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# A R&D Based Real Business Cycle Model

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# A R&D Based Real Business Cycle Model

By

Ka Wai Terence Fung

A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, the City University of New York

2011

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This manuscript has been read and accepted for the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

## A R&D Based Real Business Cycle Model

by

Terence Fung

Adviser: Professor Jonathan Conning

The New Keynesian Real Business Cycle model with staggered price adjustment is augmented with a R&D producing sector. Two sources of economic shocks are separately considered, namely random participation (perturbances to value of alternative investment opportunities in another sector) and financial intermediation (shocks to the cost of raising capital in the financial intermediation market). We find that, when comparing to the baseline model, both random participation and financial intermediation models can explain pro-cyclical R&D spending. Additionally the investment oversensitivity problem is corrected. However, only the financial intermediation model is consistent with the observed finding that the volatility of R&D is larger than that of investment and output.

## Acknowledgements

I'd like to take this opportunity to thank Professor Conning for his patience and guidance through all these years. I was very fortunate to have received such tremendous support from Professor Thurston and Professor Chow. You are always there for me when I was in trouble.

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Although I did not have much contact with Professor Neftci, no one at the Graduate Center understands me better than he does. I should have met him at the Hong Kong University of Science and Technology a couple years ago.

At last, I'd like to thank my parents for all of their love and encouragement. Though we all went through very difficult times in the past few years, today, we stay united, as a family.

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# Chapter 1

## Introduction and the Baseline Model

### 1 Introduction

There are two competing theories in the growth literature. The Solow and Swan neoclassical model, that attributes growth to the accumulation of capital and exogenous technological change. The Romer (1990), Aghion and Howitt (1997) (hereafter AH model) style models, that treat technological development as a product of deliberate human effort - the accumulation of Research and Development (R&D) effort. However, ever since the seminal work of Prescott and Kydland (1982), in which the neoclassical model with exogenous technology shocks was demonstrated to be capable of exhibiting dynamics similar to those observed U.S. data<sup>1</sup>; the Solow and Swan model reigns in business cycle theory.

This paper attempts to fill in the gap by combining the endogenous growth and business cycle theory. It focuses on ties between R&D and financing, in ways different from the standard real business cycle model. The source of growth in the Solow and Swan model is exogenous technological change. Cycles transpire when some external shocks hit the economy, reducing the aggregate productivity during recession, vice versa for boom. Since deliberate R&D effort drives the economy in the endogenous growth model, we will consider two shocks in the R&D production sector. Nonetheless, the proportion of R&D is at most 5% of GDP in the U.S. economy, is it possible to demonstrate that shocks in this smaller sector can generate dynamics similar to the standard shocks? If yes, how much new insight can we get from it? Does it explain some stylized facts that traditional models fail to replicate?

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<sup>1</sup>For a survey and comparison of real business cycle models, see Rebelo (2005).

This paper is structured as follows. In chapter 1, section 2 outlines the New Keynesian staggered price model augmented with a R&D production sector. Section 1 of chapter 2 describes the first source of shocks to the economy- one where perturbations arise from imperfections in labor matching in the R&D sector. We will discuss the parameters chosen for the calibration and examine the impulse response functions. Chapter 3 deals with another variant of the model: a surprise in the financial intermediation sector that would affect capital accumulation. Chapter 4 compares the calibrated second moments and impulse response functions to U.S. data.

## **1.1 Literature Review and Motivation**

Important endogenous growth models include Romer (1990), Aghion and Howitt (1997), and Grossman and Helpman (1991). The specific one considered in this paper is Aghion and Howitt (1997). We will examine the impact of interest rate, government and productivity shocks on R&D and output. For a nominal shock to have effect on real variables, certain forms of market imperfections have to be introduced. Thus, we synthesize the New Keynesian Real Business Cycle model (Clarida, Gali and Gertler (1999) with staggered prices, with the AH R&D producing sector. This forms the baseline model. An important underlying assumption is the existence of a perfectly competitive capital market. How does a R&D firm pay for its expense before the investment turns profitable? Aghion and Howitt (1997) assume that either the intermediate firms own it and provide the funding; or the R&D firm borrows money and repays the loan by selling the patent to the intermediate firms after success. The second assumption is maintained in this chapter.

The financing of innovation projects is not likely to be perfect. Instead of modelling complicated contractual arrangements between R&D firms and a third party, we propose two alternate sources of economic perturbations to model financial frictions: financial intermediation shocks, and separately, random participation in R&D sector. The former addresses the difficulty of funding risky R&D projects. The latter can be construed as a change in outside opportunities that drags away resources from the R&D sector, similar to the Rybczynski effect in international trade<sup>2</sup>. Both models suggest that R&D is procyclical. Our calibrated financial intermediation model is consistent with the stylized fact that the volatility of R&D is larger than that of investment and output - a result that models based on technology or interest rate shocks cannot replicate.

Combining the endogenous growth and business cycle models might add new insight. In fact, there has been no lack of criticisms of the Real Business Cycle Models (RBC). Bernanke and Parkinson (1991) argue that since the technology shock is roughly the same as Solow residual; if technology shocks are the source of economic fluctuations, then the covariance of Solow residuals and output should be positive. Since it is generally believed that the Great Depression cannot be attributed to a technology shock but the postwar recessions is, then Solow residuals and output should move together only in the postwar period. However, they find that the co-movement between Solow residuals and output has remained relatively intact. RBC theorists posit that an exogenous shock hits the economy, and it propagates over time due to workers' intertemporal substitution of leisure. To be more specific, as a positive technology shock increases the marginal product of labor and real wages, workers are more willing to substitute labor for leisure during economic booms, and vice versa for recessions. Taken literally,

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<sup>2</sup>Assume that there are two industries (labor intensive and capital intensive) using two inputs (labor and capital); Rybczynski (1955) proves that endowment increases raises the output of the industry using that factor intensively and decreases the output of the other industry. The Dutch disease is one notable example of this type of phenomenon.

this implies that there is no involuntary unemployment during a recession. Empirically, it seems difficult, if not impossible for ideas to disappear. Hence, the idea of technological regress is impossible. Two modifications have been proposed: either assume new forms of innovations, for example government expenditure surprises (King Plosser and Rebelo (1988) ), money shocks (Gertler and Gali (1999)), sunspot activity (Farmer and Guo (1994)) or departure from the neoclassical framework.

### **1.1.1 R&D and Growth**

A natural alternative to the RBC model is a business cycle model with endogenous technology. Most of endogenous growth theorists recognize the importance R&D on growth. Romer (1990) contends that " technical change arises in large part because of intentional actions taken by people who respond to market incentives." Thus, technical change is endogenous, instead of exogenous as portrayed in the standard neoclassical model. Obviously, R&D is one of those "intentional actions", that can enhance the productivity of existing capital, improve the quality of a continuum of products (Grossman and Helpman (1991)) or create new products (Young (1995)). In RBC models based on the neoclassical growth framework, R&D is simply an alternative form of capital investment. However, R&D and capital are very different in nature. First, technology and R&D are nonrival and, to a large extent, only partially excludable, but capital is not. Second, capital return is the sum of the real interest rate and depreciation. R&D is very risky and it requires a higher expected return as compensation. Third, strong spillover effect of R&D is well documented. For instance, using the coefficient estimates from regressions of total factor productivity growth of advanced economy on R&D investment, Jones and Williams (1998) show analytically that while the private return of R&D ranges 7 to 14 percent, the lower

bound of social return is 30 percent. Thus, the optimal R&D is at least two to four times actual investment. Below, we will briefly review the literature about how R&D contributes to growth at firm and macroeconomic levels.

Using data compiled by Scherer (1981) , Griliches and Lichtenberg (1984) examine the relationship between total factor productivity (TFP) growth and R&D intensity. They argue that effective quantity of intermediate input is not measured accurately. No adjustment is made for input quality improvement. Griliches and Lichtenberg (1984) derive a revised version of measured TFP which "deviates from its actual TFP growth by the weighted sum of errors in the various materials deflators, using respective cost shares as weights". Using more disaggregated data than Scherer (1981), they confirm the positive relationship between TFP and R&D intensity. The authors proceed to break down R&D into own process, own product and imported product R&D (intermediate input quality improvement). Using average TFP of different years<sup>3</sup>, they conclude that the coefficients of own process and own product R&D are largely significant.

Imitation and competition, nonetheless, can reduce the potential gains of R&D. Extending the Grossman and Helpman (1991) quality ladder model, Segerstrom (1991) develops a dynamic general equilibrium model which allows both innovative and imitative activities. The winner of next-generation highest quality ladder will make monopoly profits; however, the expected payoff is less than that of Grossman and Helpman (1991) because of imitation threat. The model predicts that the innovative intensity has to exceed a threshold value for subsidy to be welfare enhancing; thus the impact of a government subsidy on R&D is ambiguous.

Scherer and Ravenscraft (1982), argue that both imitation and competition may depress R&D returns. However, Kamien and Schwartz (1978) shows that competition and imitation

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<sup>3</sup>Only a single year R&D data (1974) is available in the Scherer (1981) data. The dependent variables are TFP growth rates in other years.

would not undermine the growth effect of R&D. By optimal control theory, Kamien and Schwartz (1978) derive the optimal R&D spending time path and the planned introduction date of innovation when the R&D race is drastic (i.e., the new technology renders the old one unprofitable); and when the projects are fully funded by internal financing. They find that the cash constraint is not so prohibitive. As long as diminishing returns, market interest rates or technology survival rate are not too high, the cash constraint will not be binding. Therefore, it will be optimal to carry out risky R&D projects.

Jaffe (1986) estimates the effect of firm level R&D expenditure on the number of patent applications, firm's market value and profits, controlling for technological opportunity and R&D spillover. However, the R&D spillover, defined as the positive external production effect among firms, is unobservable. Following the work of Griliches (1979), Jaffe (1986) assumes that firms filing the same categories of patents should have higher spillover on each other. The spillover effect on a particular firm is a weighted sum of all other firms' R&D expenditures. The weight is a measure of firm proximity, which he applies the mathematical concept of weighted norm. Each firm has a vector of relative R&D expenditure. Two firms are considered to be close if the vector product is large. The author estimates the elasticity of patent applications, profit and market value -which is approximated by Tobin's  $q$ - with respect to R&D expenditure, spillover and the interaction of these two factors. By the Three Stage Least Square estimation method, the results support positive effect of R&D expenditure on growth. If each firm increases R&D by 10 percent, total patents would increase by 20 percent. The social rate of return for R&D is 27 percent.

Decomposing public sector capital into public infrastructure and publicly-financed R&D, Nadiri and Mamuneas (1994) estimate the cost savings from public capital on twelve two-digit SIC industries. Using intermediate input as the numeraire good, Nadiri and Mamuneas (1994)

express the cost in translog functional form. The average cost determinants are labor, capital, a deterministic time trend, public infrastructure, public-financed R&D and the interaction terms of these variables. Using a panel with 360 observations over the sample period 1956-1986, they find that the cost elasticities estimates of public infrastructure range from -0.1089 to -0.2113; while the cost elasticities of R&D range from -0.0088 to -0.0557. All estimates are statistically significant except for one industry. Hence, the impact of infrastructure on cost is stronger.

The effect of R&D expenditure on profit is estimated by Connolly and Hirschey (1984). Since current R&D affects current and future profits, the strong feedback effect renders the OLS estimates biased. The authors use a measure called relative excess valuation as an instrument of profit. The market value of a firm (observable) has two components: tangible and intangible assets. A proxy of the former is book value of stocks, thus the latter can be estimated as a residual, which includes R&D and advertising. To deal with the feedback effect problem, a system of four equations are estimated simultaneously. The endogenous variables are relative excess value, R&D spending, advertising expenditure and market concentration ratio. Two Stage Least Squares estimation results strongly support a simple positive relation between R&D on relative excess value.

### **1.1.2 R&D and the Business Cycle**

Despite the importance of R&D in the growth literature, relatively less effort has been made to synthesize the theory of short-run economic fluctuations with endogenous growth theory. Howitt (1998) is one of those few who attempt, in which final good and R&D sectors compete for labor use. He contends that the arrival of new 'general technological knowledge' retards the current growth rate due to withdrawal of labor from manufacturing to the research sector. But his model has some shortcomings: the time lag between the arrival of new technological know-how

and the subsequent economic boom is unreasonably long, the business cycle is deterministic, and it predicts that output is countercyclical.

Suarez and Sussman (1997) construct a model such that cycles are driven by defaults and financial distress. In their model, producers borrow money from financial institutions. The profit of the former is unobservable, so the producers have incentive to default. The enforcement problem can be resolved by writing a contract, that specifies the interest payment and articulates the financial institution's right of liquidating the firms. When the production firms are illiquid at the end of rising asset prices (e.g. end of a boom, beginning of a downturn), the production firms may be forced to default, and the financial institutions are more likely to liquidate the firms, ending the life of a profitable firm earlier. However, the cycle is still deterministic.

Others simply augment the endogenous growth models with exogenous shocks. Schmitt-Ghrohe (1997) studies the business cycle fluctuations predicted by a two-sector endogenous growth model with sector-specific external increasing returns to scale. He focuses on the autocorrelations between output, hours and consumption. In essence, the model is a learning-by-doing business cycle model, in which aggregate investment is the source of increasing returns. The calibrated second moment of key variables is in sync with filtered time series. However, once the impulse response functions are examined, the impact of a shock is miniscule. Canton (2002) analyzes the impact of cyclical volatility on long-term growth. An i.i.d shock is introduced into the Lucas-Uzawa model's human capital accumulation equation. Stadler (1990) shows that, with endogenous technical change, both real and money models yield very similar output processes. Learning-by-doing is again the source of externality, which is constantly perturbed by an independent and identically distributed shock. Aggregate productivity is a positive function of average labor.

All of the above models share three shortcomings. First, Boldrin and Woodford (1990) contends that most endogenous business cycle models share the same spirit as the neoclassical real business cycle models, in particular, an exogenous shock hitting the aggregate production function. Second, almost all real business cycle models rely on the assumption of a steady state. It is believed that output deviates from its long-run trend by exogenous technology shocks-no matter how one interprets the shock-output eventually will return to the trend. That justifies the log-linearization procedure. However, in models like Stadler (1990), a steady state simply does not exist, not to mention stability. Bolieau and Normandin (2005) show that some international real business cycle models with incomplete financial adjustment do not possess a deterministic steady state. They also warn that theorists should ensure the choice of parameters is consistent with a stable system of stochastic differential equations. Third, the Research and Development (R&D) sector-the essential component in most endogenous growth models- is omitted.

Our model is different from the earlier works, in the way that the financial market is the source of economic fluctuation. King and Levine (1993) demonstrate that a higher degree of financial development, in particular lower agency cost of raising capital, has positive growth effects. The agency cost is randomized in our model. The idea is that when positive R&D news hits the market, it lowers the cost of raising capital, thus affecting current output. As argued above, it is hard to apprehend the concept of a negative technology shock ( for example, ideas cannot disappear). A negative R&D shock can however be interpreted easily. It can be due to unexpected delays of new invention, unsatisfactory research progress, termination of a project or competitors patent. The financial intermediation model should work for developed countries. We will compare the calibrated moments to the observed U.S. data.

## 1.2 Empirical Motivation

The above papers establish the role of R&D in the growth literature. What about the effect of R&D expenditure on output fluctuation? Using the 1953-2003 U.S. annual industry R&D expenditure from National Science Foundation and real GDP from the Bureau of Economic Analysis (BEA), we test if there exists a long run equilibrium relation between the two series by cointegration; and if it does, the adjustment coefficient will be estimated by the Error Correction Model. Figure 1.1 and 1.2 are the Autocovariance functions (ACF) of log series of R&D and GDP respectively. Obviously, the two series are highly persistent and nonstationary. The first differenced series are, however, stationary, as shown at figure 1.3 and 1.4.

We proceed to test for unit root by the Philips-Perron test of the log series<sup>4</sup>. Assuming that the trend is a constant, the Phillips Perron test statistics are -2.595 (p=0.1) and -0.76 (p=0.82) for log R&D expenditure and log real GDP respectively. The null hypothesis of unit root cannot be rejected. Then we test for the existence of cointegrating factor. Assuming that the trend is constant, the trace statistics is 17.67, which is significant at both 1% and 5% level. *Hence, there exists a long-run equilibrium relation between US R&D expenditure and GDP. The two series show similar cyclical pattern.* The normalized cointegration factor is (1, -1.1) and the Vector Error Correction Model (VECM) is reported as follows:

$$\begin{bmatrix} \Delta rd_t \\ \Delta gdp_t \end{bmatrix} = \begin{bmatrix} 0.721 \\ (0.21) \\ 0.0957 \\ (0.11) \end{bmatrix} + \begin{bmatrix} 0.16 \\ (0.046) \\ 0.014 \\ (0.024) \end{bmatrix} \begin{bmatrix} 1 & -1.1 \\ (0.08) \end{bmatrix} \begin{bmatrix} rd_{t-1} \\ gdp_{t-1} \end{bmatrix} +$$

---

<sup>4</sup>The augmented Dickey-Fuller test is not used because the test statistics is contingent on the selection of autocorrelation order. Philips -Perron test does not depend on AR order and it accounts for heteroskedasticity and serial correlation.

$$\begin{bmatrix} 0.47 & 0.52 \\ (0.11) & (0.29) \\ 0.014 & 0.043 \\ (0.024) & (0.147) \end{bmatrix} \begin{bmatrix} \Delta rd_{t-1} \\ \Delta gdp_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{bmatrix}$$

### 1.3 Model Outline

We will develop the New Keynesian business cycle model (Clarida, Gali and Gertler (1999), Gertler (1992)) which generates endogenous growth (Aghion and Howitt (1997)). The outline of the model is illustrated at diagram 1. The competitive final good sector combines a continuum of intermediate goods into a single final good, in the spirit of Dixit and Stiglitz (1977). The monopolistic intermediate sector uses capital and labor as inputs, taking the demand from the final goods sector as given. The production is a modified neoclassical Cobb-Douglas function. With the assumption of monopolistic competition, sticky prices, money and other nominal rigidities can easily be incorporated into the model. The intermediate firms charge a countercyclical price markup, as portrayed in Rotemberg and Woodford (1995). The consumers supply capital and labor to the intermediate firms. They own the shares of the intermediate firms. At the end of each market period, the consumers sell the used capital to a capital producing sector and repurchase them later. The capital producing firm uses last period's capital and new investment from the final goods sector to produce the current capital stock which to be used in the intermediate good's sector. Capital output is a function of the investment good - capital stock ratio.

An innovation production sector is added to the Gali and Gertler type model. As in Aghion and Howitt (1997), the invention of intermediate goods along the quality ladder, stemming from R&D is the source of endogenous economic growth<sup>5</sup>. In the Aghion and Howitt

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<sup>5</sup>Romer (1990), in contrast, ascribes growth to increasing varieties of intermediate goods- that is horizontal growth.

(1997) model, the R&D firms use labor and capital as inputs. We deviate from the early work and adopt a simple assumption that the final good is the sole input. Technology innovations arrive randomly with a constant Poisson arrival rate, which is a positive function of R&D input. In this way, while the stock of technology accumulates over time with a deterministic long-run trend, unlike other real business cycle models, technology regress is ruled out.

The R&D sector delivers blueprints to the intermediate firms. Their relation can be modeled in different ways. For instance, it can be portrayed in a principal-agent relation. The R&D firms enter into contractual agreement with the intermediate firms, which provided funding for innovation and promise to buy the next generation technology. Alternatively, the financing of R&D projects is supported by a perfect capital market. They repay the debt by selling the next generation patent to the intermediate firms. The former is the setting of chapter two and the latter for chapter three. We intend to model two types of economic perturbation in R&D sector related to financing. Two modifications will be made to the baseline model of chapter 1. The impulse response functions and calibrated moments will be compared in chapter 4.

That completes the outline of the New Keynesian with an augmented R&D sector. The next section is devoted to the delineation of the baseline model.

## **2 General Equilibrium Model**

### **2.1 The Representative Consumer**

This section establishes the baseline model. Section 2.1 - 2.4 follows closely the New Keynesian business cycle model with staggered prices by Clarida, Gali and Gertler (1999) and Gertler (1998)<sup>6</sup>. To extend the model, a new R&D sector will be added in the next section. Moreover, we assume that the only form of financial asset that consumers hold is a zero coupon bond,  $(B_t)$ .

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<sup>6</sup>Staggered price is necessary for nominal shocks to have impact on real variables.

Money plays no role in the utility function. A continuum of infinitely-lived consumers, indexed from zero to one, in each market period, purchase a final good ( $C_t$ ). They provide labor ( $N_t$ ) to the intermediate firms, and rent physical capital ( $K_t$ ) to them. The representative household chooses final good consumption, labor supply, capital supply and quantity of real bond holdings at each market period to maximize the following time separable utility function conditional on information available at time  $t$ :

$$E_t \left\{ \sum_{i=0}^{\infty} \beta^i \left[ \frac{1}{1-\gamma} C_{t+i}^{1-\gamma} - \frac{a_n}{1+\gamma_n} N_{t+i}^{1+\gamma_n} \right] \right\} \quad (1)$$

subject to

$$C_t = \frac{W_t}{P_t} N_t + Z_t K_{t-1} + \int_0^1 \omega(z) \pi_t(z) dz + TR_t - \frac{\frac{1}{R_{t+1}^n} B_t - B_{t-1}}{P_t} - Q_t [K_t - (1-\delta)K_{t-1}] \quad (2)$$

where  $\gamma < 1, \gamma_n > 0$  are the consumption and labor elasticity of substitution respectively.

$\beta$  is the subjective discount rate.  $a_n$  is the disutility coefficient of labor.  $\delta$  denotes the depreciation rate of capital. These are the exogenous parameters.

$\frac{W_t}{P_t}$  is the real wage.  $Z_t$  is the rental cost of capital. The consumers own the monopolistic intermediate firms and the final good production sector.  $\pi_t(z)$  is the profit from owning intermediate firm  $z$ ,  $\omega(z)$  is the number of shares of the intermediate firms<sup>7</sup>. The government transfers  $TR_t$  to the consumers each period.  $R_{t+1}^n$  is the nominal interest rate.  $\frac{1}{R_{t+1}^n}$  denotes the price of a zero coupon bond that pays one dollar next period.  $Q_t$  is the price of renewed capital.

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<sup>7</sup>Ownership of final good sector is irrelevant. Constant returns to scale guarantees zero profit, so it does not enter the consumers' budget constraint.

For simplicity, it is assumed that utility is separable in consumption ( $C_t$ ) and labor ( $N_t$ ). Unlike many other more simple setups, in this model, the consumption good is not a perfect substitute for capital. Equation (2) is the consumer's budget constraint. The representative household supplies labor ( $N_t$ ) to the intermediate good sector, earning a return of  $\frac{W_t}{P_t}$  each period. He possesses the initial capital ( $K_{t-1}$ ), rents it to the intermediate firms, making a return of  $Z_t$ . They own the intermediate firms. The total profit is  $\int_0^1 \omega(z)\pi_t(z)dz$ . They invest in the bond market, making a nominal return of  $R_{t+1}^n$ . At the end of each market period, the household enhances the capital by selling it to the capital producers at a price  $Z_t^k$ -that will be zero around steady state as shown in section 2.4 - and repurchase it at a price,  $Q_t$ .

The functional equation can be represented by the Bellman equation:

$$v\left(\frac{B_{t-1}}{P_t}, K_{t-1}\right) = \max\left[\frac{1}{1-\gamma} C_t^{1-\gamma} - \frac{a_n}{1+\gamma_n} N_t^{1+\gamma_n} + \beta E_t v\left(\frac{B_t}{P_{t+1}}, K_t\right)\right] \quad (3)$$

Substituting the budget constraint into the objective function, define real interest rate as

$$R_{t+1} \equiv R_{t+1}^n E_t \left(\frac{P_t}{P_{t+1}}\right) \quad (4)$$

we get the following first order conditions:

$$\frac{W_t}{P_t} = a_n \frac{N_t^{\gamma_n}}{C_t^{-\gamma}} \quad (5)$$

$$C_t = E_t (R_{t+1}^n \beta C_{t+1}^{-\gamma})^{\frac{1}{\gamma}} \quad (6)$$

$$1 = E_t \left\{ R_{t+1}^n E_t \left(\frac{P_t}{P_{t+1}}\right) \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \right\} \quad (7)$$

$$1 = E_t \left\{ \frac{Z_{t+1} + (1-\delta)Q_{t+1}}{Q_t} \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma} \right\} \quad (8)$$

Equation (5) is the marginal rate of substitution between consumption and leisure. Equation (6) is the intertemporal consumption substitution condition. Equation(7) represents the Euler condition, while equation (8) denotes the capital supply condition.

## 2.2 Final Goods Output

The final good is produced by combining different varieties of differentiated intermediate goods  $z \in [0,1]$ . Without loss of generality, the number of varieties is held constant, meaning all innovation is aimed at improving existing varieties<sup>8</sup>. The final good sector is perfectly competitive. The CES production is given by:

$$Y_t^f = \left( \int_0^1 Y_t^f(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (9)$$

$\frac{\varepsilon}{\varepsilon-1}$  is the constant elasticity of substitution between the intermediate goods.

A la Dixit-Stiglitz CES production exhibits diminishing marginal return, a property that will drive the firms to diversify and produce using all the intermediate goods. The final good firms choose  $Y_t^f(z)$  to minimize cost<sup>9</sup>:

$$\begin{aligned} & \text{Min}_{Y_t^f(z)} \int_0^1 P_t(z) Y_t^f(z) dz \\ & \text{s.t} \left( \int_0^1 Y_t^f(z)^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \geq \bar{Y} \end{aligned} \quad (10)$$

Using the fact that elasticity of substitution between final good and intermediate good is 1, the first order condition, and equating supply and demand, the lagrangian multiplier can be verified

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<sup>8</sup>For a treatment of increasing varieties, see Young (1995). Basically, there will be an additional equation that determines growth rate of varieties. The key implications of the model remain unchanged.

<sup>9</sup>With the assumption of constant returns to scale and perfect competition, the size of each firm is indeterminate. We, therefore, derive the input demand by cost minimization.

to be  $P_t$ , i.e. the composite price index; while the optimal demand for intermediate good is given by:

$$Y_t(z) = \left( \frac{P_t(z)}{P_t} \right)^{-\varepsilon} Y_t \quad (11)$$

The demand price elasticity is the constant  $\varepsilon$ . The composite price index is given by:

$$P_t = \left( \int_0^1 [P_t(z)]^{1-\varepsilon} dz \right)^{\frac{1}{\varepsilon-1}} \quad (12)$$

The baseline model considered here is different from Gertler (1998) in the way that, the final good is used not only for consumption; the capital producers and R&D firms purchase it as input.

### 2.3 The Intermediate Sector

There is a continuum of monopolistically competitive firms owned by consumers, with compact support  $z \in [0,1]$ . The profit is reimbursed to the consumers at the end of each period. Each firm  $z$  faces the downward sloping demand given by equation (11). Gertler (1998) assumes that the intermediate firm uses both capital and labor as inputs. The production technology takes Cobb-Douglas functional form:

$$Y_t(z) = A_t N_t(z)^\alpha K_t(z)^{1-\alpha} \quad (13)$$

where  $A_t$  is a technology parameter, and its growth rate is determined by R&D sector.

$N_t(z)$  and  $K_t(z)$  are the labor and capital input demands respectively. Consumers supply labor and rent capital to intermediate firms. The relative wage is  $\frac{w_t}{P_t}$  and the rental price is  $Z_t$ .

The intermediate goods are sold to the final good firms at  $P_t$ , which can be fixed at each market period. The price adjustment mechanisms will be discussed at next subsection.

The input demand is derived by cost minimization.

$$\text{Min}_{N_t(z), K_t(z)} \left[ \frac{W_t}{P_t} N_t(z) + Z_t K_t(z) \right] \quad (14)$$

$$\text{s.t. } A_t N_t(z)^\alpha K_t(z)^{1-\alpha} - Y_t(z) \geq 0 \quad (15)$$

The first order conditions give:

$$\alpha \frac{Y_t(z)}{N_t(z)} = (1 + \mu_t) \frac{W_t}{P_t} \quad (16)$$

$$(1 - \alpha) \frac{Y_t(z)}{K_t(z)} = (1 + \mu_t) Z_t \quad (17)$$

where the markup  $\mu_t$  is the inverse of real marginal cost, i.e.  $1 + \mu_t = \frac{1}{MC_t}$ <sup>10</sup>. Equations (16) and

(17) relate the demand for labor and capital to the mark-up wages and rental cost.

### 2.3.1 Optimal Pricing Setting

The staggered price component of Gertler (1998) follows closely the Calvo (1983) model. Price adjustment follows a Bernoulli distribution, with a probability  $\theta$  that price remains fixed. In each market period, only  $(1 - \theta)$  fraction of intermediate firms can readjust prices. Since the draw is independent of history and we do not need to keep track of firms changing prices. The expected time over which the price is fixed, i.e., the expected waiting time for the next price adjustment is  $\frac{1}{1 - \theta}$ .<sup>11</sup>

In each period, a fraction of  $\theta$  firms will be able to change price to maximize profit.

The remaining  $1 - \theta$  firms can only adjust output to meet demand. Because of constant returns to scale, the size of intermediate firms cannot be determined.

<sup>10</sup> Note that the mark-up is time-variant. Equation (16) is one of the profit maximizing condition of a monopolistic firm that marginal revenue product equal to a mark-up of marginal cost.

<sup>11</sup> Alternatively, we can assume an adjustment cost:  $AC_{j,t}^Y = \frac{\phi_y}{2} \left( \frac{P_{j,t}}{P_{j,t-1}} - \hat{\mu} \right)^2 Y_t$  where  $\phi_y$  measures the degree to which firms dislike to deviate in their price setting behavior from the steady state inflation rate  $\hat{\mu}$ . Sbordone (1998) contends that this price setting is observationally equivalent to Calvo's model.

Define  $MC_t^n$  as the intermediate firm's nominal marginal cost -  $MC_t^n = P_t MC_t$ , and

$\Lambda_{t,i} = \left(\frac{C_{t+i}}{C_t}\right)^{\gamma}$  as the ratio of the marginal utility of consumption at  $t+i$  to marginal utility at  $t$ .

A firm that is allowed to change its price at time  $t$  chooses  $P_t(z)$  to maximize:

$$E_t \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,i} \left[ \frac{P_t(z)}{P_{t+i}} Y_{t,t+i}(z) - \frac{MC_{t+i}^n}{P_{t+i}} Y_{t,t+i}(z) \right] \quad (18)$$

$$s.t. Y_{t,t+i}(z) = \left( \frac{P_t(z)}{P_{t+i}} \right)^{-\varepsilon} Y_{t,t+i}$$

where  $\beta^i \Lambda_{t,i}$  is the stochastic discount factor at  $t+i$ ,  $Y_{t,t+i}(z)$  is the demand for intermediate goods from  $t$  to  $t+i$ . The intermediate firms take  $MC_t^n, P_t$  and  $Y_t$  as given. The first order condition gives

$$P_t^* = (1 + \mu) \sum_{i=0}^{\infty} \varphi_{t,i} MC_{t+i}^n \quad (19)$$

where

$$\varphi_{t,i} = \frac{E_t \left[ (\theta\beta)^i \Lambda_{t,i} Y_{t,t+i} \left( \frac{1}{P_{t+i}} \right)^{-\varepsilon} \right]}{E_t \left[ \sum_{i=0}^{\infty} (\theta\beta)^i \Lambda_{t,i} Y_{t,t+i} \left( \frac{1}{P_{t+i}} \right)^{-\varepsilon} \right]} \quad (20)$$

Hence, desired price is a weighted average of the marginal cost in the future. The weight is a function of discounted income at  $t+i$ , and expected life time income. The perfect price adjustment case is given by setting  $\theta = 0$ . Last, given that all firms that adjust price in time  $t$  choose the same price, the average price of firms that do not adjust is simply price of last period.

The price index can be expressed as:

$$P_t = \left[ \theta P_{t-1}^{1-\varepsilon} + (1 + \theta) P_t^{*1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \quad (21)$$

which is a weighted average of lagged price and optimal price.

## 2.4 Capital Production

We take the initial level of capital as exogenous. In the Gertler (1998) setup, there are a large number of identical competitive capital producers, indexed by  $j \in [0,1]$ . Households each period rent capital to intermediate firms, and households later sell it to the capital producers at the price  $Z_t^k$  and then repurchases it at the beginning of next period at the price of  $Q_t$ <sup>12</sup>. The production function for the new capital takes the form

$$Y_t^k(j) = \phi \left[ \frac{I_t(j)}{K_t(j)} \right] K_t(j)$$

$\phi(\cdot)$  is the adjustment cost in the growth literature, with the properties:  $\phi'(\cdot) > 0, \phi''(\cdot) < 0$ , and at steady state,  $\phi(0) = 0, \phi(\frac{I}{K}) = \frac{I}{K}$ , which is the steady state ratio of investment to capital. The capital production satisfies the Inada condition. The marginal value of product is positive and strictly decreasing with higher investment - capital ratio. Investment is perfect substitute of consumption good. The zero profit condition with respect to  $I_t(j)$  gives

$$Q_t \phi' \left( \frac{I_t(j)}{K_t(j)} \right) = 1$$

All capital firms are identical. Their optimal  $\frac{I_t(j)}{K_t(j)}$  is the same. The industry first order condition is given by

$$Q_t = \Phi \left( \frac{I_t}{K_{t-1}} \right) \quad (22)$$

where

$$\Phi \left( \frac{I_t}{K_{t-1}} \right) = \frac{1}{\phi' \left( \frac{I_t}{K_{t-1}} \right)} \quad (23)$$

and  $K_{t-1} = \int_0^1 K_t(j) dj$ . By construction  $\phi(\frac{I}{K}) = \frac{I}{K}$  at steady state. Thus,  $\phi'(\cdot) = 1$  and the

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<sup>12</sup> This approach is used in Gertler (2000) building on Dixit and Stiglitz (1977).

steady state  $Q$  is 1.  $Z_t^k$  can be ignored.

**Proposition 1.** The capital price  $Z_t^k$  is zero around steady state.

**Proof.** The first order condition for  $K_t(j)$  is:

$$Q_t \left[ \phi \left( \frac{I_t(j)}{K_t(j)} \right) - \frac{I_t(j)}{K_t(j)} \phi' \left( \frac{I_t(j)}{K_t(j)} \right) \right] = Z_t^k$$

By construction, in the steady state,  $\phi \left( \frac{I}{K} \right) = \frac{I}{K}$ ,  $\phi' \left( \frac{I}{K} \right) = 1$ , and  $Q = 1$ , then

$$Q \left[ \phi \left( \frac{I}{K} \right) - \frac{I}{K} \phi' \left( \frac{I}{K} \right) \right] = \left\{ \frac{I}{K} - \frac{I}{K} \right\} = 0$$

Q.E.D

Intuitively, one can interpret  $Q_t - Z_t^k$  as net cost of reinstalling capital, with  $Z_t^k = 0$  as normalization. We have delineated the New Keynesian model with fixed price. A modified Aghion and Howitt (1992) R&D sector will be appended.

## 2.5 Research Sector

The following extension to the model is built on Aghion and Howitt (1992). There is a continuum of research firms for each intermediate good  $z$ , indexed by  $z \in [0,1]$ . Each of them innovates by building on existing cutting-edge technology in the economy,  $A_t^{max}$  -which is shared by all R&D firms. A perfect capital market provides financing to each R&D firm who repays the debt by selling the patent to the intermediate firms.

The R&D and intermediate firm relation is different from AH, in which the research sector is portrayed as in the patent-race literature that has been surveyed by Tirole (1988) and Reinganum (1989) and the creative destruction by Schumpeter (1950). The innovator will become the monopoly in the next market period. Competition is drastic in the sense that the

existing firm will be replaced by the next innovator who can set price without restrictions from the incumbent. Chapter 2 will depict a principal-agency relation; and that explains the change of assumption. Unlike the AH model, each R&D firm uses final good  $(\Xi_t(i))$  as input, instead of

labor alone. Define  $\xi_t(i) = \frac{\Xi_t(i)}{A_t^{max}}$ , the industry Poisson arrival rate,  $\rho_t$ , is given by

$$\rho_t = \lambda \rho(\xi_t) \quad (24)$$

where  $\lambda > 0, \rho'(\cdot) > 0, \rho''(\cdot) < 0, \rho(0) = 0, \xi_t = \int_0^1 \xi_t(i) di$ .

$\lambda$  is the productivity parameter of research;  $\rho(\cdot)$  measures the intensity of research, exhibiting diminishing marginal product<sup>13</sup>. Hence, technology is not exogenous as hypothesized in neoclassical model. Instead, technological growth is the result of deliberate use of final goods. There exists no diffusion barrier across sectors, the cutting-edge technology becomes a public good to all R&D firms. Each innovation at date  $t$  by any firm  $i$  permits the innovator of all firms to produce using the leading edge technology. Define  $A_{z,t}$  as the technology level at intermediate sector  $z$ . Any innovation raises  $A_{z,t}$  by a constant factor of  $\gamma^\xi > 1$  for that firm. Each innovation raises technology up along the quality ladder, as depicted in Grossman and Helpman (1991).

It can be shown that the growth rate of the economy will be<sup>14</sup>:

$$g_t = \frac{A_t^{max}}{A_t} = \lambda \rho(\xi_t) \ln \gamma^\xi \quad (25)$$

At any point in time, there will be a distribution of productivity parameters  $A_{z,t}$  across

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<sup>13</sup>In the Barro and Sala-i-Martin, X (1994) model, the Poisson arrival rate is a negative function of task complexity. It shows diminishing returns and scale effect can be eliminated.

<sup>14</sup>The proof can be found at Aghion and Howitt (1997).

the sectors of the economy, with support  $[0, A_t^{max}]$ , shifting rightward over time. However, the long-run distribution of relative productivity parameters  $a = \frac{A_{z,t}}{A_t^{max}}$  will be stationary.

**Proposition 2.** The distribution of relative productivity parameters  $a$ , is invariant over time.

The stationary distribution is given by:

$$H(a) = a^{\frac{1}{\ln \gamma}}, 0 \leq a \leq 1 \quad (26)$$

Here, we replicate the proof from Aghion and Howitt (1992).

**Proof.**

Pick any  $A > 0$  that was the leading-edge parameter at some data  $t_0 \geq 0$ , and define  $\Phi(t) = F(A, t)$

$$\Phi(t_0) = 1$$

Because all sector are less advanced than  $A$ .

$$\frac{d\Phi(t)}{dt} = -\Phi(t)\lambda\rho(\xi_t(i)) \text{ for all } t \geq t_0 \quad (27)$$

After  $t_0$ , the rate at which the mass of sectors behind  $A$  falls is the overall flow of innovations occurring in sectors currently behind  $A$ . There are  $\Phi(t)$  such sectors, each innovating with a Poisson arrival rate of  $\xi_t(i)$ . Equation (27) is a first order linear differential equation with  $\Phi(t_0)$  as the initial condition. The unique solution is:

$$\Phi(t) = e^{-\lambda \int_{t_0}^t \rho(\xi_t(i)) ds} \text{ for all } t \geq t_0$$

By definition,  $\frac{A_t^{max}}{A_t^{max}} = \lambda\rho(\xi_t(i)) \ln \gamma$ , and  $A = A_{t_0}^{max}$ ,

$$A_t^{max} = A e^{\lambda \ln \gamma \int_{t_0}^t \rho(\xi_t(i)) ds} \text{ for all } t \geq t_0 \quad (28)$$

Thus,

$$\Phi(t) = \left( \frac{A}{A_t^{max}} \right)^{\frac{1}{\ln \gamma}} \quad (29)$$

Q.E.D

Define  $V_{t+1}$  as the value of the patent. The expected profit is the product of innovation arrival rate and  $V_{t+1}$ . Assuming that final good is the sole input, a typical research firm  $i$  maximizes

$$\max_{\xi_t(i)} \left[ \frac{\lambda \rho(\xi_t)}{\xi_t} \right] \xi_t(i) V_{t+1} - (1 - \beta_n) \Xi_t(i) \quad (30)$$

where  $\beta_n$  is government subsidy to research. Notice that as  $\xi_t$  increases, the arrival time of next generation technology expected to be shorter; the value of the next generation patent will be lower. In contrast with the endogenous growth literature like Romer (1990), we assume diminishing returns in the R&D sector. The first order condition is given by

$$\left[ \frac{\lambda \rho(\xi_t)}{\xi_t} \right] v_{t+1} = (1 - \beta_n) \quad (31)$$

where  $v_t = \frac{V_t}{A_t^{max}}$  is productivity adjusted value of the firm.

The value of  $V_{t+1}$  is determined by the following asset equation <sup>15</sup>:

$$R_t^n V_{t+1} = \pi_{t+1} - \lambda \rho(\xi_{t+1}) V_{t+1} \quad (32)$$

This equation has the following meaning: the net present value of the next generation technology ( $R_t^n V_{t+1}$ ), is equal to the profit when the patent remains as the leading edge technology ( $\pi_{t+1}$ ), less the expected capital loss that will occur when it is replaced by a new generation patent  $\lambda \rho(\xi_{t+1}) V_{t+1}$ . <sup>16</sup>

Note that the research curve can be lifted up in four ways: (a) decrease in interest raises

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<sup>15</sup>If the leader innovates, as shown in Barro and Sala-i-Martin, X (1994), the asset equation becomes:

$$rV_{t+1} = \pi_{t+1} - \pi_t - \lambda n_{t+1} V_{t+1} + \lambda n_{t+1} V_{t+2}.$$

<sup>16</sup> Implicitly, it is assumed that the existing patent holder does not perform R&D. The heuristic reason is that all potential competitors have access to existing technology (business stealing effect); thus the patent holder and competitors have same chance of success. Once successful the outside research firm will monopolize the whole market; the marginal reward is  $V_{t+1}$ . This is known as the 'Arrow effect' in the industrial organization literature.

However, the marginal gain to the current patent holder is  $V_{t+1} - V_t$ .

the present discounted value of monopoly; (b) an increase in the magnitude of innovation ( $\gamma$ ) also raises monopoly profit next period; (c) an increase in labor force represents scale effect; (d) an increase in  $\lambda$  increases the marginal benefit of research on the one hand, but increases the rate of creative destruction on the other hand. AH (1992) showed that the former effect would dominate. Suppose the representative consumer is risk averse, though the capital curve will be upward sloping, all comparative static will remain intact.

## 2.6 Equilibrium

We hereby characterize the equilibrium conditions of the baseline model. All intermediate good firms are identical, the symmetric equilibria of intermediate good price, output and labor demand are characterized by the following conditions

$$\begin{aligned} P_t(z) &= P_t \quad \forall z \\ Y_t(z) &= Y_t \quad \forall z \\ N_t(z) &= N_t \quad \forall z \end{aligned}$$

At equilibrium, the consumers are indifferent between lending and borrowing, so  $B_t = 0$ . The aggregate production can be expressed as

$$Y_t = \left( \int_0^1 [A_t N_t(z)^\alpha K_t(z)^{1-\alpha}]^{\frac{\varepsilon-1}{\varepsilon}} dz \right)^{\frac{\varepsilon}{\varepsilon-1}} \quad (33)$$

It can be shown that, around the steady state, the approximate aggregate production function is

$$Y_t = A_t N_t^\alpha K_{t-1}^{1-\alpha} \quad (34)$$

The aggregate demand for the final goods is

$$Y_t = C_t + I_t + G_t + \Xi_t \quad (35)$$

where  $G_t$  is government expenditure. The government maintains a balanced budget, thus

$$G_t = TR_t$$

The capital accumulation equation is

$$K_t = \Phi\left(\frac{I_t}{K_{t-1}}\right)K_t + (1-\delta)K_{t-1} \quad (36)$$

Combing equation(4) and equation(16), the labor market condition is given by

$$\alpha \frac{Y_t}{N_t} = \frac{1}{MC_t} a_n \frac{N_t^{\gamma n - 1}}{C_t^{-\gamma}} \quad (37)$$

Combining equations(6), (7) and (17),

$$E_t \left[ \frac{MC_t (1-\alpha) \frac{Y_{t+1}}{K_t} + (1-\delta)Q_{t-1}}{Q_t}, \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \right] = E_t \left[ R_{t+1}^n \frac{P_t}{P_{t+1}}, \left(\frac{C_{t+1}}{C_t}\right)^{-\gamma} \right] \quad (38)$$

The other equilibrium conditions are equations (5) ,(21), (22) and (31).

The symmetric equilibrium is characterized by  $(Y_t, C_t, I_t, N_t, K_t, \Xi_t, Q_t, P_t, MC_t, R_t^n)$  – a vector that maximizes the constrained present value of the stream of utility of the representation household, the constrained present value of intermediate firms and expected profit of R&D firms, subject to an exogenous shock.

That completes the description of the baseline model. We will introduce new shocks and generate impulse response functions in the next three chapters.

## Chapter 2

# Outside Opportunities Changes and Impulse Response Functions

### 1 Literature Review

In this chapter, we will append a shock to the R&D production sector. The model being considered is called random participation, in the spirit of Rochet and Chon`e (2002), which is an adverse selection model with random reservation utility. The randomness is change in outside opportunities. This shock can be interpreted as a sort of Dutch Diseases-brighter prospect at an outside sector - that drags away resources from R&D- similar to the Rybczynski effect in international trade theory.

For instance, the R&D expenditure level in Hong Kong remained low before 1998, because entrepreneurs invested heavily in the thriving real estate market. After the 1998 Asian financial crisis, the economy contracted and interest rate became higher. The real estate and stock market cooled down quickly. The government and private corporations launched many new R&D projects. The explanation is that, due to the Hong Kong linked exchange rate system, the Hong Kong interest rate has to follow U.S. interest rate change. Before 1998, the U.S. interest rate was low, while the Hong Kong inflation was high. The real exchange rate appreciated as nontraded good prices rose relative to tradable. Returns of investment in real estate market were higher than that of nontradable sector; and this pulled resources and talent out of tradable goods. After the 1998 financial crisis, all those prerequisites disappeared. The relative return of R&D became higher.

There is no lack of work purporting that asymmetric information affects innovation and adoption of new ideas. James, Ickes and Samuelson (1990) examine a principal offering

contracts to agents who make unobservable efforts (yielding moral hazard), occupy jobs of differing productivity (yielding adverse selection) in a repeated relationship, concluding that low rates of adoption is a response to prohibitive costs of higher adoption rate. Basu (1989) provides a theory of technological stagnation to explain simultaneously why it will not be worthwhile for both principal and agent to innovate, in which the principal is unaware of the probability that agent would drop out of the contractual relationship.

Larson and Anderson (1994) develop a risk-sharing model to examine the impact of risk preferences and future innovation possibilities expectations on patent payments and agents' incentives to innovate under technology licensing. Gallini and Wright (1990) show how a licensor - who knows the true intrinsic value of an innovation - signals his technology type with an output-based payment. Shi (1996) identifies conditions that asymmetric information induces agents to take high-risk projects. Once successful, it will generate faster evolution of knowledge and faster economic growth.

In fact, the extension to incomplete contract framework is natural, since R&D takes place either within firms where employees-inventors are subject to contractual obligations; or between independent research units and users of their innovations or financiers.

## **2 Random Participation Model**

A modification will be made to equation (30), the R&D firms' objective. For simplicity, it is assumed that the intermediate firms contract with two types of research firms such that an inefficient research agent has higher cost of delivering blueprints<sup>17</sup>. The true type of R&D firms is hidden, i.e. there is a form of adverse selection. The intermediate firms offer two contracts to

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<sup>17</sup>The extension to continuous type is natural and can be found in Rochet and Stole (2002).

induce efforts (the amount of final good that the R&D firm allocates to innovation); and distort the inefficient firms' output downward to reveal the true firm type. The contractual payment provides financing to R&D firms during the innovation process; and the R&D firm has obligation to sell the patent, if there is any in the future, to the intermediate firm. Since the payment is lump-sum, it has no first order effect in the profit functions of the intermediate firm and R&D firm.

Moreover, the reservation utility is randomized to capture the idea of changes in outside opportunities to R&D entrepreneurs. For instance, oil discovery can drag away capital inputs from manufacturing - the classic Dutch Disease scenario; and abnormal returns in the real estate sector generated by the pegged exchange rate system like Hong Kong, might drag resources and talent out of R&D.

Another interpretation is that the R&D firm may form strategic relationships with other principals in other sectors. The random reservation utility model assumes competition amongst principals. For instance, real estate industry ( a non tradable) can compete with manufacturing or R&D industries. Business cycle can change the relative profitability of different industries. In each market period, the R&D firm has the option of renegotiating the contract or entering into a new contract with another principal. The random participation model is a simple way to formulate this kind of shifting strategic relationship.

Diagram 2 summarizes the timeline of principal-agent relation. The R&D firms, first, discover their true types. The intermediate firms offer them a contract. The R&D firms have the option to accept or reject the contract. The R&D firms engage in delivering blueprints. An outside opportunity shock hits the economy. The R&D firms weight in the impact, renegotiate with the intermediate firms at the next market period.

Now, we proceed to describe the random participation model. The participation constraints of the risk-neutral R&D firms are given by:

$$u_E = \Gamma^E - A_t^{max} \xi_t^E(i) \geq \delta_E \quad (39)$$

$$u_I = \Gamma^I - A_t^{max} \kappa \xi_t^I(i) \geq \delta_I \quad (40)$$

where  $\kappa > 1$  is the inefficiency parameter - inefficient firms require more input;  $I$  and  $E$  denote inefficient and efficient type respectively;  $\delta_i$  is the exogenous reservation participation utility;  $\Gamma^i$  is the contractual payment to research firms for delivering blueprints. The intermediate firms offer two contracts each period -  $(\Gamma^E, \xi_t^E)$  and  $(\Gamma^I, \xi_t^I)$  - to induce efforts from the R&D firms. Equation (40) indicates that the cost of an inefficient firm is higher.

We perturb the agent's participation constraint by allowing some randomness in the decision to participate. Let the following be the probability density function of participation.

$$M(u_i, \kappa) = P((\delta_i, \kappa) | \delta_i \leq u_i)$$

where  $i=I,E$ , This formulation allows the possibility that  $\delta$  and  $\kappa$  to be correlated. The inverse Mill's ratio is defined as:

$$H(u_i, \kappa) = \frac{M(u_i, \kappa)}{M_u(u_i, \kappa)} \quad (41)$$

Equation (41) is non-decreasing in  $u_i$ .  $M_u(u_i, \kappa)$  is the first derivative with respect to  $u_i$ .

Direct revelation is implied by the incentive compatibility constraints.

$$u_E = \Gamma^E - A_t^{max} \xi_t^E(i) \geq \Gamma^I - A_t^{max} \xi_t^I(i) \quad (42)$$

$$u_I = \Gamma^I - A_t^{max} \kappa \xi_t^I(i) \geq \Gamma^E - A_t^{max} \kappa \xi_t^E(i) \quad (43)$$

If there is no asymmetric information, as shown at chapter 1,  $\lambda \frac{\rho(\xi_t^E)}{\xi_t} v_{t+1} = 1$  and  $\lambda \frac{\rho(\xi_t^I)}{\xi_t} v_{t+1} = \kappa$  are the optimal first order conditions for efficient and inefficient firms respectively.

Intermediate firms in each sector, under adverse selection, maximize the expected payoff function,

$$\begin{aligned} \text{Max}_{\xi_t(i)} \quad & vM(u_E, \kappa) \left( \lambda \left[ \frac{\rho(\xi_{t+1}^E)}{\xi_{t+1}^E} \right] \xi_t^E(i) V_{t+1} - (1 - \beta_n) \Xi^E(i) \right) \\ & + (1 - v)M(u_I, \kappa) \left( \lambda \left[ \frac{\rho(\xi_{t+1}^I)}{\xi_{t+1}^I} \right] \xi_t^I(i) V_{t+1} - (1 - \beta_n) \Xi^I(i) \right) \end{aligned} \quad (44)$$

$$s.t \quad \delta_E = A_t^{max} \xi_t^I(i) (\kappa - 1) + \delta_t \quad (45)$$

$v$  is the fraction of efficient research firms, which is a known constant by assumption. Other parameters are the same as those defined at chapter 1. Comparing to equation (30), the intermediate firm can only maximize expected profit of different types of R&D firms, multiplied by the participation probability. Moreover, the R&D expenditure will be below optimum due to adverse selection. Notice that  $\Xi_{t+1}^I$  equal to  $\Xi_t^I$  at the steady state. As typical in the adverse selection literature, equations (40) and (42) are binding. Combining them gives (45). The intuitive reason is that the efficient type has incentive to pretend that he is an inefficient firm; and equations (40) and (43) imply equation (39).

The first order necessary conditions are:

$$\lambda \left[ \frac{\rho(\xi_{t+1}^E)}{\xi_{t+1}^E} \right] V_{t+1} = (1 - \beta_n) A_t^{max} \quad (46)$$

$$\lambda \left[ \frac{\rho(\xi_{t+1}^I)}{\xi_{t+1}^I} \right] V_{t+1} = (1 - \beta_n) \frac{(vM(u_E, \kappa)(\kappa - 1) + (1 - v)M(u_I, \kappa))}{(1 - v)M(u_I, \kappa)} A_t^{max} \quad (47)$$

Following Rochet and Chonè (2002), we assume that  $M(u_E) = 1 - e^{-\delta_E r}$ ,  $M(u_I) = 1 - e^{-\delta_I r}$ , i.e. exponential distribution. The participation probability does not depend on  $\kappa$ . A constant has no effect on the linearized equation. Rochet and Chonè (2002) assume that efficient firms participate with probability equal to one. We assume that their participation is a constant. Since it can be set arbitrarily,  $\delta_E$  is normalized to be one. The change is immaterial in the sense that a constant has no first order effect. The participation probability is randomized by assuming that  $\delta_t$  follows a stationary first order Markov process.

$$\delta_{t+1} = \rho_\delta \delta_t + \varepsilon_{t+1}^\delta$$

where  $\varepsilon_{t+1}^\delta \sim N(0, \sigma_\delta^2)$ . Implicitly, we are assuming that inefficient firm is the only group that would switch to another production sector. Each period, a change of  $\delta_t$  forces the intermediate firms to change the contract and the R&D payments.

The new equilibrium conditions are (5), (21), (22), (34), (35), (36), (37), (38), (46) and (47).

### 3 Calibration

To derive the quantitative calibrated results, first, the steady state values are computed; second, the nonlinear system of differential equations will be log-linearized around the steady state; finally, the system will be solved by various methods to compute the second moments of simulated time series and generate impulse response functions. Uhlig (1998) proposes using the

method of undetermined coefficients<sup>18</sup>. The advantage is that it has exact solution. Recently, the AIM iterative algorithm has become more popular. However, it provides only approximate solutions. We choose Uhlig's method in this paper.

### 3.1 Parameter Specification

This section describes the benchmark values used to compute the impulse responses of the economy to various shocks. The forth column of table 1 reports the parameter values of the random participation model. For the steady state value of labor, we set it to 1/3, i.e., a third of total endowment of time. The scaling technology factor,  $A_t$  is normalized to 1. Following Hansen(1985) and Backus, Kehoe, and Kyland (1995), the labor share of output  $(1-\alpha)$  is 0.64. We set the steady state gross interest rate equal to 1.01 per quarter (Prescott and Kydland (1982) and Backus, Kehoe, and Kyland (1995)) and 1.04 annually, which implies that  $\beta$  equal to 1/1.01. The depreciation rate, is set equal to 0.035.

The coefficient of relative risk aversion,  $\gamma$ , is 0.5; meaning the representative household is slightly risk averse. We find that moderate degree of risk aversion is sufficient to simulate the second moments of investment, output and R&D. Labor intertemporal substitution is 1. The random participation outside opportunity is assumed to be first order stationary Markov process, with AR(1) coefficients equal to 0.75. The Phillips relation is derived by log-linearizing equation

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<sup>18</sup>Let  $x(t)$  be a vector of state variables,  $y(t)$  be a vector of endogenous variables and  $z(t)$  be a vector of exogenous shocks. Write the equilibrium equations as follows:

$$\begin{aligned}
 0 &= AAx(t) + BBx(t-1) + CCy(t) + DDz(t) \\
 0 &= E_t[FFx(t+1) + GGx(t) + HHx(t-1) + JJy(t+1) + KKy(t) + LLz(t+1) + MMz(t)] \\
 z(t+1) &= NNz(t) + \text{epsilon}(t+1)
 \end{aligned}$$

where  $\text{epsilon}(t+1)$  is a vector of white noise.

Solving by method of undetermined equations, all state and endogenous variables can be expressed as lagged state variables and shocks. Then, we can generate the impulse response functions and calculate the simulated moments. For detail, please see <http://www2.wiwi.hu-berlin.de/wpol/html/toolkit/toolkit.pdf>.

(21):

$$\hat{\pi}_t = \lambda_\pi \hat{MC}_t + \beta E_t \hat{\pi}_{t+1} \quad (48)$$

Gali, Gertler, and Lopez-Salido (2001) suggests that the  $\lambda_\pi$  coefficient in Phillips is either 0.014 or 0.024; we choose the former.

The steady state consumption-output ratio, investment-output ratio, and R&D-output ratio are chosen to be 0.5, 0.4 and 0.1 respectively. The government plays no role in this setup. Thus, the  $G_t$  term in equation(35) will disappear;  $\beta_n$  at equations (44), (46) and equations (47) will equal to zero, i.e. no government subsidy.

For Y/K, Andres, Lopez-Salido D and Valles (2002) choose 1/8 and the steady state marginal cost equal to 1/1.2. They argue that a wide range choice of marginal cost can be used for calibrating their model. The labor disutility factor  $a_n$  is a free parameter in our model. We choose 1.3 to make the values of steady state variables consistent with our choice of steady state ratios. The capital production function is expressed as

$$\phi\left(\frac{I_t}{K_{t-1}}\right) = \frac{\phi_k}{2} \left(\frac{I_t}{K_{t-1}}\right)^{\frac{1}{2}} \quad (49)$$

where  $\phi_k$  can be interpreted as the adjustment cost coefficient. The Steady state probability of raising capital is set to be 0.15. The R&D rate of arrival is given by :

$$\rho(\xi_t) = \phi_\xi (\xi_t)^{\frac{1}{2}} \quad (50)$$

The Poisson arrival rate exhibits diminishing return, which is consistent with the idea that invention becomes more difficult as the degree of complexity increases over time. Parameters that have no second order effect, such as the research productivity coefficient  $\lambda$ , on the log-linearized system can be safely ignored. Finally, the steady state  $\kappa$  - the inefficiency

parameter of R&D firms -is equal to 1.5.

### 3.2 Impulse Response Functions

Every stationary vector autoregressive system has a unique Wold decomposition. We are going to investigate the impact of a structural shock on the path of economic variables.

The y axis of figure 2 measures the percent deviation from the steady state. A one standard deviation shock to  $\delta_t$  drives down the R&D spending of efficient firms by 0.7 percent at the initial period, and it returns to the long term path after 6 periods. Each period represents a year, because the predicted model moments will be compared to the actual data, which is observed annually. The impact on inefficient firms is smaller, approximately 0.3 after 1 period. One possible explanation is that the efficient firm's participation and incentive compatibility constraints are binding, so it is more sensitive to change in reservation utility. Another reason is the relative large portion of efficient firms in the model -  $\nu$  equal to 0.8.

Demand for final goods as R&D input will decrease, which reduce the demand for capital and investment inputs. Eventually, the capital price will drop. As shown in figure 2, output drops by 0.25 percent at period 0. It settles down to the steady state after 4 years. Capital price change is negative 1.2 percent. The investment drop is the biggest among the variables being considered. The initial period drop is almost 2.5 percent. Both capital price and investment overshoot after 3 years.

The R&D expenditure of exhibits hump-shaped dynamic movement, since both current period and next period R&D appear in the log-linearized equation. For the inefficient type, it can be expressed as:

$$E_t[y(t+1) - \frac{\overline{MC}}{1-\overline{MC}} mc_{t+1} - 0.5\xi_t^I - 0.5\xi_{t+1}^I] = (1 - e^{-\bar{r}})r_t + \delta_I \quad (51)$$

One contribution of this paper is that, random participation in R&D sector alone, is capable of generating enough dynamic as most real business cycle models do. Boldrin and Woodford (1990) note that many of "endogenous business cycles" studies have the same basic structure as RBC- that is, it relies on an exogenous shock which hits the aggregate production directly and then propagate through time. However, in our case, the source of shock is only confined to the R&D sector; and it is assumed that R&D makes up at most 10% of GDP. In a word, we have a shock in a small sector, but it is enough to generate sufficient deviation from steady state time series. We have calibrated different values of  $\nu$  (the percentage of efficient R&D firms) and, most of the results noted above remain invariant.

## Chapter 3

# Financial Intermediation Shock and Impulse Response

## Functions

### 1 Introduction

In the literature, financial shocks most often take the form of monetary policy surprises (for instances King and Plosser (1984), Altig, Christiano, Eichenbaum and Linde (2004), and Clarida, Gail and Gertler (2002)) or exchange rate regime changes (for instance, Baxter and Stockman (1989)). Once nominal frictions, like fixed prices and wages, are introduced, these models can generate impulse responses similar to those of technology shocks. For instance, in the Christiano and Eichenbaum (1992) model, the household allocates a fraction of money holding to consumption goods, and the remaining lent to financial intermediaries. The household is liquidity constrained, in the sense that current consumption decision is not a function of time- $t$  realization of monetary policy. In this way, they can show that the interest rate decreases as money supply increases - a result that the authors claim that most monetary RBC models fail to replicate- and that the liquidity effect is persistent.

Other researchers examine how financial market frictions (for example credit constraints) affect the ways that real variables respond to technology and money shocks (see Bernanke, Gertler and Gilchrist (1999)). An important extension is the International Real Business Cycle (IRBC) model with incomplete financial markets. Backus, Kehoe and Kyland (1995) argue that IRBC is consistent with the international stylized fact that the cross-country consumption correlation is less than the cross-country output correlation. Williamson (1987) illustrates how financial intermediation, bankruptcy costs and credit rationing propagate the effects of a

stochastic disturbance. He argues that monetary shocks generate cycles that are inconsistent with empirical evidence, lending support to RBC.

Sutherland (1997) constructs an international RBC with staggered prices and trading frictions across countries. Simulations show that volatilities of a number of variables increase with monetary shocks and increasing financial market integration. The opposite holds for real demand and supply shocks. Fuerst (1995) adds financial intermediaries into a RBC model to examine the effect of real and monetary shocks. The impulse responses are remarkably weak, so he concludes that “we have plenty of sources of business cycle shocks, but little in the way of propagation.”

Our model is different from the earlier works, in the way that the financial market is the source of economic fluctuation. King and Levin (1993) demonstrate that a higher degree of financial development, in particular lower agency cost of raising capital, has positive growth effects. The agency cost is randomized in our model. The idea is that when positive R&D news hits the market, it lowers the cost of raising capital, thus affecting current output. As argued above, it is hard to apprehend the concept of a negative technology shock ( for example, ideas cannot disappear). A negative R&D shock can however be interpreted easily. It can be due to unexpected delays of new invention, unsatisfactory research progress, termination of a project or competitors patent, which render both internal and external financing more difficult. The financial intermediation model should work for developed countries. We will compare the calibrated moments to the observed U.S. data.

## **2 The Model**

In the AH model, the consumers are risk neutral. They earn their wages and dividends from

owning intermediate firms, and pay the R&D firms their expected wage even though there is no invention. However, consumers in this paper are risk averse, thus financing R&D outlays at the time of no invention becomes an issue. It is assumed in chapter one that R&D projects are financed by a perfect capital market. Here, we add the assumption that raising capital involves an agency cost.

It is sometimes argued that technical know-how and scientific theories are abundant. The key problem is funding the experiments, applying the ideas for practical usage. King and Levine (1993) demonstrate how financial development bridges the gap. Letting  $f$  be the agency cost of identifying a capable researcher, and  $\psi_t$  be the probability of raising sufficient capital to finance the research projects, through the financial market. The new arbitrage equation (revision of equation (31) is

$$\lambda \left[ \frac{\rho(\xi_t)}{\xi_t} \right] v_{t+1} = (1 - \beta_n) \left( 1 + \frac{f}{\psi_t} \right) \quad (52)$$

The research arbitrage equation is essentially the same as Aghion and Howitt (1992). Profit depends on both labor and some exogenous parameters. The effective input cost is increased by a factor of  $\frac{f}{\psi}$  due to adverse selection. Higher financial development is represented by a lower  $f$ . A lower  $\psi_t$  raises the agency cost. It is generally believed that when the stock market collapses, the financial intermediaries lose their functions of reducing moral hazard and adverse selection. The agency cost increases sharply during economic slump. For instance, corporate profits drop significantly during recession - a macroeconomic factor unrelated to firms' characteristics. However, firms' leverage increases at those times. Banks and bond holders may have reason to worry about defaults. Loan market cannot function properly, thus increasing the cost of raising capital.

Berthelemy and Varoudakis (1996) come up with the same conclusion by augmenting Aghion and Howitt (1992) model with a banking sector, which employs labor to raise loanable funds for R&D and intermediate sector. The arbitrage condition remains unchanged, but the resources devoted to securing more savings deplete R&D. However, if only R&D sector demands for funding, then the intermediation cost will affect the arbitrage equation, and his result will be similar to King and Levine (1993).

We further assume that  $\psi_t$  - the probability of access to capital - follows a stationary Markov process, perturbed by an independent and identically distributed shock.

$$\psi_{t+1} = \rho_\psi \psi_t + \varepsilon_{t+1}^\psi \quad (53)$$

$$\varepsilon_{t+1}^\psi \sim N(0, \sigma_\psi^2) \quad (54)$$

The idea is that investors are sensitive to R&D news, for instance, new formula, new design and research progress. A negative shock can be unexpected delay of new invention, unsatisfactory research progress, termination of a project or competitors patent. The stock market before 2000 is arguably such an instance. Investment was stimulated by waves of technology break-through news. A positive shock makes fund raising easier. The capital accumulation equation can be rewritten as:

$$K_t = e^{\psi_t} \phi\left(\frac{I_t}{K_{t-1}}\right) K_t + (1 - \delta) K_{t-1} \quad (55)$$

Where  $e^{\psi_t} \phi\left(\frac{I_t}{K_{t-1}}\right) K_t$  is effective investment. The equations that characterize the equilibrium are (5) ,(21), (22), (31) (34), (35), ( 37), ( 38), (52), (53) and (55).

### 3 Parameter values

The parameter values are reported at the fifth column of table 1. They are the same values used for chapter 2, except two parameters. The coefficient of relative risk aversion,  $\gamma$ , is 0.9 for the financial intermediation model. The AR(1) coefficient of the shock is 0.5.

### 4 Impulse Response Functions

A positive shock improves the chance of raising capital (or reduction in agency cost); thus the output (R&D firms use final good as input), investment (for capital production). The R&D spending effects are all positive as shown in figure 3. The output change after a shock is mild - 0.25% increase; it settles down to the steady state after 4 periods. Investment increases by 0.75%; the hump-shaped time path is attributed to lagged adjustment of capital stock (equation (55)). Similar to the random participation model, investment shock dies out at the sixth period.

Capital price adjustment is almost one to one; it overshoots after 2 period. The propagation effect diminishes rather slowly. R&D expenditure, on the other hand, rises by 2 percent - the sharpest change among all variables considered. It returns to the long run path after 6 periods.

Notice the order of series volatilities: R&D, followed by investment and output. This can be explained by the fact that the random shock distorts R&D outlay directly, and capital indirectly through the capital accumulation equation. Output drops can be reconciled by the decrease in derived demand for final good. The calibrated moments of this model will be compared to the U.S. data at next chapter.

## Chapter 4

### Model Comparisons and Calibrations

Most economists agree that many economic series deviate away from a stationary trend due to some exogenous shock, and then propagates itself through time. Typical perturbations include the famous Prescott and Kyland technology shocks (Prescott and Kyland (1982)), government expenditure surprises (King Plosser and Rebelo (1988) ), money shocks (Gertler and Gali (1999)) and sunspot activity (Farmer and Guo (1994)). In this chapter, we will examine the traditional new Keynesian models shocks - technology, interest rate and government shocks -in the Gertler-Gali model augmented with a R&D sector - the baseline model of chapter one. The impulse response functions will be compared to those of random participation and financial intermediation models. Finally, we will compare the predicted moments to the actual economic data.

#### **1 Baseline Model and Parameter Values**

The equilibrium equations of the baseline model are (5), (21), (22), (31), (34), (35), (36), (37) and (38) from chapter 1. The parameter values are reported at the third column of table 1. Readers should notice that there are 9 equations and 10 unknowns. RBC theorists assume a stochastic process for one of the variables to complete the model. Three scenarios will be considered here: a surprise of government expenditure, a productivity shock and an unexpected interest rate change. The first two scenarios follow a stationary first order Autoregressive (AR)

process. The AR coefficients are 0.8 and 0.95 for the government expenditure and productivity shocks respectively. The steady state consumption-output ratio, investment-output ratio, government-output and R&D-output ratio are set to 0.6, 0.15, 0.2 and 0.05 respectively.

Additionally, we can assume that the interest rate is anchored by the Taylor's rule

$$R_{t+1}^n = R^n \left( \frac{P_t}{P_{t-1}} \right)^{\gamma_\pi} \left( \frac{Y_t}{Y_t^*} \right)^{\gamma_y} e^{\varepsilon_t^r} \quad (56)$$

$R^n$  is the real rate of interest at steady state;  $Y_t^*$  is the potential output level under flexible prices.  $\varepsilon_t^r$  is a sequence of uncorrelated monetary policy shock, and where  $\gamma_\pi > 1$  and  $\gamma_y > 0$ .

Under the Taylor rule, the target inflation rate is zero.

The inflation coefficient of the Taylor rule  $\gamma_\pi$  should be larger than 1, and the output coefficient  $\gamma_y$  should be non-negative number. Based on the empirical studies of Gali, Gertler, and L'opez-Salido (2005), we set  $\gamma_\pi$  and  $\gamma_y$  equal to 1.5 and 0.5 respectively. The other parameter values are the same as those considered at chapters 2 and 3.

## 2 Comparing Models

The impulse response functions of interest rate surprise, productivity shock and government expenditure surprise are presented in figure 4.1, 4.2 and 4.3 for the baseline model. One typical observation of this class of staggered price model is the countercyclical markup. The monopolistic intermediate firms set price before getting to know aggregate demand. The ex-ante profit maximizing condition is given by

$$E_{t-1}\left(\frac{P_t(z)}{P_t}\right) = E_{t-1}[(1 + \mu_t)MC_t] \quad (57)$$

where  $1 + \mu_t$  is the steady state desired markup. Ex-post, the pre-committed price cannot be changed. The firms have to readjust markup:

$$\frac{P_t(z)}{P_t} = (1 + \mu_t)MC_t \quad (58)$$

For instance, an unanticipated increase in demand for final goods increases demand for intermediate goods. The increase in derived demand drives up wages, marginal cost and thus depresses markup. A negative productivity shock, on the other hand, reduces marginal cost and thus increases markup. We find the same countercyclical pattern in all models with technology and government expenditure shocks.

In the first scenario, the monetary authority adjusts interest rate according to the Taylor rule with policy surprise. The impulse responses with respect to a one standard deviation interest rate shocks are reported in figure 4.1. The interest rate shock can explain the variation in R&D expenditure by affecting discounted profit of research project. An unexpected interest rate hike depresses output (since both capital and labor decreases) and investment. Output drops by eight percent initially and returns to the steady state after six periods. Capital price and investment decrease by eighteen and seventy percent respectively. Moreover, both series overshoot after four periods.

However, R&D slightly increases, i.e. countercyclical. The reason is that, while higher interest rate (due to crowding out effect) lowers discounted franchise profit, lower output demand depresses output price, which R&D sector uses as input. In this case, the latter effect dominates.

Figure 4.2 shows the impact of one standard deviation positive technology shock. The result is analogous to the traditional real business cycle model that it has positive effect on output, consumption and investment. The existence of staggered price mechanism explains for the declining marginal cost, since a positive technology shock increases marginal product of labor and capital. For a one standard deviation change in productivity, output increases by two percent, capital price by four percent. While research and development expenditure does not change at all, investment expenditure overshoots - for every one percent increase in technology shock, investment increases by sixteen percent.

As shown in figure 4.3, a one standard deviation government expenditure surprise crowds out consumption and investment and depresses markup. Output drops by half percent initially and returns to the steady state after four periods. Capital price and investment decrease by one and five percent respectively. Both series overshoot after four periods. The oversensitivity problem of investment is reduced. R&D expenditure, however, remains countercyclical.

Let's compare the results to chapter two and three. One virtue of the random participation and financial intermediation model is that the investment oversensitivity issue disappears. Moreover, the R&D investment is consistently procyclical. The introduction of R&D sectors and alternative shock channels improve the empirical performance of the baseline model. Note that the order of volatility is investment, output followed by R&D, which will be confronted with actual data in the next section.

### **3 Comparing to the US Data**

It is worth noting that the government, technology, interest rate shocks in baseline model, as well as the random participation, all imply that the variance of investment is largest, followed by

R&D and output. The financial intermediation model, on the other hand, predicts that R&D volatility is highest, followed by investment and output.

Column two of table 2 reports the U.S. volatilities of aggregate output, investment and R&D expenditure, using 1953-2003 Chained-type real GDP and Gross Domestic Investment data from Bureau of Economic Analysis, Department of Commerce. National Science Foundation collects data for industry, government, university R&D expenditure. We choose to report the result for industry R&D, since it makes up 70% of total expenditure and the computed volatility with other R&D categories remain the same. All data are denominated in 2000 constant dollars. Following the real business cycle literature, we compute the logged series, demeaning them by Hodrick-Prescott filter, and finally compute variances and covariance by the residual series. The results are reported in table 2.

We found that, R&D expenditure has highest variance, followed by investment and then output. Hence, *only the financial intermediation model is consistent with this finding*. The second moments of U.S. filtered series is compared to the calibrated moments of the financial intermediation model. Following Hansen and Wright (1997), we compute the relative standard deviation of investment and R&D expenditure relative to GDP. As seen in the third column, the calibrated output and R&D volatilities are relatively larger than observed U.S. data; however, the calibrated relative investment volatility is very close to actual U.S. data equal to 3.5. The predicted R&D- output standard deviation is 9.4, twice as high as the observed series. If, however, the assumed R&D share is increased from 0.1 to 0.15, the model now predicts a volatility of R&D of only 5.1, closer to the actual data. The table also examines the autocorrelations. As a whole, the signs of calibrated correlations are correct, though it over predicts them.

## 4 Conclusion

There is relatively limited work on endogenous growth and business cycle models. This paper attempts to bridge the gap in the literature. Since R&D is the core component in most of the endogenous models, we try to introduce shocks in the R&D sector and then to examine the impact on the rest of the economy. In this paper, a well-studied New Keynesian Model is augmented with a R&D production sector depicted in Aghion and Howitt (1992). Two sources of exogenous shocks to the economy are proposed, namely random participation (shocks to outside opportunities to R&D resources) and financial intermediation (financial market imperfection).

We find that the new Keynesian model, when augmented with an R&D sector; the traditional shocks like interest rate, government expenditure and productivity shocks do not perform well. Investment is very sensitive to an exogenous shock. For instance, as shown in figure 4.1, a one standard deviation shock in interest rate reduces investment by 70 percent. Moreover, R&D is either counter-cyclical or acyclical.

In each of these new models considered in this paper, R&D is procyclical, consistent with the U.S. data. Also, the financial intermediation and random participation models do not suffer from the strong investment overshooting problem of the baseline model. Finally, we find that only the financial intermediation model is consistent with the observed fact that the volatility of R&D is larger than that of investment and output.

# Appendix

Figure 1.1. U.S. R&D Time Series

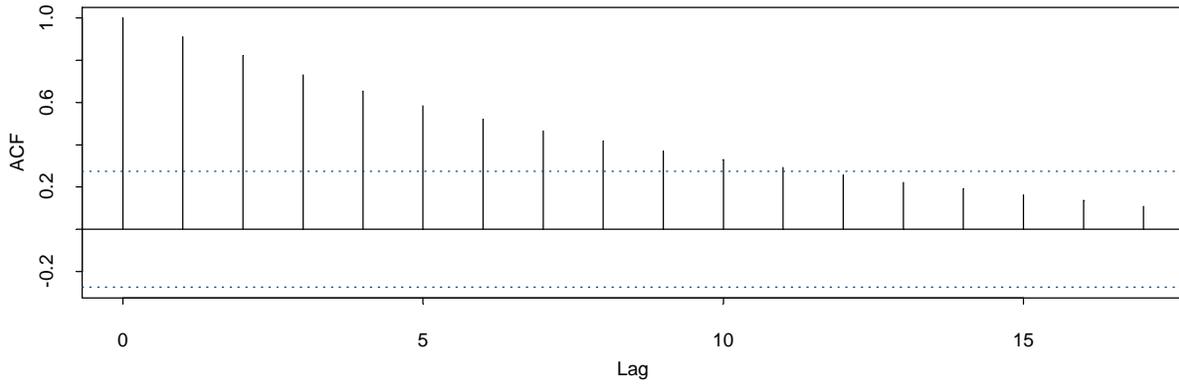


Figure 1.2. U.S. GDP Time Series

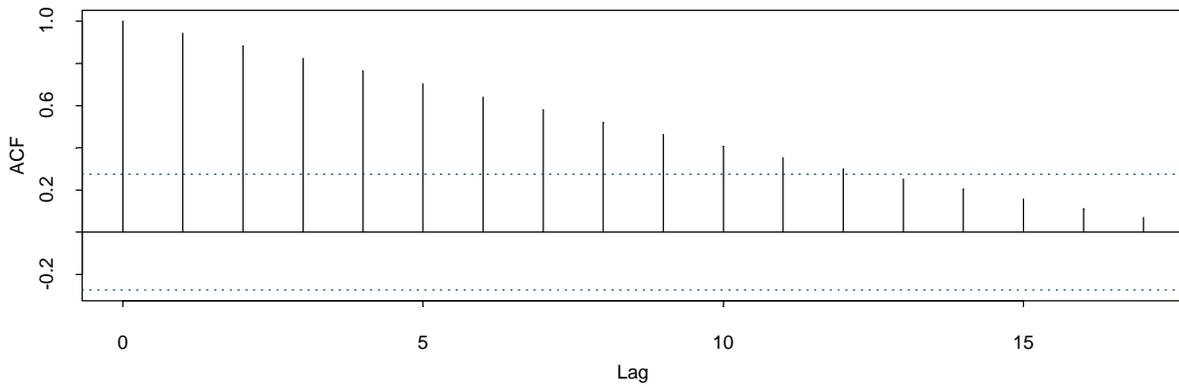


Figure 1.3. Differenced U.S. R&D Time Series

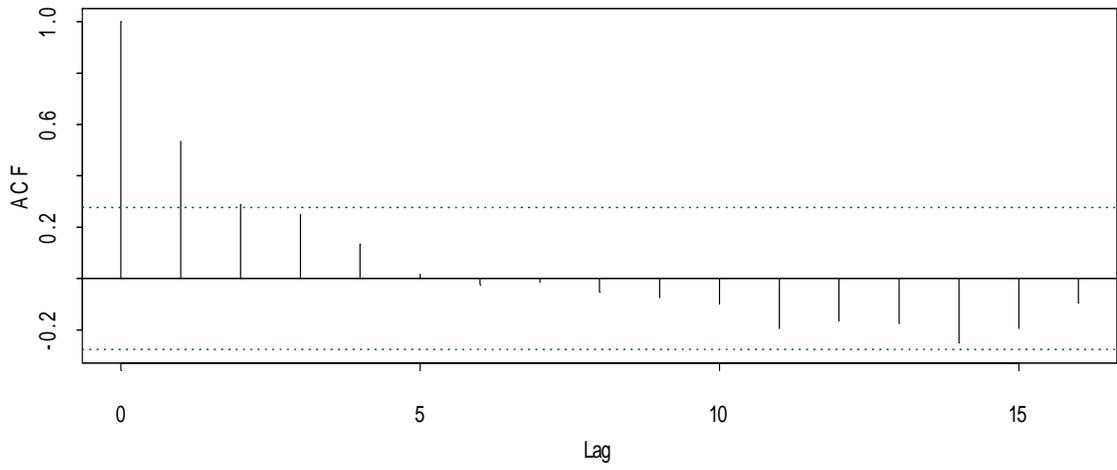
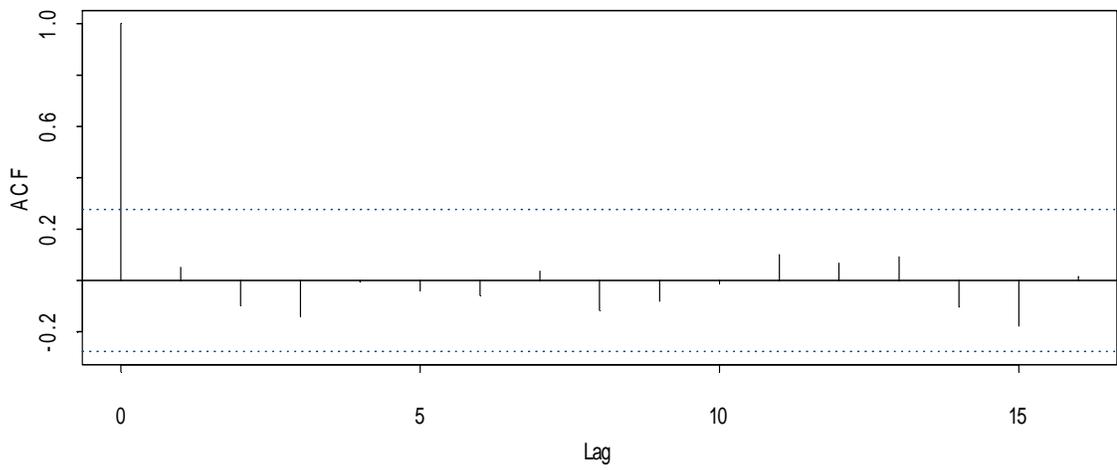
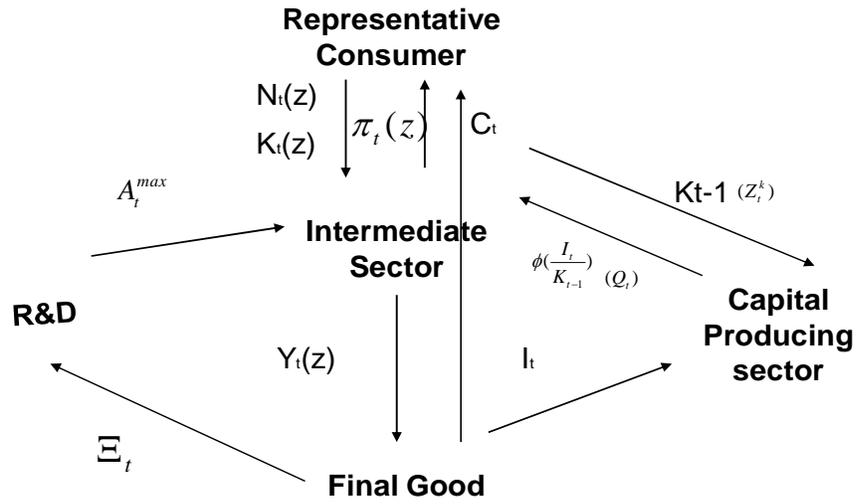


Figure 1.4. Differenced U.S. GDP Time Series

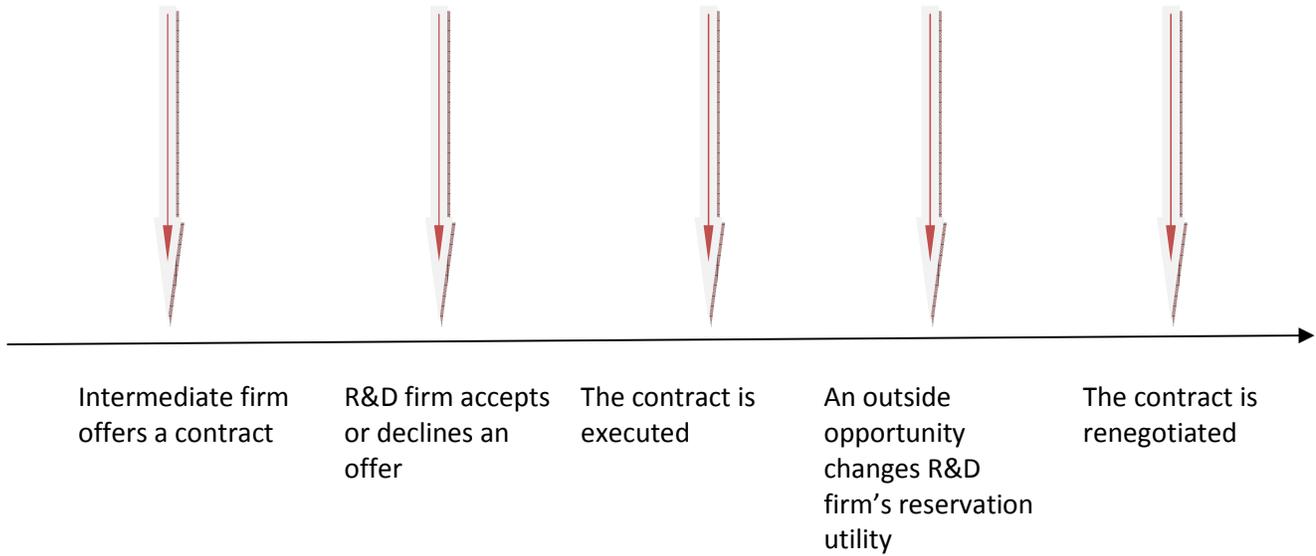


# Diagram 1 Model Outline



## Diagram 2

### Timeline of R&D Firm and Intermediate Firm Contractual Relation.



**Table 1 Parameter values of various models.**

Symbol	Definition	Gertler-Gali	Random Participation	Financial Intermediation
$\gamma$	Relative risk aversion of consumption	0.5	0.5	0.9
Autoregressive Coefficients	Technology shock	0.9		
	Interest rate shock	0.75		
	Government expenditure shock	0.8		
	Random Participation shock		0.75	
	Financial intermediation shock			0.5
	Standard deviation of shock	0.712	0.612	0.612
$\frac{C}{Y}$	Steady State Consumption Output ratio	0.6	0.5	0.5
$\frac{I}{Y}$	Steady State Investment Output ratio	0.15	0.4	0.4
$\frac{G}{Y}$	Steady State Government Expenditure Output ratio	0.2	Nil	Nil
$\frac{R\&D}{Y}$	Steady State R&D Expenditure Output ratio	0.05	0.1	0.1
L	Steady state of employment	1/3		
A	Technology parameter	1		
R	Gross interest rate per quarter	1.01		
$\frac{K}{Y}$	Steady State Capital Output ratio	1/8		
MC	Steady State Marginal Cost	5/6		
$\kappa$	Inefficiency Parameter of R&D firms	1.5		
$(1-\alpha)$	Labor income share of Output	0.64		
$\beta$	Consumer's subjective discount rate	1/1.01		
$a_n$	Labor Disutility Coefficient	1.3		
$\delta$	Depreciation rate per quarter	0.035		
$\nu$	Fraction of efficient R&D firms	0.8		
$\gamma_n$	Labor intertemporal rate of	1		

	substitution	
$\gamma_{\pi}$	Inflation coefficient of Taylor Rule	1.5
$\gamma_y$	Ouput coefficient of Taylor Rule	0.5
$\lambda_{\pi}$	Inflation Coefficient in Phillips equation	0.024
$\Psi$	Steady state probability of raising sufficient capital	0.15

Figure 2 Impulse Responses to Random Participation Shock

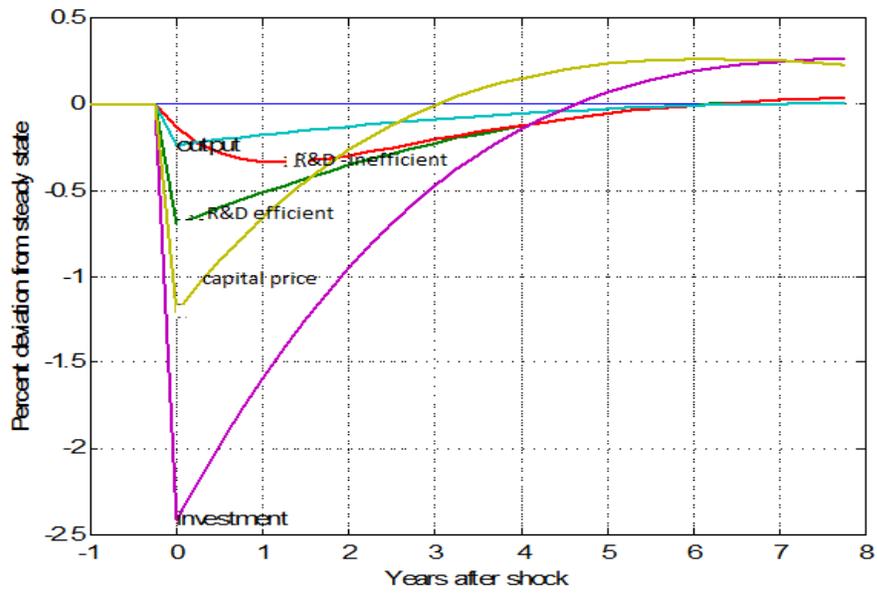


Figure 3 Impulse Responses to a Financial Intermediation Shock

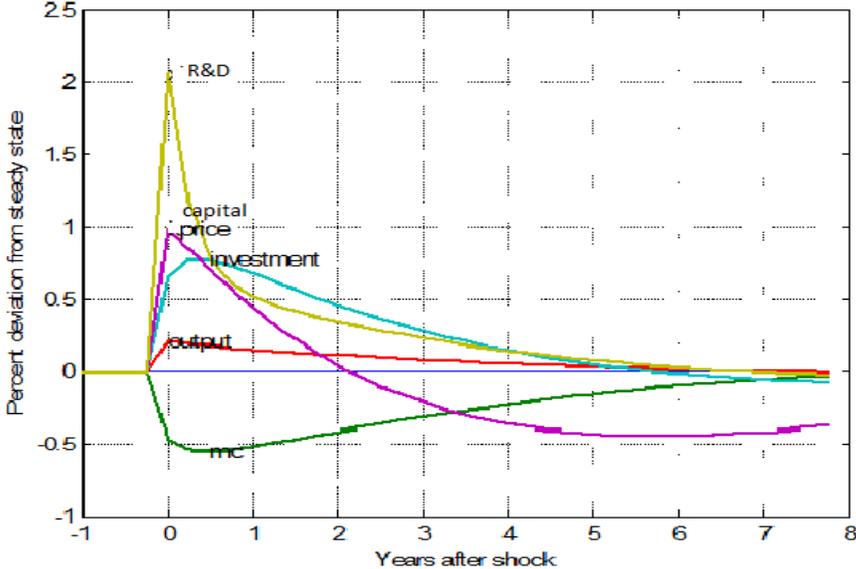


Figure 4.1 Impulse Responses to a Shock in Interest Rate Policy

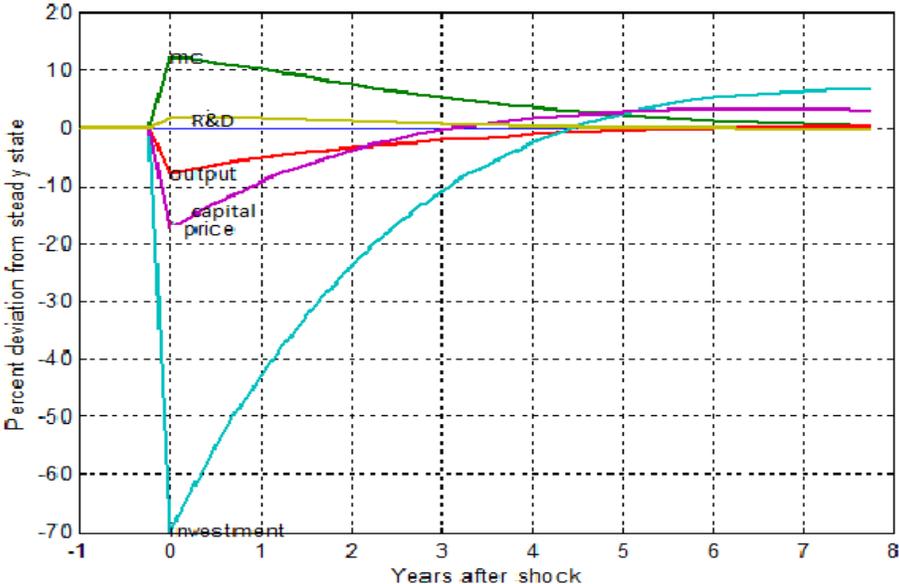


Figure 4.2 Impulse Responses to a Shock in Technology

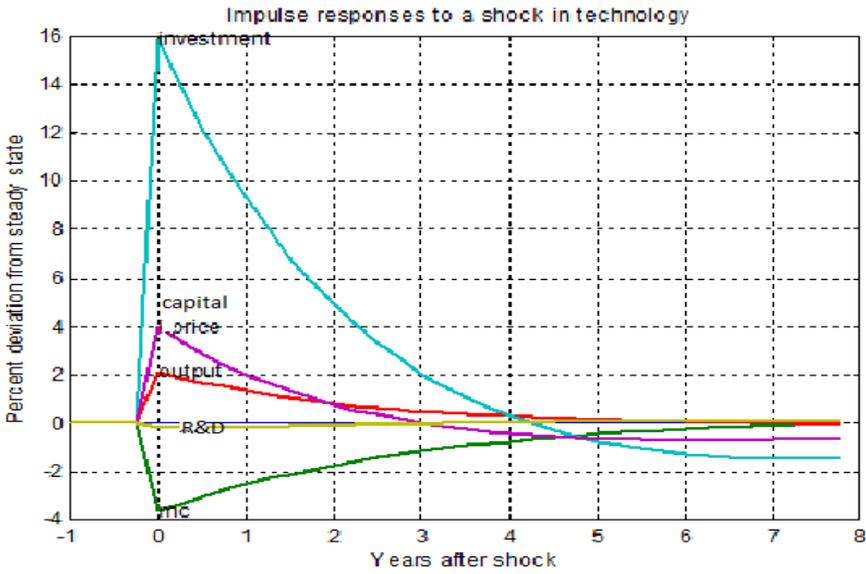
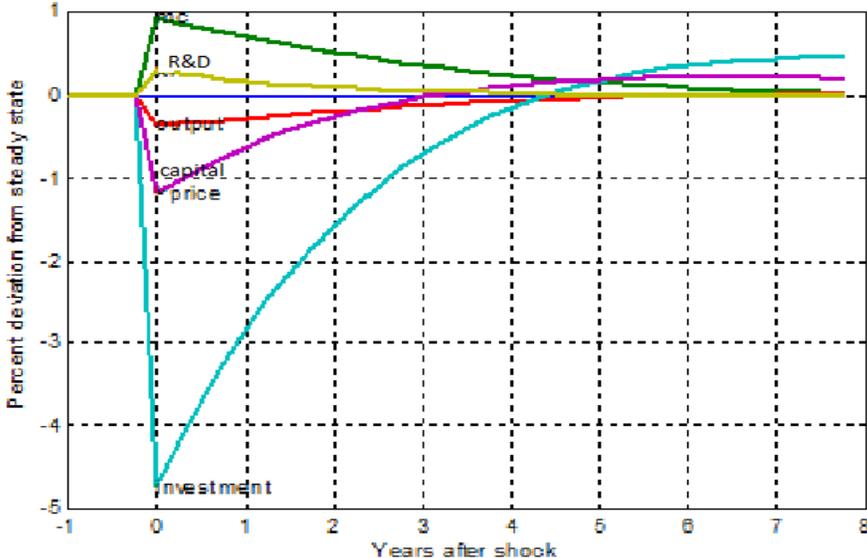


Figure 4.3 Impulse Responses to a Shock in Government Expenditure



**Table 2. U.S and Calibrated Second Moments of Financial Intermediation Model**

	<b>U.S</b>	<b>Calibrated</b>	
		$\frac{C}{Y}=0.5; \frac{I}{Y}=0.4; \frac{R\&D}{Y}=0.1$	$\frac{C}{Y}=0.5; \frac{I}{Y}=0.35; \frac{R\&D}{Y}=0.15$
$\sigma_y$	0.026	0.062	0.19
$\frac{\sigma_I}{\sigma_y}$	3.5	3.7	4.8
$\frac{\sigma_{rd}}{\sigma_y}$	4.0	9.4	5.1
$Corr(y_t, RD_{t-1})$	0.35	0.64	0.62
$Corr(y_t, RD_t)$	0.37	0.97	0.86
$Corr(y_t, RD_{t+1})$	0.26	0.28	0.25
$Corr(y_t, I_{t-1})$	0.39	0.65	0.66
$Corr(y_t, I_t)$	0.78	0.95	0.99
$Corr(y_t, I_{t+1})$	0.69	0.86	0.75

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