3.2. For lack of a better name and to avoid confusion with the myriad of other re-coloring algorithms, this algorithm will be henceforth referred to as the Orange-Gray-Azure (OGA) re-coloring algorithm. Reddish and bluish colors are plotted in their respective regions of the gamut. Greenish colors are converted to grayscale through desaturation. Because the algorithm works on a pixel-wise basis, color re-assignment is universal (pixels of the same color are always assigned the same new color).

The OGA re-coloring algorithm preserves the lightness of the original color by utilizing two opponent color spaces. Opponent spaces are color models consisting of an achromatic channel of black/white and primary colors channels of yellow/blue and magenta-red/green-
cyan. Recall that the second stage in the two-step model of human color vision is the opponent stage (refer to Figure 1.4). Why are two different opponent spaces necessary? Color spaces are essentially projections of color. Like a geographic projection, they have advantages and disadvantages. The CIELAB opponent space’s achromatic channel $L^*$ is based on human perception and thus has a very accurate measurement of lightness (see Figure 2.4c). The CIELAB chromatic channels, however, present a problem in that they are nonlinear making scaling problematic (see Figure 3.3a). The opponent space oRGB (Bratkova et al., 2009) has two linear chromatic channels, $C'_1$ (which is positive for yellow and negative for blue) and $C'_2$ (which is positive for magenta-red and negative for green-cyan), which have a range of -1.00 to 1.00 (see Figure 3.3b). These attributes make the oRGB space easy to scale. The oRGB achromatic channel $L'$, however, is based on luma encoding which may result in scaled colors appearing darker or lighter than the original color (see Figure 2.4b). To continue with the geographic projection analogy, CIE 1976 space is to an area preserving projection as oRGB space is to a scale preserving projection. In order to utilize the advantageous attributes of both color spaces, the OGA re-coloring algorithm scales the oRGB achromatic channel by the CIELAB achromatic channel.

Color modifications, particularly ones carried out in color spaces other than RGB, can result in colors located outside the RGB gamut. There are several approaches to remap a color back within the gamut. The method used here was simple linear scaling as recommended by Bratkova et al. (2009). Figure 3.4 provides a detailed explanation of how the algorithm works. For additional information on conversion between color spaces, refer to Appendix A.
Figure 3.1. Figure 1.6 map re-colored by the OGA algorithm with simulation of dichromatic vision.

(a) Normal vision.

(b) Protan vision.

(c) Deutan vision.

(d) Tritan vision.
Figure 3.2. The OGA re-coloring algorithm gamut was constructed by combining the blue region of the red-green CVD gamut with the red region of the blue-green CVD gamut. These regions were then rotated until they appeared uniformly saturated to both red-green and blue-green CVD types resulting in a reddish-orange, azure blue, and monochromatic gamut.

(a) Red-green CVD gamut.  
(b) Blue-green CVD gamut.  
(c) OGA algorithm gamut.

Figure 3.3. The CIELAB and oRGB opponent color spaces are used by the OGA algorithm to preserve lightness. The CIELAB has an accurate achromatic channel but its chromatic channels are nonlinear making scaling problematic. The oRGB chromatic channels are linear and thus easy to scale but its achromatic channel is not as accurate as that of CIELAB.

(a) CIELAB $(a^*, b^*)$.
(b) oRGB $(C'_1, C'_2)$.
The RGB ($R_\gamma, G_\gamma, B_\gamma$) channels of the pixel from the original map image are obtained. The original RGB channels are then converted to both CIELAB ($L^*_{\gamma}, a^*_\gamma, b^*_\gamma$) and oRGB ($L'_{\gamma}, C'_{1\gamma}, C'_{2\gamma}$) opponent color spaces:

$$\text{RGB}(R_\gamma, G_\gamma, B_\gamma) \rightarrow CIELAB(L^*_{\gamma}, a^*_\gamma, b^*_\gamma) \quad (3.1a)$$
$$\text{RGB}(R_\gamma, G_\gamma, B_\gamma) \rightarrow oRGB(L'_{\gamma}, C'_{1\gamma}, C'_{2\gamma}) \quad (3.1b)$$

The original color is mapped onto the algorithm gamut:

$$R_\alpha = \min(0, R_\gamma - 0.5G_\gamma) \quad (3.1c)$$
$$B_\alpha = \min(0, B_\gamma - 0.5G_\gamma) \quad (3.1d)$$
$$G_\alpha = 0.42R_\alpha + 0.58B_\alpha \quad (3.1e)$$

The following steps are necessary to retain the lightness of the original image. The mapped color is converted from RGB ($R_\alpha, G_\alpha, B_\alpha$) to oRGB ($L'_\alpha, C'_{1\alpha}, C'_{2\alpha}$) color space:

$$\text{RGB}(R_\alpha, G_\alpha, B_\alpha) \rightarrow oRGB(L'_\alpha, C'_{1\alpha}, C'_{2\alpha}) \quad (3.1f)$$

In order to obtain the intermediate lightness channel, the oRGB luma from the original color ($L'_\gamma$) and the chromatic channels from the remapped color ($C'_{1\alpha}, C'_{2\alpha}$) are converted to CIELAB ($L^*_\beta, a^*_\beta, b^*_\beta$):

$$oRGB(L'_\gamma, C'_{1\alpha}, C'_{2\alpha}) \rightarrow CIELAB(L^*_\beta, a^*_\beta, b^*_\beta) \quad (3.1g)$$

The original CIELAB lightness channel ($L^*_{\gamma}$) is divided by the intermediate CIELAB lightness channel ($L^*_\beta$) and used to scale the original colors oRGB luma ($L'_\gamma$):

$$L'_\gamma = L'_\gamma \frac{L^*_{\gamma}}{L^*_\beta} \quad (3.1h)$$

Using the scaled oRGB luma channel ($L'_\gamma$) and the remapped chromaticity channels ($C'_{1\alpha}, C'_{2\alpha}$), the final color is converted from oRGB to RGB ($R_\gamma, G_\gamma, B_\gamma$):

$$oRGB(L'_\gamma, C'_{1\alpha}, C'_{2\alpha}) \rightarrow RGB(R_\gamma, G_\gamma, B_\gamma) \quad (3.1i)$$
3.2 Map Production

This study uses methods based on those established by Olson and Brewer (1997) where participants evaluate six pairs of maps and one practice map. Each map pair is composed of a map that uses a color scheme that is potentially confusing to viewers with CVD (original rendition) and a map that is re-colored in a manner to accommodate viewers with CVD (re-colored rendition). In order to avoid biasing map comparison, map rendition order for each pair is randomly assigned. Refer to Appendix B for screenshots of the maps as well as simulations of how they are perceived by color vision impaired viewers. All maps were produced using the JavaScript visualization library D3.js (Bostock and Davies, 2013). The following is a list of the map schemes produced for this study:

- **Practice Map**: *Sequential monochrome* color schemes represent a sequence of quantitative values using differing lightness and saturation of a single hue.

- **Maps 1 – 2**: *Diverging* color schemes highlight a single variable’s extremes where the critical mid or zero point is represented by a neutral or transitional hue.

- **Maps 3 – 4**: *Qualitative dot* color schemes use variations in hue to represent categorical variables at the point level.

- **Maps 5 – 6**: *Two variable* color schemes visualize two variables at once by overlaying the representative hue of one variable onto the hue of another variable.

- **Maps 7 – 8**: *Sequential polychrome* color schemes represent a sequence of quantitative values using ordered hue and lightness variation.
• Maps 9 – 10: *Balance* color schemes visualize two complementary quantitative variables with two hues which blend into a mixture at the mid-point.

• Maps 11 – 12: *Qualitative area* color schemes use variations in hue to represent categorical variables at the polygon level.

### 3.3 Color Scheme Selection

In order to test the re-coloring algorithm’s capacity, color schemes that are confusing to the red-green and blue-green CVD types but still understandable to normal viewers were constructed. This study employed methodology developed by Olson and Brewer (1997) involving confusion lines plotted in the CIE 1931 xyY chromaticity diagram (see Figure 3.5). The CIE 1931 xyY color space represents colors using three parameters. The \( x \) and \( y \) parameters represent the chromaticity (hue) of a color while the \( Y \) parameter represents luminance (brightness) of a color. Confusion lines converge at a point specific to CVD type (see Table 1.1 for convergence point coordinates). Colors that fall along a confusion line are perceived as indistinguishable when of similar luminance while colors that straddle confusion lines are perceived as distinguishable. The CIE 1931 xyY coordinates of the colors selected for this study are represented graphically and numerically in Appendix C.

A color schemes minimum CIEDE2000 color difference (\( \Delta E_{00\text{min}} \)) was used to verify that the original color scheme would be confusing to CVD subjects while the re-colored color scheme would be accommodating. CIEDE2000 color difference (\( \Delta E_{00} \)) is a means of quantifying the difference or perceptual distance between two colors. As \( \Delta E_{00} \) is a perceptual measure, it is influenced by an individual’s color vision. \( \Delta E_{00\text{min}} \) represents the weakest point in a color scheme (the two colors that are perceptually closest). In order to determine
ΔE_{00min} for each color scheme for each vision type, ΔE_{00} was calculated among all potential color pair combinations and the minimum value was taken. When determining ΔE_{00min} for CVD types, dichromatic vision was simulated using the BVM color appearance model (Brettel et al., 1997; Viénot et al., 1999; Viénot and Brettel, 2001). As mentioned in the preceding paragraph, the original color schemes should be indistinguishable to CVD viewers but understandable to viewers with normal color vision. As the goal of a re-coloring algorithm is to make a confusing image more distinguishable to color vision impaired viewers, the re-colored map renditions should have higher ΔE_{00min} values than the original map renditions for the three CVD types. Normal color vision viewers, however, may experience a decrease in ΔE_{00min} from original to re-colored map renditions. (Olson and Brewer, 1997) note that since adjustment of colors means working with a more restricted gamut, there is less of an opportunity for selecting highly contrasting colors that would appeal to viewers with normal color vision. They suggest that accommodating renditions of maps, while an improvement for color vision impaired viewers, need not be used for normal color vision viewers.

Figure 3.6a provides a visual comparison of ΔE_{00min} values for both the original and re-colored renditions subdivided by map scheme and color vision type. The three CVD types have larger ΔE_{00min} values for the re-colored renditions compared to those of the original renditions. The inverse is true for normal color vision. When comparing the accommodating/re-colored renditions of the two methods (see Figure 3.6b), Olson and Brewer’s (1997) method has higher ΔE_{00min} values for the diverging and qualitative dot map schemes while the OGA algorithm has higher ΔE_{00min} values for the balance, qualitative area, sequential polychrome and two variable map schemes. The minimum CIEDE2000 color difference (ΔE_{00min}) values used to produce these figures can be found in Appendix C Table C.8.
Figure 3.5. CIE 1931 chromaticity diagram with confusion lines and simulation of dichromatic vision.

(a) Normal vision.  
(b) Protan vision.  
(c) Deutan vision.  
(d) Tritan vision.
Figure 3.6. Slopegraphs of minimum CIEDE2000 color difference $\Delta E_{00\text{min}}$.

(a) Comparison of original and OGA re-colored renditions.

(b) Comparison of Olson and Brewer (1997) accommodating and OGA re-colored renditions.
3.4 Questionnaire Design

Questionnaires, often used in the social sciences, allow researchers to garner a more in-depth understanding of a participant's experiences and behavior (Suchan and Brewer, 2000). Olson and Brewer (1997) utilized this tool in their groundwork laying inquiry into map reading and color vision. This project will draw from their research model in which participants evaluate map renditions by answering three types of questions. Matching questions involve identifying an element in the legend which corresponds to a selected map feature. Content questions require the participant to interpret the map by identifying a spatial trend in the underlying data. Preference questions determine which map rendition participants find most accommodating. Data on participants’ map reading abilities, factors that affect color vision, and demographic information were gathered prior to the map component of the survey. Refer to the Appendix D for a complete list of the questions as well as the correct answers.

3.5 Participant Selection

The selection of qualified participants is an important in that it enables researchers to gain knowledge of a rich and complex segment of the population. Eighty-four adults (persons of 18 years of age and older) participated. Participants were recruited through the color blindness subreddit, an online forum for both individuals with CVD as well as those with normal vision that are interested in the topic (Reddit, 2016). Participants’ color vision status was determined using secure online versions of the Farnsworth D-15 and Lanthony D-15DS arrangement tests which were scored using methods developed by Vingrys and King-Smith (1988) and Foutch et al. (2011) (see Section 1.2.3) using the CVD package in R (Gama et al., 2015). The control group is composed of 40 individuals with normal color vision (passed
both D-15 and D-15DS tests). The case group is composed of 40 individuals that suffer from inherited or acquired color vision confusion (failed either the D-15 or D-15DS test). Four individuals with incomplete achromatic vision were removed from the study as the algorithm was not designed to address the needs of this color vision impairment. Both the control and case groups were from similar age, gender and map use cohorts (see Table 3.1).

While online color vision tests provide an anonymous and convenient means of assessing one’s color vision, there may be a question as to their accuracy. In a clinical situation, the D-15 and D-15DS panel tests are administered in a controlled environment with consistent lighting. If the tests are administered on a computer, the monitor is professionally color calibrated. In order to ensure some level of accuracy in the online color vision tests employed in this research, an informal experiment was conducted. Several of the author’s colleagues were given the physical panel tests in a controlled environment and then asked to complete the online version of the panel tests on their home computers. The results of both test formats were identical in all cases for both normal and CVD individuals.

3.6 Data Collection

Olson and Brewer (1997) used a standardized environment to insure consistent lighting and color calibration. Participants in Kröger et al.’s (2013) study on accommodating color schemes for OpenStreetMap, however, took completed their surveys “under usual working conditions” ... in other words, from their home computer via the Internet. This approach allows participants anonymity and access to a familiar computing environment. In addition, the pool of potential participants is much larger as geographic proximity is no longer a limiting factor. In fact, Olson and Brewer (1997) recruited from Michigan State University
Table 3.1. Summary of participant background information.

<table>
<thead>
<tr>
<th></th>
<th>Normal Vision (Control Group)</th>
<th>CVD (Case Group)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Age (in years)</td>
<td>18-40</td>
<td>33</td>
</tr>
<tr>
<td></td>
<td>41-60</td>
<td>35</td>
</tr>
<tr>
<td>Map use</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Once a week</td>
<td>30</td>
<td>29</td>
</tr>
<tr>
<td>Once a month</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>Rarely</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>Gender</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Female</td>
<td>11</td>
<td>3</td>
</tr>
<tr>
<td>Male</td>
<td>29</td>
<td>37</td>
</tr>
<tr>
<td>Ever had CVD test</td>
<td>Yes</td>
<td>32</td>
</tr>
<tr>
<td></td>
<td>No</td>
<td>8</td>
</tr>
<tr>
<td></td>
<td>35</td>
<td>5</td>
</tr>
<tr>
<td>Color vision type</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(according to D–15 and</td>
<td>Normal</td>
<td>40</td>
</tr>
<tr>
<td>D–15DS panel tests)</td>
<td>Deutan</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Protan</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>Tritan</td>
<td>0</td>
</tr>
</tbody>
</table>

and found that their pool of CVD participants showed less experience with maps than that of their normal color vision participants. In addition, the authors pointed out that the conditions of their study were “artificial”. In light of Olson and Brewer’s (1997) and Kröger et al.’s (2013) findings, the survey for this research was administered via the Internet.

The survey for this research was hosted on a secure website. Upon completion of online versions of the Farnsworth D-15 and Lanthony D-15DS color vision tests, participants were asked a series of questions about their age, gender and map reading experience. Participants then answered matching, content, and preference questions (described in section 3.4) about the thirteen maps (see Appendix B). Participants’ responses to the survey questions were saved in a Google Sheet stored on a secure Google Drive. Participation was completely anonymous. Participants provided their initials to give informed consent and name or any other identifying attributes were not collected.
3.7 Data Analysis

Figure 3.7 presents participants’ responses to content and matching questions on six map schemes in box plot form. The distribution of the data is highly skewed with a pronounced ceiling effect and outliers are present. Due to these factors, non parametric analyses based on ranking were selected. These tests do not require a normal distribution and are much less sensitive to outliers than other tests such as linear models and t-tests.

This research presents three hypotheses. The first hypothesis is that the CVD group (case group) performed better when using the re-colored maps than when using the original maps. The second hypothesis is that the normal color vision group’s (control group) performance while using the original maps was the same as when using the re-colored maps. These two hypotheses were evaluated for each map scheme using a Wilcoxon signed-rank test, a non parametric test which compares two matched groups (Wilcoxon, 1945). P-values are determined using the Pratt method where zero-differences (tied values across groups) are included in the ranking process. The Wilcoxon test statistic $W$ is the proportion of pairs where one group has a higher rank than the other group and is calculated by dividing a group’s sum of ranks by the total sum of ranks ($n(n + 1)/2$). $W_{RC}$ indicates the probability that the score of the re-colored rendition will be higher than that of the original rendition. The reverse is true for $W_{OR}$. Due to the presence of tied values across groups, $W_{RC}$ and $W_{OR}$ do not always sum to 1.00. $W_{RC}$ and $W_{OR}$ can be interpreted as the probability of an outcome. The rank-biserial correlation is a directional effect size that ranges from -1.00 (indicative of an extremely strong negative correlation) to 1.00 (indicative of an extremely strong positive correlation). The rank-biserial correlation for the Wilcoxon signed-rank tests
(represented by the notation \( r_W \)) expresses the influence of the re-colored map renditions on performance by group and was computed by taking the difference between the test statistics \( (r_W = W_{RC} - W_{OR}) \) (Kerby, 2014). The Wilcoxon signed-rank test was executed using the stats (R Core Team, 2015) and coin (Hothorn et al., 2015) packages in R.

The third hypothesis is that when using the re-colored maps, the performance of the CVD group (case group) is equal to that of the normal color vision group (control group). This hypothesis was evaluated for each map scheme using a Mann-Whitney rank sum test, a non parametric test which compares two independent groups (Mann and Whitney, 1947). The Mann-Whitney test statistic \( U \) is the proportion of pairs where one group has a higher rank than the other group and is calculated by dividing a group’s sum of ranks by the total sum of ranks \( (n_1n_2) \). \( U_{\text{case}} \) indicates the probability that the scores of the case group will be higher than those of the control. The reverse is true for \( U_{\text{ctrl}} \). Tied values across groups are divided between the groups thus \( U_{\text{case}} \) and \( U_{\text{ctrl}} \) sum to 1.00. \( U_{\text{case}} \) and \( U_{\text{ctrl}} \) can be interpreted as the odds or probability of an outcome. The rank-biserial correlation for the Mann-Whitney rank sum tests (represented by the notation \( r_U \)) expresses the influence of CVD on performance by map rendition and was computed by taking the difference between the test statistics \( (r_U = U_{\text{case}} - U_{\text{ctrl}}) \) (Kerby, 2014). The Mann-Whitney rank sum test was executed using the stats (R Core Team, 2015) and coin (Hothorn et al., 2015) packages in R.

Statistical significance tests indicate the likelihood that an effect or relationship is due to chance where the p-value is the basis for rejection of the null hypothesis. When using a non parametric method to investigate multiple measurements taken on the same individual are not independent, an increase in the number of hypotheses tested results in an increase
in the chance of wrongly rejecting the null hypothesis (Dunn, 1959). This effect can be countered using the Holm-Bonferroni method where hypotheses are rejected sequentially until no further rejections are possible (Holm, 1979). P-values generated by the Wilcoxon signed rank and Mann-Whitney rank sum tests were Holm-Bonferroni corrected using the stats package in R (R Core Team, 2015).

Preference responses were analyzed according to map scheme and participant group using a binomial test from the stats (R Core Team, 2015) package in R.
Figure 3.7. Box plots of questionnaire scores illustrating the data’s highly skewed distribution, pronounced ceiling effect, and presence of outliers.
Chapter 4. Results

4.1 Wilcoxon Signed Rank Tests

The results of the Wilcoxon signed-rank tests are presented by individual map scheme in Tables 4.6 through 4.4 and aggregated over all map schemes in Tables 4.1. These results are further broken down by group, map rendition and question type. All questions refers to the aggregation over all question types. All schemes refers to the aggregation over all map schemes. When referring to a map scheme and question type combination (for example, matching questions about the qualitative dot map scheme), the following format is used: qualitative dot/matching.

4.1.1 Summary Statistics

A visual inspection of case group summary statistics reveals that mean ($\bar{x}$) and mean rank ($\bar{R}$) values are greater and standard deviation ($s$) values are lesser for re-colored rendition scores compared to those of original rendition scores. Median ($Md$) values among the case group were typically a perfect score of 100 for both original and re-colored renditions except for all schemes/all questions, all schemes/content, all schemes/matching, qualitative dot/all questions, qualitative dot/content, and two variable/all questions combinations. Control group summary statistics do not vary greatly between renditions.

4.1.2 Significance Testing

An examination of the Holm–Bonferroni corrected p-values at a significance level of $\alpha = 0.05$ verifies both the first and second hypotheses. Among the case group, original rendition scores differ significantly from re-colored rendition scores for all map scheme/question type combinations. Among the control group, however, original rendition scores did not differ
significantly from re-colored rendition scores for any map scheme/question type combination.

4.1.3 Effect Size

The rank-biserial correlation effect size $r_W$ for all map scheme/question type combinations are presented graphically in Figure 4.1 where interpretation threshold values were obtained from Maher et al. (2013). $r_W$ expresses the influence of the re-colored map renditions on performance by group. $r_W$ ranges from -1.00 (indicative of an extremely strong negative correlation) to 1.00 (indicative of an extremely strong positive correlation). The case group and control group are symbolized in orange and blue, respectively.

4.1.3.1 Control Group Comparisons

The $r_W$ for all control group map scheme/question type combinations show a negligible effect ($-0.1 \leq r_W \leq 0.1$).

4.1.3.2 Case Group Comparisons

Among all case group map scheme/question type combinations, $r_W$ is positive. The $r_W$ for the all schemes/all questions and all schemes/content combinations show a very large positive effect ($r_W > 0.7$). The $r_W$ for the all schemes/matching combination shows a large positive effect ($0.5 < r_W \leq 0.7$). The $r_W$ for the qualitative dot/all questions and qualitative dot/content combinations show a medium positive effect ($0.3 < r_W \leq 0.5$). The $r_W$ for the balance/content, diverging/content, diverging/matching, qualitative area/matching, sequential polychrome/content and sequential polychrome/matching combinations show a negligible positive effect ($r_W \leq 0.1$). The $r_W$ for the remaining combinations show a small positive effect ($0.1 < r_W \leq 0.3$).
Table 4.1. Wilcoxon signed-rank test, maps 1 – 12: all schemes.

<table>
<thead>
<tr>
<th>Question Type</th>
<th>Group</th>
<th>Map Rendition</th>
<th>Summary Statistics</th>
<th>Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\bar{x}$ $Md$ $s$ $R$</td>
<td>$p - value$ $W_{OR} W_{RC}$ $r_W$</td>
</tr>
<tr>
<td>All</td>
<td>Control</td>
<td>Original</td>
<td>96.47 100 6.4 41.7</td>
<td>1 0.11 0.1 -0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>96.42 98.5 5.87 39.3</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>75.9 80 17.41 25.4</td>
<td>&lt; 0.0001* 0.01 0.9 0.89 (+XL)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>95.5 98.5 7.3 55.6</td>
<td></td>
</tr>
<tr>
<td>Content</td>
<td>Control</td>
<td>Original</td>
<td>93.38 100 12.4 41.11</td>
<td>1 0.04 0.07 0.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>93.83 100 10.45 39.89</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>71.75 75 19.9 24.9</td>
<td>&lt; 0.0001* 0.01 0.85 0.84 (+XL)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>94.6 100 11.29 56.1</td>
<td></td>
</tr>
<tr>
<td>Matching</td>
<td>Control</td>
<td>Original</td>
<td>98.4 100 4.1 41.55</td>
<td>1 0.06 0.04 -0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>98 100 4.15 39.45</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>78.58 83 17.01 27.6</td>
<td>&lt; 0.0001* 0.02 0.71 0.69 (+L)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>95.9 100 5.92 53.4</td>
<td></td>
</tr>
</tbody>
</table>

*p - value significant at $\alpha = 0.05$.

†descriptive interpretation thresholds of effect size from Maher et al. (2013).

Table 4.2. Wilcoxon signed-rank test, maps 1 – 2: diverging.

<table>
<thead>
<tr>
<th>Question Type</th>
<th>Group</th>
<th>Map Rendition</th>
<th>Summary Statistics</th>
<th>Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\bar{x}$ $Md$ $s$ $R$</td>
<td>$p - value$ $W_{OR} W_{RC}$ $r_W$</td>
</tr>
<tr>
<td>All</td>
<td>Control</td>
<td>Original</td>
<td>98.5 100 5.33 40.5</td>
<td>NA 0.01 0.01 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>98.5 100 5.33 40.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>82 100 24.31 34.02</td>
<td>0.0113* 0.02 0.17 0.15 (+S)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>95.5 100 16 46.98</td>
<td></td>
</tr>
<tr>
<td>Content</td>
<td>Control</td>
<td>Original</td>
<td>97.5 100 11.04 41</td>
<td>1 0.01 0.01 0</td>
</tr>
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<td></td>
<td>Re-Colored</td>
<td>96.25 100 13.34 40</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>81.25 100 29.28 35.11</td>
<td>0.0352* 0.02 0.11 0.09</td>
</tr>
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<td></td>
<td></td>
<td>Re-Colored</td>
<td>96.25 100 17.5 45.89</td>
<td></td>
</tr>
<tr>
<td>Matching</td>
<td>Control</td>
<td>Original</td>
<td>99.17 100 5.22 40</td>
<td>1 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>100 100 0 41</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>82.5 100 26.21 35.5</td>
<td>0.0352* 0.01 0.07 0.06</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>95 100 16.14 45.5</td>
<td></td>
</tr>
</tbody>
</table>

*p - value significant at $\alpha = 0.05$.

†descriptive interpretation thresholds of effect size from Maher et al. (2013).
Table 4.3. Wilcoxon signed-rank test, maps 3 – 4: qualitative dot.

<table>
<thead>
<tr>
<th>Question Type</th>
<th>Group</th>
<th>Map Rendition</th>
<th>Summary Statistics</th>
<th>Test Statistics</th>
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<tr>
<td></td>
<td></td>
<td></td>
<td>( \bar{x} ) ( M_d ) ( s ) ( \bar{R} )</td>
<td>( p-value ) ( W_{OR} ) ( W_{RC} ) ( r_W )</td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>Original</td>
<td>98 100 9.92 39.99</td>
<td>1 0 0.01 0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>99.5 100 3.16 41.01</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>67 80 30.23 29.35</td>
<td>&lt;0.0001* 0 0.43 0.43 (+M†)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>93.5 100 17.77 51.65</td>
<td></td>
</tr>
<tr>
<td>Content</td>
<td>Control</td>
<td>Original</td>
<td>97.5 100 15.81 40.49</td>
<td>1 0 0 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>98.75 100 7.91 40.51</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>60 50 37.89 30.4</td>
<td>&lt;0.0001* 0.01 0.35 0.34 (+M†)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>93.75 100 20.22 50.6</td>
<td></td>
</tr>
<tr>
<td>Matching</td>
<td>Control</td>
<td>Original</td>
<td>98.35 100 7.28 39.5</td>
<td>1 0 0 0</td>
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<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>100 100 0 41.5</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>71.6 100 32.64 33.3</td>
<td>0.0002* 0 0.19 0.19 (+S†)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>93.33 100 18.85 47.7</td>
<td></td>
</tr>
</tbody>
</table>

\( *p-value \) significant at \( \alpha = 0.05 \).

†descriptive interpretation thresholds of effect size from Maher et al. (2013).

Table 4.4. Wilcoxon signed-rank test, maps 5 – 6: two variable.

<table>
<thead>
<tr>
<th>Question Type</th>
<th>Group</th>
<th>Map Rendition</th>
<th>Summary Statistics</th>
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<td></td>
<td>( \bar{x} ) ( M_d ) ( s ) ( \bar{R} )</td>
<td>( p-value ) ( W_{OR} ) ( W_{RC} ) ( r_W )</td>
</tr>
<tr>
<td></td>
<td>All</td>
<td>Original</td>
<td>88 100 23.88 40.35</td>
<td>1 0.08 0.08 0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>87.5 100 22.55 40.65</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>73 80 31.23 32.46</td>
<td>0.0008* 0.02 0.29 0.27 (+S†)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>93 100 14 48.54</td>
<td></td>
</tr>
<tr>
<td>Content</td>
<td>Control</td>
<td>Original</td>
<td>80 100 35.45 38.77</td>
<td>1 0.01 0.05 0.04</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>87.5 100 27.15 42.23</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>63.75 100 42.35 32.6</td>
<td>0.0016* 0.01 0.18 0.17 (+S†)</td>
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<td></td>
<td>Re-Colored</td>
<td>93.75 100 23.17 48.4</td>
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</tr>
<tr>
<td>Matching</td>
<td>Control</td>
<td>Original</td>
<td>93.35 100 22.88 42.9</td>
<td>1 0.05 0.02 -0.03</td>
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<td>Re-Colored</td>
<td>88.35 100 24.54 38.1</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>79.2 100 31.77 36.45</td>
<td>0.0352* 0.03 0.16 0.13 (+S†)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>92.55 100 15.93 44.55</td>
<td></td>
</tr>
</tbody>
</table>

\( *p-value \) significant at \( \alpha = 0.05 \).

†descriptive interpretation thresholds of effect size from Maher et al. (2013).
Table 4.5. Wilcoxon signed-rank test, maps 7 – 8: sequential polychrome.

<table>
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<th>Summary Statistics</th>
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<td>All</td>
<td>Control</td>
<td>Original</td>
<td>99.5</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>98.5</td>
<td>100</td>
<td>5.33</td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>83.5</td>
<td>100</td>
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<tr>
<td></td>
<td>Re-Colored</td>
<td>98.5</td>
<td>100</td>
<td>5.33</td>
</tr>
<tr>
<td>Content</td>
<td>Control</td>
<td>Original</td>
<td>98.75</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>96.25</td>
<td>100</td>
<td>13.34</td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>97.5</td>
<td>100</td>
<td>11.04</td>
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<td>Matching</td>
<td>Control</td>
<td>Original</td>
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<td>100</td>
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<td></td>
<td>Re-Colored</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
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<td></td>
<td>Case</td>
<td>Original</td>
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<td>100</td>
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<tr>
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<td>Re-Colored</td>
<td>99.17</td>
<td>100</td>
<td>5.22</td>
</tr>
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</table>

*p-value significant at $\alpha = 0.05$.  
†descriptive interpretation thresholds of effect size from Maher et al. (2013).

Table 4.6. Wilcoxon signed-rank test, maps 9 – 10: balance.

<table>
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<td></td>
<td>$\bar{x}$</td>
<td>Md</td>
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<tr>
<td>All</td>
<td>Control</td>
<td>Original</td>
<td>98.5</td>
<td>100</td>
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<tr>
<td></td>
<td>Re-Colored</td>
<td>99.5</td>
<td>100</td>
<td>3.16</td>
</tr>
<tr>
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<td>Case</td>
<td>Original</td>
<td>73.5</td>
<td>100</td>
</tr>
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<td>Re-Colored</td>
<td>98.5</td>
<td>100</td>
<td>7</td>
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<tr>
<td>Content</td>
<td>Control</td>
<td>Original</td>
<td>96.25</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>98.75</td>
<td>100</td>
<td>7.91</td>
</tr>
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<td>Case</td>
<td>Original</td>
<td>75</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>97.5</td>
<td>100</td>
<td>15.81</td>
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<td>Control</td>
<td>Original</td>
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<td>100</td>
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<td>Re-Colored</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
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<td></td>
<td>Case</td>
<td>Original</td>
<td>72.42</td>
<td>100</td>
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<tr>
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<td>Re-Colored</td>
<td>99.17</td>
<td>100</td>
<td>5.22</td>
</tr>
</tbody>
</table>

*p-value significant at $\alpha = 0.05$.  
†descriptive interpretation thresholds of effect size from Maher et al. (2013).
Table 4.7. Wilcoxon signed-rank test, maps 11 – 12: qualitative area.

<table>
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<td>All</td>
<td>Control</td>
<td>Original</td>
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<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>94</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>76</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>93.5</td>
<td>100</td>
</tr>
<tr>
<td>Content</td>
<td>Control</td>
<td>Original</td>
<td>90</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>85</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>70</td>
<td>100</td>
</tr>
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<td></td>
<td></td>
<td>Re-Colored</td>
<td>88.75</td>
<td>100</td>
</tr>
<tr>
<td>Matching</td>
<td>Control</td>
<td>Original</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Case</td>
<td>Original</td>
<td>79.92</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Re-Colored</td>
<td>96.65</td>
<td>100</td>
</tr>
</tbody>
</table>

\*\( p-value \) significant at \( \alpha = 0.05 \).
\†Descriptive interpretation thresholds of effect size from Maher et al. (2013).
Figure 4.1. Graphical representation of Wilcoxon signed-rank rank-biserial effect size ($r_W$) with descriptive interpretation thresholds from Maher et al. (2013). $r_W$ expresses the influence of the re-colored map renditions on performance by group. The case group and control group are symbolized in orange and blue, respectively.
4.2 Mann-Whitney Rank Sum Test

The results of the Mann-Whitney rank sum tests are presented by individual map scheme in Tables 4.13 through 4.11 and aggregated over all map schemes in Tables 4.8. These results are further broken down by group, map rendition and question type. All questions refers to the aggregation over all question types. All schemes refers to the aggregation over all map schemes. When referring to a map scheme and question type combination (for example, matching questions about the qualitative dot map scheme), the following format is used: qualitative dot/matching.

4.2.1 Summary Statistics

A visual inspection of original rendition summary statistics reveals that mean ($\bar{x}$) and mean rank ($\bar{R}$) values are greater and standard deviation ($s$) values are lesser for control group scores compared to those of case group scores. Median ($Md$) values among the original rendition were typically a perfect score of 100 for both control and case groups except for all schemes/all questions, all schemes/content, all schemes/matching, qualitative dot/all questions, qualitative dot/content, and two variable/all questions combinations. Re-colored rendition summary statistics do not vary greatly between groups.

4.2.2 Significance Testing

Re-colored rendition summary statistics do not vary greatly between groups. An examination of the Holm–Bonferroni corrected p-values at a significance level of $\alpha = 0.05$ verifies the third hypothesis in all cases. Among the re-colored renditions, case group scores did not differ significantly from control group scores for all map scheme and question type comparisons.
4.2.3 Effect Size

The rank-biserial correlation effect size $r_U$ for all map scheme/question type combinations are presented graphically in Figure 4.2 where interpretation threshold values were obtained from Maher et al. (2013). $r_U$ expresses the influence of CVD on performance by rendition (i.e., disparity in performance between groups). $r_U$ ranges from -1.00 (indicative of an extremely strong negative correlation) to 1.00 (indicative of an extremely strong positive correlation). The original rendition and re-colored rendition are symbolized in green and pink, respectively.

4.2.3.1 Original Rendition Comparisons

The $r_U$ for the all schemes/all questions, all schemes/content, and all schemes/matching combinations show a very large negative effect ($r_U < -0.7$). The $r_U$ for the qualitative dot/all questions and qualitative dot/content show large negative effect ($-0.7 \leq r_U < -0.5$). The $r_U$ for the balance/content, diverging/content, sequential polychrome/content, sequential polychrome/matching, two variable/content, and two variable/matching combinations show a small negative effect ($r_U < -0.1$). The $r_U$ for the remaining combinations show medium negative effect ($-0.5 \leq r_U < -0.3$).

4.2.3.2 Re-Colored Rendition Comparisons

The $r_U$ for the all schemes/matching, qualitative dot/all questions, qualitative dot/matching, and diverging/matching combinations show a small negative effect ($r_U < -0.1$). The $r_U$ for the two variable/content combination shows a small positive effect ($r_U > 0.1$). The $r_U$ for the remaining combinations show a negligible effect ($-0.1 \leq r_U \leq 0.1$).
<table>
<thead>
<tr>
<th>Question Type</th>
<th>Map Rendition</th>
<th>Group</th>
<th>Summary Statistics</th>
<th>Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>( \bar{x} ) ( Md ) ( s ) ( \bar{R} )</td>
<td>( p - value ) U_{ctrl} U_{case} ( r_U )</td>
</tr>
<tr>
<td>All</td>
<td>Original</td>
<td>Control</td>
<td>96.47 100 6.4 56.58</td>
<td>&lt; 0.0001* 0.9 0.1 -0.8 (-XL( ^\dagger ))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>96.42 98.5 5.87 41.25</td>
<td>1 0.52 0.48 -0.04</td>
</tr>
<tr>
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<td></td>
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<td>95.5 98.5 7.3 39.75</td>
<td></td>
</tr>
<tr>
<td>Content</td>
<td>Original</td>
<td>Control</td>
<td>93.38 100 12.4 55.14</td>
<td>&lt; 0.0001* 0.87 0.13 -0.74 (-XL( ^\dagger ))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>71.75 75 19.9 25.86</td>
<td></td>
</tr>
<tr>
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<td>Re-Colored</td>
<td>Control</td>
<td>93.83 100 10.45 38.77</td>
<td>1 0.46 0.54 0.08</td>
</tr>
<tr>
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<td></td>
<td>Case</td>
<td>94.6 100 11.29 42.23</td>
<td></td>
</tr>
<tr>
<td>Matching</td>
<td>Original</td>
<td>Control</td>
<td>98.4 100 4.1 55.59</td>
<td>&lt; 0.0001* 0.88 0.12 -0.76 (-XL( ^\dagger ))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>78.58 83 17.01 25.41</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>98 100 4.15 44.26</td>
<td>1 0.59 0.41 -0.18 (-S( ^\dagger ))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>95.9 100 5.92 36.74</td>
<td></td>
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\( ^* \) \( p - value \) significant at \( \alpha = 0.05 \).

\( ^\dagger \) descriptive interpretation thresholds of effect size from Maher et al. (2013).

Table 4.9. Mann-Whitney rank sum test, maps 1 – 2: diverging.

<table>
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<tr>
<th>Question Type</th>
<th>Map Rendition</th>
<th>Group</th>
<th>Summary Statistics</th>
<th>Test Statistics</th>
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<td></td>
<td></td>
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<td>( p - value ) U_{ctrl} U_{case} ( r_U )</td>
</tr>
<tr>
<td>All</td>
<td>Original</td>
<td>Control</td>
<td>98.5 100 5.33 47.95</td>
<td>0.0010* 0.69 0.31 -0.38 (-M( ^\dagger ))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>98.5 100 5.33 41.08</td>
<td>1 0.51 0.49 -0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>95.5 100 16 39.92</td>
<td></td>
</tr>
<tr>
<td>Content</td>
<td>Original</td>
<td>Control</td>
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</tr>
<tr>
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<td></td>
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</tr>
<tr>
<td></td>
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<td>Control</td>
<td>96.25 100 13.34 40.04</td>
<td>1 0.49 0.51 0.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
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<td></td>
</tr>
<tr>
<td>Matching</td>
<td>Original</td>
<td>Control</td>
<td>99.17 100 5.22 47.09</td>
<td>0.0018* 0.66 0.34 -0.32 (-M( ^\dagger ))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>82.5 100 26.21 33.91</td>
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</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>100 100 0 42.5</td>
<td>1 0.55 0.45 -0.1 (-S( ^\dagger ))</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
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\( ^* \) \( p - value \) significant at \( \alpha = 0.05 \).

\( ^\dagger \) descriptive interpretation thresholds of effect size from Maher et al. (2013).
### Table 4.10. Mann-Whitney rank sum test, maps 3 – 4: qualitative dot.

<table>
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<th>Test Statistics</th>
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<td></td>
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<td>$Md$</td>
</tr>
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<td>100</td>
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<td></td>
<td></td>
<td>Case</td>
<td>67</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>99.5</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>93.5</td>
<td>100</td>
</tr>
<tr>
<td>Content</td>
<td>Original</td>
<td>Control</td>
<td>97.5</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>60</td>
<td>50</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>98.75</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>93.75</td>
<td>100</td>
</tr>
<tr>
<td>Matching</td>
<td>Original</td>
<td>Control</td>
<td>98.35</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>71.6</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>93.33</td>
<td>100</td>
</tr>
</tbody>
</table>

* $p-value$ significant at $\alpha = 0.05$.

† Descriptive interpretation thresholds of effect size from Maher et al. (2013).

### Table 4.11. Mann-Whitney rank sum test, maps 5 – 6: two variable.

<table>
<thead>
<tr>
<th>Question Type</th>
<th>Map Rendition</th>
<th>Group</th>
<th>Summary Statistics</th>
<th>Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>$\bar{x}$</td>
<td>$Md$</td>
</tr>
<tr>
<td>All</td>
<td>Original</td>
<td>Control</td>
<td>88</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>73</td>
<td>80</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>88</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>93</td>
<td>100</td>
</tr>
<tr>
<td>Content</td>
<td>Original</td>
<td>Control</td>
<td>80</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>63.75</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>87.5</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>93.75</td>
<td>100</td>
</tr>
<tr>
<td>Matching</td>
<td>Original</td>
<td>Control</td>
<td>93.35</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>79.2</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>88.35</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Case</td>
<td>92.55</td>
<td>100</td>
</tr>
</tbody>
</table>

* $p-value$ significant at $\alpha = 0.05$.

† Descriptive interpretation thresholds of effect size from Maher et al. (2013).
Table 4.12. Mann-Whitney rank sum test, maps 7 – 8: sequential polychrome.

<table>
<thead>
<tr>
<th>Question Type</th>
<th>Map Rendition</th>
<th>Group</th>
<th>Summary Statistics</th>
<th>Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
<td>Control</td>
<td>$\overline{x}$</td>
<td>Md</td>
</tr>
<tr>
<td>All</td>
<td>Original</td>
<td>Case</td>
<td>99.5</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>98.5</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Case</td>
<td>98.5</td>
<td>100</td>
</tr>
<tr>
<td>Content</td>
<td>Original</td>
<td>Control</td>
<td>98.75</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>96.25</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Case</td>
<td>97.5</td>
<td>100</td>
</tr>
<tr>
<td>Matching</td>
<td>Original</td>
<td>Control</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Case</td>
<td>99.17</td>
<td>100</td>
</tr>
</tbody>
</table>

*p - value significant at $\alpha = 0.05$.  
\textsuperscript{†}descriptive interpretation thresholds of effect size from Maher et al. (2013).

Table 4.13. Mann-Whitney rank sum test, maps 9 – 10: balance.

<table>
<thead>
<tr>
<th>Question Type</th>
<th>Map Rendition</th>
<th>Group</th>
<th>Summary Statistics</th>
<th>Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Original</td>
<td>Control</td>
<td>$\overline{x}$</td>
<td>Md</td>
</tr>
<tr>
<td>All</td>
<td>Original</td>
<td>Case</td>
<td>98.5</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>99.5</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Case</td>
<td>98.5</td>
<td>100</td>
</tr>
<tr>
<td>Content</td>
<td>Original</td>
<td>Control</td>
<td>96.25</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>98.75</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Case</td>
<td>97.5</td>
<td>100</td>
</tr>
<tr>
<td>Matching</td>
<td>Original</td>
<td>Control</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Case</td>
<td>99.17</td>
<td>100</td>
</tr>
</tbody>
</table>

*p - value significant at $\alpha = 0.05$.  
\textsuperscript{†}descriptive interpretation thresholds of effect size from Maher et al. (2013).
Table 4.14. Mann-Whitney rank sum test, maps 11 – 12: qualitative area.

<table>
<thead>
<tr>
<th>Question Type</th>
<th>Map Rendition</th>
<th>Group</th>
<th>Summary Statistics</th>
<th>Test Statistics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All</td>
<td>Original CONTROL CASE</td>
<td>$\bar{x}$ 110.28 $Md$ 49.14</td>
<td>$R$ 0.0018* $U_{ctrl}$ 0.69 $U_{case}$ 0.31 $r_U$ -0.38 (-M†)</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control CASE</td>
<td>$\bar{x}$ 94.15 $Md$ 40.11</td>
<td>$R$ 1.049 0.51 0.02</td>
</tr>
<tr>
<td></td>
<td>Content</td>
<td>Original CONTROL CASE</td>
<td>$\bar{x}$ 28.19 $Md$ 46.7</td>
<td>$R$ 0.0165* $U_{ctrl}$ 0.66 $U_{case}$ 0.34 $r_U$ -0.32 (-M†)</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control CASE</td>
<td>$\bar{x}$ 30.38 $Md$ 39.81</td>
<td>$R$ 1.048 0.52 0.04</td>
</tr>
<tr>
<td></td>
<td>Matching</td>
<td>Original CONTROL CASE</td>
<td>$\bar{x}$ 100 0 47</td>
<td>$R$ 0.0010* $U_{ctrl}$ 0.66 $U_{case}$ 0.34 $r_U$ -0.32 (-M†)</td>
</tr>
<tr>
<td></td>
<td>Re-Colored</td>
<td>Control CASE</td>
<td>$\bar{x}$ 100 0 41.5</td>
<td>$R$ 1.052 0.48 -0.04</td>
</tr>
</tbody>
</table>

* $p - value$ significant at $\alpha = 0.05$.
† Descriptive interpretation thresholds of effect size from Maher et al. (2013).
Figure 4.2. Graphical representation of Mann-Whitney rank sum rank-biserial effect size ($r_U$) with descriptive interpretation thresholds from Maher et al. (2013). $r_U$ expresses the influence of CVD on performance by rendition (i.e., disparity in performance between groups). The original rendition and re-colored rendition are symbolized in green and pink, respectively.
4.3 Binomial Analysis of Map Preference

Tables 4.15 presents the results of the binomial analysis to determine if preference question scores differed from 50% at a significance level of $\alpha = 0.05$. Among the case group, all re-colored renditions were significantly favored. Among the control group, the original renditions of the two variable and qualitative dot schemes as well as all original rendition schemes overall were significantly favored.

Table 4.15. Results of binomial analysis on map preference.

<table>
<thead>
<tr>
<th>Map Color Scheme</th>
<th>Group</th>
<th>Original</th>
<th>Re-colored</th>
<th>Sample Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>All</td>
<td>Normal (Control)</td>
<td>59%*</td>
<td>41%</td>
<td>n=240</td>
</tr>
<tr>
<td></td>
<td>CVD (Case)</td>
<td>23%</td>
<td>77%*</td>
<td>n=240</td>
</tr>
<tr>
<td>Balance</td>
<td>Normal (Control)</td>
<td>50%</td>
<td>50%</td>
<td>n=40</td>
</tr>
<tr>
<td></td>
<td>CVD (Case)</td>
<td>10%</td>
<td>90%*</td>
<td>n=40</td>
</tr>
<tr>
<td>Diverging</td>
<td>Normal (Control)</td>
<td>50%</td>
<td>50%</td>
<td>n=40</td>
</tr>
<tr>
<td></td>
<td>CVD (Case)</td>
<td>25%</td>
<td>75%*</td>
<td>n=40</td>
</tr>
<tr>
<td>Qualitative Area</td>
<td>Normal (Control)</td>
<td>60%</td>
<td>40%</td>
<td>n=40</td>
</tr>
<tr>
<td></td>
<td>CVD (Case)</td>
<td>30%</td>
<td>70%*</td>
<td>n=40</td>
</tr>
<tr>
<td>Qualitative Dot</td>
<td>Normal (Control)</td>
<td>75%*</td>
<td>25%</td>
<td>n=40</td>
</tr>
<tr>
<td></td>
<td>CVD (Case)</td>
<td>22%</td>
<td>78%*</td>
<td>n=40</td>
</tr>
<tr>
<td>Sequential Polychrome</td>
<td>Normal (Control)</td>
<td>52%</td>
<td>48%</td>
<td>n=40</td>
</tr>
<tr>
<td></td>
<td>CVD (Case)</td>
<td>35%</td>
<td>65%*</td>
<td>n=40</td>
</tr>
<tr>
<td>Two Variable</td>
<td>Normal (Control)</td>
<td>65%*</td>
<td>35%</td>
<td>n=40</td>
</tr>
<tr>
<td></td>
<td>CVD (Case)</td>
<td>15%</td>
<td>85%*</td>
<td>n=40</td>
</tr>
</tbody>
</table>

* Significantly greater than 50% at $\alpha = 0.05$. 
Chapter 5. Discussion

5.1 Interpretation of Statistical Tests

The OGA algorithm performed well when tested on a control group composed of individuals with normal color vision and a case group composed of individuals with red-green and blue-green CVD. The results of the Wilcoxon signed rank tests support the first two hypotheses of this research. Among the case group, original rendition scores differed significantly from re-colored rendition scores for all map scheme/question type combinations. Among the control group, however, original rendition scores did not differ significantly from re-colored rendition scores for any map scheme/question type combination. The rank-biserial correlation for the Wilcoxon signed-rank tests $r_W$ is a directional effect size which expresses the influence of the re-colored map renditions on performance by group. Very large positive effect sizes on the part of the case group were observed in overall map performance and performance on content questions. A large positive effect size on the part of the case group was observed in the performance on matching questions. Algorithm efficacy on the part of the case group varied among the individual schemes where the largest and smallest effect sizes were observed in the qualitative dot and sequential polychrome schemes, respectively.

The Mann-Whitney rank sum test results support the third hypothesis of this research. Among the re-colored renditions, case group scores did not differ significantly from control group scores for all map scheme and question type comparisons. The rank-biserial correlation for the Mann-Whitney rank sum tests $r_U$ is a directional effect size which expresses the influence of CVD on performance by map rendition (i.e., disparity in performance between groups). Very large negative effect sizes for the original rendition comparisons were observed
in overall map performance as well as in performance on content and matching questions. Among the individual schemes for the original rendition comparisons, the largest and smallest negative effect sizes were observed in the qualitative dot and sequential polychrome schemes, respectively.

The binomial tests on the map preference questions revealed that the case group significantly favored all re-colored map renditions over the original map renditions. The control group significantly favored two of the six original map renditions as well as all original rendition schemes overall. These results support the claim that image re-coloring is a valid approach to rendering post production maps accessible to color vision impaired viewers suffering from red-green and blue-green type defects.

5.2 Additional Participant Details

Only four of the forty participants in the case group had the tritan (blue-green) type of color vision impairment which is typically an acquired form of CVD. There are several potential reasons for the small response rate by individuals with acquired CVD. One cause might be the relatively young age of the participants (all participants were under 62 years of age). Acquired CVD often results from health issues affecting the elderly population. Likely, the age of the participants is related to using the ColorBlind subreddit as a recruiting tool. Another cause might be that individuals with acquired CVD are unaware of their color vision impairment. Indeed, many of the participants in both the case and control groups reported taking some form of color vision test in the past (see Table 3.1). While the sample size of the tritan type CVD group is too small to conduct a valid statistical test, a visual examination of their results revealed that all four individuals performed better when using the re-colored
map renditions than when using the original map renditions.

Four individuals with incomplete achromatism were not considered for this study as the OGA re-coloring algorithm was not intended to address this type of color vision impairment. As mentioned earlier, this form of CVD can cause confusion in distinguishing desaturated colors. While the sample size of this group is too small to conduct a valid statistical test, a visual examination of their results revealed that all four individuals performed better when using the original map renditions than when using the re-colored map renditions. This result is not surprising as the colors in the original map renditions have a higher saturation than those in the re-colored map renditions (see Appendix C).

Several participants provided feedback on the ColorBlind subreddit. For the most part, participants found the survey to be easy. One participant even timed themselves and reported that the survey took them about 20 minutes to complete. Two participants noted that the colors in the original map versions were often impossible to interpret. One participant felt confused by the format of the two variable map scheme. Indeed, Tufte (2001) questions whether two variable maps are an effective means of visualizing data.

5.3 Features of the OGA Algorithm

The OGA algorithm has several features that make it ideal for re-coloring maps. First, while the re-coloring algorithms described in section 2.3 produce separate color images for each CVD type, the OGA algorithm produces a single color image that is accessible to red-green and blue-green CVD observers. Applying Kuhn et al.’s (2008a) algorithm to a single maps results in four maps; a map for protans, a map for deutsans, a map for tritans, and the original map for viewers with normal color vision. The OGA algorithm produces
just one map accessible to both normal and CVD observers. Second, although several of the previously described algorithms have global color reassignment within the image; color re-mapping may vary based on the original image’s color composition. For example, cobalt blue may be re-colored to grayish-red in one image and cornflower blue in another. The OGA algorithm results in consistent color reassignment regardless of the color composition of the original image. Cobalt blue, for instance, will always be re-colored cornflower blue. This universal color reassignment is important when recoloring atlases or map series that rely on a standard color scheme, particularly those where the legend is located on a separate page. Third, the OGA algorithm is straightforward and, thus, can be easily adapted for use in a variety of platforms. While originally developed in C# .NET, the algorithm has been implemented in JavaScript, Python, MatLab, and R.

5.4 Limitations of the OGA Algorithm

The Orange-Gray-Azure algorithm has several limitations. First, the algorithm works by collapsing an image with a three dimensional gamut containing millions of colors into a two dimensional gamut containing thousands of colors. By using just variations in saturation and two hues to represent a spectrum of colors, certain color pairs are bound to be problematic. This occurs when one or both of the colors have a low saturation. One such pair is pale pink and bright yellow where both are re-mapped to pale orange. Another is grayish cyan and pale violet where both are re-mapped to grayish azure. Second, the algorithm re-colors green, a hue confusing to both red-green and blue-green deficient observers, by desaturation resulting in gray. As a consequence, maps that contain both gray and green may lose visual information. Third, in order to produce a gamut that is visible to both red-green and blue-
green dichromats, some saturation was sacrificed. The resulting re-colored maps are much less vibrant than the originals. In his evaluation of the ColorBrewer color schemes, Gardner (2005) found that CVD participants had difficulty distinguishing the pastel schemes. One method to address the above limitations is to compare the minimum CIEDE2000 color difference values of CVD simulations of the original and re-colored renditions of the map in question (as described in Section 3.3). The rendition with the largest minimum CIEDE2000 color difference value will be the most accessible. Finally, the OGA algorithm is intended to re-color maps already in circulation that are inaccessible to observers with color vision deficiencies. It is not a replacement for good cartographic design.

5.5 Conclusion

This research introduces a post publication method that addresses accessibility for map readers with inherited and acquired CVD. The OGA algorithm utilizes a combination of gamut re-mapping and desaturation methodologies to produce a re-colored map where confusing colors are rendered distinguishable to red-green and blue-green CVD observers. The results of this research are favorable. CVD participants performed better when using re-colored map renditions and felt they were easier to understand than the original map renditions. The re-colored map renditions did not have a significantly adverse effect on participants with normal color vision.
Appendix A. Formulae

A.1 Dichromatic Vision Simulation Equations

The following equations have been adapted from Brettel et al. (1997); Viénot et al. (1999); Viénot and Brettel (2001).

Gamma decoding of nonlinear \( R'G'B' \) values \([0,255]\) to linear \( RGB \) values \([0,1]\).

\[
R = (R'/255)^{2.2} \quad (A.1a)
\]
\[
G = (G'/255)^{2.2} \quad (A.1b)
\]
\[
B = (B'/255)^{2.2} \quad (A.1c)
\]

Transform color space from \( RGB \) to \( XYZ \).

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = [M_{RGB \rightarrow XYZ}] \begin{bmatrix} R \\
G \\
B \end{bmatrix} \quad (A.1d)
\]

\[
= \begin{bmatrix}
40.9568 & 35.5041 & 17.9167 \\
21.3389 & 70.6743 & 7.98680 \\
1.86297 & 11.4620 & 91.2367
\end{bmatrix} \begin{bmatrix} R \\
G \\
B \end{bmatrix}
\]

Transform color space from \( XYZ \) to \( LMS \).

\[
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix} = [M_{XYZ \rightarrow LMS}] \begin{bmatrix} X \\
Y \\
Z \end{bmatrix} \quad (A.1e)
\]

\[
= \begin{bmatrix}
0.15514 & 0.54312 & -0.03286 \\
-0.15514 & 0.45684 & 0.03286 \\
0.00000 & 0.00000 & 0.01608
\end{bmatrix} \begin{bmatrix} X \\
Y \\
Z \end{bmatrix}
\]
Transform color space from $RGB$ to $LMS$.  
\[
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix} = [M_{RGB \to XYZ}] [M_{XYZ \to LMS}] 
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

\[
= [M_{RGB \to LMS}] 
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

\[
= \begin{bmatrix}
17.8824 & 43.5161 & 4.11935 \\
3.45565 & 27.1554 & 3.86714 \\
0.02996 & 0.18431 & 1.46709
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

(A.1f)

Transform color space from $LMS$ to $RGB$.  
\[
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix} = [M_{RGB \to LMS}]^{-1} 
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix}
\]

\[
= [M_{LMS \to RGB}] 
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0.08094 & -0.13050 & 0.11672 \\
-0.01025 & 0.05402 & -0.11361 \\
-0.00037 & -0.00412 & 0.69351
\end{bmatrix}
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix}
\]

(A.1g)

Equations for the reduced dichromatic color domain in $LMS$ color space using white ($w$) and an anchor ($n$) stimuli. For red/green type CVD, $n = \text{blue}$. For yellow/blue type CVD, $n = \text{red}$.  

\[
\alpha = M_w S_n - M_n S_w
\]

\[
\beta = S_w L_n - S_n L_w
\]

\[
\gamma = L_w S_n - L_n S_w
\]

(A.1h) (A.1i) (A.1j)
Linear transformation in $LMS$ color space from normal color domain to protan color domain.

\[
\begin{bmatrix}
L_p \\
M_p \\
S_p
\end{bmatrix} = \begin{bmatrix}
M_{LMS\rightarrow pLMS}
\end{bmatrix}
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix} = \begin{bmatrix}
0 & -\beta/\alpha & -\gamma/\alpha \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix} = \begin{bmatrix}
0 & 2.02344 & -2.52581 \\
0 & 1 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix}
\]

(A.1k)

Linear transformation in $LMS$ color space from normal color domain to deutan color domain.

\[
\begin{bmatrix}
L_d \\
M_d \\
S_d
\end{bmatrix} = \begin{bmatrix}
M_{LMS\rightarrow dLMS}
\end{bmatrix}
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 1 \\
-\alpha/\beta & 0 & -\gamma/\beta \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0.49421 & 0 & 1.24827 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix}
\]

(A.1l)

Linear transformation in $LMS$ color space from normal color domain to tritan color domain.

\[
\begin{bmatrix}
L_t \\
M_t \\
S_t
\end{bmatrix} = \begin{bmatrix}
M_{LMS\rightarrow tLMS}
\end{bmatrix}
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 1 \\
0 & 1 & 0 \\
-\alpha/\gamma & -\beta/\gamma & 0
\end{bmatrix}
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
-0.01224 & 0.07203 & 0
\end{bmatrix}
\begin{bmatrix}
L \\
M \\
S
\end{bmatrix}
\]

(A.1m)
Linear transformation of protan color domain from \( LMS \) to \( RGB \) color space.

\[
\begin{bmatrix}
R_p \\
G_p \\
B_p
\end{bmatrix}
= \left[M_{RGB\rightarrow LMS}\right]^{-1}
\left[M_{LMS\rightarrow pLMS}\right]
\left[M_{RGB\rightarrow LMS}\right]
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\tag{A.1n}
\]

\[
= \left[M_{RGB\rightarrow pRGB}\right]
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0.11 & 0.89 & 0 \\
0.11 & 0.89 & 0 \\
0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

Linear transformation of deutan color domain from \( LMS \) to \( RGB \) color space.

\[
\begin{bmatrix}
R_d \\
G_d \\
B_d
\end{bmatrix}
= \left[M_{RGB\rightarrow LMS}\right]^{-1}
\left[M_{LMS\rightarrow dLMS}\right]
\left[M_{RGB\rightarrow LMS}\right]
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\tag{A.1o}
\]

\[
= \left[M_{RGB\rightarrow dRGB}\right]
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

\[
= \begin{bmatrix}
0.29 & 0.71 & 0 \\
0.29 & 0.71 & 0 \\
-0.02 & 0.02 & 1
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

Linear transformation of tritan color domain from \( LMS \) to \( RGB \) color space.

\[
\begin{bmatrix}
R_t \\
G_t \\
B_t
\end{bmatrix}
= \left[M_{RGB\rightarrow LMS}\right]^{-1}
\left[M_{LMS\rightarrow tLMS}\right]
\left[M_{RGB\rightarrow LMS}\right]
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\tag{A.1p}
\]

\[
= \left[M_{RGB\rightarrow tRGB}\right]
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

\[
= \begin{bmatrix}
1 & 0.14 & -0.14 \\
0 & 0.86 & 0.14 \\
0 & 0.86 & 0.14
\end{bmatrix}
\begin{bmatrix}
R \\
G \\
B
\end{bmatrix}
\]

Gamma encoding of linear \( RGB \) values \([0,1]\) to nonlinear \( R'G'B' \) values \([0,255]\).

\[
R' = 255R^{1/2.2}
\tag{A.1q}
\]

\[
G' = 255G^{1/2.2}
\tag{A.1r}
\]

\[
B' = 255B^{1/2.2}
\tag{A.1s}
\]
A.2 Color space conversion from CIE 1931 \((R, G, B)\) to CIE 1931 \((x, y, Y)\)

The following equations have been adapted from Schanda (2007).

Scale CIE 1931 \((R,G,B)\) range from \([0,255]\) to \([0,1]\).

\[
\begin{align*}
R' &= R/255 \quad (A.2a) \\
G' &= G/255 \quad (A.2b) \\
B' &= B/255 \quad (A.2c)
\end{align*}
\]

Perform gamma decoding.

\[
\begin{align*}
R'' &= \begin{cases} 
100\left(\frac{(R' + 0.055)/1.055}{1.055}\right)^2 & R' > 0.04045 \\
100(R'/12.92) & \text{otherwise}
\end{cases} \\
G'' &= \begin{cases} 
100\left(\frac{(G' + 0.055)/1.055}{1.055}\right)^2 & G' > 0.04045 \\
100(G'/12.92) & \text{otherwise}
\end{cases} \\
B'' &= \begin{cases} 
100\left(\frac{(B' + 0.055)/1.055}{1.055}\right)^2 & B' > 0.04045 \\
100(B'/12.92) & \text{otherwise}
\end{cases}
\end{align*}
\]

Conversion to CIE 1931 \((X, Y, Z)\) color space.

\[
\begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix} = 
\begin{bmatrix}
0.4124 & 0.3576 & 0.1805 \\
0.2126 & 0.7152 & 0.0722 \\
0.0193 & 0.1192 & 0.9505
\end{bmatrix} \begin{bmatrix}
R'' \\
G'' \\
B''
\end{bmatrix}
\]

Conversion from CIE 1931 \((X, Y, Z)\) color space to chromaticity coordinates \((x, y, Y)\).

\[
\begin{align*}
x &= X/(X + Y + Z) \quad (A.2h) \\
y &= Y/(X + Y + Z) \quad (A.2i) \\
Y &= Y \quad (A.2j)
\end{align*}
\]
A.3 Color space conversion from CIE 1931 \((X, Y, Z)\) to CIELAB \((L^*, a^*, b^*)\)

The following equations have been adapted from Schanda (2007).

Divide CIE 1931 \((X, Y, Z)\) channels by reference standards (observer= 2°, illuminant= D65).

\[
X' = \frac{X}{95.047} \quad \text{(A.3a)}
\]
\[
Y' = \frac{Y}{100.000} \quad \text{(A.3b)}
\]
\[
Z' = \frac{Z}{108.883} \quad \text{(A.3c)}
\]

Perform gamma encoding.

\[
X'' = \begin{cases} 
X'^{1/3} & \text{if } X' > 0.008856 \\
7.787X' + (16/116) & \text{otherwise}
\end{cases} \quad \text{(A.3d)}
\]
\[
Y'' = \begin{cases} 
Y'^{1/3} & \text{if } Y' > 0.008856 \\
7.787Y' + (16/116) & \text{otherwise}
\end{cases} \quad \text{(A.3e)}
\]
\[
Z'' = \begin{cases} 
Z'^{1/3} & \text{if } Z' > 0.008856 \\
7.787Z' + (16/116) & \text{otherwise}
\end{cases} \quad \text{(A.3f)}
\]

Conversion to CIELAB \((L^*, a^*, b^*)\) color space.

\[
L^* = 116Y'' - 16 \quad \text{(A.3g)}
\]
\[
a^* = 500(X'' - Y'') \quad \text{(A.3h)}
\]
\[
b^* = 200(Y'' - Z'') \quad \text{(A.3i)}
\]
A.4  Color space conversion from CIELAB \((L^*, a^*, b^*)\) to CIE 1931 \((X, Y, Z)\)

The following equations have been adapted from Schanda (2007).

Convert CIELAB \((L^*, a^*, b^*)\) channels into intermediary form.

\[
Y'' = (L^* + 16)/116 \quad \text{(A.4a)}
\]
\[
X'' = (a^*/500) + Y'' \quad \text{(A.4b)}
\]
\[
Z'' = (Y'' - b^*)/200 \quad \text{(A.4c)}
\]

Perform gamma decoding.

\[
X' = \begin{cases} 
X''^3 \\
(X''/7.787) - (16/116) 
\end{cases} 
\text{ otherwise} \quad \text{(A.4d)}
\]
\[
Y' = \begin{cases} 
Y''^3 \\
(Y''/7.787) - (16/116) 
\end{cases} \quad \text{ otherwise} \quad \text{(A.4e)}
\]
\[
Z' = \begin{cases} 
Z''^3 \\
(Z''/7.787) - (16/116) 
\end{cases} \quad \text{ otherwise} \quad \text{(A.4f)}
\]

Multiply intermediate channels by reference standards (observer= 2°, illuminant= D65).

\[
X = 95.047X' \quad \text{(A.4g)}
\]
\[
Y = 100.000Y' \quad \text{(A.4h)}
\]
\[
Z = 108.883Z' \quad \text{(A.4i)}
\]
A.5 Color space conversion from CIE 1931 \((x, y, Y)\) to CIE 1931 \((R, G, B)\)

The following equations have been adapted from Schanda (2007).

Conversion from chromaticity coordinates \((x, y, Y)\) to CIE 1931 \((X, Y, Z)\) color space.

\[
X = x(Y/y) \tag{A.5a}
\]
\[
Y = Y \tag{A.5b}
\]
\[
Z = (1 - x - y)(Y/y) \tag{A.5c}
\]

Conversion to intermediary CIE 1931 \((R'', G'', B'')\) color space.

\[
\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 3.2406 & -1.5372 & -0.4986 \\ -0.9689 & 1.8758 & 0.0415 \\ 0.0557 & -0.2040 & 1.0570 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} \tag{A.5d}
\]

Perform gamma encoding.

\[
R' = \begin{cases} 
((1.055(R'' - 0.055))^{1/2.4})/100 & R'' > 0.31308 \\
(12.92R'')/100 & \text{otherwise} 
\end{cases} \tag{A.5e}
\]
\[
G' = \begin{cases} 
((1.055(G'' - 0.055))^{1/2.4})/100 & G'' > 0.31308 \\
(12.92G'')/100 & \text{otherwise} 
\end{cases} \tag{A.5f}
\]
\[
B' = \begin{cases} 
((1.055(B'' - 0.055))^{1/2.4})/100 & B'' > 0.31308 \\
(12.92B'')/100 & \text{otherwise} 
\end{cases} \tag{A.5g}
\]

Scale range from \([0,1]\) to CIE 1931 \((R,G,B)\) \([0,255]\).

\[
R = 255R' \tag{A.5h}
\]
\[
G = 255G' \tag{A.5i}
\]
\[
B = 255B' \tag{A.5j}
\]
A.6  Color space conversion from CIE 1931 \((R, G, B)\) to oRGB \((L', C'_1, C'_2)\)

The following equations have been adapted from Bratkova et al. (2009).

Scale CIE 1931 \((R,G,B)\) range from \([0,255]\) to \([0,1]\).

\[
R' = R/255 \quad (A.6a) \\
G' = G/255 \quad (A.6b) \\
B' = B/255 \quad (A.6c)
\]

Conversion to a parallelepiped space via linear transformation.

\[
\begin{bmatrix}
L' \\
C'_1 \\
C'_2
\end{bmatrix}
= \begin{bmatrix}
0.2990 & 0.5870 & 0.1140 \\
0.5000 & 0.5000 & 1.0000 \\
0.8660 & 0.8660 & 0.0000
\end{bmatrix}\begin{bmatrix}
R' \\
G' \\
B'
\end{bmatrix} \quad (A.6d)
\]

A.7  Color space conversion from oRGB \((L', C'_1, C'_2)\) to CIE 1931 \((R, G, B)\)

The following equations have been adapted from Bratkova et al. (2009).

Conversion from a parallelepiped space via linear transformation.

\[
\begin{bmatrix}
R' \\
G' \\
B'
\end{bmatrix}
= \begin{bmatrix}
1.0000 & 0.1140 & 0.7436 \\
1.0000 & 0.1140 & -0.4111 \\
1.0000 & -0.8660 & 0.1663
\end{bmatrix}\begin{bmatrix}
L' \\
C'_1 \\
C'_2
\end{bmatrix} \quad (A.7a)
\]

Scale range from \([0,1]\) to CIE 1931 \((R,G,B)\) \([0,255]\).

\[
R = 255R' \quad (A.7b) \\
G = 255G' \quad (A.7c) \\
B = 255B' \quad (A.7d)
\]
A.8 CIEDE2000 Color Difference Equations

The following equations have been adapted from Sharma et al. (2005); Witt (2007).

Transform hue and chroma into cylindrical space.

\[ C_{i,ab}^* = \sqrt{a_i^* + b_i^*} \quad i = [1, 2] \]  
\[ C_{ab}^* = \frac{C_{1,ab}^* + C_{2,ab}^*}{2} \]  
\[ G = 0.5 \left( 1 - \sqrt{\frac{C_{ab}^*}{C_{ab}^* + 25}} \right) \]  
\[ a_i' = (1 + G) a_i^* \quad i = [1, 2] \]  
\[ C_i' = \sqrt{a_i'^2 + b_i'^2} \quad i = [1, 2] \]  
\[ h_i' = \begin{cases} 0 & b_i^* = a_i' = 0 \\ \tan^{-1}(b_i^*, a_i') & \text{otherwise} \end{cases} \quad i = [1, 2] \]

Calculate lightness, chroma and hue angle differences.

\[ \Delta L' = L_2^* - L_1^* \]  
\[ \Delta C' = C_2' - C_1' \]

Calculate CIELAB hue angle difference (\( \Delta h' \)).

\[ \Delta h' = \begin{cases} 0 & C_1'C_2' = 0 \\ h_2' - h_1' & C_2'C_1' \neq 0; \ |h_2' - h_1'| \leq 180^\circ \\ (h_2' - h_1') - 360^\circ & C_2'C_1' \neq 0; \ |h_2' - h_1'| > 180^\circ \\ (h_2' - h_1') + 360^\circ & C_2'C_1' \neq 0; \ |h_2' - h_1'| < -180^\circ \end{cases} \]  
\[ \Delta H' = 2\sqrt{C_1'C_2'} \sin \left( \frac{\Delta h'}{2} \right) \]
Calculate mean lightness, hue angle and chroma.

\[ L' = \frac{(L_1' - L_2')}{2}, \quad (A.8l) \]
\[ C' = \frac{(C_1' - C_2')}{2}, \quad (A.8m) \]
\[ h' = \begin{cases} \frac{h_1' - h_2'}{2} & |h_2' - h_1'| \leq 180^\circ; \quad C'_1 C'_2 \neq 0 \\ \frac{(h_1' - h_2') + 360^\circ}{2} & |h_2' - h_1'| > 180^\circ; \quad (h_1' - h_2') < 360^\circ; \quad C'_1 C'_2 \neq 0; \\ \frac{(h_1' - h_2') - 360^\circ}{2} & |h_2' - h_1'| > 180^\circ; \quad (h_1' - h_2') \geq 360^\circ; \quad C'_1 C'_2 \neq 0; \\ (h_1' - h_2') & C'_1 C'_2 = 0 \end{cases}, \quad (A.8n) \]

Calculate the \( T \) function which addresses complex hue angle dependence.

\[ T = 1 - 0.17 \cos (\bar{h}' - 30^\circ) + 0.24 \cos (2\bar{h}') + 0.32 \cos (3\bar{h}' + 6^\circ) - 0.20 \cos (4\bar{h}' + 63^\circ) \quad (A.8o) \]
\[ \Delta \theta = 30 \left\{ - \left[ \frac{\bar{h}' - 275^\circ}{25} \right]^2 \right\} \quad (A.8p) \]
\[ R_C = 2 \sqrt{\frac{C'^7}{C'^7 + 25^7}} \quad (A.8q) \]

Calculate the lightness \((S_L)\), chroma \((S_C)\) and hue \((S_H)\) weighting functions.

\[ S_L = 1 + \frac{0.015 (L' - 50)^2}{\sqrt{20 + (L' - 50)^2}}, \quad (A.8r) \]
\[ S_C = 1 + 0.045 C' \quad (A.8s) \]
\[ S_H = 1 + 0.015 C'T \quad (A.8t) \]

Calculate the hue rotation term \((R_T)\) which addresses hue angles in the neighborhood of 275° (blue region).

\[ R_T = -\sin (2\Delta \theta) R_C \quad (A.8u) \]
The complete color difference formula CIEDE2000 is as follows.

\[
\Delta E_{00}^{12} = \Delta E_{00}^0 (L_1^*, a_1^*, b_1^*, L_2^*, a_2^*, b_2^*)
\]

\[
= \sqrt{\left( \frac{\Delta L'}{k_{LS}} \right)^2 + \left( \frac{\Delta C'}{k_{SC}} \right)^2 + \left( \frac{\Delta H'}{k_{SH}} \right)^2 + R_T \left( \frac{\Delta C'}{k_{SC}} \right) \left( \frac{\Delta H'}{k_{SH}} \right)}
\]

(A.8v)

A.9 Color space conversion from CIE 1931 \((R, G, B)\) to HSV \((H, S, V)\)

The following equations have been adapted from Travis (1991).

Calculate saturation as percentage.

\[
S = \frac{\max(R, G, B) - \min(R, G, B)}{\max(R, G, B)}
\]

(A.9a)

Calculate value as percentage.

\[
V = \frac{\max(R, G, B)}{255}
\]

(A.9b)

Calculate hue in degrees.

\[
H = \begin{cases} 
   60 \left( 5 + \frac{\max(R, G, B) - B}{\max(R, G, B) - \min(R, G, B)} \right) & R = \max(R, G, B); \ G = \min(R, G, B); \\
   60 \left( 1 - \frac{\max(R, G, B) - B}{\max(R, G, B) - \min(R, G, B)} \right) & R = \max(R, G, B); \ G \neq \min(R, G, B); \\
   60 \left( 1 + \frac{\max(R, G, B) - R}{\max(R, G, B) - \min(R, G, B)} \right) & G = \max(R, G, B); \ B = \min(R, G, B); \\
   60 \left( 3 - \frac{\max(R, G, B) - G}{\max(R, G, B) - \min(R, G, B)} \right) & G = \max(R, G, B); \ B \neq \min(R, G, B); \\
   60 \left( 3 + \frac{\max(R, G, B) - G}{\max(R, G, B) - \min(R, G, B)} \right) & R = \max(R, G, B); \\
   60 \left( 5 - \frac{\max(R, G, B) - R}{\max(R, G, B) - \min(R, G, B)} \right) & \text{otherwise}
\end{cases}
\]

(A.9c)
Appendix B. Maps

The following maps are at 25% scale. Dichromatic vision was simulated using the BVM color appearance algorithm.

Figure B.1. Practice map: sequential monochrome scheme depicting percentage of 2010 population under 18 years of age in California by county (U.S. Census Bureau, 2010b).

(a) Normal vision.  (b) Protan vision.

(c) Deutan vision.  (d) Tritan vision.
Figure B.2. Map 1: original rendition of diverging scheme depicting change in population since 2000 in Mississippi by county (U.S. Census Bureau, 2010a, 2000).

(a) Normal vision.  
(b) Protan vision.  
(c) Deutan vision.  
(d) Tritan vision.
Figure B.3. Map 2: re-colored rendition of diverging scheme depicting change in population since 2000 in Louisiana by county (U.S. Census Bureau, 2010a, 2000).

(a) Normal vision.  
(b) Protan vision.  
(c) Deutan vision.  
(d) Tritan vision.
Figure B.4. Map 3: original rendition of qualitative dot scheme depicting 311 service requests during June 2013 for Bronx, New York (OpenData, 2013).

(a) Normal vision.  
(b) Protan vision.  
(c) Deutan vision.  
(d) Tritan vision.
Figure B.5. Map 4: re-colored rendition of qualitative dot scheme depicting 311 service requests during June 2013 for Brooklyn, New York (OpenData, 2013).

(a) Normal vision.  
(b) Protan vision.

(c) Deutan vision.  
(d) Tritan vision.
Figure B.6. Map 5: original rendition of two variable scheme depicting obesity and produce consumption for New York City (NYC DOHMH, 2011).

(a) Normal vision.

(b) Protan vision.

(c) Deutan vision.

(d) Tritan vision.
Figure B.7. Map 6: re-colored rendition of two variable scheme depicting obesity and sugar-sweetened beverage consumption for New York City (NYC DOHMH, 2011).

(a) Normal vision.

(b) Protan vision.

(c) Deutan vision.

(d) Tritan vision.
Figure B.8. Map 7: original rendition of sequential polychrome scheme depicting 2011 mean travel time to work in New Mexico by county (U.S. Census Bureau, 2013).

(a) Normal vision.
(b) Protan vision.
(c) Deutan vision.
(d) Tritan vision.
Figure B.9. Map 8: re-colored rendition of sequential polychrome scheme depicting 2011 mean travel time to work in Colorado by county (U.S. Census Bureau, 2013).

(a) Normal vision.

(b) Protan vision.

(c) Deutan vision.

(d) Tritan vision.
Figure B.10. Map 9: original rendition of balance scheme depicting 2010 sex ratio in Mississippi by county (U.S. Census Bureau, 2010b).

(a) Normal vision.  
(b) Protan vision.  
(c) Deutan vision.  
(d) Tritan vision.
Figure B.11. Map 10: re-colored rendition of balance scheme depicting 2010 sex ratio in Louisiana by county (U.S. Census Bureau, 2010b).

(a) Normal vision.  

(b) Protan vision.  

(c) Deutan vision.  

(d) Tritan vision.
Figure B.12. Map 11: original rendition of qualitative area scheme depicting 2012 land use in Manhattan by tax lot (NYC DCP, 2012).

(a) Normal vision.  
(b) Protan vision.

(c) Deutan vision.  
(d) Tritan vision.
Figure B.13. Map 12: re-colored rendition of qualitative area scheme depicting 2012 land use in Manhattan by tax lot (NYC DCP, 2012).

(a) Normal vision.  
(b) Protan vision. 

(c) Deutan vision.  
(d) Tritan vision.
Appendix C. Color Schemes

The following chromaticity diagrams and attribute tables were produced using equations in Appendix A. Color names were produced using the NTC color name library (Mehta, 2016).

**Figure C.1.** Chromaticity diagram, practice map: sequential monochrome.

![Chromaticity Diagram](image)

**Table C.1.** Color attributes, practice map: sequential monochrome.

<table>
<thead>
<tr>
<th>ID</th>
<th>Color Name</th>
<th>HEX</th>
<th>HSV</th>
<th>RGB</th>
<th>Lab</th>
<th>orRGB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>H</td>
<td>S</td>
<td>R</td>
<td>G</td>
</tr>
<tr>
<td>OR-1</td>
<td>pale cobalt</td>
<td>#BFD5FF</td>
<td>219°</td>
<td>25%</td>
<td>191</td>
<td>213</td>
</tr>
<tr>
<td>OR-2</td>
<td>lt. bril. cobalt</td>
<td>#7FA9FF</td>
<td>220°</td>
<td>50%</td>
<td>127</td>
<td>169</td>
</tr>
<tr>
<td>OR-3</td>
<td>lt. bril. phthalo</td>
<td>#3C7CFF</td>
<td>220°</td>
<td>76%</td>
<td>60</td>
<td>124</td>
</tr>
<tr>
<td>OR-4</td>
<td>viv. cobalt</td>
<td>#004DEA</td>
<td>220°</td>
<td>100%</td>
<td>0</td>
<td>77</td>
</tr>
<tr>
<td>OR-5</td>
<td>str. cobalt</td>
<td>#003AB2</td>
<td>220°</td>
<td>100%</td>
<td>0</td>
<td>58</td>
</tr>
</tbody>
</table>
Figure C.2. Chromaticity diagrams, maps 1 – 2: diverging.

(a) Original rendition.  
(b) Re-colored rendition.

Table C.2. Color attributes, maps 1 – 2: diverging.

<table>
<thead>
<tr>
<th>ID</th>
<th>Color Name</th>
<th>HEX</th>
<th>HSV</th>
<th>RGB</th>
<th>Lab</th>
<th>oRGB</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>OR-1</td>
<td>lum. viv. orange</td>
<td>#FF7F00</td>
<td>29° 100%</td>
<td>255 127 0</td>
<td>66 43 73</td>
<td>0.59 0.75 0.43</td>
</tr>
<tr>
<td>OR-2</td>
<td>lum. viv. amber</td>
<td>#FFBF00</td>
<td>44° 100%</td>
<td>255 191 0</td>
<td>81 10 83</td>
<td>0.74 0.87 0.22</td>
</tr>
<tr>
<td>OR-3</td>
<td>lum. viv. yellow</td>
<td>#FFFF00</td>
<td>60° 100%</td>
<td>255 255 0</td>
<td>97 -21 94</td>
<td>0.89 1.00 0.00</td>
</tr>
<tr>
<td>OR-4</td>
<td>lum. viv. green</td>
<td>#00FF00</td>
<td>120° 100%</td>
<td>255 0</td>
<td>87 -86 83</td>
<td>0.59 0.50 -0.87</td>
</tr>
<tr>
<td>OR-5</td>
<td>lum. viv. opal</td>
<td>#00FFD4</td>
<td>169° 100%</td>
<td>255 212 89 59</td>
<td>0.68 -0.33 -0.87</td>
<td></td>
</tr>
<tr>
<td>OR-6</td>
<td>lum. viv. cerulean</td>
<td>#00BFFF</td>
<td>195° 100%</td>
<td>91 255 72 -17 -42</td>
<td>0.55 -0.63 -0.65</td>
<td></td>
</tr>
<tr>
<td>RC-1</td>
<td>bril. tangelo</td>
<td>#F38437</td>
<td>24° 77%</td>
<td>243 132 55</td>
<td>66 37 57</td>
<td>0.61 0.52 0.38</td>
</tr>
<tr>
<td>RC-2</td>
<td>lt. bril. orange</td>
<td>#FAAF7A</td>
<td>24° 51%</td>
<td>250 175 122 77 21 38</td>
<td>0.75 0.35 0.25</td>
<td></td>
</tr>
<tr>
<td>RC-3</td>
<td>pale tangelo</td>
<td>#FEDAC0</td>
<td>25° 24%</td>
<td>254 218 192 89</td>
<td>8 17 0.89 0.17 0.12</td>
<td></td>
</tr>
<tr>
<td>RC-4</td>
<td>gray</td>
<td>#D4D4D4</td>
<td>0° 0%</td>
<td>212 212 212 82 0</td>
<td>0 0.83 0.00 0.00</td>
<td></td>
</tr>
<tr>
<td>RC-5</td>
<td>lt. cornflower</td>
<td>#B0DFFF</td>
<td>204° 31%</td>
<td>176 223 255 86 -7 -20 -0.22 -0.16 -0.08</td>
<td></td>
<td></td>
</tr>
<tr>
<td>RC-6</td>
<td>lt. azure</td>
<td>#58B5F6</td>
<td>204° 64%</td>
<td>88 181 246 70 -8 -40 0.63 -0.44 -0.32</td>
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Figure C.3. Chromaticity diagrams, maps 3 – 4: qualitative dot.

(a) Original rendition.
(b) Re-colored rendition.

Table C.3. Color attributes, maps 3 – 4: qualitative dot.

<table>
<thead>
<tr>
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<th>Lab</th>
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<td></td>
<td></td>
<td></td>
<td>H</td>
<td>S</td>
<td>R</td>
<td>G</td>
</tr>
<tr>
<td>OR-1</td>
<td>lum. viv. tangelo</td>
<td>#FF6A00</td>
<td>24°</td>
<td>100%</td>
<td>255</td>
<td>106</td>
</tr>
<tr>
<td>OR-2</td>
<td>str. pistachio</td>
<td>#36A500</td>
<td>100°</td>
<td>100%</td>
<td>54</td>
<td>165</td>
</tr>
<tr>
<td>OR-3</td>
<td>lum. viv. cornflower</td>
<td>#2592FF</td>
<td>210°</td>
<td>85%</td>
<td>37</td>
<td>146</td>
</tr>
<tr>
<td>RC-1</td>
<td>bril. vermilion</td>
<td>#EB7624</td>
<td>24°</td>
<td>85%</td>
<td>235</td>
<td>118</td>
</tr>
<tr>
<td>RC-2</td>
<td>gray</td>
<td>#8D8D8D</td>
<td>0°</td>
<td>0%</td>
<td>141</td>
<td>141</td>
</tr>
<tr>
<td>RC-3</td>
<td>bril. azure</td>
<td>#2D97E0</td>
<td>204°</td>
<td>80%</td>
<td>45</td>
<td>151</td>
</tr>
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</table>
**Figure C.4.** Chromaticity diagrams, maps 5 – 6: two variable.

(a) Original rendition.  
(b) Re-colored rendition.

**Table C.4.** Color attributes, maps 5 – 6: two variable.

<table>
<thead>
<tr>
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<th>Color Name</th>
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<th>HSV $S$</th>
<th>RGB $R$</th>
<th>RGB $G$</th>
<th>RGB $B$</th>
<th>Lab $L^*$</th>
<th>Lab $a^*$</th>
<th>Lab $b^*$</th>
<th>oRGB $L'$</th>
<th>oRGB $C_1'$</th>
<th>oRGB $C_2'$</th>
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<tbody>
<tr>
<td>OR-1</td>
<td>lt. bril. cerulean</td>
<td>#80D5FF</td>
<td>199°</td>
<td>50%</td>
<td>128</td>
<td>213</td>
<td>255</td>
<td>81</td>
<td>-14</td>
<td>-28</td>
<td>0.75</td>
<td>-0.33</td>
<td>-0.29</td>
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<tr>
<td>OR-2</td>
<td>bril. azure</td>
<td>#2198D3</td>
<td>199°</td>
<td>84%</td>
<td>33</td>
<td>152</td>
<td>211</td>
<td>59</td>
<td>-10</td>
<td>38</td>
<td>0.48</td>
<td>-0.46</td>
<td>-0.40</td>
</tr>
<tr>
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<td>str. azure</td>
<td>#125C81</td>
<td>200°</td>
<td>86%</td>
<td>18</td>
<td>92</td>
<td>129</td>
<td>36</td>
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<td>-26</td>
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<td>-0.29</td>
<td>-0.25</td>
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<tr>
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<td>bril. malachite</td>
<td>#64DA78</td>
<td>130°</td>
<td>54%</td>
<td>100</td>
<td>218</td>
<td>120</td>
<td>78</td>
<td>-53</td>
<td>38</td>
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<td>-0.40</td>
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<tr>
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<td>str. emerald</td>
<td>#00A81B</td>
<td>129°</td>
<td>100%</td>
<td>0</td>
<td>168</td>
<td>27</td>
<td>59</td>
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<td>0.22</td>
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<tr>
<td>OR-6</td>
<td>deep emerald</td>
<td>#006610</td>
<td>129°</td>
<td>100%</td>
<td>0</td>
<td>102</td>
<td>16</td>
<td>37</td>
<td>-43</td>
<td>37</td>
<td>0.24</td>
<td>0.14</td>
<td>-0.35</td>
</tr>
<tr>
<td>OR-7</td>
<td>bril. gold</td>
<td>#E1CC67</td>
<td>49°</td>
<td>54%</td>
<td>225</td>
<td>204</td>
<td>103</td>
<td>81</td>
<td>-5</td>
<td>52</td>
<td>0.78</td>
<td>0.44</td>
<td>0.07</td>
</tr>
<tr>
<td>OR-8</td>
<td>str. gold</td>
<td>#AA8E00</td>
<td>50°</td>
<td>100%</td>
<td>170</td>
<td>142</td>
<td>0</td>
<td>59</td>
<td>-1</td>
<td>64</td>
<td>0.53</td>
<td>0.61</td>
<td>0.10</td>
</tr>
<tr>
<td>OR-9</td>
<td>deep gold</td>
<td>#685600</td>
<td>49°</td>
<td>100%</td>
<td>104</td>
<td>86</td>
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<td>37</td>
<td>0</td>
<td>44</td>
<td>0.32</td>
<td>0.37</td>
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<tr>
<td>RC-1</td>
<td>very lt. azure</td>
<td>#8CCFFD</td>
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<td>45%</td>
<td>140</td>
<td>207</td>
<td>253</td>
<td>80</td>
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<td>-29</td>
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<td>mod. cerulean</td>
<td>#4594CB</td>
<td>204°</td>
<td>66%</td>
<td>69</td>
<td>148</td>
<td>203</td>
<td>58</td>
<td>-6</td>
<td>-35</td>
<td>0.51</td>
<td>-0.37</td>
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<tr>
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<td>#2A5A7B</td>
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<td>66%</td>
<td>42</td>
<td>90</td>
<td>123</td>
<td>36</td>
<td>-5</td>
<td>-23</td>
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<td>-0.22</td>
<td>-0.16</td>
</tr>
<tr>
<td>RC-4</td>
<td>cyanish gray</td>
<td>#BBC1C6</td>
<td>207°</td>
<td>6%</td>
<td>187</td>
<td>193</td>
<td>198</td>
<td>77</td>
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<td>-3</td>
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<td>0%</td>
<td>142</td>
<td>142</td>
<td>142</td>
<td>59</td>
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<td>dark gray</td>
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<td>88</td>
<td>88</td>
<td>88</td>
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<td>very lt. tangelo</td>
<td>#FDBA8B</td>
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<td>45%</td>
<td>88</td>
<td>88</td>
<td>88</td>
<td>37</td>
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<td>0</td>
<td>0.35</td>
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<tr>
<td>RC-8</td>
<td>mod. orange</td>
<td>#BC825A</td>
<td>24°</td>
<td>52%</td>
<td>253</td>
<td>186</td>
<td>139</td>
<td>80</td>
<td>18</td>
<td>33</td>
<td>0.79</td>
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<td>52%</td>
<td>115</td>
<td>80</td>
<td>55</td>
<td>37</td>
<td>11</td>
<td>20</td>
<td>0.34</td>
<td>0.17</td>
<td>0.12</td>
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</table>
Figure C.5. Chromaticity diagrams, maps 7 – 8: sequential polychrome.

(a) Original rendition.  
(b) Re-colored rendition.

Table C.5. Color attributes, maps 7 – 8: sequential polychrome.

<table>
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<td></td>
<td></td>
<td></td>
<td>$H$</td>
<td>$S$</td>
<td>$R$</td>
<td>$G$</td>
</tr>
<tr>
<td>OR-1</td>
<td>lum. viv. gold</td>
<td>#FFE921</td>
<td>54°</td>
<td>87%</td>
<td>255 233 33</td>
<td>91 -10</td>
</tr>
<tr>
<td>OR-2</td>
<td>lum. viv. green</td>
<td>#00F700</td>
<td>120°</td>
<td>100%</td>
<td>0 247 0</td>
<td>85 -84</td>
</tr>
<tr>
<td>OR-3</td>
<td>viv. artic blue</td>
<td>#00D1E5</td>
<td>185°</td>
<td>100%</td>
<td>0 209 229 76</td>
<td>-35 -21</td>
</tr>
<tr>
<td>OR-4</td>
<td>lum. viv. azure</td>
<td>#0180FF</td>
<td>210°</td>
<td>100%</td>
<td>1 128 255 54</td>
<td>18 -70</td>
</tr>
<tr>
<td>OR-5</td>
<td>viv. violet</td>
<td>#6E00DD</td>
<td>269°</td>
<td>100%</td>
<td>110 0 221 35 74</td>
<td>-83</td>
</tr>
<tr>
<td>RC-1</td>
<td>very lt. orange</td>
<td>#FFCFAD</td>
<td>24°</td>
<td>32%</td>
<td>255 207 173 86</td>
<td>12 23</td>
</tr>
<tr>
<td>RC-2</td>
<td>gray</td>
<td>#CECECE</td>
<td>0°</td>
<td>0%</td>
<td>206 206 206 82</td>
<td>0 0</td>
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<td>bril. cerulean</td>
<td>#78C0F2</td>
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<td>50%</td>
<td>120 192 242 74</td>
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</tr>
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<td>204°</td>
<td>89%</td>
<td>23 134 211 53</td>
<td>-1 -47</td>
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<td>str. azure</td>
<td>#135380</td>
<td>204°</td>
<td>85%</td>
<td>19 83 128 33</td>
<td>-2 -30</td>
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</table>
Figure C.6. Chromaticity diagrams, maps 9 – 10: balance.

(a) Original rendition. 
(b) Re-colored rendition.


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<td>$S$</td>
<td>$R$</td>
<td>$G$</td>
</tr>
<tr>
<td>OR-1</td>
<td>vivid. crimson</td>
<td>#EA003F</td>
<td>343°</td>
<td>100%</td>
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<tr>
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<td>very lt. amaranth</td>
<td>#FF8CAB</td>
<td>343°</td>
<td>45%</td>
<td>255</td>
<td>140</td>
</tr>
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<td>OR-3</td>
<td>gray</td>
<td>#AAAAAA</td>
<td>0°</td>
<td>0%</td>
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<td>170</td>
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<td>OR-4</td>
<td>lt. brl. blue violet</td>
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<td>50%</td>
<td>166</td>
<td>127</td>
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<td>lum. viv. blue violet</td>
<td>#6623FF</td>
<td>258°</td>
<td>86%</td>
<td>102</td>
<td>35</td>
</tr>
<tr>
<td>RC-1</td>
<td>str. orange</td>
<td>#AF4C07</td>
<td>24°</td>
<td>96%</td>
<td>175</td>
<td>76</td>
</tr>
<tr>
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<td>lt. tangelo</td>
<td>#D6A583</td>
<td>24°</td>
<td>39%</td>
<td>214</td>
<td>165</td>
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<tr>
<td>RC-3</td>
<td>gray</td>
<td>#AAAAAA</td>
<td>0°</td>
<td>0%</td>
<td>170</td>
<td>170</td>
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<tr>
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<td>mod. cerulean</td>
<td>#699DC1</td>
<td>204°</td>
<td>46%</td>
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<td>193</td>
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<td>#0962A0</td>
<td>204°</td>
<td>94%</td>
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Figure C.7. Chromaticity diagrams, maps 11 – 12: quantitative area.

(a) Original rendition. 

(b) Re-colored rendition.

Table C.7. Color attributes, maps 11 – 12: quantitative area.

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<th>G</th>
<th>B</th>
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<th>a*</th>
<th>b*</th>
<th>oRGB L'</th>
<th>C_1</th>
<th>C_2</th>
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<td>#FF0000</td>
<td>0°</td>
<td>100%</td>
<td>255</td>
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<td>0</td>
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<td>0.87</td>
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<td>120°</td>
<td>100%</td>
<td>147</td>
<td>0</td>
<td>52</td>
<td>-57</td>
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<td>34</td>
<td>0.34</td>
<td>0.29</td>
<td>-0.50</td>
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<td>#5571FF</td>
<td>230°</td>
<td>67%</td>
<td>85</td>
<td>113</td>
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<td>#00E9FF</td>
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<td>100%</td>
<td>84</td>
<td>233</td>
<td>84</td>
<td>-38</td>
<td>-23</td>
<td>0</td>
<td>0.65</td>
<td>-0.54</td>
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<td>lt. bril. gamboge</td>
<td>#FFCD37</td>
<td>45°</td>
<td>78%</td>
<td>255</td>
<td>205</td>
<td>55</td>
<td>4</td>
<td>75</td>
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<td>0.80</td>
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<td>#00F700</td>
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<td>100%</td>
<td>247</td>
<td>0</td>
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<td>81</td>
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<td>0.57</td>
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<td>24°</td>
<td>98%</td>
<td>194</td>
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<td>48</td>
<td>41</td>
<td>57</td>
<td>0.42</td>
<td>0.53</td>
<td>0.38</td>
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<td>0%</td>
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<td>35</td>
<td>134</td>
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Table C.8. Minimum CIEDE2000 color difference $\Delta E_{00min}$ by map color scheme, method, rendition, and color vision type.

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<th>Protan</th>
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</tr>
</tbody>
</table>

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Appendix D. Survey Questions

D.1 Background Questions

1. Are you aware of any problems with your color vision?
   (a) No
   (b) Yes

2. Have you ever been given a color vision test and if so, what were the results?
   (a) No
   (b) Yes, normal color vision
   (c) Yes, mild red-green confusion
   (d) Yes, mild blue-green confusion
   (e) Yes, severe red-green confusion
   (f) Yes, severe blue-green confusion
   (g) Yes, do not remember

3. When did you become aware of problems with your color vision?
   (a) As a child (under 18 years of age)
   (b) During early adulthood (between 18 and 40 years of age)
   (c) During middle adulthood (between 40 and 60 years of age)
   (d) During older adulthood (over 60 years of age)
   (e) I have no problems with my color vision

4. Over the course of your life, have you noticed any change in your color vision?
   (a) No, I have notice no change in my color vision
   (b) Yes, my color vision has worsened
   (c) Yes, my color vision has improved
5. Over the course of your life, have you noticed any change in your eyesight? This does not include changes in color vision.
   (a) No, I have notice no change in my eyesight
   (b) Yes, my eyesight has worsened
   (c) Yes, my eyesight has improved

6. How often do you use maps (e.g., Google Maps, maps in apps, maps in games, maps in the media)?
   (a) At least once a week
   (b) At least once a month
   (c) At least once a year
   (d) Rarely
   (e) Never

7. Have you ever found a map confusing because of its color scheme?
   (a) No
   (b) Yes

8. Do you suffer from cataracts?
   (a) No
   (b) Yes

9. Do you suffer from type 1 diabetes?
   (a) No
   (b) Yes

10. Do you suffer from type 2 diabetes?
    (a) No
    (b) Yes
11. Do you suffer from glaucoma?
   (a) No
   (b) Yes

12. Do you suffer from macular degeneration?
   (a) No
   (b) Yes

13. How old are you?
   (a) Between 18 and 40 years of age
   (b) Between 41 and 60 years of age
   (c) Between 61 and 79 years of age
   (d) Over 79 years of age

14. What is your gender?
   (a) Female
   (b) Male

D.2 Practice Map

Content Question

15. Which of the following most closely describes the overall trend in percentage of population under 18 years of age from Glenn County along the arrow to Shasta County?
   (a) increase in the percentage of population under 18 years of age
   (b) decrease in the percentage of population under 18 years of age
   (c) no change in the percentage of population under 18 years of age
   (d) not answerable; relevant colors too similar
Matching Questions

16. The percentage of population under 18 years of age in Imperial County is
   (a) < 19.0%
   (b) 19.0% to 22.9%
   (c) 23.0% to 26.9%
   (d) 27.0% to 30.9%
   (e) > 31.0%
   (f) not answerable; relevant colors too similar

17. The percentage of population under 18 years of age in San Luis Obispo County is
   (a) < 19.0%
   (b) 19.0% to 22.9%
   (c) 23.0% to 26.9%
   (d) 27.0% to 30.9%
   (e) > 31.0%
   (f) not answerable; relevant colors too similar

D.3 Map 1: Diverging, Rendition 1

Content Questions

18. Which of the following most closely describes the overall trend in change in population since 2000 from Panola County along the arrow to Tate County?
   (a) increase in change in population
   (b) decrease in change in population
   (c) no change in change in population
   (d) not answerable; relevant colors too similar

19. Which of the following most closely describes the trend in change in population since 2000 from Tallahatchie County along the arrow to Yalobusha County?
   (a) increase in change in population
   (b) decrease in change in population
   (c) no change in change in population
   (d) not answerable; relevant colors too similar
Matching Questions

20. The change in population since 2000 in Oktibbeha County is
   (a) > 20% (GAIN)
   (b) 10% to 20% (GAIN)
   (c) 0% to 10% (GAIN)
   (d) -10% to 0% (LOSS)
   (e) -20% to -10% (LOSS)
   (f) < -20% (LOSS)
   (g) not answerable; relevant colors too similar

21. The change in population since 2000 in Jackson County is
   (a) > 20% (GAIN)
   (b) 10% to 20% (GAIN)
   (c) 0% to 10% (GAIN)
   (d) -10% to 0% (LOSS)
   (e) -20% to -10% (LOSS)
   (f) < -20% (LOSS)
   (g) not answerable; relevant colors too similar

22. The change in population since 2000 in Jefferson Davis County is
   (a) > 20% (GAIN)
   (b) 10% to 20% (GAIN)
   (c) 0% to 10% (GAIN)
   (d) -10% to 0% (LOSS)
   (e) -20% to -10% (LOSS)
   (f) < -20% (LOSS)
   (g) not answerable; relevant colors too similar
D.4  Map 2: Diverging, Rendition 2

Content Questions

23. Which of the following most closely describes the trend in change in population since 2000 from Richland County along the arrow to Ouachita County?

(a) increase in change in population
(b) decrease in change in population
(c) no change in change in population
(d) not answerable; relevant colors too similar

24. Which of the following most closely describes the trend in change in population since 2000 from Rapides County along the arrow to Grant County?

(a) increase in change in population
(b) decrease in change in population
(c) no change in change in population
(d) not answerable; relevant colors too similar

Matching Questions

25. The change in population since 2000 in Plaquemines County is

(a) > 20% (GAIN)
(b) 10% to 20% (GAIN)
(c) 0% to 10% (GAIN)
(d) -10% to 0% (LOSS)
(e) -20% to -10% (LOSS)
(f) < -20% (LOSS)
(g) not answerable; relevant colors too similar
26. The change in population since 2000 in Bossier County is

(a) > 20% (GAIN)
(b) 10% to 20% (GAIN)
(c) 0% to 10% (GAIN)
(d) -10% to 0% (LOSS)
(e) -20% to -10% (LOSS)
(f) < −20% (LOSS)
(g) not answerable; relevant colors too similar

27. The change in population since 2000 in Terrebonne County is

(a) > 20% (GAIN)
(b) 10% to 20% (GAIN)
(c) 0% to 10% (GAIN)
(d) -10% to 0% (LOSS)
(e) -20% to -10% (LOSS)
(f) < −20% (LOSS)
(g) not answerable; relevant colors too similar

Preference Question

28. You just answered questions about Map 1 and Map 2. Which map’s color scheme was easiest to understand:

(a) Map 1
(b) Map 2
D.5 Map 3: Qualitative Dot, Rendition 1

Content Questions

29. The most common 311 service request within the outlined square area is
   (a) Graffiti
   (b) Noise
   (c) Rodent
   (d) not answerable; relevant colors too similar

30. The most common 311 service request within the outlined square area is
   (a) Graffiti
   (b) Noise
   (c) Rodent
   (d) not answerable; relevant colors too similar

Matching Questions

31. The 311 service request of the circled point is
   (a) Graffiti
   (b) Noise
   (c) Rodent
   (d) not answerable; relevant colors too similar

32. The 311 service request of the circled point is
   (a) Graffiti
   (b) Noise
   (c) Rodent
   (d) not answerable; relevant colors too similar

33. The 311 service request of the circled point is
   (a) Graffiti
(b) Noise
(c) Rodent
(d) not answerable; relevant colors too similar

D.6 Map 4: Qualitative Dot, Rendition 2

Content Questions

34. The most common 311 service request within the outlined square area is
(a) Graffiti
(b) Noise
(c) Rodent
(d) not answerable; relevant colors too similar

35. The most common 311 service request within the outlined square area is
(a) Graffiti
(b) Noise
(c) Rodent
(d) not answerable; relevant colors too similar

Matching Questions

36. The 311 service request of the circled point is
(a) Graffiti
(b) Noise
(c) Rodent
(d) not answerable; relevant colors too similar

37. The 311 service request of the circled point is
(a) Graffiti
(b) Noise
38. The 311 service request of the circled point is

(a) Graffiti
(b) Noise
(c) Rodent
(d) not answerable; relevant colors too similar

Preference Question

39. You just answered questions about Map 3 and Map 4. Which map’s color scheme was easiest to understand:

(a) Map 3
(b) Map 4

D.7 Map 5: Two Variable, Rendition 1

Content Questions

40. Which of the following most closely describes the trend in percent of obese individuals and percent of individuals who ate no servings of fruit or vegetables on day prior to survey from SE Queens along the arrow to Jamaica?

(a) % Obese: Low to Low, % Ate No Fruit/Vegetables: High to High
(b) % Obese: Low to Medium, % Ate No Fruit/Vegetables: High to High
(c) % Obese: Low to High, % Ate No Fruit/Vegetables: High to High
(d) % Obese: Medium to Low, % Ate No Fruit/Vegetables: High to High
(e) % Obese: Medium to Medium, % Ate No Fruit/Vegetables: High to High
(f) % Obese: Medium to High, % Ate No Fruit/Vegetables: High to High
(g) % Obese: High to Low, % Ate No Fruit/Vegetables: High to High
(h) % Obese: High to Medium, % Ate No Fruit/Vegetables: High to High
(i) % Obese: High to High, % Ate No Fruit/Vegetables: High to High
(j) not answerable; relevant colors too similar
41. Which of the following most closely describes the trend in percent of obese individuals and percent of individuals who ate no servings of fruit or vegetables on day prior to survey from South Bronx along the arrow to Pelham - Throgs Neck?

(a) % Obese: Low to Low, % Ate No Fruit/Vegetables: High to High
(b) % Obese: Low to Medium, % Ate No Fruit/Vegetables: High to High
(c) % Obese: Low to High, % Ate No Fruit/Vegetables: High to High
(d) % Obese: Medium to Low, % Ate No Fruit/Vegetables: High to High
(e) % Obese: Medium to Medium, % Ate No Fruit/Vegetables: High to High
(f) % Obese: Medium to High, % Ate No Fruit/Vegetables: High to High
(g) % Obese: High to Low, % Ate No Fruit/Vegetables: High to High
(h) % Obese: High to Medium, % Ate No Fruit/Vegetables: High to High
(i) % Obese: High to High, % Ate No Fruit/Vegetables: High to High
(j) not answerable; relevant colors too similar

Matching Questions

42. The percent of obese individuals and percent of individuals who ate no servings of fruit or vegetables on day prior to survey in Sunset Park is

(a) % Obese: Low, % Ate No Fruit/Vegetables: Low
(b) % Obese: Low, % Ate No Fruit/Vegetables: Medium
(c) % Obese: Low, % Ate No Fruit/Vegetables: High
(d) % Obese: Medium, % Ate No Fruit/Vegetables: Low
(e) % Obese: Medium, % Ate No Fruit/Vegetables: Medium
(f) % Obese: Medium, % Ate No Fruit/Vegetables: High
(g) % Obese: High, % Ate No Fruit/Vegetables: Low
(h) % Obese: High, % Ate No Fruit/Vegetables: Medium
(i) % Obese: High, % Ate No Fruit/Vegetables: High
(j) not answerable; relevant colors too similar
43. The percent of obese individuals and percent of individuals who ate no servings of fruit or vegetables on day prior to survey in Canarsie - Flatlands is

(a) % Obese: Low, % Ate No Fruit/Vegetables: Low
(b) % Obese: Low, % Ate No Fruit/Vegetables: Medium
(c) % Obese: Low, % Ate No Fruit/Vegetables: High
(d) % Obese: Medium, % Ate No Fruit/Vegetables: Low
(e) % Obese: Medium, % Ate No Fruit/Vegetables: Medium
(f) % Obese: Medium, % Ate No Fruit/Vegetables: High
(g) % Obese: High, % Ate No Fruit/Vegetables: Low
(h) % Obese: High, % Ate No Fruit/Vegetables: Medium
(i) % Obese: High, % Ate No Fruit/Vegetables: High
(j) not answerable; relevant colors too similar

44. The percent of obese individuals and percent of individuals who ate no servings of fruit or vegetables on day prior to survey in Southern SI is

(a) % Obese: Low, % Ate No Fruit/Vegetables: Low
(b) % Obese: Low, % Ate No Fruit/Vegetables: Medium
(c) % Obese: Low, % Ate No Fruit/Vegetables: High
(d) % Obese: Medium, % Ate No Fruit/Vegetables: Low
(e) % Obese: Medium, % Ate No Fruit/Vegetables: Medium
(f) % Obese: Medium, % Ate No Fruit/Vegetables: High
(g) % Obese: High, % Ate No Fruit/Vegetables: Low
(h) % Obese: High, % Ate No Fruit/Vegetables: Medium
(i) % Obese: High, % Ate No Fruit/Vegetables: High
(j) not answerable; relevant colors too similar
D.8 Map 6: Two Variable, Rendition 2

Content Questions

45. Which of the following most closely describes the trend in percent of obese individuals and percent of individuals who drink one or more sugar-sweetened beverage daily from Pelham - Throgs Neck along the arrow to South Bronx?

(a) % Obese: Low to Low, % Drink Sugary Drinks: High to High
(b) % Obese: Low to Medium, % Drink Sugary Drinks: High to High
(c) % Obese: Low to High, % Drink Sugary Drinks: High to High
(d) % Obese: Medium to Low, % Drink Sugary Drinks: High to High
(e) % Obese: Medium to Medium, % Drink Sugary Drinks: High to High
(f) % Obese: Medium to High, % Drink Sugary Drinks: High to High
(g) % Obese: High to Low, % Drink Sugary Drinks: High to High
(h) % Obese: High to Medium, % Drink Sugary Drinks: High to High
(i) % Obese: High to High, % Drink Sugary Drinks: High to High
(j) not answerable; relevant colors too similar

46. Which of the following most closely describes the trend in percent of obese individuals and percent of individuals who drink one or more sugar-sweetened beverage daily from Jamaica along the arrow to Southeast Queens?

(a) % Obese: Low to Low, % Drink Sugary Drinks: High to High
(b) % Obese: Low to Medium, % Drink Sugary Drinks: High to High
(c) % Obese: Low to High, % Drink Sugary Drinks: High to High
(d) % Obese: Medium to Low, % Drink Sugary Drinks: High to High
(e) % Obese: Medium to Medium, % Drink Sugary Drinks: High to High
(f) % Obese: Medium to High, % Drink Sugary Drinks: High to High
(g) % Obese: High to Low, % Drink Sugary Drinks: High to High
(h) % Obese: High to Medium, % Drink Sugary Drinks: High to High
(i) % Obese: High to High, % Drink Sugary Drinks: High to High
(j) not answerable; relevant colors too similar
Matching Questions

47. The percent of obese individuals and percent of individuals who drink one or more sugar-sweetened beverage daily in Southern SI is

(a) % Obese: Low, % Drink Sugary Drinks: Low
(b) % Obese: Low, % Drink Sugary Drinks: Medium
(c) % Obese: Low, % Drink Sugary Drinks: High
(d) % Obese: Medium, % Drink Sugary Drinks: Low
(e) % Obese: Medium, % Drink Sugary Drinks: Medium
(f) % Obese: Medium, % Drink Sugary Drinks: High
(g) % Obese: High, % Drink Sugary Drinks: Low
(h) % Obese: High, % Drink Sugary Drinks: Medium
(i) % Obese: High, % Drink Sugary Drinks: High
(j) not answerable; relevant colors too similar

48. The percent of obese individuals and percent of individuals who drink one or more sugar-sweetened beverage daily in Bensonhurst - Bay Ridge is

(a) % Obese: Low, % Drink Sugary Drinks: Low
(b) % Obese: Low, % Drink Sugary Drinks: Medium
(c) % Obese: Low, % Drink Sugary Drinks: High
(d) % Obese: Medium, % Drink Sugary Drinks: Low
(e) % Obese: Medium, % Drink Sugary Drinks: Medium
(f) % Obese: Medium, % Drink Sugary Drinks: High
(g) % Obese: High, % Drink Sugary Drinks: Low
(h) % Obese: High, % Drink Sugary Drinks: Medium
(i) % Obese: High, % Drink Sugary Drinks: High
(j) not answerable; relevant colors too similar
49. The percent of obese individuals and percent of individuals who drink one or more sugar-sweetened beverage daily in Northern SI is

(a) % Obese: Low, % Drink Sugary Drinks: Low
(b) % Obese: Low, % Drink Sugary Drinks: Medium
(c) % Obese: Low, % Drink Sugary Drinks: High
(d) % Obese: Medium, % Drink Sugary Drinks: Low
(e) % Obese: Medium, % Drink Sugary Drinks: Medium
(f) % Obese: Medium, % Drink Sugary Drinks: High
(g) % Obese: High, % Drink Sugary Drinks: Low
(h) % Obese: High, % Drink Sugary Drinks: Medium
(i) % Obese: High, % Drink Sugary Drinks: High
(j) not answerable; relevant colors too similar

Preference Question

50. You just answered questions about Map 5 and Map 6. Which map’s color scheme was easiest to understand:

(a) Map 5
(b) Map 6
D.9 Map 7: Sequential Polychrome, Rendition 1

Content Questions

51. Which of the following most closely describes the trend in mean travel time to work in minutes from Colfax County along the arrow to Union County?

   (a) Low to Low
   (b) Low to Med
   (c) Low to Med-Low
   (d) Med to Low
   (e) Med to Med
   (f) Med to Med-Low
   (g) Med-Low to Low
   (h) Med-Low to Med
   (i) Med-Low to Med-Low
   (j) not answerable; relevant colors too similar

52. Which of the following most closely describes the trend in mean travel time to work in minutes from Socorro County along the arrow to Cibola County?

   (a) Low to Low
   (b) Low to Med
   (c) Low to Med-Low
   (d) Med to Low
   (e) Med to Med
   (f) Med to Med-Low
   (g) Med-Low to Low
   (h) Med-Low to Med
   (i) Med-Low to Med-Low
   (j) not answerable; relevant colors too similar

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Matching Questions

53. The mean travel time to work in minutes in Catron County is
   (a) Low (< 15)
   (b) Med-Low (15 to 20)
   (c) Med (20 to 25)
   (d) Med-High (25 to 30)
   (e) High (> 30)
   (f) not answerable; relevant colors too similar

54. The mean travel time to work in minutes in Sierra County is
   (a) Low (< 15)
   (b) Med-Low (15 to 20)
   (c) Med (20 to 25)
   (d) Med-High (25 to 30)
   (e) High (> 30)
   (f) not answerable; relevant colors too similar

55. The mean travel time to work in minutes in Lea County is
   (a) Low (< 15)
   (b) Med-Low (15 to 20)
   (c) Med (20 to 25)
   (d) Med-High (25 to 30)
   (e) High (> 30)
   (f) not answerable; relevant colors too similar
D.10  Map 8: Sequential Polychrome, Rendition 2

Content Questions

56. Which of the following most closely describes the trend in mean travel time to work in minutes from Kit Carson County along the arrow to Yuma County?

(a) Low to Low  
(b) Low to Med  
(c) **Low to Med-Low**  
(d) Med to Low  
(e) Med to Med  
(f) Med to Med-Low  
(g) Med-Low to Low  
(h) Med-Low to Med  
(i) Med-Low to Med-Low  
(j) not answerable; relevant colors too similar

57. Which of the following most closely describes the trend in mean travel time to work in minutes from Moffat County along the arrow to Routt County?

(a) Low to Low  
(b) Low to Med  
(c) **Low to Med-Low**  
(d) Med to Low  
(e) Med to Med  
(f) Med to Med-Low  
(g) Med-Low to Low  
(h) Med-Low to Med  
(i) Med-Low to Med-Low  
(j) not answerable; relevant colors too similar
58. The mean travel time to work in minutes in Hinsdale County is
   (a) Low (< 15)
   (b) Med-Low (15 to 20)
   (c) Med (20 to 25)
   (d) Med-High (25 to 30)
   (e) High (> 30)
   (f) not answerable; relevant colors too similar

59. The mean travel time to work in minutes in Las Animas County is
   (a) Low (< 15)
   (b) Med-Low (15 to 20)
   (c) Med (20 to 25)
   (d) Med-High (25 to 30)
   (e) High (> 30)
   (f) not answerable; relevant colors too similar

60. The mean travel time to work in minutes in San Miguel County is
   (a) Low (< 15)
   (b) Med-Low (15 to 20)
   (c) Med (20 to 25)
   (d) Med-High (25 to 30)
   (e) High (> 30)
   (f) not answerable; relevant colors too similar

Preference Question

61. You just answered questions about Map 7 and Map 8. Which map’s color scheme was easiest to understand:
   (a) Map 7
   (b) Map 8
D.11 Map 9: Balance, Rendition 1

Content Questions

62. Which of the following most closely describes the trend in sex ratio of children under five years of age from Neshoba County along the arrow to Newton County?

(a) increase in Males per 100 Females
(b) decrease in Males per 100 Females
(c) no change in Males per 100 Females
(d) not answerable; relevant colors too similar

63. Which of the following most closely describes the trend in sex ratio of children under five years of age from George County along the arrow to Jackson County?

(a) increase in Males per 100 Females
(b) decrease in Males per 100 Females
(c) no change in Males per 100 Females
(d) not answerable; relevant colors too similar

Matching Questions

64. The sex ratio of children under five years of age in Franklin County is

(a) > 110.0 Males
(b) 105.0 to 110.0 Males
(c) 95.0 to 104.9 Males
(d) 90.0 to 94.9 Males
(e) < 90.0 Males
(f) not answerable; relevant colors too similar
65. The sex ratio of children under five years of age in Newton County is
(a) > 110.0 Males
(b) 105.0 to 110.0 Males
(c) 95.0 to 104.9 Males
(d) 90.0 to 94.9 Males
(e) < 90.0 Males
(f) not answerable; relevant colors too similar

66. The sex ratio of children under five years of age in Pearl River County is
(a) > 110.0 Males
(b) 105.0 to 110.0 Males
(c) 95.0 to 104.9 Males
(d) 90.0 to 94.9 Males
(e) < 90.0 Males
(f) not answerable; relevant colors too similar

D.12 Map 10: Balance, Rendition 2

Content Questions

67. Which of the following most closely describes the trend in sex ratio of children under five years of age from Vernon County along the arrow to Rapides County?
(a) increase in Males per 100 Females
(b) decrease in Males per 100 Females
(c) no change in Males per 100 Females
(d) not answerable; relevant colors too similar

68. Which of the following most closely describes the trend in sex ratio of children under five years of age from Cameron County along the arrow to Calcasieu County?
(a) increase in Males per 100 Females
(b) decrease in Males per 100 Females
(c) no change in Males per 100 Females
(d) not answerable; relevant colors too similar
Matching Questions

69. The sex ratio of children under five years of age in Concordia County is
   (a) > 110.0 Males
   (b) 105.0 to 110.0 Males
   (c) 95.0 to 104.9 Males
   (d) 90.0 to 94.9 Males
   (e) < 90.0 Males
   (f) not answerable; relevant colors too similar

70. The sex ratio of children under five years of age in Terrebonne County is
   (a) > 110.0 Males
   (b) 105.0 to 110.0 Males
   (c) 95.0 to 104.9 Males
   (d) 90.0 to 94.9 Males
   (e) < 90.0 Males
   (f) not answerable; relevant colors too similar

71. The sex ratio of children under five years of age in St. Tammany County is
   (a) > 110.0 Males
   (b) 105.0 to 110.0 Males
   (c) 95.0 to 104.9 Males
   (d) 90.0 to 94.9 Males
   (e) < 90.0 Males
   (f) not answerable; relevant colors too similar

Preference Question

72. You just answered questions about Map 9 and Map 10. Which map’s color scheme
   was easiest to understand:
   (a) Map 9
   (b) Map 10
D.13  Map 11: Qualitative Area, Rendition 1

Content Questions

73. How many lots with the land use category 'public facilities' are located within the outlined block?
   (a) 0
   (b) 3
   (c) 6
   (d) 9
   (e) 12
   (f) not answerable; relevant colors too similar

74. How many lots with the land use category 'residential, elevator' are located within the outlined block?
   (a) 0
   (b) 3
   (c) 6
   (d) 15
   (e) 17
   (f) not answerable; relevant colors too similar

Matching Questions

75. The land use category of the lot containing the dot is
   (a) Residential, Walk-Up
   (b) Residential, Elevator
   (c) Mixed Use
   (d) Commercial
   (e) Industrial
   (f) Public Facilities
   (g) not answerable; relevant colors too similar
76. The land use category of the lot containing the dot is

(a) Residential, Walk-Up
(b) Residential, Elevator
(c) Mixed Use
(d) Commercial
(e) Industrial
(f) Public Facilities
(g) not answerable; relevant colors too similar

77. The land use category of the lot containing the dot is

(a) Residential, Walk-Up
(b) Residential, Elevator
(c) Mixed Use
(d) Commercial
(e) Industrial
(f) Public Facilities
(g) not answerable; relevant colors too similar

D.14 Map 12: Qualitative Area, Rendition 2

Content Questions

78. How many lots with the land use category ‘public facilities’ are located within the outlined block?

(a) 0
(b) 1
(c) 3
(d) 6
(e) 8
(f) not answerable; relevant colors too similar
79. How many lots with the land use category ‘residential, elevator’ are located within the outlined block?

(a) 0
(b) 3
(c) 6
(d) 12
(e) 28
(f) not answerable; relevant colors too similar

Matching Questions

80. The land use category of the lot containing the dot is

(a) Residential, Walk-Up
(b) Residential, Elevator
(c) Mixed Use
(d) Commercial
(e) Industrial
(f) Public Facilities
(g) not answerable; relevant colors too similar

81. The land use category of the lot containing the dot is

(a) Residential, Walk-Up
(b) Residential, Elevator
(c) Mixed Use
(d) Commercial
(e) Industrial
(f) Public Facilities
(g) not answerable; relevant colors too similar
82. The land use category of the lot containing the dot is

(a) Residential, Walk-Up
(b) Residential, Elevator
(c) Mixed Use
(d) Commercial
(e) Industrial
(f) Public Facilities
(g) not answerable; relevant colors too similar

Preference Question

83. You just answered questions about Map 11 and Map 12. Which map’s color scheme was easiest to understand:

(a) Map 11
(b) Map 12
Bibliography


R Core Team (2015), *stats: R statistical functions*. R package version 3.2.2.


