Economic Convergence — The German 1990 Economic and Monetary Union

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ECONOMIC CONVERGENCE —
THE GERMAN 1990 ECONOMIC AND MONETARY UNION

by

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the requirements for the degree of Doctor of Philosophy, The City University of New
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THE CITY UNIVERSITY OF NEW YORK
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INTRODUCTION.

The study of the German's 1990 unification experience is quite interesting because of the different yet similar characteristics of the two national economies. The old German Democratic Republic (GDR) or East Germany, who acceded to the Federal Republic of Germany (FRG) or West Germany, share the same official language, heritage, and were one country before WWII. The differences that existed between the two Germanys prior to their unification were in the social, political and economic arena. Yet irrespective of their differences, there is no doubt that monetary unification took place previous to any economic convergence of the two nations. Will this monetary union between these unequal economic participants lead to economic convergence?

Period of Study.

After Germany's defeat in World War II, Germany was divided into four occupation zones which resulted in the French occupying the southwest; the British, the northwest; the Americans, the south; and the Soviets, the east.

---

1 I specify the 1990 unification as Germany also underwent a prior unification in 1870. Hereafter, Germany's unification will signify their 1990 unification.

2 The specific similarities and differences in their characteristics will be discussed in light of the variables analyzed in this study. For Section I, the post-union variables are: population, per capita GDP and savings growth rates. (See Section I, Chapter 1.1 for definitions and source of data.) For Section II, the pre-union variables are: deaths, divorces, employment in manufacturing, foetal mortalities, output per person employed, infant mortalities, live births, marriages, migration, population and labor productivity in manufacturing. The post-union variables are: deaths, employment in manufacturing, output per person employed, population and labor productivity in manufacturing. (See Appendix A.10 for definitions and sources of data.)

3 Unequal economic participants is measured by the economies' long-run steady-state per capita GDP levels; per capita GDP in Section I and output per person employed in Section II.

4 These zones did not follow any natural features and Berlin, its capital, was also divided into the four zones.
Two years later, the French zone of occupation merged with the other Western (American-British) zone which resulted in the proclamation on 23 May 1949 of the *Bundesrepublik Deutschland* (FRG) with Bonn as its capital. And, on 7 October, the Soviet zone by constitution was proclaimed as the *Deutsche Demokratische Republik* (GDR) with East Berlin as its capital—two separate German nations.

In this paper, the analysis of the pre-unification period will start with the year 1950 as it was the first full year the FRG and the GDR legally became separate countries.

After forty years of separation, on 18 May 1990 West German Chancellor Helmut Kohl and East German Prime Minister Lothar de Maiziere, signed a state treaty that would merge the two economies and 1 July 1990 was proclaimed as the day for economic and social union—a step toward political union.

Thus, the analysis for the unification period will cover the years 1990 until the latest date available on the data.\(^5\)

---

\(^5\) For Study I, the latest year available is 1999 for the three variables of interest: the population, per capita GDP and savings growth rates. (See Section I, Chapter 1.1 for definitions and source of data.) For Study II, the latest year available for the variables of interest are: 1996 for the labor productivity in manufacturing, output per person employed and employment in manufacturing; 1994 for the population; and 1992 for the deaths variables. (See Appendix A10 for definitions and sources of data.)
Scope of Study.

This paper is divided into two studies with one common goal: After Germany's unification has the monetary union between these two unequal economic participants led to economic convergence, measured by per capita GDP in Section I and output per person employed in Section II?

The quest to define and measure economic convergence as well as a measurement of its speed is found in the literature of economic growth. No study on economic convergence can be complete without exploring this body of literature. This is the focus of my first study (Section I) beginning with Germany's unification period.

The neoclassical model of Solow & Swan in 1956 primarily impacted the proliferation of economic growth theories. Economic growth theories model the long-run growth rate of an economy. When comparing the growth rate of several countries, these models can be used to explain if the gap between the growth rates of the economies is growing closer to one another or to a specified constant (i.e., converging.) In other words, in its broadest sense, a mathematical textbook definition of convergence is the limit of the difference between two series to be close to zero or a specific number: \( \lim_{t \to \infty} (X - Y) = \delta \) (a constant.)

---

6 See a more detailed description of this model in Section I, Chapter 2.

7 These extend to the endogenous growth models of the mid-1980s, the inclusion of human capital, the role of returns to scale and the diffusion of technology. However, Barro & Sala-i-Martin (1992) find that the introduction of technology into the neoclassical production function does not affect the results of Beta.

8 Or for a random series, mathematical textbook definition of weak convergence is the stochastic convergence extended to include the expected value of such limit: \( E \{\lim_{t \to \infty} (X - Y) = \delta\} \).
convergence among countries or regions employ the following two hypotheses about convergence processes: Beta Convergence—Absolute and Conditional—and Sigma Convergence.

Thus, in the first section of my paper, the founding exogenous economic growth model of Solow and Swan with these processes will be used to test whether the same long-run equilibrium steady-state path is evidenced for the two German regions utilizing the notion of “clubs.” The idea of club convergence\(^9\) sparked by Baumol & Wolff and later used by Barro & Sala-i-Martin (1995) will be used to place the former GDR and FRG German regions into a club and answer the question of whether convergence has occurred among each club ten years since their unification.

A more detailed study of the regions themselves is the focus of my second study. Section II takes a statistical, graphical dispersion measurement of convergence and the Dickey & Fuller unit-root approach in analyzing the economic convergence of East and West Germany as well as reviewing ten other data series\(^{10}\) for the pre-union period and four other data series\(^{11}\) for the post-union period. Analyses from these three approaches and possible explanations for the regions’ results and differences are discussed.

---

\(^9\) Economies that are similar in their structural characteristics and have similar conditions will converge to one another so that the developed, developing and underdeveloped economies would be an example of (constitute) three clubs. See Section I, Chapter 3.2 for further discussion on club convergence.

\(^{10}\) These ten data series are: deaths, divorces, employment in manufacturing, foetal mortalities, infant mortalities, live births, marriages, migration, population and labor productivity in manufacturing.

\(^{11}\) These four data series are: deaths, employment in manufacturing, population and labor productivity in manufacturing.
As separate data were not readily available for the former two nations, a database encapsulating these eleven data series for each region, over the pre- and post-union periods, was created. Chapter 1.1 in Section II describes my data collection efforts as well as the sources and definitions of the variables used in this section of study.

The statistical calculations\textsuperscript{12} for each data series explored, in natural logarithms (\( \ln \)) are the mean, standard deviation, variance, coefficient of variation, continuous and cumulative growth rates.

The graphical dispersion measurement of convergence\textsuperscript{13} measures the standard deviation of the natural logarithm (\( \ln \)) of the data series of the two German regions over time. (See Section II, Chapter 1.2 for methodology.) For convergence to exist, the standard deviation of the cross-regions' \( \ln \) of the variable must decrease over time (\( \lim_{t \to \infty} \sigma_t \to 0 \) as \( t \to \infty \)). When graphed time/year on the horizontal axis and dispersion (calculated as the standard deviation of the cross-regions' \( \ln \) of the variable over time) on the vertical axis, it should exhibit a downward-sloping curve.

The test that will be used to measure whether a data series has a stationary or non-stationary trend, implying convergence, or non-convergence, respectively, is the Dickey & Fuller unit-root (DF)\textsuperscript{14} test. The DF test will be performed on each of the eleven

\textsuperscript{12} Results shown in Tables 6 and 8 for the pre- and post-union periods, respectively.

\textsuperscript{13} See Graphs G3 – G17.

\textsuperscript{14} The Augmented Dickey-Fuller (ADF) approach controls for higher-order correlation by adding lagged difference terms of the dependent variable to the right-hand side of the regression. This test, as opposed to the DF test, is the norm in most literature with time series data. However, in order to perform the
individual time series for both regions. If I can reject the null hypothesis of a unit root, in favor of the alternative that its root is less than one, this would imply that the series has a stationary, deterministic trend where the shocks are temporary and over time will dissipate. The series will eventually revert to its long-run mean level—convergence. If, on the other hand, the series fails to reject the null hypothesis of a unit root, this would imply a non-stationary series—a stochastic trend with an increasing variance over time. However, this does not imply non-convergence. Taking first differences of the series and trying to fit a stationary model to the differences, I can then test if the first difference of a unit root process is covariance stationary. When comparing the two regions’ series, if the variables have different orders of integration, in levels—are not moving to a constant differential from one another—this would imply non-convergence of the two series. However, if the variables have the same order of integration, in levels, convergence of the two series cannot be ruled out. Thus, a form of co-integration is needed to test whether a linear combination of the two series is stationary. I check for co-integration by testing the differences between the two regions’ data series; the E-W Difference series referred to in my analysis. Thus for each of the eleven variables, I take the difference between the two nations’ series and test this E-W Difference series for stationarity. If I again fail to reject the null hypothesis of a unit root in favor of the alternative, in levels, this would lead to the additional evidence of non-convergence.

ADF test, you have to specify the number of lagged first difference terms to add to the test regression or include lags sufficient to remove any serial correlation in the residuals. As I have short-sample data especially for the post-unification period (at most seven data points) I have opted to use the DF test with the methodology briefly described above and in more detail in Section II, Chapter 1.3 “Unit-Root Methodology.”

15 Testing each series for stationarity is needed for comparing the order of integration between the two series. Results can be found in Tables 7 and 9 for the pre- and post-union periods, respectively.

16 See Section II, Chapter 1.3 “Unit-Root Methodology.”
Analyses from the combined three approaches (statistical, graphical and unit-root) for the pre-union output per person employed data series will show that economic convergence had not occurred by year-end 1989 (see Section I, Chapter 2.6.) For the remaining ten variables, there is no a priori reason why these data series should converge, especially for the pre-unification period. Additionally, even if the dispersion between the two regions’ rates, say deaths to population growth rates did converge, the larger country (FRG) would still have a permanently higher death level than the smaller country (GDR.) Thus, convergence of the dispersion of growth rates does not necessarily imply convergence in levels for these ten variables.\textsuperscript{17} Therefore, these variables are reported with two purposes in mind. One, aiming to show the differences of these two nations and where possible offer reasons for their differences, analysis of the differences in territory and government (for the pre-unification period) and the remaining ten statistical variables\textsuperscript{18} and their inter-relationships will be explored. Second, fertility, migration, mortality and the population growth rates (live births, migration, mortality—deaths, infant and foetal—and the population growth rates serving as proxies, respectively) are variables analyzed in the neoclassical growth theory model (Study I) that affect the per capita GDP growth rates. Thus, I reconcile the results of my findings for these data series in Study II within the context of the neoclassical growth theory discussed in Study I.\textsuperscript{19}

\textsuperscript{17} However this is not true for the GDP PP variable as neoclassical growth theory does predict convergence of per capita income levels. See Section I, Chapter 2.1.3.

\textsuperscript{18} Results are shown in Tables 6, 7 and Graphs G3-G13 for the pre-unification period and in Tables 8, 9 and Graphs G14-17 for the post-unification period.

\textsuperscript{19} Results of findings can be found in Section II, Chapter 2.6.
Analyses from the combined three approaches (statistical, graphical and unit-root) for the post-union output per person employed eastern and western Germany's data series will show whether economic convergence after Germany's monetary unification, as of the end of the study period, 1996, has occurred.

Furthermore, a comparison of the pre- and post- standard deviation and arithmetic mean signifying convergence and 'catch-up' respectively, is performed. In comparing the pre- and post-union periods, if I find a decrease in the standard deviation of the $ln$ of the variable between the two periods, this would imply a tendency toward convergence. And, if I find an increase in the arithmetic mean of the $ln$ of the variable between the two periods, this would imply a tendency for that region to 'catch up' (to the other.) Thus, for the five variables that span through both periods, I compare the pre- and post-union results utilizing the above methodologies to see whether a monetary union resulted in a change, if any, in the post-union data series (as compared to their pre-union positions.)

The analysis of Study I (post-union convergence testing) and the analysis of Study II (the GDR and FRG data series' statistical, graphical measurement of convergence and unit-root pre- and post-union testing) will show the economic impact of a monetary union of two unequal economic participants.

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20 See Section II, Chapter 4 for full discussion.

21 See Tables 6 and 8 for the pre- and post-union results, respectfully.

22 The variables that span through both the pre- and post-periods are: deaths, employment in
Roadmap.

This thesis is divided into two sections—one for each study—with a common conclusion.

Section I focuses on growth theory, the Solow-Swan Model\(^2\)\(^3\) and convergence testing for the post-unification period—1990 to 1999.

Chapter 1 discusses the data, source\(^2\)\(^4\) and definition of variables used in this study; the categorization of economies by income (Table 1); and the assumptions made about the income categorization, club, of eastern and western Germany. It also introduces the concept and rationale of the three convergence-testing processes: Absolute Beta Convergence (ABC) Conditional Beta Convergence (CBC) and Sigma Convergence (SC.)

Chapter 2 is devoted to the concept and testing of ABC. It delves into detail the exogenous growth Solow-Swan model, defines neoclassical and the Solow-Swan model with ABC. Included in this chapter are the literature reviews, criticisms and the ABC testing results (Table 2.)

\(^2\)\(^3\) As an addendum to the theories of growth but not applied in this study, Appendix A3 offers a brief discussion of the extensions of the neoclassical Solow-Swan model and convergence.

\(^2\)\(^4\) Only one data source was used for this section of study. The data was extracted from the 2001 World Development Indicators (WDI) database on CD-ROM.
Chapter 3 is the focus of the Conditional Beta Convergence (CBC) concept. It discusses the neoclassical Solow-Swan model with CBC, literature reviews, criticisms and the CBC testing results (Tables 3-5.)

In Chapter 4, the relationship with Sigma Convergence (SC) and the neoclassical Solow-Swan model as well as the relationship with SC and Absolute Beta Convergence are introduced. This chapter concludes with the literature reviews, criticisms and the SC testing results (Graphs G1-G2.)

Chapter 5 is the post-unification period’s summary chapter for Section I of the Absolute Beta Convergence, Conditional Beta Convergence and Sigma Convergence testing.

Section II of this study, focuses on the statistical, graphical dispersion measurement of convergence and unit-root testing of the pre- and post-unification data series of both German nations. The format of this study, Section II, parallels the format of study in Section I.

Chapter 1 discusses the data sources, database creation, graphical measurement of dispersion, and the Dickey Fuller unit-root test measuring stationarity—possibility of convergence/divergence for each separate and comparative data series.

In Chapter 2, the focus is on the individual economies of East and West Germany from 1950 to 1989—the pre-unification period. It analyses and reports the statistical data differences between the GDR and the FRG (Table 6) the results of unit-root testing for each separate and comparative data series (Table 7) and the analyses of the graphical
dispersion measurement of convergence (Graphs G3-G13.) Discussed are the differences in territory and governments, differences in the population as well as the possible reasons for the differences in the population—Berlin Wall, migration, live births, and marriage/divorce rates. It further explores the differences in deaths, foetal mortalities and infant mortalities in addition to the differences in output per person employed, employment in manufacturing and labor productivity in manufacturing. Other miscellaneous statistical differences\(^{25}\) are provided for East-West comparative purposes. The chapter concludes with a summary section for the pre-unification period.

The analysis of Chapter 3 begins with the unification period of study, 1990. For the five data series\(^{26}\) — deaths, employment in manufacturing, output per person employed, population and labor productivity in manufacturing — for the former-FRG and former-GDR nations, presented are: the statistical findings (Table 8); the results of the Dickey-Fuller unit-root test (Table 9) to determine if stationarity in these variables, separately and jointly are found; and the analyses of the graphical dispersion measurement of convergence (Graphs G14-G17.)

Chapter 4 is the summary chapter for Section II of the statistical, graphical dispersion measurement of convergence and unit-root testing results for the pre- and post-union periods with respect to the deaths, employment in manufacturing, output per person employed, population and labor productivity in manufacturing variables.

\(^{25}\) Real GDP per worker, price level GDP, degree of openness, investment share of GDP and consumption share of GDP. See Section II, Chapter 2.5 for figures and definitions.

\(^{26}\) Of the eleven variables in the database for the study of Section II, these are the only data series that span through both the pre- and post- East and West periods. See Appendix A10.
The last section in this paper is the Summary & Implications. It summarizes and integrates the findings of both studies, Sections I and II, and attempts to answer the question of whether a monetary union of two unequal economic participants (dissimilar economies) that were once one country, has led to economic convergence—defined as per capita GDP in Section I and output per person employed in Section II—as of the ending period of study, 1999. Included is a short discussion on possible future research.
SECTION I.
GROWTH THEORY, SOLOW-SWAN MODEL &
CONVERGENCE TESTING

No study on convergence testing can be complete without studying the founding exogenous growth theory of the Solow-Swan model with Absolute Beta Convergence (ABC) Conditional Beta Convergence (CBC) and Sigma Convergence (SC.)

The data used for this section of study—per capita GDP, population and savings growth rates—as well as the methodology and the concepts of convergence testing of the High-OECD-Income Club (HOIC) and the Upper-Middle-Income Club (UMC) economies, representing the former West and East German nations, respectively, follow.
Chapter 1. Data and Convergence Methodology

1.1 Data Source & Definition of Variables

The variables utilized in the Beta—Absolute and Conditional—and Sigma Convergence processes are the variables cited by the neoclassical model of Solow & Swan that characterize differing steady states for each economy. Thus, the variables under analysis for the post-union period, 1990-1999, that may determine the characteristics of each economy, whereby having the economies converge to their own steady-state equilibrium, are the population rate, per capita GDP and the savings rate. The population growth rate, per capita GDP, and the savings rate as a percent of GDP, proxy for the above variables, respectively. These three variables are extrapolated from the 2001 World Development Indicators (WDI) CD-ROM database and are defined as follows.

POP GROWTH RATE is the population growth rate reported as an annual percent. The “population is based on the de facto definition of population, which counts all residents regardless of legal status or citizenship—except for refugees not permanently settled in the country of asylum, who are generally considered part of the population of the country of origin.”

GDP PER CAPITA is the per capita GDP expressed in constant 1995 U.S. Dollars. It is defined as gross domestic product divided by the mid-year population. “GDP is the sum of gross value added by all resident producers in the economy plus any product taxes and

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27 World Development Indicators: \Definitions\Population Growth Rate.
minus any subsidies not included in the value of the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural resources.\textsuperscript{28} This variable will be converted in natural logs for the analyses in Tables 2-5.

SAVINGS RATE is the savings rate defined as $1 - \text{consumption rate}$. In this case, final consumption expenditure expressed as a percent of GDP is used as a proxy for consumption. It is the sum of household final-consumption expenditure (private consumption) and general government final-consumption expenditure (general government consumption.) "This estimate includes any statistical discrepancy in the use of resources relative to the supply of resources."\textsuperscript{29}

Furthermore, the WDI database categorizes countries by four income levels. High-income (HIC) countries include 27 OECD (HOIC) economies and 23 non-OECD (HNOIC) economies. For both groups, high-income economies are those in which the 1999 gross net income (GNI) per capita was $9,266 or more. The 38 economies that comprise the upper-middle-income (UMC) group are those in which the 1999 GNI per capita was between $2,996 and $9,265. The lower-middle-income (LMC) group countries are comprised of 55 economies in which the 1999 GNI per capita was between $755 and $2,995. Lastly, the 64 low-income (LIC) economies are those in which the 1999 GNI per capita was $755 or less. Due to missing data on the WDI database, some

\textsuperscript{28} World Development Indicators: Definitions\GDP Per Capita.

\textsuperscript{29} World Development Indicators: Definitions\Final Consumption Expenditure Expressed as a Percent of GDP.
countries are excluded from the analyses. Appendices A5 through A9, representing these
five income groups, list the countries that are included and excluded from the above
categories. This income categorization will be used in the analysis of club
convergence.\textsuperscript{30}

The statistics on the 2001 WDI database are reported up to 1999. Thus all post-
unification analyses in this study, Section I, for the three variables, POP GROWTH
RATE, GDP PER CAPITA and the SAVINGS RATE for the HOIC, HNOIC, UMC,
LMC and LIC income categories, are assessed on ten years of data.

Based on the five income categories from the WDI database described above, the
following table reports the average value for the population, per capita GDP, and savings
rate variables for the period 1990-1999—the post-unification period. Again, the
neoclassical Solow-Swan model\textsuperscript{31} prescribes these variables that determine the
characteristics of each economy, whereby having the economies converge to their own
steady-state equilibrium.

\textsuperscript{30} See Baumol, Blackman & Wolf and Section I, Chapter 3.1: Neoclassical Solow-Swan Model & CBC.

\textsuperscript{31} The neoclassical Solow-Swan model is fully described in this section of study in Chapter 2.
Table 1. ALL WDI ECONOMIES AVERAGE VALUES FROM 1990-1999

<table>
<thead>
<tr>
<th>Economic Groups</th>
<th>POP Growth Rate</th>
<th>Per Capita GDP</th>
<th>Savings Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>HOIC</td>
<td>0.599029</td>
<td>$25,946.66</td>
<td>0.230815</td>
</tr>
<tr>
<td>HNOIC</td>
<td>1.774129</td>
<td>$16,583.00</td>
<td>0.288635</td>
</tr>
<tr>
<td>UMC</td>
<td>1.223903</td>
<td>$5,170.29</td>
<td>0.225850</td>
</tr>
<tr>
<td>LMC</td>
<td>1.523027</td>
<td>$1,558.95</td>
<td>0.153236</td>
</tr>
<tr>
<td>LIC</td>
<td>2.347337</td>
<td>$450.30</td>
<td>0.764098</td>
</tr>
</tbody>
</table>

Notes: POP Growth Rate is reported as an annual percent. Per Capita GDP is expressed in constant 1995 U.S. Dollars. Savings Rate is defined as 1 – consumption rate. In this case final consumption expenditure expressed as a percent of GDP is used as a proxy for consumption.

Expanded definition of variables & economies can be found in Section I.1.1.

HOIC – High OECD Income Economic Club
HNOIC – High Non-OECD Income Economic Club
UMC – Upper-Middle Income Economic Club
LMC – Lower-Middle Income Economic Club
LIC – Low Income Economic Club

List of countries within each economic group can be found in Appendices A5-A9; respectively.


Data on these three variables reported in Table 1, cannot be separately retrieved for eastern and western Germany from the World Development Indicators (WDI) database. This is because WDI does not report statistics on eastern Germany; data on Germany prior to 1990 is not available; and data on Germany beginning from 1990 are reported in the aggregate (both regions.) Thus for the club convergence study, in Section I, the
former West Germany will be considered a member of the HOIC income-club as reported by WDI, and I will make the assumption that former East Germany will be considered a member of the UMC income-club. I base this later assumption on the following. The GDR more closely resembles the economies of the UMC with respect to the variables in question. East Germany's per capita GDP as of 1988\(^{32}\) was already up to $11,209. Additionally, even though some Eastern Bloc economies are listed as members of the LMC category (such as Bulgaria, Kazakhstan, Latvia, Lithuania, Romania and Yugoslavia) East Germany was considered to be the most developed country of the Eastern Bloc. Of the countries that comprise the Eastern Bloc, the more advanced economies of Estonia, Hungary, Poland, Czech and Slovak Republics are categorized as UMC by the WDI database.

For the remainder of this section of study and for the club convergence discussions to follow (Section I, Chapter 3.1) the LIC, LMC and the HNOIC income categories are not relevant for the study of eastern and western Germany and are therefore excluded from the convergence testing. Thus, for the post-unification period, I will restrict my analyses of convergence testing, to the two economic (income) groups: HOIC and UMC of which western and eastern Germany, respectively, are by assumption, members.

1.2 Convergence Methodology

Beta Convergence—Absolute Beta Convergence (ABC) and Conditional Beta Convergence (CBC)—and Sigma Convergence (SC) are features of statistical processes;
dependent and theory-free concepts that show whether a given series, over a period of time, is negatively related to its beginning period.

Once these concepts are incorporated in context with a theoretical growth model, these models can be used to test whether the same long-run equilibrium steady-state path is evidenced among all (or a group of specified) countries with each other and answer the question of whether convergence exists among each club.

Each of the three (Absolute Beta Convergence, Conditional Beta Convergence and Sigma Convergence) concepts in the context of the neoclassical exogenous growth model of Solow and Swan, measuring convergence are described in sequence (in Chapters 2-4) including their corresponding literature reviews, criticisms and my test results.

Specifically, in Chapter 2, convergence testing is performed across the High-OECD Income (HOIC) and the Upper-Middle Income (UMC) economies, to test whether cross-country convergence—a mean revision in the level of per capita GDP across the economies of HOIC and UMC, or Absolute Beta Convergence—exists.

As I do not suspect convergence (ABC) to exist amongst both income-group economies, Conditional Beta Convergence (CBC) the concept of club convergence the concept of club convergence will be explored in Chapter 3. If CBC is found, this implies that the economies within

---

33 Indeed, ABC for the HOIC and UMC economies was not found; see Table 2. ABC was also tested on all five income clubs which also proved non-convergence; results not shown.

34 Baumol & Wolff and Barro & Sala-i-Martin (1995.)
each income club will converge to their club’s own steady-state long-run equilibrium incomes. Thus, if Absolute Beta Convergence is not found and CBC club convergence is found, this would support the hypothesis that the two clubs have not converged to one another—the steady-state growth rates are not homogeneous across the two clubs during the period of study.

In any case, if convergence is found for Absolute Beta Convergence or CBC, the speed of such convergence is calculated as prescribed in Appendix A1.

Sigma Convergence (SC) is discussed in Chapter 4. SC will be said to exist if the dispersion, measured as the unweighted standard deviation for the $\ln$ of the GDP PER CAPITA over the post-unification period decreases. Note that Beta Convergence is a necessary but not a sufficient condition for SC as the dispersion not only depends on the long-run steady-state equilibrium values but also depends on the variance of the error or shock term. (See Section I, Chapter 4.1) This is why I have included this test in addition to the analyses of Beta Convergence.
Chapter 2. Absolute Beta Convergence

The concept of Beta Convergence asks if the growth rate of a country’s variable (measured by per capita GDP) over a given period, is negatively related to the level of a country’s variable at the beginning of the period.

If you apply this definition to analyze the convergence of a single economy, this would imply that the lower the beginning per capita income, the higher the growth rate of that economy’s per capita income over a given period of time. In other words, as the base year’s per capita income increases with time, the differences between the initial and base year’s per capita incomes will decrease over time.

If you apply this definition to analyze the convergence of many economies, this would imply that at a specified beginning and ending year, economies with a lower starting per capita income, will experience a higher growth rate of that economy’s per capita income measured as the difference between the ending and beginning period of time. In other words, for all countries, as the base year’s per capita income increases the differences between the ending and base year’s per capita incomes will decrease.

Mathematically, the above concept for each economy (ignoring the subscript \( i \) representing each economy) in its general form in discrete time can be expressed as:

\[
ln \left( \frac{Y_t}{L_t} \right) - ln \left( \frac{Y_{t0}}{L_{t0}} \right) = \alpha - \beta ln \left( \frac{Y_{t0}}{L_{t0}} \right) \quad \text{[Equation 1]}
\]
where: \( Y \) = GDP\(^{35} \)

\( L \) = population

\( \ln(Y/L) \) = per capita income\(^{36} \)

\( t \) = latest period/year of analysis

\( t_0 \) = beginning period/year of analysis

\( \alpha \) = a positive constant

\( \beta \) = Beta coefficient

To ensure this negative connection and in order for convergence as defined above to exist, the assumption is that Beta must be a positive integer and lie between 0 and 1. (See Appendix A2 for the Extension of the Beta Convergence Concept.)

Absolute Beta Convergence (ABC) is defined as convergence by different countries to the same level of a variable (i.e., per capita GDP.) Put differently, it asks whether there is mean reversion in the level of that variable across economies.

The mathematical properties and relationships that exist in Equation 1 hold true for Absolute Beta Convergence. However this concept assumes that the economies in question share a common steady state—a situation where the quantities of the variable(s) being analyzed grow at a constant rate, \( dx/dt = 0 \), where \( x \) is a variable under analysis, \( d \) is the first derivative and \( dt \) is the time derivative.

---

\(^{35}\) Output implies income only when net foreign assets are zero. (Obstfeld & Rogoff, pg. 455) This discussion is also addressed in Appendix A3 “Neoclassical Solow-Swan Model Extension & Convergence.”

\(^{36}\) Output per worker implies output per capita only when countries have identical labor force participation.
Example: This implies that countries will have the same steady-state per capita incomes, where the variable \( x \) represents per capita incomes. Convergence of this type, when applied to various growth models asks whether initially poor (low-income per capita) economies tend to grow faster than initially rich (high-income per capita) economies.

The most prominent exogenous growth model is the neoclassical Solow-Swan model.

### 2.1 Neoclassical Defined

"Neoclassical" refers to the production function. If we ignore technological progress,\(^3\) then the production function that has as its inputs only physical capital \( (K) \) and labor \( (L) \) can take the form:

\[
Y = f(K, L)
\]

which in its intensive form, can be written:

\[
y = f(k) \quad \text{[Equation 2]}
\]

where \( y \) is output per effective unit of labor and \( k \) is capital per effective unit of labor.

However, in order for it to be labeled "neoclassical" the following three properties must be met. (See Barro & Sala-i-Martin (1995) Chapter 1)

1. For positive \( K \) and \( L \) \( (K > 0 \text{ and } L > 0) \) the production function exhibits positive and diminishing marginal products to each input so that:

\[ \]

\(^3\) Barro & Sala-i-Martin (1992) find that if technological progress, represented by parameter \( A \), was used in the production function, the result does not affect \( \beta \). "Thus the convergence coefficient \( \beta \) can be similar across economies that differ greatly in levels of per capita product because of differences in the available technique (or in government policies or natural resources that amount to differences in the parameter \( A \))." pg. 226.
\( \frac{\partial f}{\partial K} > 0, \quad \frac{\partial f}{\partial L} > 0 \)

\( \frac{\partial^2 f}{\partial K^2} < 0, \quad \frac{\partial^2 f}{\partial L^2} < 0 \)

2. The production function exhibits constant returns to scale:

\( f(\lambda K, \lambda L) = \lambda f(K, L) \) for all \( \lambda > 0 \).

3. The Inada conditions are satisfied: the marginal product of each input approaches infinity as each input goes to zero and approaches zero as each input goes to infinity.

\[
\lim (f(K)) = \infty \text{ as } K \to 0, \quad \lim (f(L)) = \infty \text{ as } L \to 0
\]

\[
\lim (f(K)) = 0 \text{ as } K \to \infty, \quad \lim (f(L)) = 0 \text{ as } L \to \infty
\]

The Cobb-Douglas production function: \( Y = AK^aL^{1-a} \) is an example of a neoclassical production function.\(^{38}\)

In a neoclassical model, the standard condition is that of a closed economy whereby output is equal to income and the amount invested is equal to the amount saved in the economy, in equilibrium. (Barro & Sala-i-Martin (1995) pg. 15) The relaxation of this assumption and its implications for convergence are discussed in Appendix A3.

### 2.1.1 Solow-Swan Model

The Solow-Swan model is a growth model with all long-run exogenously-set elements and a constant and positive saving-rate specification. Its fundamental differential

\(^{38}\) Barro & Sala-i-Martin (1995) show, on page 17, the mathematical proof of how the Cobb-Douglas production function meets the neoclassical criteria.
equation, derived from the production function and the definition of the stock of physical
capital at a point in time in a closed economy,\textsuperscript{39} is defined as:

$$\frac{dk}{dt} = s \cdot f(k) - (n + \delta) \cdot k$$

[Equation 3]

where \(d\) = first derivative,

\(s\) = saving rate and equals \(1 - c\) where \(c\) is the rate of consumption,

\(k\) = \(K/L\),

\(n\) = population growth (of \(L\))

\(\delta\) = depreciation rate (of \(K\))

\(n + \delta\) = effective depreciation rate for \(k\).

This nonlinear equation depends only on \(k\).

\textbf{2.1.2 The Neoclassical Solow-Swan Model}

Combining the neoclassical properties to the Solow-Swan model, this implies that the per
capita, quantities \(k\), \(y\), and \(c\), do not grow in steady state\textsuperscript{40} but the levels of the variables
\(K\), \(Y\) and \(C\) grow in steady state at the rate of the population growth, \(n\). Put differently,
"changes in the level of technology, the saving rate, the rate of population growth and the
depreciation rate do not affect the steady-state growth rates of per capita output, capital
and consumption" which are all equal to zero. (Barro & Sala-i-Martin (1995) pg. 19)

\textsuperscript{39} A net increase in the stock of physical capital at a point in time for a closed economy is defined as gross
investment minus depreciation. This can be written as: \(dK/dt = I - \delta K\) where \(I\) is gross investment. As
amount invested = amount saved in a closed economy, then gross investment = amount saved times the
production function so that \(dK/dt = S \cdot f(K,L) - \delta K\). Dividing both sides by \(L\), to obtain the intensive
form, and setting \(n = dL/L\), yields Equation 3.

\textsuperscript{40} The steady-state values, denoted by an asterisk, for \(y\) and \(c\) are: \(y^* = f(k^*)\) and \(c^* = (1 - s) \cdot f(k^*)\);
respectively.

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Example: if via government policy the savings rate is increased (decreased) permanently, this will only exhibit short-run positive (negative) per capita growth rates but in the long-run, the per capita growth rates will return to zero with the levels of \( k \) and \( y \) being permanently higher (lower.)

If we divide both sides of the above equation [Equation 3] by \( k \) then:

\[
\frac{dk}{dt} = \frac{s \cdot f(k)}{k} = (n + \delta) \tag{Equation 4}
\]

where: \([(dk/dt) / k]\) is the growth rate of capital per worker

The left-side of the equation \((s \cdot f(k) / k)\) is the downward-sloping saving rate curve while the right-side of the equation \((n + \delta)\) is a constant number and is invariant with \( k \).

If we were to graph Equation 4, \( k \) would be measured on the horizontal axis. The intersection of \([s \cdot f(k) / k]\) representing the downward-sloping savings rate curve; and \((n + \delta)\) representing the horizontal, constant and invariant curve with \( k \); would connote steady-state status \((k^*)\). Thus if \( k < k^* \), the growth rate of \( k \) is positive and \( k \) would increase toward \( k^* \) while if \( k^* < k \), the growth rate of \( k \) is negative and \( k \) would monotonically approach (decrease toward) \( k^* \).

\[\text{Again, steady-state is defined as the various quantities growing at a constant rate whereby } \frac{dk}{dt} = 0 \text{ so that from Equation 3: } s \cdot f(k) = (n + \delta) \cdot k = k^* \text{ where } k^* \text{ refers to the steady-state growth rate equal to zero.}\]
If the production function is Cobb-Douglas (neoclassical) then Equation 4 = $\gamma_k$; the growth rate of the K/L ratio. So, Equation 4 can be rewritten as:

$$\gamma_k = s \cdot f(k) / k - (n + \delta)$$  \[Equation 5\]

When we incorporate the above equation with the Cobb-Douglas production function, in its intensive form and ignore the technological variable (i.e., $y = k^\alpha$) then the above equation equals,

$$\gamma_k = s \cdot k^{1-\alpha} - (n + \delta)$$  \[Equation 6\]

In addition, we find that the derivative of $\gamma_k$ with respect to $k$ is negative.

$$\frac{\partial \gamma_k}{\partial k} = s \cdot [f'(k) - f(k) / k] / k < 0$$

Thus, smaller values of $k$ are associated with larger values of $\gamma_k$. Does this necessarily mean convergence?

### 2.1.3 Neoclassical Solow-Swan Model & Convergence

In measuring convergence in this model, the model does not predict convergence in all cases, which implies that the poor (low-income) economies will not as a priori grow faster than the rich (high-income) ones; Absolute Beta Convergence need not exist. Thus in order to preserve this inverse relationship between growth rates and initial positions is to hold constant the savings rate, the level of the production function and all government policy variables which may shift the position of the production function. The transitional

---

$^{42}$ The growth rate of the level of $y$ is given by $\gamma_k = \gamma_k + n$. 

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dynamics of this model does however show how an economy's per capita income converges toward its own steady state and to those of other economies; CBC is assured.

To do this, we must look at the behavior of output along the transition and its relation to the growth rate of the K/L ratio, $\gamma_k$.

If we let $\gamma_y$ represent the growth rate of output per capita, then,

$$\gamma_y = \frac{\frac{dy}{dt}}{y} = \frac{f'(k)}{[(\frac{dk}{dt})/f(k)]} = \frac{k \cdot f'(k)}{f(k)} \cdot \gamma_k$$

where $[k \cdot f'(k)/f(k)]$ is the capital share, $a$, of the Cobb-Douglas production function, so that the above equation is reduced to:

$$\gamma_y = a \cdot \gamma_k$$

[Equation 7]

Thus, the behavior of $\gamma_y$ follows the same pattern of $\gamma_k$.

2.1.4 Neoclassical Solow-Swan Model & Absolute Beta Convergence

Specifically, to test for the existence of ABC in the neoclassical Solow-Swan model, if we take the log linear approximation of Equation 3 around the steady state, $\gamma^*$,

$$\gamma_k = \frac{d(ln k)}{dt} \equiv -\beta ln (k/k^*)$$

[Equation 8]

where $\beta = (1 - a) \cdot (n + \delta)$

[Equation 9]

and determines the speed of convergence from $k$ to $k^*$.44


44 The true speed of convergence for $k$ and $y$ is not constant; it depends on the distance from its steady-
As $\gamma_k$ also applies to $\gamma_y$, as shown by Equation 7, then

$$\ln \gamma / \gamma^* = a \cdot \ln (k / k^*)$$

If we substitute the above equation into Equation 8,

$$\gamma_y \equiv -(1 - a) \cdot (n + \delta) \ln (\gamma / \gamma^*) \quad \text{or} \quad \gamma_y \equiv -\beta \ln (\gamma / \gamma^*) \quad \text{[Equation 10]}$$

where $\beta = (1 - a) \cdot (n + \delta)$; same convergence coefficient as $k$; Equation 9.

This indicates how rapidly an economy's output per capita $\gamma$ approaches its steady-state value $\gamma^*$. This implies that for a neoclassical production function, Beta is independent of $s$, the saving rate and $A$, the level of technology. So, for given values of the parameters, $n$ and $\delta$, Beta is determined by the capital share parameter, $a$. The higher (lower) the $a$, the lower (higher) the $\beta$.

Equation 10 is a differential equation in $\ln (\gamma_t)$ with the solution:

$$\ln (\gamma_t) = (e^{-\beta}) \ln (\gamma_0) + (1 - e^{-\beta}) \ln (\gamma^*) \quad \text{[Equation 11]}$$

where time $t$ in $\ln (\gamma_t)$ is halfway between $\ln (\gamma_0)$ and $\ln (\gamma^*)$.

So the average growth rate of $\gamma$ over the interval between years $0$ and $T$ would equal:

$$(1/T) \cdot \ln \left( [(\gamma_T) / (\gamma_0)] \right) = (1/T) \cdot (1 - e^{-\beta}) \ln (\gamma^*/\gamma_0) \quad \text{[Equation 12]}$$

state so that $\beta$ is really equal to $(1 - a) \cdot (n + \delta) \cdot [(y/y^*) \exp -(1-a)/a]$. But at steady-state, $y = y^*$ and thus the last term drops out.
The higher the Beta, the greater the responsiveness of the average growth rate to the gap between \( \ln(\gamma^*) \) and \( \ln(\gamma_0) \),\(^{45}\) the more rapid the convergence to the steady state.

If we express Equation 12, in discrete periods to economy \(^{46}\) and add a random disturbance term, the following equation can be written:

\[
\ln \left( \frac{\gamma_{i,t}}{\gamma_{i,t-1}} \right) = \alpha_i - (1 - e^{-\beta}) \cdot \ln \gamma_{i,t-1} + \varepsilon_{i,t}
\]

[Equation 13]

where \( \alpha_i = (1 - e^{-\beta}) \ln \gamma_{i,*} \)

and \( \alpha_i = \alpha \) as \( \alpha_i \) is the same for all \( i \) so that the steady-state value of \( \gamma_{i,*} \) does not differ across economies.

From the above, this implies that the average growth rate over the interval between any two points in time, \( t_0 \) and \( t_0 + T \) will equal:

\[
(1/T) \cdot \ln \left( \frac{\gamma_{i,t_0+T}}{\gamma_{i,t_0}} \right) = \alpha - (1/T) \cdot (1 - e^{-\beta T}) \ln \gamma_{i,t_0} + \varepsilon_{i,t_0,t_0+T}
\]

[Equation 14]

where: \( \alpha \) is a constant and equal to \( (1/T) \cdot (1 - e^{-\beta T}) \ln (\gamma^*) \) which is independent of all \( i \) and assumes \( \gamma_{i,*} = \gamma^* \). As \( T \) goes to infinity, \( (1 - e^{-\beta T}) \) goes to 0. The left-hand expression is the annualized rate of growth of relative per capita GDP in each country (the \( i \) country) over the study period between time \( t_0 \) and \( t_0 + T \).

\(^{45}\) Where \( \gamma_0 \) represents the conditional variable.

\(^{46}\) The subscript \( i \) represents an individual economy so that \( i = 1 \ldots N \).

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$\varepsilon_{i,t_0,t_0+T}$, is the disturbance term. This error term $(\varepsilon_{i,t_0,t_0+T})$ is a distributed lag of the error terms $\varepsilon_{i,t}$ between dates $t_0$ and $t_0+T$. It is an average of the shocks over the interval. If there is a serial persistence in the error term, $\varepsilon_{i,t}$, then the correlation between $\varepsilon_{i,t_0,t_0+T}$ and $ln y_{i,t_0}$ is likely to be large for small $T$ but negligible for large $T$.

One way to solve this problem, in estimating $\beta$, is to use nonlinear least square regression so as to avoid the differences in the length of interval chosen. This is the method used by Barro and Sala-i-Martin (1992.)

However, OLS is used to estimate the regressions for the HOIC and UMC participants, as the length of interval, 1990-1999 (10 years) is constant for all economies.

If, after running the regression, Equation 14, $\beta$, the Beta coefficient, has a positive value between zero and one, then the data set exhibits Absolute Beta Convergence. It signifies the tendency for per capita incomes to equalize across economies, same steady-state growth rates.

---

47 This is equal to $(1/T) \sum \tau$ (from 0 to $T$) $e^{-\beta(T+\tau)}$

48 The assumption is the absence of capital market imperfections (Obstfeld & Rogoff, pg. 454.)
2.2 Literature Review

In applying the test of Absolute Beta Convergence to the neoclassical Solow-Swan model, most studies if they find ABC, find a strong tendency toward Absolute Convergence at a slow rate of 2% per year which translates to approximately 35 years for half the gap in two countries’ per capita incomes to close.

For the OECD countries, Obstfeld & Rogoff plotted the $\ln$ of 1990 real GDP per worker – $\ln$ of 1950 real GDP per worker on the vertical axis and the $\ln$ of 1950 GDP per worker on the horizontal axis and found a negative relationship—meaning that poorer (low-income) economies grew at a faster rate than higher per capita income countries. Their regression model yielded a coefficient of 6.47 and a slope of 0.58 (both statistically significant and an $R^2$ of 0.83.) More than half the absolute difference in initial country per capita incomes had dissipated by the end of the sample period. Thus, they find a strong tendency toward ABC at a slow rate of 2% per year.

Additionally, Baumol, de Long and Romer agree that if there is convergence per worker across countries, the rate is likely to be low similar to Barro & Sala-i-Martin’s many studies among states in the US, prefectures in Japan and 90 regions across 8 countries in Europe that also find a growth rate of about 2% per annum.

Cashin & Sahay examined the 20 states of India for the period 1961-1991 using the real per-capital-state-net-domestic product. They found evidence of ABC however; the speed of convergence was slower, at 1.5% of the gap to close within each year, as opposed to the 2% reported by other studies of regional convergence in developed countries.
Furthermore, this slow speed of convergence contradicts the hierarchy of convergence. That is regions within a given country should exhibit the strongest convergence, followed by similar national economies (such as OECD countries) and followed by national economies.

Yet, despite all the studies that find ABC, Romer and DeLong point out the 'natural sample-selection bias' that tends to overstate the case for convergence. They caution that choosing countries that are already wealthy as of the ending period of study to show that those same economies have converged will overstate the Beta coefficient. This is because if countries defined as of the current period of studies are wealthy, then by definition, they have already converged. The solution would be to redo the analysis but choose the countries that were rich as of the base period; not the current period. In this case, $\beta_2$ drops significantly and will not be overstated.

In contrast to the above findings, Obstfeld & Rogoff's study for data samples covering 'more heterogeneous economic groupings' analyzed 55 countries covering the period 1950-1990, including developing and developed countries which resulted in no evidence of ABC at any speed.

Similarly, Barro & Sala-i-Martin (1995) graphed for the period 1960-1985 the per capita growth rate on the vertical axis against the $\ln$ of real per capita GDP on the horizontal axis for a cross section of 118 countries and found their relationship slightly positive—ABC did not apply.
2.3 ABC Test Results

Regression Equation 14 was run against the 57 high-income (HOIC) and upper-middle-income (UMC) club countries (of which western and eastern Germany, respectively, by assumption, are members) for the period 1990 to 1999. I tested for the existence of Absolute Beta Convergence, cross-country convergence, among these two distinct income groups based on per capita GDP. The OLS results are shown in Table 2.

If, the Beta coefficients have a positive value between zero and one, it signifies the tendency of the per capita GDP, to equalize across economies; same steady-state growth rates among both income-club economies. (See Appendix A2 for further exposition)

Table 2. OLS RESULTS OF ABC REGRESSION
HOIC & UMC ECONOMIES
1990 - 1999

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>In Per Capita GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(beta2)</td>
<td></td>
</tr>
<tr>
<td>Coefficient</td>
<td>0.0148248</td>
<td>0.0046988</td>
</tr>
<tr>
<td>Std Error</td>
<td>0.0048839</td>
<td>0.0053289</td>
</tr>
<tr>
<td>t-stat</td>
<td>3.0354709</td>
<td>0.8817597</td>
</tr>
<tr>
<td>Sig. Level</td>
<td>0.0025265</td>
<td>0.3783296</td>
</tr>
</tbody>
</table>

Notes: In Per Capita GDP is expressed in constant 1995 U.S. Dollars.
HOIC – High OECD Income Economies Club, former-FRG member.
UMC – Upper-Middle Income Economies Club, former-GDR member.
List of country members in the HOIC and UMC groups can be found in Appendices A5 and A7, respectively.
Expanded definition of variable & economies can be found in Section I.1.1.


49 See discussion under Chapter 1.1 in Section I.
50 Variable defined in Section I, Chapter 1.1.
Absolute Beta Convergence (ABC) the tendency for the per capita incomes to equalize across the above economies—same steady-state growth rates—is not found. This result is similar to past studies combining less homogeneous economic groupings.

The measurement of ABC is not without criticism. The first is the restriction of economies having a common steady state. Second, the drawback of the Solow-Swan model is that it provides no explanations of the determinants of the long-run per capita growth rates of per capita output, capital and consumption. Martin & Sunley state that this approach is merely descriptive and says nothing about the mechanisms at work. It fails to account for the interrelationship that exists among the different regions. Absolute Beta Convergence only relates a region’s growth to its own history and then only by averaging across the trends for all regions. The implication of ABC is that it assumes that the underlying convergence-generating process is “identical across space”—that is, through time. This need not be the case as growth trends from one region may depend crucially on the growth path of others. The rate of convergence may vary from region to region within a country and different regions may converge to different long-run relative income levels that may reflect persistent local differences in their structural characteristics. In other words, we can ask the question: Can Beta Convergence be evidenced after controlling for certain variables that account for the differences in each region’s structural characteristics?

---

51 This result was also true when I combined all (HOIC, HNOIC, UMC, LMC and LIC) WDI economic groups; results not shown.

52 Obstfeld & Rogoff and Barro and Sala-i-Martin (1995.)
In an attempt to answer the above question and counteract some of the criticisms cited in the literature review above, the neoclassical model with Conditional Beta Convergence, for the HOIC and UMC clubs, is explored.
Chapter 3. Conditional Beta Convergence

In response to the restriction of economies having a common steady state, the concept of Conditional Beta Convergence (CBC) arose.

CBC is said to exist if economies converge to their own steady-state long-run (equilibrium) incomes. This means that steady-state growth rates are not homogeneous across economies (i.e., $dx/dt \neq 0$) and convergence is conditional on the different structural characteristics of each economy (i.e., societal references, technologies, rate of population growth, government policies and the savings rate.) Example: This implies that countries will have different steady states, relative to their steady state, per capita incomes.

Convergence of this type, when applied to various growth models asks whether the economies that are far below their steady-state position will grow faster than economies that are closer to their steady-state position.

How does this concept of Beta Convergence fit with the mathematical properties and relationships expressed in Equation 1?

Two complementary methods are used to explain the concept of CBC.

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Method 1:
Method 1 suggests that if we restrict the analysis of convergence to sets of economies with similar characteristics, this bypass the problem of differing steady states. In this case, the mathematical properties of Equation 1 need not be modified, as you would be applying the concept of Absolute Beta Convergence to each set of similar economies or clubs. As a result, you would perform the same test and the same ABC interpretations can be applied to each club; test results shown in Tables 3 and 4 for the HOIC and UMC clubs, respectively.

Method 2:
On the other hand, method 2 states that if we did not want to bother choosing countries with similar characteristics, or wanted to allow for differing steady-states in the equation itself, then Equation 1 would need to be modified to incorporate the addition of this(these) structural/conditional variable(s); test results shown in Table 5.

This can be accomplished in the following way. If we allow Z to represent a structural variable which proxy the individual steady-state in each country, in other words, allow the Beta Convergence to be conditioned on this Z variable, and assume that Z in the base year is inversely related to the rate of growth of the economy (similar analogy as in the other right-hand variable Y) then Equation 1 would be transformed to:

\[
\ln \left( \frac{Y_t}{L_t} \right) - \ln \left( \frac{Y_t \circ}{L_t \circ} \right) = \alpha - \beta_2 \ln \left( \frac{Y_t \circ}{L_t \circ} \right) - \beta_3 \ln \left( \frac{Z_t \circ}{L_t \circ} \right)
\]

[Equation 15]
where: $\frac{Z_{10}}{L_{10}} = \text{any structural-type variable, expressed in per capita, in the initial year of analysis}$,

$\beta_2 = \beta$ coefficient in Equation 1.

$\beta_3 = \text{coefficient on the above variable}$.

All Betas are restricted to be positive integers between 0 and 1 inclusive.

Appendix A4 shows the extensions of the CBC concept.

These two methods for controlling the variables that account for the differences in each nation's structural characteristics (CBC) will be tested in Section I, Chapter 3.3 and the results are reported in Tables 3-5.

3.1 Neoclassical Solow-Swan Model & Conditional Beta Convergence

With respect to Conditional Beta Convergence, the neoclassical model does predict convergence so that once we control for the determinants of the steady state, then a country with a lower real per capita income at the beginning year of study will exhibit a higher per capita growth rate.

This (CBC) implies that different countries will have different steady-state relative per capita incomes and that economies that are far below their steady-state position will grow faster than economies that are closer to their steady-state position.

One of the causes for differing steady states is differing saving rates. From Equation 4, in Section I, Chapter 2.1.1, if we had two economies with different savings rates, this would imply that there would exist two separate downward-sloping saving rate curves,


\((s \cdot f(k) / k)\) where the slope for the higher-income economy would be steeper and the slope for the lower-income economy would be flatter. This would mean that the richer (higher-income) economy would be further away from its steady state than the poorer (low-income) economy.

To see this algebraically, if from Equation 3, we replace \(k\) with its steady-state value \(k^*\), and at steady-state \(dk/dt = 0\), then

\[s \cdot f(k^*) = (n + \delta) \cdot k^*\]

isolating \(s\) yields,

\[s = (n + \delta) \cdot k^* / f(k^*)\]

If we now replace \(s\) in Equation 5 and rearranging the terms,

\[\gamma_k = (n + \delta) \cdot \left[\left(f(k) / k\right) / \left(f(k^*) / k^*\right) - 1\right]\]  \[\text{[Equation 16]}\]

Given Equation 16, if \(k = k^*\), then \(\gamma_k = 0\). Furthermore, as \(k\) decreases, the average product of \(k\), \((f(k) / k)\) increases which increases \(\gamma_k\). However, \((f(k) / k)\) must be high in relation to \((f(k^*) / k^*)\) in order for a country to grow rapidly.

\[3.2 \ \text{Literature Review}\]

Baumol & Wolff demonstrate that if countries are grouped by low-income, medium-income and high-income, then there is evidence of convergence within each group. This theory conforms to the notion of club convergence.
The idea of club convergence arose due to the fact that Absolute Beta Convergence was not evident when trying to measure the cross-country speed of Beta Convergence. Instead when they restricted their data to include only the OECD countries, then, ABC was found. This sparked the idea that maybe these OECD countries represented a club.

This club convergence by definition is similar to the theory of ABC in that economies that are similar in their structural characteristics and have similar initial conditions will converge to one another. The developed/rich economies, the developing economies and the underdeveloped economies are examples of three ‘convergence clubs.’ This theory suggests that we should foresee convergence within each club but that convergence across the clubs need not be present. Thus, even though this theory does not rule out that the inequalities among the different clubs may decrease in time (i.e., convergence) but most likely, the inequalities will persist, or even increase in time. (Martin & Sunley, pg. 204)

In contrast to their earlier studies, when Barro & Sala-i-Martin (1995) restricted the input data to include only the ‘relatively advanced’ OECD countries in 1960 (the beginning of the sample period) their study showed that ABC did hold—“the initially poorer countries did experience significantly higher per capita growth rates.” (Pg. 27) ABC was even stronger when they used each state (to represent a different economy) of the continental US. They plotted the annual growth rate of per-capita income from 1880-1990 against the log of per-capita personal income from 1880, and found the relationship to be negative with a β value of 0.0174. When they included regional dummies, the β value

53 The four main census regions were Northeast, South, Midwest and West.
increased slightly to 0.0177 signifying that the speed of convergence for the regions was not that much stronger than that of the individual states (pg. 388.)

### 3.3 CBC Test Results

As Absolute Beta Convergence was not found for the combined HOIC and UMC\(^{54}\) economies, results shown in Table 2, Conditional Beta Convergence will be tested for each income-club; HOIC and UMC.

As stated previously in this Chapter (3.1) in order to test for CBC, we must first hold constant the steady state of each economy. This can be accomplished, via two similar means.

I began by employing the concepts of Method 1 in testing CBC advanced by Martin & Sunley to treat differing steady states. I thus restricted the analysis of convergence to sets of economies with similar characteristics. Using this method, Equation 14 the ABC regression equation, did not need to be modified, as I applied this regression equation to each set of economies/club—the HOIC and UMC clubs.

Relying on the WDI classification of these economies, I test for CBC, method 1, regression Equation 14, on the HOIC and UMC economies for the period 1990-1999. OLS produced the results shown in Tables 3 and 4, respectively. If Beta is found to be positive between 0 and 1, then the countries within each club are said to display

\(^{54}\) ABC was also not found when all 207 economies in all income categories were used from the WDI database.
Conditional Beta Convergence. (See Appendix A4 for the extension of the interpretations of the Beta results.)

Table 3. OLS RESULTS OF CBC REGRESSION
HOIC CLUB ECONOMIES
1990 - 1999

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>In Per Capita GDP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.0792316</td>
<td>0.0669344</td>
</tr>
<tr>
<td>Std Error</td>
<td>0.0152594</td>
<td>0.0151236</td>
</tr>
<tr>
<td>t-stat</td>
<td>5.1923223</td>
<td>4.4258176</td>
</tr>
<tr>
<td>Sig. Level</td>
<td>5.009E-07</td>
<td>1.564E-05</td>
</tr>
</tbody>
</table>

Notes: In Per Capita GDP is expressed in constant 1995 U.S. Dollars.
HOIC – High OECD Income Economies Club, former-FRG member.
List of country members in the HOIC can be found in Appendix A5.
Expanded definition of variable & economy can be found in Section I.1.1.


Beta measures the speed of convergence—the strength of the conversion effect. The closer the Beta is to 1, the faster the convergence. From Table 3, Beta has the correct sign and is statistically significant. As convergence is evident, calculating its speed, as defined in Appendix A1, it would take a little over 10 years for half the gap between the HOIC economies’ per capita incomes to close and 20 ½ years to eliminate three-quarters of the initial gap from the steady-state position.
Beta is found to be three times higher than in past studies on convergence. This may be due, as Romer and DeLong suggest, to the fact that the Beta coefficient will be overstated if we pick countries that are similar in characteristics as of the ending period of study. In this study, the club members of the HOIC club are high-income, high-developed economies that are being studied in the most recent decade and only over the recent ten-year time span (1990-1999.) Additionally, with the onset of the European Union and the fact that 15 out of the 23 HOIC members are part of the European Union, it is more likely that these economies will grow closer in their per capita GDP incomes; and thus a higher Beta coefficient is expected.

Table 4. OLS RESULTS OF CBC REGRESSION UMC CLUB ECONOMIES 1990 - 1999

<table>
<thead>
<tr>
<th></th>
<th>Constant</th>
<th>In Per Capita GDP (βeta)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient</td>
<td>0.0773454</td>
<td>0.0800079</td>
</tr>
<tr>
<td>Std Error</td>
<td>0.0137660</td>
<td>0.0162637</td>
</tr>
<tr>
<td>t-stat</td>
<td>5.6185664</td>
<td>4.9194123</td>
</tr>
<tr>
<td>Sig. Level</td>
<td>4.455E-08</td>
<td>1.445E-06</td>
</tr>
</tbody>
</table>

Notes: In Per Capita GDP is expressed in constant 1995 U.S. Dollars.
UMC – Upper-Middle Income Economies Club, former-GDR member.
List of country members in the UMC can be found in Appendix A7.
Expanded definition of variable & economy can be found in Section I.1.1.


55 Obstfeld & Rogoff, Baumol, Baumol & Wolff, deLong, Romer and Barro & Sala-i-Martin's studies.

56 The Intergovernmental Conference and European Union began in December 1990 two months after Germany’s unification. The Maastricht Treaty was written in December 1991, signed in February 1992 and became effective as of 1 January 1993. Fifteen of the 23 HOIC economies, listed in Appendix A5, are members of the European Union.
Table 4 is the OLS result of Equation 14 for the upper-middle-income (UMC) club economies. Beta has the correct sign and is highly statistically significant. Again, as convergence is evident, calculating its speed, it would take 8 ½ years for half the gap between the UMC economies’ per capita incomes to close and a little over 17 years to eliminate three-quarters of the initial gap from the steady-state position.

Again, this Beta result is found to be higher (by four times) than the 2% growth rate reported in past studies on convergence. However (same explanation as with the high-OECD-income (HOIC) study) these UMC economies are being studied in the most recent decade and only over the ten-year time span 1990-1999; thus a higher Beta is expected.

Furthermore a comparison of Tables 3 and 4 show that a higher growth rate exists for the UMC, lower income club, versus the HOIC, higher income club economies, as predicted by the neoclassical Solow-Swan convergence model. See the discussion under Section I, Chapter 3.1.

Method 2 testing of Conditional Beta Convergence (CBC) controlling for differing steady states for the high-OECD-income (HOIC) and upper-middle (UMC) income-economies follow.

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57 Obstfeld & Rogoff, Baumol, Baumol & Wolff, deLong, Romer and Barro & Sala-i-Martin’s studies.

58 See Table 4.

59 See Table 3.
The most common test of CBC takes the same form as the test of ABC Equation 14 except that it adds a structural-type variable to the regression equation. This structural-type variable would then explain the individual steady state in each country. This method incorporates the concept of CBC of Equation 15 and applies it to the regression equation derived from Equation 14 yielding:

\[
\left(\frac{1}{T}\right) \cdot \ln \left[\frac{(Y_{i,t_0+T})}{(Y_{i,t_0})}\right] = \\
\alpha - \left(\frac{1}{T}\right) \cdot (1 - e^{-\beta_2 t}) \ln (Y_{i,t_0}) - \left(\frac{1}{T}\right) \cdot (1 - e^{-\beta_3 t}) \ln (Z_{1i,t_0}) + \varepsilon_{i,t_0,t_0+T}
\]

[Equation 17]

Controlling for two structural variables, the population, and savings rates (assuming a negative relationship exist between these variables and the explanatory variable) in the regression equation itself would mean adding another \(Z\) term on the right-hand side of Equation 17.

\[
\left(\frac{1}{T}\right) \cdot \ln \left[\frac{(Y_{i,t_0+T})}{(Y_{i,t_0})}\right] = \alpha - \left(\frac{1}{T}\right) \cdot (1 - e^{-\beta_2 t}) \ln (Y_{i,t_0}) - (1/T) \cdot (1 - e^{-\beta_3 t}) \ln (Z_{2i,t_0}) + \varepsilon_{i,t_0,t_0+T}
\]

[Equation 17']

I thus employ Conditional Beta Convergence Method 2 Equation 17' on the high-OECD-income (HOIC) and upper-middle-income (UMC) economies for the period 1990-1999 using the population growth rate \((Z_{1i})\) and the savings rate \((Z_{2i})\) as the structural variables. OLS produced the results reported in Table 5.
If the coefficients are found to be positive, conditional upon these Z variables and lie between 0 and 1, then the economies in question are said to display Conditional Beta Convergence. (See Appendix A4 for an extension and interpretation of the Beta results using the CBC concept with structural variables.)

Table 5. OLS RESULTS OF CBC REGRESSION HOIC & UMC CLUB ECONOMIES (STRUCTURAL VARIABLES: POPULATION & SAVINGS GROWTH RATES) 1990 - 1999

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>In Per Capita GDP</th>
<th>Population Growth Rate</th>
<th>Savings Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>Coefficient</td>
<td>Std Error</td>
<td>t-stat</td>
</tr>
<tr>
<td>0.0266989</td>
<td>0.0050706</td>
<td>5.2654682</td>
<td>2.128E-07</td>
</tr>
<tr>
<td>0.0121558</td>
<td>0.0054986</td>
<td>2.2107219</td>
<td>6.1799609</td>
</tr>
<tr>
<td>0.0323681</td>
<td>0.0052376</td>
<td>1.387E-09</td>
<td>1.387E-09</td>
</tr>
<tr>
<td>0.07728859</td>
<td>0.04504522</td>
<td>1.71580000</td>
<td>1.71580000</td>
</tr>
</tbody>
</table>

Notes: In Per Capita GDP is expressed in constant 1995 U.S. Dollars. Population Growth Rate is reported as an annual percent. Savings Rate is defined as 1 – consumption rate. In this case final consumption expenditure expressed as a percent of GDP is used as a proxy for consumption.

HOIC – High OECD Income Economies Club, former-FRG member.
UMC – Upper-Middle Income Economies Club, former-GDR member.
List of country members in the HOIC and UMC groups can be found in Appendices A5 and A7, respectively.

Expanded definition of variables & economies can be found in Section 1.1.1.


The coefficients from Table 5 have the correct signs and are statistically significant. Thus, once we control for these structural variables, convergence is evident among the two clubs, but it takes longer for the economies to converge to their own steady state—
evidenced by the per capita GDP growth rate where $\beta_2$ has dropped in value as expected. (Discussion found in Appendix A4)

In calculating its speed, it would take more than 56 $\frac{1}{2}$ years for half the gap between the current high-income and upper-middle income economies' per capita incomes to close and approximately 113 $\frac{3}{4}$ years to eliminate three-quarters of the initial gap from the steady-state position.

In sum, ten years after the German 1990 unification, we see that the former economies of eastern and western Germany, members of the upper-middle-income (UMC) and high-OECD-income (HOIC) clubs, respectively, have not converged to one another as of 1999. Absolute Beta Convergence (ABC) is not found for the combined two clubs, (Table 2) but convergence is evidenced among each club (Tables 3 and 4.) Once I controlled for the determinants of the steady state (savings and population growth rates for the HOIC and the UMC clubs) then Conditional Beta Convergence (CBC) was found (Table 5.) Note that CBC on the UMC club, as expected by theory, did indeed exhibit a higher per capita growth rate compared to the HOIC club ($\beta_2=0.0800$ in Table 4 versus $\beta_2=0.0669$ in Table 3.)

In addition to the non-convergence ABC findings for the combined income clubs (Table 2) and to support the CBC in Tables 3-5, the steady-state growth rate for each club analysis follows. From Equation 4 of the Neoclassical Solow-Swan model, the growth
rate of income per capita connotes steady-state status.\textsuperscript{60} As the average per capita GDP steady-state growth rate for the post-unification period is 1.395\% for the HOIC economies and 1.858\% for the UMC economies, it would appear that these two clubs have different long-run steady-state values and will not converge to one another—confirming prior findings.

As Beta Convergence does not guarantee a decline in the dispersion of real per capita income with time (discussed in Section I, Chapter 4.1) Sigma Convergence testing, the third statistical concept measuring convergence, within the context of the exogenous neoclassical Solow-Swan model for the HOIC and UMC club economies is explored.

\textsuperscript{60} Barro & Sala-i-Martin (1995) pg. 19.
Chapter 4. Sigma Convergence

SC is an independent and theory-free concept that shows whether a given series, over a period of time, is negatively related to its beginning period. It is said to exist if the dispersion (variance or standard deviation) of the cross-region economies’ relative\(^{61}\) variable (X) tends to decrease over time.

The variance of cross-region economies’ variable (X) can be written as:

\[
D_t = \frac{1}{n-1} \cdot \sum_{i=1}^{n} \left[ \ln(X_{i,t}) - \mu_t \right]^2
\]

where:
- \(D_t\) = the dispersion
- \(\mu_t\) = the sample mean of the \(\ln(X_{i,t})\)
- \(n\) = number of countries/regions

For a large number \(n\), the sample variance is close to the population variance so the above equation by definition is:

\[
\sigma_t^2 = D_t = \frac{1}{N} \cdot \sum_{i=1}^{N} \left[ \ln(Y_{i,i}) - \mu_t \right]^2
\]

[Equation 18]

If \(\sigma_t^2\) is to decrease over time, then in the limit, \(\sigma_t^2\) tends to zero as time \(t\) goes to infinity \((\lim \sigma_t^2 \to 0\) as \(t \to \infty\)).

Moreover, this technique asks whether the variance of the logarithm of a variable, say per capita GDP, is shrinking across economies over time. This must mean that,

\(^{61}\) Relative in this case means relative across a group of economies—in this case the economies within the HOIC and UMC clubs.
\[ \sigma^2_{t+\tau} < \sigma^2_t  \]^

where: \( \sigma^2_t \) = the variance of the \( ln \ X \) of country \( i \) at a beginning time \( t \).

\[ \sigma^2_{t+\tau} \] = the variance of the \( ln \ X \) of country \( i \) at a later period of time than at time \( t \).

The same properties hold if the dispersion is measured by the standard deviation of the \( ln \) of the variable instead of its variance.

When graphed, time/year on the horizontal axis and the variable’s dispersion (standard deviation for the \( ln \) of the per capita GDP variable for the club economies) on the vertical axis, it should exhibit a downward-sloping curve. Graphs G1 and G2 show the results of Sigma Convergence testing for the post-unification per capita GDP, for the high-OECD-income (HOIC) and upper-middle-income (UMC) economies, respectively, for this study in Section I.

Specifically, when testing Sigma Convergence (SC) in the neoclassical framework, as already mentioned, a higher Beta coefficient implies a greater tendency toward convergence. However, the condition that the coefficient is less than one, “rules out leapfrogging or overshooting effect.” (Barro & Sala-i-Martin (1995) pg. 31) This means that the trailing (lower per-capita) economy cannot surpass the leading (higher per-capita) economy.

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62 This assumption was shown by Martin & Sunley, appendix 2.
If we incorporate Equation 18 with Equation 13, and from Equation 13 assume \( \varepsilon_{i,t} \) has zero mean, the same variance \( (\sigma_{\varepsilon}^2) \) for all economies and is distributed independently over time and across economies, then the evolution of \( D_t \) or the cross-economy variance of \( \ln(\gamma_{i,t}) \) denoted as \( \sigma_t^2 \) evolves over time as:

\[
\sigma_t^2 \equiv (e^{-2\beta})\sigma_{t-1}^2 + \sigma_{\varepsilon}^2
\]  

[Equation 19]

The steady-state first-order difference equation for dispersion is then given by:

\[
D^* = \sigma_{\varepsilon}^2 / [1 - (e^{-2\beta})]
\]  

[Equation 20]

From both the above equations, it is evident that \( \sigma_t^2 \) and \( D^* \) increase with \( \sigma_{\varepsilon}^2 \) and fall with \( \beta \).

Does this necessarily mean that \( \sigma_t^2 \) will fall over time?

### 4.1 Neoclassical Solow-Swan Model, SC & ABC

Sigma Convergence (SC) is closely related to Absolute Beta Convergence (ABC) as the existence of ABC generates declining dispersion in SC. However, the existence of ABC is a necessary but not a sufficient condition for SC. In other words, even if Absolute Beta Convergence holds, the dispersion of real per capita income, \( \sigma_t^2 \) need not decline with time. This is because even though the long-run (steady-state) dispersion, \( D^* \), falls with Beta, as seen from Equation 20, SC also depends on the variance of the error term or “shocks,” \( \sigma_{\varepsilon}^2 \).

---

63 See Martin & Sunley appendix 1 for its exposition.
To see the relationship between $\sigma_t^2$ and $D^*$ (and $\sigma_e^2$) the evolution of $\sigma_t^2$ is given by:

$$\sigma_t^2 = D^* + (e^{-2B}) \cdot (\sigma_{t-1}^2 - D^*) \quad [\text{Equation } 21]$$

which equals,

$$\sigma_t^2 = D^* + (e^{-2B}) \cdot (\sigma_{t-1}^2 - D^*), \quad \text{or}$$

$$\sigma_t^2 = \left[ \sigma_e^2 / (1 - e^{-2B}) \right] + \left[ \sigma_{t-1}^2 - \sigma_e^2 / (1 - e^{-2B}) \right] \cdot e^{-2Bt}$$

where: $\sigma_{t-1}^2 = \text{dispersion at time } t$.  

To conclude, the dispersion, $\sigma_t^2$, may fall, rise or remain constant depending upon whether the initial value of the variance, $\sigma_{t-1}^2$, is above, below, or the same as its steady state value, $D^*$.  

As an example, borrowing the terms from Equation 13, a positive Beta would decrease the dispersion of $ln\ (\gamma_{i,t})$ for a given distribution of the disturbance term in country $i$ for period $t$. But new exogenous shocks to the disturbance term will tend to increase the dispersion of $ln\ (\gamma_{i,t})$ at least temporarily above its steady-state value of sigma rendering the outcome of Sigma Convergence ambiguous (that is, a decrease in the dispersion and a rise in the variance of the error term.) 

However, given that the steady-state distribution of the error term in country $i$ for

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64 Mathematically is accomplished by isolating $\sigma_e^* \text{ in Equation 20 and inserting its value in Equation 19. Opening the parenthesis and re-arranging its terms will yield Equation 21.
period $t$ remains constant over time, then for any given temporary shock the dispersion of $\ln (y_{i,t})$ approaches its steady-state value of the dispersion over time (i.e., $\sigma^2$ monotonically approaches the steady-state value, $D^*$ over time.)

In short, a positive $\beta$ coefficient does not ensure a falling $\sigma^2$ and even though Absolute Beta Convergence is assumed, the concept of SC implying that $\sigma^2$ falls over time is only true if $\sigma^2$ is greater than $D^*$.

4.2 Literature Review

In Barro & Sala-i-Martin's study of Japanese prefectures from 1930-1990, when they graphed time/year on the horizontal axis and income dispersion (unweighted standard deviation for the $\ln$ of per capita income for all economies) on the vertical axis, they found a downward-sloping curve—evidence of Sigma Convergence.

Boyle & McCarthy's study of OECD countries for the period 1950-1988 also found evidence of Sigma Convergence (and Beta Convergence by default) over the full period of 1.3%. However after 1972, the data shows no evidence of Beta Convergence. They attribute this discrepancy with that of Barro & Sala-i-Martin in that they use Kendall's index of rank concordance (Siegel, pp. 229-239) which is a direct measure of Beta Convergence—taken as a measure of the extent of leapfrogging—while Barro's regressions are an indirect measure of Beta Convergence.
Cashin & Sahay (1996) examined the dispersion of per capita incomes across the 20 Indian states for the period 1961-1991 and even though they found evidence of ABC, they also found an overall widening of the per capita income dispersion—no evidence of Sigma Convergence.

4.3 Sigma Convergence Test Results

Sigma Convergence (SC) defined as the decrease in the dispersion of the economies’ per capita income levels over time, reinforces the results of Beta Convergence testing in the previous chapter for the high-OECD-income (HOIC) and upper-middle-income (UMC) economies. Graphs G1 and G2 show the result of Sigma Convergence for these two clubs of which the former-West and former-East German nations are members, respectively. The HOIC economies show Sigma Convergence for all the years following the unification period, Graph G1. SC was also evidenced for the UMC economies, Graph G2, starting with the year 1993.

As discussed in the earlier part of this Chapter (4.1) Beta Convergence is a necessary but not a sufficient condition for SC. An explanation for the lack of SC for the first three years for the UMC club may be the fact that the initial value of the variance, σ0², is not above its steady state value, D*. The cause of the dispersion, one of the causes for differing steady states, discussed in Section I, Chapter 3.1, is differing saving rates. I tested for CBC on the UMC economies utilizing the population and savings growth rates as structural variables. The savings rate variable was not found to be statistically significant for these economies, however both structural variables were found statistically significant for the HOIC economies. (Results of OLS testing not shown)
Chapter 5. Section I. Conclusion of the German 1990 Unification

Convergence Test Results

For the German 1990 unification, the results of convergence testing, Beta Convergence—Absolute and Conditional—and Sigma Convergence, show that the former West (FRG) and East (GDR) German nations have not converged, expressed by their per capita GDP levels, by end of year 1999.

Absolute Beta Convergence, the tendency for the per capita incomes to equalize across all economies—same steady-state growth rates—is not found (Section I, Chapter 2, Table 2.)

Conditional Beta Convergence, the tendency for the per capita incomes to equalize across homogeneous economies/clubs—where convergence is conditional on the different structural characteristics of each economy—is found for the High-OECD-Income-Club (HOIC) and the Upper-Middle-Income-Club (UMC) and of which western and eastern Germany are members (Tables 3 and 4) respectively. Furthermore, the two (HOIC and UMC) clubs are found to have different long-run steady-state equilibrium values: average per capita GDP growth rates of 1.395% and 1.858% for the HOIC and UMC economies, respectively. See my discussion in Section I, Chapter 3.3 and Barro & Sala-i-Martin (1995) pg. 19. Thus, these two clubs have not converged to one another by end of year 1999.
Sigma Convergence—the decrease in the dispersion of the cross-region economies’ relative per capita incomes over time—reinforces the findings of Beta Convergence for the two clubs. Specifically, the HOIC economies show Sigma Convergence for all the years following the unification period while the UMC economies also show Sigma Convergence following the unification period but convergence begins with the year 1993 (Graphs G1 and G2, respectively.) This lack of convergence for the first three years may be explained as it is found that the savings growth rate for the UMC economies is not a statistically significant structural variable that can explain the individual steady state of each UMC country; while this is not the case for the HOIC economies.

However, the club convergence results of Beta and Sigma Convergence Testing in this section do not attempt to answer the question of will (or should) the former two German nations converge to one another when viewed individually and not part of a club. This question arises due to the implications made from the assumptions in the Solow-Swan model with Absolute Beta Convergence. When economies share similar levels of technology, similar tastes/preferences and institutions—no barriers to the flow of capital and labor across borders, we should see more evidence of ABC within regions as opposed to among countries as regions share one central government, legal system and institutional set-up.66

This would seem especially true of the former Eastern and western German regions as they are now one nation and also share the same culture; tend to have smaller differences

66 See discussions by Cashin & Sahay (1996) and Barro & Sala-i-Martin (1995.)
in language; have access and can share technologies; and be more apt to migrate among regions within the same country (as opposed to another country.)

In following this line of thought, then even though I did not find Absolute Beta Convergence for the combined two income clubs, we may be able to see evidence of convergence, a closing of the gap, only when examining the per capita GDP of the former two German regions; and not their respective income clubs. This is the scope of Section II.

\[\text{In Section II, Chapter 2.2.A2, it is shown that for the pre-unification period, the migration variable in East Germany was positive (in-migration found) for only ten years, 1968-1977, inclusive, and positive (in-migration found) for West Germany beginning with the year 1971. However, over the period, as seen from Table 6, the arithmetic mean of the two nations were found to be negative (out-migration.) Specifically, the } \ln \text{ Migration of West Germany was } -0.12234 \text{ and the } \ln \text{ Migration of East Germany was } -0.09145. \text{ Additionally for the post-unification period, for the years 1990-1992, the } \ln \text{ Migration of western Germany was again found to be negative (out-migration.)}\]
SECTION II.
STATISTICAL, GRAPHICAL MEASUREMENT OF CONVERGENCE & UNIT-ROOT TESTING

A more detailed study of the former Eastern and western German regions utilizing a statistical, graphical dispersion measurement of convergence (Sigma Convergence) and the Dickey and Fuller unit-root tests are explored.

My data collection efforts and the variables used for this section of study—deaths, divorces, employment in manufacturing, foetal mortalities, output per person employed, infant mortalities, live births, marriages, migration, population and labor productivity in manufacturing—as well as the methodology and concepts in measuring convergence among the former Eastern and western German nations follow.
Chapter 1. Data, Graphical and Unit-Root Methodology

1.1 Data Collection Efforts, Sources & Definition of Variables

Accessibility (of separate data for the two German regions for both their pre- and post-unification periods) and reliability of data (especially with respect to the reporting of accurate statistical data for the GDR region) were my two main concerns. Thus, my persistence in the search for accurate and separate regional data, led to three data sources used in this section of study: Demographic Yearbooks of the United Nations, Groningen Growth & Development Centre\(^6\) and Summers & Heston’s Penn World Tables. The compilation of the eleven data series for both regions in the creation of my database, extracted from each data source, is explained in turn.

Manually searching through 52 volumes of the Demographic Yearbooks of the United Nations for common reported variables between the two regions, I created the database used in this section of study. The final extractions of the seven common reported variables for the two regions from each of the 46 volumes (1950-1996) are deaths, divorces, foetal mortalities, infant mortalities, live births, marriages and population—

DEATHS, German Tables, 1950-1992, Issues 10-52,
DIVORCE, German Tables, 1950-1990, Issues 10-50,
F-MORT, German Tables, 1950-1990, Issues 10-50,
I-MORT, German Tables, 1950-1990, Issues 10-50,
L-BIRTHS, German Tables, 1950-1992, Issues 10-52,
MARRIAG, German Tables, 1950-1991, Issues 10-51, and

\(^6\) Data reproduced in Van Ark.
The variable migration (MIGR) is computed from the data variables from my database. Originally extracted from the Demographic Yearbooks of the United Nations, German Tables, 1951-1992, Issues 11-52. It is defined as the current population at time (in year) $t$, minus the number of births at time $t$, plus the number of deaths at time $t$, minus the previous year's population:

$$MIGR_t = POPUL_t - [L-BIRTHS_t - DEATHS_t + POPUL_{t-1}]$$

If the MIGR variable is found to be positive, this would imply in-migration and if it is found to be negative, this would imply out-migration. Note that this computed variable’s data series for East and West Germany begins with the year 1951, as the 1950 population figure is needed in the initial calculation of the $POPUL_{t-1}$ variable.

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69 See Appendix A10 for data availability and definitions.

70 The statistics on Saarland were reported separately for East and West Germany until 1956; thereafter, they became incorporated with the West German statistics.

71 See Appendix A10 for data availability and definitions for the variables used in the compilation of this migration variable.
The employment in manufacturing, output per person employed, and the labor productivity in manufacturing (EMPLOY, GDP PP and PROD, respectively) variables were extracted from the Groningen Growth & Development Centre, and added to my database.

All eleven data series for East and West Germany were separated into two groups: pre-unification period 1950-1989 and post-unification period 1990-until the latest date available: 1996 for the PROD, GDP PP and EMPLOY; 1994 for the POP; and 1992 for the DEATHS variable. The data for the rest of the six variables do not span into the unification period. Then the series were converted to natural logs (ln) and all statistical calculations reported in this study (Sections II) are in ln.

The database statistics were then reviewed for input accuracy and compared against the Penn World Tables data for discrepancies. Additionally, a few data points from the Penn World Tables' data source, were directly extracted and interspersed in this paper: real GDP per worker, price level GDP, degree of openness, investment share of GDP, consumption share of GDP, and real per capita GDP.

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72 Figures are reproduced in Van Ark, pg. 38.
73 See Appendix A10 for data availability and definitions.
74 See Appendix A10 for data definitions and exact reporting years for each East and West data series.
76 These 5 variables are defined and discussed in Section II, Chapter 2.5.
77 This variable is defined and discussed in Section I, Chapter 1.1.
2001 World Development Indicators on CD-ROM: \Definitions\GDP Per Capita
In sum, my database used for Section II contains eleven data series/variables for East and West Germany from 1950 until the latest available date—see above referencing or Appendix A10 for variable definitions and time frames.

1.2 Graphical Measurement of Convergence—Sigma Convergence

A graphical measurement of convergence can be seen by the theory-free statistical concept of Sigma Convergence (SC) explained in Section I, Chapter 4.1.

Again, the variance of cross-region economies’ variable \( X \) can be written as:

\[
D_t = \frac{1}{(n-1)} \cdot \sum_{i=1}^{n} \left[ ln(X_{i,t}) - \mu_t \right]^2
\]

where: \( D_t = \) the dispersion

\( \mu_t = \) the sample mean of the \( ln(X_{i,t}) \)

\( n = \) number of countries/regions; in this case \( n=2 \).

But, from the above equation, since we are just comparing East to West Germany, the first term in the equation, \( 1/(n-1) \), would be equal to 1 and thus I can re-write the equation as:

\[
\sigma_t^2 = D_t = \sum_{i=1}^{N} [ln(X_{i,t}) - \mu_t]^2 \quad [\text{Equation 22}]
\]

Again, the same properties hold if the dispersion is measured by the standard deviation of the \( ln \) of the variable instead of its variance.
Thus, I graph time/year on the horizontal axis and the variable's dispersion (standard deviation for the ln of the variable for both the GDR & FRG economies) on the vertical axis. For convergence to exist, the graph should exhibit a downward-sloping curve. Graphs G3-G13 and G14-17 show the results of the Sigma Convergence testing for the pre- and post-unification variables, respectively, for this study in Section II.

1.3 Unit-Root Methodology

As stated in the Introduction, a test that measures whether a data series has a stationary or non-stationary trend is the Dickey-Fuller (DF) unit-root test. A stationary time series has three distinct characteristics.\(^7\)\(^8\) It exhibits a mean reversion (fluctuates around a constant long-run mean) it has a finite variance that is time-invariant, and has a theoretical correlogram that diminishes as lag length increases. A non-stationary time series also possesses three distinct characteristics:\(^7\)\(^9\) no long-run mean reversion, its variance is time-dependent and tends to infinity as time approaches infinity and the theoretical autocorrelations do not decay (but in finite samples, the same correlogram decays slowly.)

In order to formally test the presence of a unit root—a deterministic, stationary trend—if one of the roots of its autoregressive polynomial, AR(p) is 1, a unit root, then the data series becomes an AR(p-1) process in differences. The higher-order correlations are performed by summing the lagged difference terms of the dependent variable on the right-hand side of the regression.

\(^{78}\) McAllister, Treacy & Hanvey.

\(^{79}\) Ibid.
Consider specifically an AR(1) autoregressive model where the current value of a series is linearly related to its past values plus a stochastic shock with the following three forms,

\[
Y_t = b_1 Y_{t-1} + E_t \quad [\text{Equation 23}]
\]

\[
Y_t = b_0 + b_1 Y_{t-1} + E_t \quad [\text{Equation 24}]
\]

\[
Y_t = b_0 + b_1 Y_{t-1} + b_2 \text{Time}_t + E_t \quad [\text{Equation 25}]
\]

where \(b_0\), \(b_1\) and \(b_2\) are parameters and for all three equations, \(b_1\) is the parameter of interest. Equation 23 would be considered a pure random walk (if \(b_1 = 1\)—an AR(1) with a unit coefficient—and \(E_t\), the shock, is assumed to be white noise and normally distributed with a mean of zero and a variance of \(\sigma^2\)). Equations 24 and 25 have the same properties as Equation 23 but Equation 24 is Equation 23 with the addition of an intercept or a drift term (\(b_0\)—essentially a stochastic trend model that on average grows each period by the drift. Equation 25 is Equation 23 with both a drift and a linear time trend (\(b_0\) and \(b_2\) Time \(t\), respectively.) As my study examines whether the variables are converging to a common value or to one another, this theory does not permit drifts or trends. As such, Equations 24 and 25 are not used for this study.

Estimating Equation 23 above subtracting \(Y_{t-1}\) from both sides of the equation, it becomes a regression of the first difference of \(Y\) on the first lag of \(Y\):

\[
\Delta y_t = B_1 Y_{t-1} + E_t \quad [\text{Equation 23'}]
\]
Where \( B_t = b_t - 1 \). The null hypothesis (\( H_0 \)) is that \( b_t = 1 \) so that we are testing in this first difference [Equation 23'], that \( B_t = 0 \). The alternative hypothesis (\( H_1 \)) is that \( b_t < 1 \) so that we are testing in this first difference [Equations 23'] that \( B_t < 0 \).

Thus in Equation 23' we test for the \( H_0 \): of a random walk (if \( B_t = 0 \)) against the alternative of a zero mean and a covariance stationary AR(1) process.

For each series, EViews3\(^{80}\) compares the OLS results of the t-test against the MacKinnon levels for unit root tests; as Dickey and Fuller showed that the distribution under the null hypothesis is nonstandard. If we can reject the null hypothesis of a unit root, in favor of the alternative \((-1 < b_t < 1)\) this would imply that \( Y \) is a stationary series; it has a deterministic trend where the series will eventually revert to its long-run mean level—convergence.

Alternatively, if the series fails to reject the null hypothesis of a unit root (\( H_0: b_t = 1 \)) within a certain statistical confidence interval, this would signify that we cannot reject that \( Y \) is a non-stationary series; it has a stochastic trend and the variance will increase with time. If the absolute value of \( b_t \) is greater than one \(|b_t > 1|\) the series is explosive—variance of \( Y \) goes to infinity.

Furthermore, if for example, we fail to reject the \( H_0 \): in Equation 23' of a random walk, even though the series fails to revert to any population mean, we can again difference the series and try to fit a stationary model to the differences. We can then test if this new

first difference of a unit root process is covariance stationary and invertible; zero-mean, white noise and well-behaved series.

An equivalent form of the AR(p-1) model can be written as an ARMA, autoregressive moving average, (p-1,q) process where q=0; so that an AR(1) model is equivalent to an ARMA(1,0) model. Once we difference the model, we can write the process as an ARIMA (p-1,d,q) model; an autoregressive integrated moving average is a stationary and invertible ARMA process in first differences after differencing d times. Following with my earlier example of an AR(1) process, in first differences, it would be equivalent to an ARIMA(1,1,0) model. The order of integration corresponds to the number of autoregressive roots of the series. If d = 0, then Y is covariance stationary or integrated of order 0, I(0) white noise process. The series is in levels. If d = 1, first differenced, Y is a random walk and integrated of order 1, I(1.). Shocks to a random walk have completely permanent effects; a unit shock moves the expected path of the series by one unit. If we fail to reject the Ho: in first differences, we can re-test in second differences for stationarity. In this case, d = 2, the series would contain 2 unit roots and we say the series is integrated of order 2, I(2.) Higher orders of integration are seldom found in the literature. For the pre-unification period, I will test for stationarity in levels and in first differences. For the post-unification period, my data is extremely short (3-7 years) and so I will test for stationarity in levels only.

81 Unit-Root test results are shown in Table 7.
82 Unit-Root test results are shown in Table 9.
The above analyses measure whether a single data series has a stationary or non-stationary trend implying convergence or non-convergence, respectively. When comparing two series, if the variables have different orders of integration, that would imply non-convergence of the two series. However, if the variables have the same order of integration, convergence of the two series cannot be ruled out. Thus, a form of co-integration is needed to test whether a linear combination of the two series is stationary. I check for co-integration by testing the differences between the East and West series; the E-W Difference series referred to in my analysis. Thus for each variable, I take the difference between the two nations’ series and test this E-W Difference series for stationarity. If I again fail to reject the null hypothesis of a unit root in favor of the alternative, in levels, this would lead to the additional evidence of non-convergence. For the pre-unification and post-unification periods, I will test the E-W Difference series for stationarity in levels.83

83 Unit-Root test results are shown in Table 7 and 9 for the pre- and post-union periods, respectively.
Chapter 2. The Pre-Unification Period

The East and West German nations prior to their unification differed from one another in many areas including: territory, government, population, migration, births, marriages, divorces, deaths, foetal mortalities, infant mortalities, output per person employed, employment in manufacturing, and labor productivity in manufacturing. Aiming to show the diversities of these two nations during their pre-union period (1950-1989) and where possible offer reasons for their differences, analysis of the differences in territory and government as well as the above-mentioned eleven statistical variables (whose results are shown in Tables 6-7 and Graphs G3-G13) are discussed.

Additionally, the live births, migration, mortality—deaths, infant and foetal—and the population growth rates are variables analyzed in the neoclassical growth theory model (Study I) that affect the per capita GDP growth rates. Thus, in an effort to reconcile these variables’ pre-union statistics within the context of the neoclassical growth model, their inter-relationships will be discussed later in this section in Chapter 2.6.

Table 6 shows summary statistics—standard deviation, arithmetic mean, variance, coefficient of variation, continuous growth rate, and cumulative growth rate—exclusive of territory and government, for each of the eleven variables, in ln form.84

84 See Appendix A10 for definitions and sources of data.
The DEATHS, DIVORCE, F-MORT, I-MORT, L-BIRTHS, MARRIAG and MIGR
variables are year-end totals measured as the actual year-end number of deaths, divorces, foetal mortalities, infant mortalities, live births, marriages and migration, respectively, for the year. The POPUL is the mid-year estimate for the actual number of people/population during the year. The EMPLOY, GDP PP AND PROD are defined as employment, output and productivity in manufacturing indices in the base year 1975, so that 1975 = 100. The index is calculated as \[1-\{(\text{base-year value} - \text{current-year value}) / \text{base-year value}\} \times 100.\]
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<th>WEST GERMANY</th>
<th>EAST GERMANY</th>
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<td><strong>DEATHS:</strong></td>
<td>0.099709</td>
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<td>27.73222</td>
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</tr>
<tr>
<td>Cum. Growth Rate (%)</td>
<td></td>
<td></td>
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</table>

| **DIVORCE:**   | 0.385074     | 0.291603     |
| Standard Deviation | 11.21201    | 10.45980     |
| Arithmetic Mean   | 0.144575    | 0.082907     |
| Variance          | 3.434473    | 2.787846     |
| Coefficient of Variation (%) | 1.004160 | 0.010160     |
| Cont. Growth Rate (%) | 40.16659    | 0.406313     |
| Cum. Growth Rate (%) |            |              |

|**EMPLOY** (index 1975=100): | 0.102382     | 0.089598     |
| Standard Deviation | 4.587071    | 4.544405     |
| Arithmetic Mean   | 0.010220    | 0.007827     |
| Variance          | 2.231970    | 1.971603     |
| Coefficient of Variation (%) | 0.664830 | 0.878620     |
| Cont. Growth Rate (%) | 26.59308    | 35.14490     |
| Cum. Growth Rate (%) |            |              |

|**F-MORT:**     | 0.754818     | 0.632030     |
| Standard Deviation | 8.899649    | 7.855098     |
| Arithmetic Mean   | 0.555506    | 0.389475     |
| Variance          | 8.481435    | 8.046112     |
| Coefficient of Variation (%) | -5.01932 | -5.09221     |
| Cont. Growth Rate (%) | -200.773    | -203.688     |
| Cum. Growth Rate (%) |            |              |

|**GDP PP** (index 1975=100): | 0.535210     | 0.484206     |
| Standard Deviation | 4.281953    | 4.320232     |
| Arithmetic Mean   | 0.279289    | 0.228594     |
| Variance          | 12.49921    | 11.20788     |
| Coefficient of Variation (%) | 4.644390 | 4.445630     |
| Cont. Growth Rate (%) | 185.7754    | 177.8251     |
| Cum. Growth Rate (%) |            |              |
Table 6 (cont’d)

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<th>EAST GERMANY</th>
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<tr>
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<td><strong>In MARRIAG:</strong></td>
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<td>In PROD <em>(index 1975=100):</em></td>
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<td>Cum. Growth Rate (%)</td>
<td>159.3339</td>
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**Notes:**

The datum for the MIGR variable starts in 1951 and the data for all other variables start in 1950.

The DEATHS, DIVORCE, F-MORT, I-MORT, L-BIRTHS, MARRIAG and MIGR variables are year-end totals measured as the actual year-end number of deaths, divorces, foetal mortalities, infant mortalities, live births, marriages and migration, respectively for the year.

The POPUL is the mid-year estimate for the actual number of people/population during the year.

The EMPLOY, GDP PP and PROD are defined as employment, output and productivity in manufacturing indices in the base year 1975, so that 1975 = 100. Each index is calculated as:

\[1-\frac{\text{current-year value} - \text{base-year value}}{\text{base-year value}} \times 100.\]

Definitions and sources of data can be found in Appendix A10.

Cont. (Continuous) Growth Rate is calculated as:

\[\frac{X(1989) - X(1950)}{N} \times 100, \text{ where } N = 40 \text{ and } X \text{ is the analyzed variable in ln.}\]

* Cont. (Continuous) Growth Rate for the MIGR variable is calculated as:

\[\frac{X(1989) - X(1951)}{N} \times 100, \text{ where } N = 39 \text{ and } X \text{ is the analyzed variable in ln.}\]

Cum. (Cumulative) Growth Rate is calculated as: \(\sum (X_t - X_{t-1}) \times 100, \text{ for } t=1951 \text{ through } t=1989, t-1 \text{ is the previous year, and } X \text{ is the analyzed variable in ln.}\)
With respect to the Dickey-Fuller test, for each of the East and West German time series in levels and in first differences, the regression equation tested is the null hypothesis of a pure random walk. And, for the E-W Difference series, the regression equation tested is the null hypothesis of a pure random walk, in levels.

Table 7 shows the results of these regression equations. Reported are the t-statistics for each of the eleven variables, the number of observations (N) for each regression equation, as well as the critical values at the 1-, 5- and 10% significance levels corresponding to each N value.

The decision for each series (East, West and E-W Difference) is: If the t-statistic is less than (lies to the left of) the critical value, we can reject the null hypothesis of a unit root (Ho: $B_i = 0$ in Equations 23' referenced in Section II, Chapter 1.3) in favor of the alternative hypothesis that its root is less than one (H1: $B_i < 0$ in Equations 23' referenced in Section II, Chapter 1.3.) This (rejection of the null hypothesis) would imply that the series is stationary; it follows a deterministic trend where the series will eventually revert to its long-run mean level—convergence.
<table>
<thead>
<tr>
<th></th>
<th><strong>WEST</strong> t-statistic</th>
<th><strong>EAST</strong> t-statistic</th>
<th><strong>E-W DIFF.</strong> t-statistic</th>
<th><strong>N</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In DEATHS:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levels</td>
<td>1.681664</td>
<td>-0.327849</td>
<td>1.33319</td>
<td>39</td>
</tr>
<tr>
<td>First Differences</td>
<td>-7.052100 *</td>
<td>-9.596090 *</td>
<td></td>
<td>38</td>
</tr>
<tr>
<td><strong>In DIVORCE:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levels</td>
<td>0.238156</td>
<td>-0.014067</td>
<td>-0.54840</td>
<td>39</td>
</tr>
<tr>
<td>First Differences</td>
<td>-6.723151 *</td>
<td>-5.150143 *</td>
<td></td>
<td>38</td>
</tr>
<tr>
<td><strong>In EMPLOY (index 1975=100):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levels</td>
<td>1.395476</td>
<td>2.803563</td>
<td>-0.65273</td>
<td>39</td>
</tr>
<tr>
<td>First Differences</td>
<td>-3.511191 *</td>
<td>-5.243710 *</td>
<td></td>
<td>38</td>
</tr>
<tr>
<td><strong>In F-MORT:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levels</td>
<td>-6.380782 *</td>
<td>-4.324231 *</td>
<td>-0.19188</td>
<td>39</td>
</tr>
<tr>
<td>First Differences</td>
<td>-2.300299 *</td>
<td>-3.283017 *</td>
<td></td>
<td>38</td>
</tr>
<tr>
<td><strong>In GDP PP (index 1975=100):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levels</td>
<td>4.752095</td>
<td>6.664129</td>
<td>-1.71596 *</td>
<td>39</td>
</tr>
<tr>
<td>First Differences</td>
<td>-2.707455 *</td>
<td>-3.389658 *</td>
<td></td>
<td>38</td>
</tr>
<tr>
<td><strong>In I-MORT:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levels</td>
<td>-8.221137 *</td>
<td>-6.526105 *</td>
<td>0.59716</td>
<td>39</td>
</tr>
<tr>
<td>First Differences</td>
<td>-2.204462 *</td>
<td>-2.651072 *</td>
<td></td>
<td>38</td>
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<tr>
<td><strong>In L-BIRTHS:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levels</td>
<td>-0.661864</td>
<td>-1.399981</td>
<td>0.58814</td>
<td>39</td>
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<tr>
<td>First Differences</td>
<td>-2.965905 *</td>
<td>-3.230920 *</td>
<td></td>
<td>38</td>
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<tr>
<td><strong>In MARRIAG:</strong></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levels</td>
<td>-1.340579</td>
<td>-1.351836</td>
<td>0.26195</td>
<td>39</td>
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<tr>
<td>First Differences</td>
<td>-4.579503 *</td>
<td>-4.886288 *</td>
<td></td>
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<tr>
<td><strong>In MIGR:</strong></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>Levels</td>
<td>-1.273499</td>
<td>-2.084632 *</td>
<td>-1.86362 *</td>
<td>38</td>
</tr>
<tr>
<td>First Differences</td>
<td>-4.420149 *</td>
<td>-5.080682 *</td>
<td></td>
<td>37</td>
</tr>
<tr>
<td><strong>In POPUL:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levels</td>
<td>5.100758</td>
<td>-3.579040 *</td>
<td>5.30828</td>
<td>39</td>
</tr>
<tr>
<td>First Differences</td>
<td>-2.483048 *</td>
<td>-3.911095 *</td>
<td></td>
<td>38</td>
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<tr>
<td><strong>In PROD (index 1975=100):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Levels</td>
<td>6.628736</td>
<td>6.240193</td>
<td>-1.54874</td>
<td>39</td>
</tr>
<tr>
<td>First Differences</td>
<td>-2.487224 *</td>
<td>-2.772634 *</td>
<td></td>
<td>38</td>
</tr>
</tbody>
</table>
Table 7 (cont’d)

Notes:

The datum for the MIGR variable starts in 1951 and the data for all other variables start in 1950.

The DEATHS, DIVORCE, F-MORT, I-MORT, L-BIRTHS, MARRIAG and MIGR variables are year-end totals measured as the actual year-end number of deaths, divorces, foetal moralities, infant moralities, live births, marriages and migration, respectively for the year.

The POPUL is the mid-year estimate for the actual number of people/population during the year.

The EMPLOY, GDP PP and PROD are defined as employment, output and productivity in manufacturing indices in the base year 1975, so that 1975 = 100.

Each index is calculated as:
[1-{(base-year value - current-year value) / base-year value} x 100.]

Definitions and sources of data can be found in Appendix A10.

* Rejects the null hypothesis of a unit root (t-statistic < critical values) at the 1-, 5- or 10% significance levels.

<table>
<thead>
<tr>
<th>Critical Values for:</th>
<th>N=39</th>
<th>N=38</th>
<th>N=37</th>
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<tr>
<td></td>
<td>1%</td>
<td>5%</td>
<td>10%</td>
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<td>-2.6227</td>
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<td>-1.6202</td>
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<td></td>
<td>-2.6243</td>
<td>-1.9498</td>
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</tr>
<tr>
<td></td>
<td>-2.6261</td>
<td>-1.9501</td>
<td>-1.6205</td>
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</tbody>
</table>

Tables 6 and 7 report the statistical and unit-root results (respectively) for the eleven variables in the database: DEATHS, DIVORCE, EMPLOY, F-MORT, GDP PP, I-MORT, L-BIRTHS, MARRIAG, MIGR, POPUL and PROD for the period 1950-1989.

The results reported in these tables along with these eleven variables’ graphical dispersion measurement of convergence shown in Graphs G3-G13, will be used to better understand the diversities of the East and West German nations prior to their unification.

Where possible, I offer and/or speculate on the reasons for the regions’ differences.
The differences in territory and government are also discussed with no statistical implications made.

Later in Chapter 2.6, I reconcile the directional relationships, predicated in the neoclassical growth theory of Study I, between the live births, migration, mortality—deaths, infant and foetal—and the population growth rates and their affect on the per capita GDP growth rates.

2.1 Differences in Territory and Government

The East and West German nations’ territorial differences are as follows: The FRG consisted of ten Länder, or states, plus the territory of West Berlin. They occupied a surface area of 248.58 km² with an average density of 240. The GDR consisted of five historic Länder, which were now resection into 14 regions, or Bezirke, plus the territory of East Berlin. It occupied a surface area of 108.18 km² (52% less than the size of the FRG’s surface area) and had an average density of 153 (64% of the FRG’s average density.)


86 This is the population per square kilometre of surface area, calculated as population divided by surface area. Demographic Yearbook of the U.N. 1989. German Tables, 49th Issue, p. 101.

87 Mecklenburg-Western Pomerania, Brandenburg, Saxony-Anhalt, Saxony, and Thuringia.

88 After unification, the 14 regions reverted back to the former 5 states of 1945.

89 Rostock, Schwerin, Neubrandenburg, Potsdam, Frankfurt/Oder, Cottbus, Magdeburg, Halle, Erfurt, Gera, Suhl, Dresden, Leipzig and Karl-Marx Stadt (Chemnitz.)

90 This is the population per square kilometre of surface area, calculated as population divided by surface area. Demographic Yearbook of the U.N. 1989. German Tables, 49th Issue, p. 101.
As for their governmental differences, West Germany was, and is, a social market economy with a democratic, multi-political party\(^{91}\) where a minister-president or Prime Minister who was elected for a term of five years by an appointed Federal Assembly headed most state governments. The president's main task was to represent the country in matters concerning international relations. The head of the government was the chancellor, who was advised by a cabinet of ministers. FRG’s constitution was known as the *Grundgesetz*, or Basic Law and its legislature consisted of two houses, the *Bundestag*, or Federal Diet, whose members were elected by the public, and the *Bundesrat*, or Federal Council, composed of representatives of state governments. Each state sent delegates to the federal legislature only on a consultative and nonvoting basis and the number of deputies from each state varied from three to five according to size and population. The judiciary system had and still has separate administrative, labor, social, and tax court systems. The Federal Court of Justice is the highest court for civil and criminal cases and the Federal Constitutional Court rules in disputes between the federal government and the *Länder* and between individual federal bodies.

This differed drastically from the GDR. Following World War II and the Soviet occupation of East Germany, a Communist-style planned economy was introduced. The GDR’s government had a 500-member parliament, or *Volkskammer* (People’s Chamber) elected by the people. The ceremonial head of state was the chairman of the Council of State who was elected by the *Volkskammer*. The head of government was the chairman of the Council of Ministers (Prime Minister) who was the leader of the party that held the

\(^{91}\) The main political parties of the FRG were the Christian Democratic Union, Social Democratic Party of Germany, Free Democratic Party, Christian Social Union, and the Greens (a party devoted to environmental issues.)
most seats in parliament. Thus from 1950 to 1989 the German Democratic Republic was primarily a unitary, one-party command economy.

2.2 Differences in Population

The culmination of the statistical, graphical and unit-root testing results show that the POPUL variable, in $ln$ form, for these two regions, during this period of study, were quite different from one another with the West experiencing higher growth than the East.

The statistics from Table 6 report that the West German population average growth rate was higher but that the East German population growth rate was less variable. Additionally, for this period, the average spread around the mid-year estimate for the actual population mean of 16.59 million for East Germany was lower than the average spread around the mid-year estimate for the actual mean population for West Germany of 55.76 million by 0.187% (see Coefficient of Variation in Table 6.)

The unit-root DF test results, from Table 7, show that the East German POPUL trend was deterministic in levels; rejects the null hypothesis of a unit root at the 1% significance level. For West Germany and the E-W Difference series, the POPUL trends had the wrong sign. Thus, this result of non-convergence reinforces the sigma convergence graphical testing results found in Graph G3 which measures the standard deviation over

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92 The political parties of the GDR were the Christian Democratic Union, German Social Union, Democratic Awakening and the Social Democratic party. The Party of Democratic Socialism was later renamed the Socialist Unity, or Communist party. However, it was this later party that dominated the GDR for the last 40 years.

93 See Section II, Chapter 1.3 Unit-Root Methodology for the interpretations of the results.
time of the \( \ln \) of the population of East and West Germany\(^9^4\)—curve is found to be upward sloping during the pre-unification period.

Furthermore, subdividing the population by gender, both regions’ male population was always less than the female population during the pre-unification period while the male/female ratio difference increased over the period. The FRG’s male population was at an overall average of 89.957\% of the female population (a difference of 2.9M) and the GDR’s male population was at an overall average of 84.54\% of the female population (a difference of 1.4M.)\(^9^5\)

I posit four different reasons, either in isolation or in combination, as partial explanation of the population movement: Berlin Wall, migration, live births, and marriages/divorces.

2.2. Possible Reasons for the Population Movement

2.2.A1 Berlin Wall

One possible explanation that may account for the decline in population of the West and the stabilization of the population in the East was the construction of the Berlin Wall in 1961—erected at the onset of the Cold War period and a symbol of the East-West division. It was built by the GDR to halt the exodus of their nationals. Therefore, from 1961 to 1989 the number of arrivals to West Germany from East Germany dwindled partly due to the barrier of the Berlin Wall. The population growth findings of stabilized

\(^9^4\) Graph methodology is further discussed in Section II, Chapter 1.2.

growth rates from 1961-1989 for East Germany and the precipitous decrease in growth rates for West Germany from 1964-1980 renders some support to this movement pattern.

2.2.A2 Migration

Migration statistics also support, in part, this decline in population. Migration is defined as the current population at time (in year) \( t \), minus births at time \( t \), plus deaths at time \( t \), minus the previous year’s population:

\[
\text{MIGR}_t = \text{POPUL}_t - \text{[L-BIRTHS}_t - \text{DEATHS}_t + \text{POPUL}_{t-1}]
\]

If the MIGR variable is found to be positive, this would imply in-migration and if it is found to be negative, this would imply out-migration.

The MIGR variable in East Germany was positive (in-migration found) for only ten years, 1968-1977, inclusive, and positive (in-migration found) for West Germany beginning with the year 1971. However, over the period, as seen from Table 6, the arithmetic mean of the two nations were found to be negative—out-migration.

Graph G4 measures the standard deviation over time of the \( \ln \) of this variable for the two nations.\(^{96}\) The graph shows high volatility; non-convergence.\(^{97}\)

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\(^{96}\) Graph methodology is further discussed in the “Scope of Study” section of the paper.

\(^{97}\) This result corresponds with the analysis of Barro & Sala-i-Martin (1992) in their findings of the relationship with net in-migration and per capita GDP. I explain this relationship in Section 2.6 “Summary of Pre-Unification Results.”
The unit-root DF results (reported in Table 7) reinforce the above graphical findings of non-convergence. For West Germany, the DF test fails to reject the null hypothesis of a unit root in levels. Once first differences were taken, the null can be rejected at the 1% significance level. In contrast, for East Germany, the null hypothesis is rejected in levels at the 5% significance level. As the two series have different orders of integration, this would imply non-convergence.

2.2.A3 Live-Births

Alternatively, a third partial explanation for the movement of the population—higher mean and lower continuous growth rates for West Germany as compared to East Germany for the pre-union period—may lie in the birth rates where West Germany also experienced a higher mean and substantially lower continuous growth rates.

Specifically, West Germany's arithmetic mean of live births over the pre-union period was higher by 1.12417 percentage points compared to East Germany. And, relative to the mean, the live-birth growth rate of West Germany was more variable, by a little over a quarter of a percentage point, than the live-birth growth rate of East Germany (see Coefficient of Variation in Table 6.)

In reviewing the actual data series, in their levels, the East German ln L-BIRTHS series only surpassed the West during the years 1970-1975 and 1987-1989. However, West Germany's live births were always about those of East Germany.
Table 7 shows the result of the unit-root DF test of all three series: East, West and E-W Difference series, for this variable. For these three series, the test fails to reject the null hypothesis of a unit root in levels; even though once first differences were taken the East and West German series’ null hypothesis can be rejected at the 1% significance level.

Graph G5 that measures the standard deviation for both nations over time of the \( \ln \) of this variable\(^{99}\) concurs with the findings of non-convergence in the unit-root test; the curve is volatile and upward sloping over the period.

In sum, for West Germany, the L-BIRTHS are found to be higher and more volatile than the East German series; similar results as the POPUL variable (Table 6.) Additionally, similar to the POPUL series, non-convergence is found from 1950-1989 (Graph G5 and Table 7.)

**2.2. A4 Marriages/Divorces**

A plausible explanation for the decrease in live births (negative cumulative growth rates) with a corresponding lower proportion of male to female population, may be the decrease in marriages (negative cumulative growth rates) and increase in divorces (positive cumulative growth rates.)

\(^{98}\) Arithmetic mean of \( \ln \) (L-BIRTHS) from 1950-1989 in Table 6.

\(^{99}\) Graph methodology is further discussed in the Section II, Chapter 1.2.
Analysis of the statistical, graphical and unit-root test results for the MARRIAG and DIVORCE variables follow.

MARRIAG

The unit-root DF test results show non-convergence of the MARRIAG variable (see Table 7.) Specifically, for the individual series, the results show that the MARRIAG trends fail to reject the null hypothesis of a pure random walk, in levels. But once first differences were taken, the series become deterministic at the 1% level. The test of the E-W Difference series supports non-convergence as it fails to reject the null hypothesis of a unit root, in levels. Additionally, as can be seen from Graph G6\textsuperscript{100} that measures the standard deviation over time of the $ln$ of marriages for both nations, sigma convergence over the pre-unification period is not found.\textsuperscript{101}

The statistics on the MARRIAG variable reinforce the above results. The data show that over the period 1950-1989 marriage growth rates have been a decreasing function of time for East Germany and also a decreasing function of time for West Germany except for the years 1974-1978. Even though, the mean growth in the West over the period always surpassed the East, relative to the mean, the marriage growth rate of West Germany was almost identical with the marriage growth rate of East Germany (see Coefficient of Variation in Table 6, 1.134218 vs. 1.31701, respectively.)

\textsuperscript{100} Graph methodology is further discussed in Section II, Chapter 1.2.

\textsuperscript{101} Graphical measurement of convergence is found only for the years 1967-1978.
DIVORCE

Statistics on the DIVORCE variable (Table 6) show that the divorce cumulative growth rates have been an increasing function of time for both Germanys with the West series always surpassing the East series during each time period. Relative to the mean, the divorce growth rate of West Germany is more variable by 0.65 percentage points than the divorce growth rate of East Germany for the pre-unification period (see Coefficient of Variation in Table 6.)

Graph G7\textsuperscript{102} measures the standard deviation over time, of the ln of the number of divorces for both nations. Overall, non-convergence is found. I am not aware of any reason that could explain the huge dip in the standard deviation between the East and West German series in 1978. This, what appears to be a discrepancy in data, coupled with the unreliability of the East German data series for this variable,\textsuperscript{103} renders the results suspect.

The unit-root DF test results show non-convergence of the DIVORCE variable for all three series (see Table 7.)

In summary, divorce and marriage patterns reinforce birth patterns. West Germany's live birth, divorce and marriage mean growths were consistently above the East in every year

\textsuperscript{102} Ibid.

\textsuperscript{103} Even though the Demographic Yearbooks of the UN reported this series, they caution as to the reliability (for lack of verification) of the East German divorce reported statistics.
of study and convergence of these three data series for East and West Germany cannot be found as of 1989.

### 2.3 Differences in Deaths, Foetal Mortalities & Infant Mortalities

The three mortality growth rates are the Deaths (DEATHS), Foetal Mortalities (F-MORT) and Infant Mortalities (I-MORT).

Over the pre-union period, the mean growth rates for all three variables are higher for West Germany (as compared to East Germany) and East-West convergence, in levels, is not found. Analysis of the statistical, graphical and unit-root test results for each variable follow (see Tables 6-7 and Graphs G8-10.)

**DEATHS**

Looking at the individual data series, in levels, the West German DEATHS is consistently above the East in every year of study. Statistics on the DEATHS variable (Table 6) show that the cumulative death growth rate has been positive and more volatile for the FRG and negative and less volatile for the GDR; similar to the POPUL outcome.

As seen from Graph G8, which measures the standard deviation over time of the \( \ln \) of the DEATHS series for the FRG and GDR regions, sigma convergence is not found; in fact, the curve is upward sloping.

---

104 Graph methodology is further discussed in Section II, Chapter 1.2.
Results of the unit-root DF test show a deterministic trend for the individual deaths series only when first differences are taken (Table 7.) Thus, these results concord with the above statistical results for this variable—convergence is not found.

**F-MORT**

A review of the individual data series, in levels, on the F-MORT variable show that foetal mortalities have consistently been higher for West Germany up to 1978. Relative to the mean, the foetal mortality growth rate of West Germany is slightly more variable, by less than half a percentage point, than the foetal mortality growth rate of East Germany for the pre-unification period (see Coefficient of Variation in Table 6.)

The unit root DF test results (Table 7) on the F-MORT variable rejects the null hypothesis of a unit root in levels at the 1% significance level for both nations. However, with respect to the E-W Difference series, it fails to reject in levels. Thus, convergence of this variable cannot be assured. This result is re-enforced by the graphical measurement of convergence seen from Graph G9.\(^{105}\) Up to 1974, the standard deviation of the \(ln\) of the variable for the 2 regions was increasing, signifying non-convergence. From 1974 to 1981, convergence is seen, but then after, non-convergence is once again found. In sum, over the entire pre-unification period, sigma convergence is not found.

**I-MORT**

Statistics on the I-MORT variable (Table 6) show that even though the arithmetic mean value for this variable in East Germany was lower than that of West Germany, relative to

\(^{105}\) Ibid.
the mean, the infant mortality growth rate of East Germany is more variable, by 1.57 percentage points, than the infant mortality growth rate of West Germany (see Coefficient of Variation in Table 6.)

Graph G10\textsuperscript{106} measures the standard deviation over time of the \textit{ln} of the I-MORT variable for both nations. Convergence can be seen only from 1974-1985.\textsuperscript{107}

Furthermore, the unit-root DF results reported in Table 7 show that the individual I-MORT series for East and West Germany can be rejected in levels at the 1% significance level. However, for the E-W Difference series, the null hypothesis cannot be rejected in levels. Thus, both unit-root testing and the sigma testing cannot support convergence of this variable over the entire pre-unification period.

Similar to the East MARRIAG and East DIVORCE series, the validity of the findings for the pre-unification East German I-MORT series is suspect as even though the Demographic Yearbooks of the UN reported these series, they caution as to their reliability due to lack of verification.

\section*{2.4 Differences in Output per Person Employed, Employment & Labor Productivity in Manufacturing}

Results of the statistical, graphical and unit-root test results for the output per person employed (GDP PP) employment in manufacturing (EMPLOY) and labor productivity in

\textsuperscript{106} Ibid.

\textsuperscript{107} Similar patterns emerge even when I explore this variable as a percent of the population; results not shown.
manufacturing (PROD) showing non-convergence (evidenced from Table 6-7 and Graphs G11-G13) over the pre-unification period, are discussed.

**GDP PP**
Statistics on the output per person employed (GDP PP) variable show that from 1950 to 1989, the continuous output per person employed growth rates declined for both nations (Table 6.) Relative to the mean, the output per person employed growth rate of the FRG was more variable, by one 1.3 percentage points, than the output per person employed growth rate of the GDR (see Coefficient of Variation in Table 6.)

The East and West German DF test results (Table 7) show that in levels, the GDP PP series have the wrong signs. Thus, stationarity cannot be found in levels for each individual series. But, in examining the co-integrated E-W Difference series, the unit-root null hypothesis is rejected in levels. Graph G11 reinforces these results. Even though the general direction of the standard deviation of the $ln$ of the GDP PP for the 2 nations is downward sloping, the curve is too volatile. Thus convergence cannot be assured.

**EMPLOY**
The West German employment in manufacturing mean growth was slightly higher and relative to the mean, more variable, by a quarter of a percentage point, than the employment growth rate of East Germany (see Coefficient of Variation in Table 6.)

---

108 See Section II, Chapter 1.3 Unit-Root Methodology for the interpretations of the results.
Table 7, results of the unit-root DF test for the East and West EMPLOY data series, show that the series follow a deterministic trend once first differences were taken. However, with respect to the individual and E-W Difference series, I find that in levels, all three series fail to reject the null hypothesis of a unit root. In sum, non-convergence is found.

The above finding of non-convergence for these two series is supported by the graphical measurement of convergence. Graph G12\textsuperscript{109} measures the standard deviation over time of the \( \ln \) of the EMPLOY variable for both nations, which show non-convergence from 1950 to 1989.

**PROD**

The statistics in Table 6 show that West Germany experienced a lower arithmetic mean and relative to the mean, the PROD growth rate of West Germany was more variable by 2.15 percentage points, than the PROD growth rate of East Germany (see Coefficient of Variation in Table 6.)

As seen from Graph G13\textsuperscript{110} that measures the standard deviation over time of the \( \ln \) of the labor productivity in manufacturing (PROD) variable for both nations, while the general direction from 1963-1989 is downward sloping, sigma convergence cannot be assured.

\textsuperscript{109} Graph methodology is further discussed in Section II, Chapter 1.2.

\textsuperscript{110} Ibid.
The unit-root DF test results (Table 7) show that the East and West PROD data series, reject the null hypothesis of a unit root at the 1% significance level once first differences are taken. As for its individual and E-W Difference series, the unit root test fails to reject the null hypothesis in levels. These results support the graphical measurement of convergence above and parallel the EMPLOY variable result of non-convergence.

In sum, for the output per person employed (GDP PP) labor productivity in manufacturing (PROD) and the employment in manufacturing (EMPLOY) variables, I find that the FRG region experienced a lower mean value for the first two variables and a higher volatility in all three variables. Non-convergence over the pre-unification period cannot be assured for all three variables.

Other national statistical differences between the two nations, not discussed up to this point in the paper, are introduced in the next section. However, due to lack of statistical data available for both economies for these data series, only five data points are offered for comparison purposes and thus no explanations are offered with respect to their convergence/divergence patterns over time.

2.5 Other Miscellaneous Differences\textsuperscript{111}

Aside the ten variables analyzed in this section in Chapters 2.1 through 2.4, five additional data points are presented in this section that may further exhibit the differences between these nations prior to their unification. These five variables are real GDP per

\textsuperscript{111} All the statistics reported in this section are 1988 figures and were extracted from the Penn World Tables, available on-line. See Summers & Heston.
worker, price level, degree of openness, investment share of GDP, and consumption share of GDP. Unless noted otherwise, these figures are as of 1988.

East Germany’s real GDP per worker (1985 international prices) was 62.7% of West Germany (18,292 vs. 29,152.) Furthermore, price levels were also dramatically different. East Germany’s price level GDP (measured as PPP GDP divided by exchange rate relative to U.S.) was 75.28% while West Germany’s price level GDP was 124.19%. The degree of openness (defined as exports plus imports divided by real per capita GDP at current international prices) also varied. East Germany’s openness was 86.51 while West Germany’s openness was 53.9.

Moreover, differences between East and West Germany, by a smaller margin, are evidenced in their Investment Share of GDP as well as in their Consumption Share of GDP, both at 1985 international prices. East Germany’s Investment Share of GDP was 29.4% while their counterpart was 24.3%. East Germany’s Consumption Share of GDP was 61% while their counterpart was 59%.

2.6 Summary of Pre-Unification Results

The data during the pre-unification period for the former-East and former-West German nations, as evidenced by the reported statistics in Table 6 (standard deviation, arithmetic mean, variance, coefficient of variation, continuous and cumulative growth rates) graphical dispersion measurement of convergence in Graphs G3-G13 (standard deviation of the ln of the variable over time) and unit-root DF test results in Table 7 (for the East, West and E-W Difference series) show that these two nations are dissimilar in all the
analyzed eleven variables: deaths, divorces, employment in manufacturing, foetal mortalities, output per person employed, infant mortalities, live births, marriages, migration, population and labor productivity in manufacturing\textsuperscript{112} as well as in territory and government.

However, the East German infant mortality and marriage data series are suspect due to lack of verification of the reported data from the UN Yearbooks. Thus, I present the results for these two series with the same voice of caution.

Of the eleven variables studied, West Germany had over the pre-unification period a higher arithmetic mean as well as a higher volatility in all the analyzed variables except for the output per person employed, infant mortality and migration variables. West Germany experienced a lower arithmetic mean and a higher volatility in their output per person employed and migration variables and a higher arithmetic mean and a lower volatility in the their infant mortality variable. The inverse finding is true of East Germany.

The inter-relationship findings (again) even though suspect for the Marriages/Divorces variables, for both nations, show that Marriages/Divorces are consistent with the results on birthrates. More families stay married the higher the birthrate (live-birth growth rates.)

\textsuperscript{112} DEATHS, DIVORCE, EMPLOY, F-MORT, GDP PP, I-MORT, L-BIRTHS, MARRIAG, MIGR, POPUL and PROD, respectively.
Neoclassical growth theory, fully discussed in the first section of this paper (Section I, Chapter 2.1.1) describes the negative relationship that should exist between the GDP PP and DEATHS, F-MORT, I-MORT, L-BIRTHS and POP variables as well as the positive relationship that should exist between the GDP PP and MIGR variable.

In particular, the theory predicts that fertility rates tend to fall with per capita income for the poorest (in terms of low-income) countries. If the live-birth series are used as a proxy for fertility rates, the above relationship does hold for East and West Germany. During the pre-unification period, I find that while the mean growth for live births was greater for West Germany, the arithmetic mean output per person employed was greater for East Germany.\(^{113}\)

Second, prediction of the Solow-Swan model is that in the absence of continuing improvements in technology, per capita growth must eventually cease. This comes from the assumption of diminishing returns to capital (Section I, Chapter 2.1.) I use mortality growth rates—deaths, foetal mortalities and infant mortalities—as proxies for medical/technical advancements (discussed in Section II, Chapter 2.3) and find lower mortality rates corresponding to higher per capita growth rates; an inverse relationship exists\(^ {114}\). Thus, similar directional results as with the live-birth growth rates and output per person employed growth rates, are found for the mortality growth rates and the output per person employed growth rates.

\(^{113}\) Results can be found in Table 6 (in Section II.)

\(^{114}\) Ibid.
Third, my finding on migration growth rates concord with Barro and Sala-i-Martin's (1992) finding that net in-migration is positively correlated with per capita income;\textsuperscript{115} even though the estimated convergence coefficients, the Betas, are marginally affected when the net in-migration is inserted as an explanatory variable in the neoclassical growth rate equation regression model.

Lastly, the growth rate of population is a key exogenous parameter in the neoclassical growth model and does reconcile with the theory of neoclassical growth rates that a higher rate of population growth lowers the steady-state level of capital and output per worker (Section I, Chapter 2.1.1.) Even though both East and West Germany's output per person employed continuous growth rates were decreasing over the period, East Germany's output per person employed growth rate was decreasing less which renders support to the purported negative population growth rate of West Germany and the positive population growth rate of East Germany.\textsuperscript{116}

\textsuperscript{115} Ibid.
\textsuperscript{116} Ibid.
Chapter 3. The Post-Unification Period

On 1 July 1990, full monetary unification took place with the West German Mark, *Deutsche Mark*, becoming the sole legal tender for the united Germany.

With the challenges in trying to unite any two nations, especially when one nation gives up its legal tender, can these regions reach economic convergence?

Analyses of the statistical (Table 8) graphical dispersion measurement of convergence (Graphs G14-17) and unit-root testing results (Table 9) for the data on the five post-unification (DEATHS, EMPLOY, GDP PP, POPUL and PROD) variables, follow.\(^\text{117}\)

3.1 Statistical, Graphical & Unit-Root Tests

Table 8, reports the statistical data—standard deviation, arithmetic mean, variance, coefficient of variation, continuous and cumulative growth rates—for the deaths, employment in manufacturing, output per person employed, population and productivity in manufacturing (DEATHS, EMPLOY, GDP PP, POPUL and PROD, respectively) for the two nations. Each series' analysis begins with the year 1990 until the latest available date; the span of years available for analysis for each variable is reported in parenthesis following the variable name.

\(^{117}\) The other variables in the database do not contain data for the post-unification period for both regions.
Table 8. **UNIFICATION STATISTICS**

<table>
<thead>
<tr>
<th></th>
<th>WESTERN GERMANY</th>
<th>EASTERN GERMANY</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>In DEATHS</strong>: (1990-1992)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.013643</td>
<td>0.053285</td>
</tr>
<tr>
<td>Arithmetic Mean</td>
<td>13.46371</td>
<td>12.19356</td>
</tr>
<tr>
<td>Variance</td>
<td>0.000124</td>
<td>0.001893</td>
</tr>
<tr>
<td>Coefficient of Variation (%)</td>
<td>0.101330</td>
<td>0.436992</td>
</tr>
<tr>
<td>Cont. Growth Rate (%)</td>
<td>-0.90853</td>
<td>-3.55048</td>
</tr>
<tr>
<td>Cum. Growth Rate (%)</td>
<td>-2.72559</td>
<td>-10.6514</td>
</tr>
<tr>
<td><strong>In EMPLOY</strong>: (index 1975=100; 1990-1996)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.074656</td>
<td>0.321916</td>
</tr>
<tr>
<td>Arithmetic Mean</td>
<td>4.518478</td>
<td>4.029120</td>
</tr>
<tr>
<td>Variance</td>
<td>0.004777</td>
<td>0.088826</td>
</tr>
<tr>
<td>Coefficient of Variation (%)</td>
<td>1.652236</td>
<td>7.989733</td>
</tr>
<tr>
<td>Cont. Growth Rate (%)</td>
<td>-2.43683</td>
<td>-10.4893</td>
</tr>
<tr>
<td>Cum. Growth Rate (%)</td>
<td>-17.0578</td>
<td>-73.4249</td>
</tr>
<tr>
<td><strong>In GDP PP</strong>: (index 1975=100; 1990-1996)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.041462</td>
<td>0.232202</td>
</tr>
<tr>
<td>Arithmetic Mean</td>
<td>4.844234</td>
<td>4.240037</td>
</tr>
<tr>
<td>Variance</td>
<td>0.001474</td>
<td>0.046215</td>
</tr>
<tr>
<td>Coefficient of Variation (%)</td>
<td>0.855908</td>
<td>5.476419</td>
</tr>
<tr>
<td>Cont. Growth Rate (%)</td>
<td>-0.77604</td>
<td>-1.72789</td>
</tr>
<tr>
<td>Cum. Growth Rate (%)</td>
<td>-5.43227</td>
<td>-12.0953</td>
</tr>
<tr>
<td><strong>In POPUL</strong>: (1990-1994)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.026549</td>
<td>0.013940</td>
</tr>
<tr>
<td>Arithmetic Mean</td>
<td>17.95722</td>
<td>16.59551</td>
</tr>
<tr>
<td>Variance</td>
<td>0.000564</td>
<td>0.000155</td>
</tr>
<tr>
<td>Coefficient of Variation (%)</td>
<td>0.147843</td>
<td>0.083998</td>
</tr>
<tr>
<td>Cont. Growth Rate (%)</td>
<td>-0.66808</td>
<td>-0.04189</td>
</tr>
<tr>
<td>Cum. Growth Rate (%)</td>
<td>-3.34039</td>
<td>-0.20949</td>
</tr>
<tr>
<td><strong>In PROD</strong>: (index 1975=100; 1990-1996)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Standard Deviation</td>
<td>0.047841</td>
<td>0.421321</td>
</tr>
<tr>
<td>Arithmetic Mean</td>
<td>4.931223</td>
<td>4.815728</td>
</tr>
<tr>
<td>Variance</td>
<td>0.001962</td>
<td>0.152152</td>
</tr>
<tr>
<td>Coefficient of Variation (%)</td>
<td>0.970169</td>
<td>8.748847</td>
</tr>
<tr>
<td>Cont. Growth Rate (%)</td>
<td>1.662300</td>
<td>8.751900</td>
</tr>
<tr>
<td>Cum. Growth Rate (%)</td>
<td>11.63607</td>
<td>61.26327</td>
</tr>
</tbody>
</table>

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Table 8 (cont’d)

**Notes:**

Date (year’s) availability is written in parenthesis after the variable name.

The DEATHS variable is the actual year-end number of deaths.

The POPUL is the mid-year estimate for the actual number of people/population during the year.

The EMPLOY, GDP PP and PROD are defined as employment, output and productivity in manufacturing indices in the base year 1975, so that 1975 = 100. Each index is calculated as:

\[1-\{(\text{base-year value} - \text{current-year value}) / \text{base-year value}\} \times 100.\]

Definitions and sources of data can be found in Appendix A10.

The Cont. (Continuous) Growth Rate is calculated as:

\[\frac{X (\text{latest available date}) - X (1990)}{N} \times 100,\]

where \(N\) is the number of years, and \(X\) is the analyzed variable in \(ln\).

The Cum. (Cumulative) Growth Rate is calculated as:

\[\sum (X_t - X_{t-1}) \times 100\]

for \(t=1990\) until the latest available date for each analyzed variable, \(t-1\) is the previous year, and \(X\) is the analyzed variable in \(ln\).

Table 9 shows the results of the Dickey-Fuller unit-root test of stationarity. Reported, are the t-statistics for the same five variables reported in Table 8, for each of the three time series (i.e., East, West and E-W Difference.) The regression equation tested is the null hypothesis of a pure random walk in levels. The number of observation (\(N\)) and corresponding critical values at the 1-, 5- and 10% significance levels are reported.

The decision to reject the null hypothesis of a unit root in favor of the alternative that its root is less than one for each series (East, West and E-W Difference) is the same as in the pre-unification decision, Table 7 found in Section II, Chapter 2. If the t-statistic is less than (lies to the left of) the critical value, we can reject the null hypothesis of a unit root.
(Ho: $B_i = 0$ in Equations 23' referenced in Section II, Chapter 1.3) in favor of the alternative hypothesis that its root is less than one (H1: $B_i < 0$ in Equations 23' referenced in Section II, Chapter 1.3.) This rejection of the null hypothesis for each East, West and E-W Difference series, would imply that the series is stationary; it follows a deterministic trend where the series will eventually revert to its long-run mean level—convergence.
Table 9. **UNIFICATION UNIT-ROOT RESULTS**

<table>
<thead>
<tr>
<th></th>
<th>WESTERN GERMANY</th>
<th>EASTERN GERMANY</th>
<th>E-W DIFF.</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>t-statistic</td>
<td>t-statistic</td>
<td>t-statistic</td>
<td></td>
</tr>
<tr>
<td><strong>ln DEATHS</strong></td>
<td>-0.144270</td>
<td>-1.457143</td>
<td>3.102868</td>
<td>3</td>
</tr>
<tr>
<td><strong>ln EMPLOY</strong></td>
<td>-1.644966*</td>
<td>-2.305246*</td>
<td>0.390298</td>
<td>7</td>
</tr>
<tr>
<td>(index 1975=100)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>ln GDP PP</strong></td>
<td>-0.017318</td>
<td>-0.786196</td>
<td>-0.505688</td>
<td>7</td>
</tr>
<tr>
<td>(index 1975=100)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>ln POPUL</strong></td>
<td>-0.207601</td>
<td>-0.478372</td>
<td>0.054250</td>
<td>5</td>
</tr>
<tr>
<td><strong>ln PROD</strong></td>
<td>1.745801</td>
<td>0.322274</td>
<td>-1.181872*</td>
<td>7</td>
</tr>
<tr>
<td>(index 1975=100)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Notes:**

All data calculations begin from 1990 until the latest available date—found under number of years, N column, above.

The DEATHS variable is the actual year-end number of deaths.

The POPUL is the mid-year estimate for the actual number of people/population during the year.

The EMPLOY, GDP PP and PROD are defined as employment, output and productivity in manufacturing indices in the base year 1975, so that 1975 = 100. Each index is calculated as:

\[1 - \left(\frac{\text{base-year value} - \text{current-year value}}{\text{base-year value}}\right) \times 100.\]

The definitions and sources of data can be found in Appendix A10.

* Rejects the null hypothesis of a unit root (t-statistic < critical values) at the 1-, 5- or 10% significance levels.

**Critical Values for:**

- \(N=3\) 1\% = -4.3347 5\% = -2.0720 10\% = -1.6759
- \(N=5\) 1\% = -3.3594 5\% = -2.0189 10\% = -1.6518
- \(N=7\) 1\% = -3.0507 5\% = -1.9962 10\% = -1.6415

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The following summarizes the results of the five post-unification variables with respect to
the statistical and unit-root DF test results from Tables 8 and 9, respectively, as well as
the analyses on the graphical dispersion measurement of convergence,\textsuperscript{118} Graphs G14-17.

**DEATHS**

DEATHS' statistics (Table 8) for the FRG show a higher mean growth, lower volatility
and lower continuous growth rate as compared with the GDR region. Over the three-year
period, the growth rates for East Germany were almost 4 times the growth rates of West
Germany. The results of the unit-root DF test on the individual series (Table 9) show that
in levels, the series fail to reject the null hypothesis of a unit root. The same result is
found for the unit-root DF test on the nations' differences (E-W Difference series.)
However, since only three observations are available for the post-union period, the results
are highly suspect (with respect to statistical validity) and no graph showing the standard
deviation of the $ln$ of this variable over time is produced.

**EMPLOY**

From Table 8, even though the EMPLOY growth rates of the GDR were more than four
times that of the FRG, its mean growth was lower and its volatility was substantially
higher. Graph G14\textsuperscript{119} measures the standard deviation over time of the $ln$ of this variable
for both nations. Visible sign of convergence is not found. With respect to the unit-root
DF test results found in Table 9, we can reject the unit root hypothesis in levels at the 5%
and 10% significance levels for East and West Germany, respectively. This would seem

\textsuperscript{118} Graph methodology is discussed in Section II, Chapter 1.2.

\textsuperscript{119} Ibid.
to imply that individually, the series have a long-run equilibrium steady-state position and that East Germany given its rate of growth is trying to 'catch up'. However, in testing the E-W Difference series, this variable fails to reject the null hypothesis in levels. This result may be attributed to the higher EMPLOY growth rate for the FRG during its first three years of unification. Thus, convergence cannot be assured.

**GDP PP**

While East Germany’s continuous GDP PP growth rate is higher than its counterpart, this variable's mean growth is lower and its volatility is higher (Table 8.) Graph G15\(^{120}\) measures the standard deviation over time of the \(ln\) of this variable for both nations and shows that convergence can be found after 1991. But, East and West Germany as well as their E-W Difference series, fail to reject the unit-root DF test in levels (see Table 9); thus non-convergence cannot be ruled out.

**POPUL**

From Table 8, not only is the arithmetic mean and volatility for the POPUL variable higher in West Germany; its continuous growth rate is also substantially higher than the East. The results of the unit-root DF test, reported in Table 9, show non-convergence for the East, West and E-W Difference series in levels. Even though only five observations are available, Graph G16 shows the standard deviation of the \(ln\) of this variable over time that indicates non-convergence.

---

\(^{120}\) Ibid.
PROD

PROD’s arithmetic mean is higher for the FRG region and its volatility is lower while its continuous growth rate is lower as compared to the GDR region; seen in Table 8. Graph G17\textsuperscript{121} measures the standard deviation over time of the $\ln$ of this variable for both nations’ series, which show that Sigma Convergence cannot be found. Furthermore, this variable has a positive DF test statistic in levels, wrong sign.\textsuperscript{122} However, the E-W Difference series shows that convergence is found in levels at the 10% significance level (Table 9.)

3.2 Summary of Post-Unification Results

Five variables were analyzed for Germany’s post-unification period: deaths, employment in manufacturing, output per person employed, population, and labor productivity in manufacturing (DEATHS, EMPLOY, GDP PP, POPUL and PROD, respectively.)

The statistics (standard deviation, arithmetic mean, variance, coefficient of variation, continuous and cumulative growth rates reported in Table 8) show that for the post-unification period, former-West Germany has a higher arithmetic mean in all the analyzed variables; a lower overall volatility and continuous growth rates for the DEATHS, EMPLOY and GDP PP variables; and a lower volatility but a higher continuous growth rate for the PROD variable. For the POPUL variable, West

\textsuperscript{121} Ibid.

\textsuperscript{122} However, once first differences were taken, the null hypothesis of a unit root for both the East and West German individual series can be rejected at the 5% and 10% significance levels, respectively. Results of differencing the two data series are speculative as the series are too short (7 observations) and thus the test results are not shown in Table 9.
Germany’s statistics are higher in all the six measurements (exclusive of the cumulative growth rate.)

The graphical dispersion measurement of convergence (Graphs G14-17)\(^{123}\) show that the two nations’ EMPLOY, POPUL and PROD (measured by the standard deviation of the \(\ln\) of the variables over time) are not consistently downward-sloping/negative over the period; thus convergence cannot be assured. However, the GDP PP graph does show convergence after 1991.

For the Dickey and Fuller test, even though these variables show sign of non-convergence (as evidenced from Table 9) after seven years following their union, too few observations are available for these data series to render any conclusive decisions about their stationarity.

\(^{123}\) Graph methodology is further discussed in Section II, Chapter 1.2.
Chapter 4. Section II. Conclusion of the German 1990 Unification
Statistical, Graphical & Unit-Root Test Results

Economic convergence, measured by the output per person employed, was found for the German 1990 unification after 1991. Of the eleven variables\textsuperscript{124} examined in this section of study (Section II) only five available variables span through both the pre- and post-unification periods: deaths, employment in manufacturing, output per person employed, population, and labor productivity in manufacturing (DEATHS, EMPLOY, GDP PP, POPUL and PROD, respectively.)

In comparing the pre- and post-union results of Tables 6 and 8, respectively, the statistical measurements—standard deviation and arithmetic mean—will have the following interpretation. If I find a decrease in the standard deviation of the $ln$ of the variable between the two (pre- and post-) periods, this would imply a tendency toward convergence. And, if I find an increase in the arithmetic mean (arithmetic mean of the $ln$ of the variable) between the two periods, this would imply a tendency for that region to ‘catch up’ (to the other.)

Summary analyses of the comparative pre- and post-union results for these five variables,\textsuperscript{125} inclusive of the statistical (Tables 6 and 8, respectively) graphical dispersion

\textsuperscript{124} Deaths, divorces, employment in manufacturing, foetal mortalities, output per person employed, infant mortalities, live births, marriages, migration, population and labor productivity in manufacturing. See Appendix A10 for their definitions and sources of data.

\textsuperscript{125} Summary analyses for the other six (and all eleven) pre-unification variables can be found in Section II, Chapter 2.6, “Summary of Pre-Unification Results.”
measurement of convergence (Graphs G3, G11-13 and G14-17, respectively) and the results of the unit-root Dickey-Fuller tests (Tables 7 and 9, respectively) follow.

DEATHS

The statistics on the DEATHS variable show that while prior to unification, both nations’ continuous growth rates were negative, they are now positive.\textsuperscript{126} Western Germany series’ volatility compared to its pre-union period is lower for the post-union period (the tendency toward convergence) while its arithmetic mean is higher (a tendency to ‘catch up’).\textsuperscript{127} Opposite results are found for the eastern German DEATHS series.\textsuperscript{128} Thus, for the DEATHS variable, if the goal is for the gap/variability between the two former-economies to decrease with time, then my findings are inconsistent with the concept of convergence as the gap between the two former nations seems to be widening. Additionally, unit-root DF test results for the post-union period (even though highly suspect due to short sample) parallel the evidence found in the pre-union period of non-convergence.\textsuperscript{129}

EMPLOY

The EMPLOY series before unification showed negative continuous growth rates for both regions, but after 1990, both regions’ growth rates are positive with eastern

\textsuperscript{126} Statistics found in Tables 6 and 8 for the pre- and post-union periods, respectively.

\textsuperscript{127} Ibid.

\textsuperscript{128} Ibid.

\textsuperscript{129} Unit-Root test results found in Tables 7 and 9 for the pre- and post-union periods, respectively.
Germany having growth rates of 4.3 times that of its western counterpart.\textsuperscript{130} However, this growth is not enough for eastern Germany to 'catch up', as its arithmetic mean was lower during the post-union period (as compared with its pre-union period.)\textsuperscript{131} For both the pre- and post-union periods, the unit-root DF test results do not show that the three series (eastern, western and E-W Difference series) are stationary in levels\textsuperscript{132} and the graphical measurement of dispersion for the pre-union period shows that Sigma Convergence is not evidenced.\textsuperscript{133} The graphical measurement of dispersion for the post-union period shows similar results as its pre-union period\textsuperscript{134} and the standard deviation of the \textit{ln} of the EMPLOY variable is higher for eastern Germany in the post-union period as compared to its pre-union period.\textsuperscript{135} This tendency toward non-convergence re-enforces the above unit-root DF test results.

**GDP PP**

For the GDP PP and the PROD variables, where before unification, the statistics showed that western Germany's arithmetic mean was less than the eastern nation, while the variable's volatility was higher; the situation reversed itself after 1990.\textsuperscript{136}

\textsuperscript{130} Statistics found in Tables 6 and 8 for the pre- and post-union periods, respectively.

\textsuperscript{131} Ibid.

\textsuperscript{132} Unit-Root test results found in Tables 7 and 9 for the pre- and post-union periods, respectively.

\textsuperscript{133} See Graph G12 which measures the standard deviation of the \textit{ln} of the EMPLOY variable over time from 1950-1989 for both the East and West German regions. Graph methodology is further discussed in Section II, Chapter 2.1.

\textsuperscript{134} See Graph G14 which measures the standard deviation of the \textit{ln} of the EMPLOY variable over time from 1990-1996 for both the East and West German regions. Graph methodology is further discussed in Section II, Chapter 2.1.

\textsuperscript{135} Statistics found in Tables 6 and 8 for the pre- and post-union periods, respectively.

\textsuperscript{136} Ibid.
Specifically, where prior to their union, the GDP PP continuous growth rates were purely negative for both nations, their growth rates are now positive with eastern Germany having a higher growth rate by 2.2 times that of western Germany.\textsuperscript{137} However, eastern Germany’s higher growth since unification may be sufficient to ‘catch up’, as its arithmetic mean is lower than its pre-union period.\textsuperscript{138} Even though the standard deviation of the $ln$ of the GDP PP variable has decreased from the pre- to the post-union period for both regions,\textsuperscript{139} as shown by the results of the DF test, for the post-union period, all three series, eastern, western and E-W Differences, do not converge in levels.\textsuperscript{140} However, these non-convergence results for the pre- and post-union periods are reinforced by the graphical measurement of dispersion for the pre-union period but not for the post-union period. The pre-union graph does not support Sigma Convergence\textsuperscript{141} but the post-union graph does show sign of Sigma Convergence of the two nations starting from 1991.\textsuperscript{142}

PROD

The labor productivity in manufacturing variable, PROD, showed similar percentage decline in their pre-union period’s growth rates while after unification, these growth rates

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\textsuperscript{137} Ibid.

\textsuperscript{138} Ibid.

\textsuperscript{139} Ibid.

\textsuperscript{140} Post-union unit-root DF test results can be found in Table 9.

\textsuperscript{141} See Graph G11 which measures the standard deviation of the $ln$ of the GDP PP variable over time from 1950-1989 for both the East and West German regions. Graph methodology is further discussed in Section II, Chapter 1.2.

\textsuperscript{142} See Graph G15 which measures the standard deviation of the $ln$ of the GDP PP variable over time from 1990-1996 for both the East and West German regions. Graph methodology is further discussed in Section II, Chapter 1.2.
are still negative but by a much wider margin between the two regions. Non-convergence was found for the pre-union period evidenced both from the unit-root DF E-W Difference series test results and by the graphical measurement of dispersion. In contrast, the E-W Difference series is found stationary in levels for the post-union period but the individual eastern and western series are not found stationary in levels. Additionally, from the graphical measurement of dispersion, we see that Sigma Convergence is not evidenced even though the tendency toward convergence, measured as a decrease in the standard deviation of the ln of the PROD variable, from pre- to post-unification as well as a tendency to 'catch-up', measured as an increase in its arithmetic mean, from pre- to post-unification, is found for both nations. Unfortunately, too few data points are available to make any conclusive assertions of convergence for the post-union period.

**POPUL**

Lastly, for the POPUL variable, where during the pre-union period, the continuous growth rates were negative for western Germany, their growth rate for the post-union period is now positive (and higher by 15.95 times that of their counterpart during the pre-union period).

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143 Ibid.

144 Unit-Root test results found in Tables 7 for the pre-union period.

145 See Graph G13 which measures the standard deviation of the ln of the PROD variable over time from 1950-1989 for both the East and West German regions. Graph methodology is further discussed in Section II, Chapter 1.2.

146 Post-union unit-root test results can be found in Tables 9.

147 See Graph G17 which measures the standard deviation of the ln of the PROD variable over time from 1990-1996 for both the East and West German regions. Graph methodology is further discussed in Section II, Chapter 1.2.

148 Statistics found in Tables 6 and 8 for the pre- and post-union periods, respectively.
The opposite directional result can be found for eastern Germany. Where during the pre-union period, the POPUL continuous growth rate was positive, their growth rate for the post-union period is still positive but by a lower rate. Furthermore, unit-root DF test results for the pre- and post-union period show no sign of convergence in levels for these three series. However, convergence, measured as a decrease in the standard deviation of the ln of the POPUL variable from the pre- to the post-union period is evidenced. But, the eastern German series does not show sign of ‘catch-up’, as its arithmetic mean is lower during the post-union period as compared with its pre-union period (and as compared to western Germany’s arithmetic mean.) Specifically, starting with 1990, non-convergence is found up to 1992 and from 1992 to 1994 the standard deviation of the ln of the population for the two regions has been decreasing; indicating convergence. However, as only five data points are available for these data series, the results (indicating convergence from 1992 to 1994) may be premature.

In sum, since Germany’s 1990 unification, convergence in the deaths, employment in manufacturing, population, and productivity is not found. However, the output per person employed variable does show sign of convergence after 1991.

\[149\] Statistics found in Tables 6 and 8 for the pre- and post-union periods, respectively.

\[150\] Ibid.

\[151\] Unit-Root test results found in Tables 7 and 9 for the pre- and post-union periods, respectively.

\[152\] Statistics found in Tables 6 and 8 for the pre- and post-union periods, respectively.

\[153\] Ibid.

\[154\] See Graph G16 which measures the standard deviation of the ln of the POPUL variable over time from 1990-1994 for both the eastern and western German regions. Graph methodology is further discussed in Section II, Chapter 1.2.
With a short post-union sample (at most seven observations) the above results are
presented with a low degree of statistical confidence. However, what we can conclude, is
that from 1950 (the first full-year the East and West German nations were proclaimed
republics) until 1989 (the year prior to their economic convergence) these two economies
were dissimilar. But, a monetary union of these two unequal economic participants as of
1990 (the year of economic and monetary convergence) to 1996 (the latest data available
date for both regions) in levels, the output per person employed (GDP PP) of eastern and
western Germany have begun to show sign of convergence.
SUMMARY & IMPLICATIONS FOR FUTURE STUDIES

Conclusions.

This study utilized several approaches—Absolute Beta Convergence, Conditional Beta Convergence, Sigma Convergence (in Section I) and statistical, graphical dispersion measurement of convergence, unit-root Dickey-Fuller testing (in Section II)—to analyze the economic impact, measured by per capita GDP levels, of a monetary union among two unequal economic participants: eastern and western Germany.

The variables utilized in Section I are the population, per capita GDP and savings growth rates for the study of club convergence and I created a database of eleven variables utilized in Section II: deaths, divorces, employment in manufacturing, foetal mortalities, output per person employed, infant mortalities, live births, marriages, migration, population and labor productivity in manufacturing.

Since the day of monetary unification the former nations of eastern and western Germany, who had been legally separated for 40 years and forced to live under different social, political and economic conditions, are seen to be growing closer to one another—

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155 For the first study, the definitions and sources of variables can be found in Section I, Chapter 1.1. Output per person employed was used as a proxy for per capita GDP in the second study, Section II. (See Appendix A10 for definition and source of data.)

156 Unequal economic participants is measured by the economies' long-run steady-state per capita GDP levels; per capita GDP in Section I and output per person employed in Section II.

157 These variables were extracted from the 2001 WDI database. See Section I, Chapter 1.1 "Data Source & Definition of Variables."

158 My data collection efforts are discussed in Section II, Chapter 1.1 and the definitions and sources for these eleven variables can be found in Appendix A10.
convergence to the same long-run equilibrium values are found in their per capita GDP levels.\textsuperscript{159}

Specifically, with respect to Study I beginning with Germany’s 1990 unification until 1999, Absolute Beta Convergence, the tendency for the per capita incomes to equalize across all economies—same steady-state growth rates—is not found.\textsuperscript{160}

Conditional Beta Convergence, the tendency for the per capita incomes to equalize across homogeneous economies/clubs—where convergence is conditional on the different structural characteristics of each economy—is found for the High-OECD-Income-Club (HOIC) and the Upper-Middle-Income-Club (UMC) of which western and eastern Germany are members, respectively.\textsuperscript{161} Furthermore, the two (HOIC and UMC) clubs are found to have different long-run steady-state equilibrium values: average per capita GDP growth rates of 1.395\% and 1.858\% for the HOIC and UMC economies, respectively.\textsuperscript{162} Thus, these two clubs have not converged to one another by end of year 1999.

Sigma Convergence—the decrease in the dispersion of the cross-region economies’ relative per capita incomes over time—reinforces the findings of Beta Convergence for the two clubs. Specifically, the HOIC economies show Sigma Convergence for all the

\textsuperscript{159} Test results are discussed in Section II, Chapter 2.2.

\textsuperscript{160} See Section I, Chapter 2 and Table 2.

\textsuperscript{161} See Section I, Chapter 3 and Tables 3 and 4.

\textsuperscript{162} Barro & Sala-i-Martin (1995) state that the growth rate of capital per worker connotes steady-state status in the neoclassical Solow-Swan model (pg. 19.) See my discussion in Section I, Chapter 3.3.
years following the unification period while the UMC economies also show Sigma Convergence following the unification period but convergence begins with the year 1993. However, this lack of convergence for the first three years may be explained as it is found that the savings growth rate variable is not statistically significant in accounting for the structural characteristics of the UMC economies.

Summarizing the individual East and West German data series from 1950 to 1989 in Study II, as expected, the former nations are found to be statistically different (i.e., do not converge to the same long-run equilibrium) with respect to their deaths, employment in manufacturing, foetal mortalities, output per person employed, live births, population, migration and labor productivity in manufacturing data series. The divorce, infant mortalities and marriage growth rates, while also showing non-convergence, are suspect due to the lack of verifiable data for the East Germany series.

Additionally, in an effort to reconcile the pre-unification statistics of Study II within the context of the neoclassical growth theory of Study I, the findings of the live births, migration, mortality—death, infant and foetal—and the population growth rates, which affect the per capita GDP growth rates, show that as predicted by theory, fertility rates tend to fall with per capita income for the poorest (in terms of low-income) countries.

See Section 1, Chapter 4.3 and Graphs G1 and G2 for the HOIC and UMC clubs, respectively.

See Section I, Chapter 3.3.

Section II, Chapter 2.6 offers a more detailed summary for the pre-unification period combining the statistical, graphical measurement of convergence and unit-root Dickey-Fuller test results of the eleven variables.

Results can be found in Section II, Table 6 (L-BIRTHS, GDP PP variables) where live-births are used as a proxy for fertility rates; see Section II, Chapter 2.6 for full discussion.
lower mortality rates correspond to higher per capita growth rates; an inverse relationship exists;\textsuperscript{167} and higher rate of population growth lowers the steady-state level of capital and output per worker.\textsuperscript{168}

Furthermore, my finding on migration growth rates concord with Barro and Sala-i-Martin’s (1992) finding that net in-migration is positively correlated with per capita income.\textsuperscript{169}

However, beginning with Germany’s 1990 unification, I find that convergence cannot be assured for the deaths, employment in manufacturing, population, and labor productivity in manufacturing of the former two nations\textsuperscript{170} but that the output per person employed variable does show sign of convergence after 1991.

Absolute Beta Convergence, Conditional Beta Convergence and Sigma Convergence—testing post-union results (Study I) and the analyses of the former GDR and FRG data series’ statistical, graphical and unit-root pre- and post-union results (Study II)—have led to similar conclusions about the economic impact of a monetary union of two unequal

\textsuperscript{167} Results can be found in Section II, Table 6 (DEATHS, I-MORT, F-MORT, GDP PP variables); see Section II, Chapter 2.6 for full discussion.

\textsuperscript{168} Results can be found in Section II, Table 6 (POPUL, GDP PP variables); see Section II, Chapter 2.6 for full discussion.

\textsuperscript{169} Results can be found in Section II, Table 6 (MIGR variable); see Section II, Chapter 2.6 for full discussion.

\textsuperscript{170} For these four variables, 1996 is the ending period of study for the employment in manufacturing, and the labor productivity in manufacturing variables, 1994 for the population variable and 1992 for the deaths variable. See Section II, Chapter 3.1 for full details or Section II, Chapter 3.2 for a summary of the post-unification results of Study II combining the statistical, graphical measurement of convergence and unit-root findings.
economic participants: convergence expressed by per capita GDP in Study I and by output per person employed in Study II, cannot be ruled out.

**Implications for Future Studies.**

A sequence to this study would be to repeat the analyses after at least another 10 years thus allowing for a longer post-unification sampling period; more robust findings.

Alternatively, an analysis of the regional differences within the former nations\(^{171}\) pending data availability, to test for regional economic convergence may not only prove useful in testing the impact of a monetary union upon the economic convergence among the individual regions but also prove useful to test whether a 'mezzogiorno' problem exists; as seen among other regions within Italy and Spain as two examples.

Furthermore, an additional application of this study can be extended to other monetary unions of dissimilar economies, to test whether a monetary union leads to economic convergence. Two possible applications of monetary unions of dissimilar economies may include the former North and South Korea, or possibly, Belgium and France (prior to the E.U. when Belgium was accepting the former French franc currency.)

\(^{171}\) Similar to the study done by Barro & Sala-i-Martin (1995.)
From [Equation 11],

\[ \ln (\gamma_t) = (e^{-\beta \cdot t}) \ln (\gamma) + (1 - e^{-\beta \cdot t}) \ln (\gamma^*) \]

where time \( t \) in \( \ln (\gamma_t) \) is halfway between \( \ln (\gamma) \) and \( \ln (\gamma^*) \) and satisfies the condition \( (e^{-\beta \cdot t}) = \frac{1}{2} \).

The half-life is therefore calculated as:

\[ \ln (2) / \beta = x \]

Or 0.69 (which = \( \ln (2) \)) divided by the calculated Beta in [Equation 1] which should be equal to the regression Beta coefficient of [Equation 14] if testing for ABC or CBC for sets of economies with similar characteristics, or [Equation 17] for CBC with differing steady-state economies.

The variable \( x \) would then be interpreted as the years it would take for the gap between the two economies to close.

Example, if \( \beta = .02 \), then \( x = 35 \). Thus, it would take roughly 35 years for \( \frac{1}{2} \) the gap between the 2 countries' per capita incomes to close! Then if we additionally assume that the capital share, \( a \), is 0.75, then we can state, “The time needed to eliminate \( \frac{3}{4} \) of an initial gap from the steady-state position is about 70 years.” (Barro & Sala-i-Martin (1995) pg. 38)
EXTENSION OF THE BETA CONVERGENCE CONCEPT

[Equation 1], \[ \ln \left( \frac{Y_t}{L_t} \right) - \ln \left( \frac{Y_{10}}{L_{10}} \right) = \alpha - \beta \ln \left( \frac{Y_{10}}{L_{10}} \right) \]
equals:

\[
\frac{\ln \left( \frac{Y_t}{L_t} \right)}{\ln \left( \frac{Y_{10}}{L_{10}} \right)} = \alpha - \beta \frac{\ln \left( \frac{Y_{10}}{L_{10}} \right)}{\ln \left( \frac{Y_{10}}{L_{10}} \right)}
\]

Let \( Y/L = \gamma \) so that,

\[ \ln \left( \frac{\gamma_t}{\gamma_{10}} \right) = \alpha - \beta \ln \gamma_{10} \]

When applying this equation to many economies, in the limit, if the economies have converged then the difference between the two periods goes to zero, thus \( \gamma_t = \gamma_{10} \),

\( (\gamma_t/\gamma_{10}) = 1 \) and \( \ln (\gamma_t/\gamma_{10}) = 0 \). The above equation is reduced to:

\[ 0 = \alpha - \beta \ln \gamma_{10} \]

where,

\[ \alpha = \beta \ln \gamma_{10} \]

and

\[ \alpha/\beta = \ln \gamma_{10} \]

From this we can summarize:

Beta measures the speed of convergence—the strength of the conversion effect. This equation implies that the closer the Beta is to 1, the faster the convergence.

If \( \beta = 0 \), the relationship collapses as \( \alpha/\beta \) which equals \( \ln \gamma_{10} \) is undefined.

If \( \beta = 1 \), then \( \alpha - \ln \gamma_{10} = 0 \) and \( \alpha = \ln \gamma_{10} \) implying that the initial level of per worker per capita income is completely explained by the differences between the ending and
initial per worker capita income—complete convergence. The interpretation is that for every percent a country’s per capita income was below average in the initial year of analysis, its cumulative growth rate was one percent higher over the entire period of time, $(T - t o + 1.)$

If $\beta < 0$, then, $0 = \alpha + a$ positive fraction of $ln \gamma_{to}$. This would imply a positive slope; that the initial level of per worker per capita income is an increasing function of the differences between the ending and initial per worker capita income. This would mean that the economies are diverging—the tendency for the initially richer economies to grow faster than the poorer economies after the initial base year of analysis.

If $0 > \beta > 1$, then, $0 = \alpha -$ fraction of $ln \gamma_{to}$, signifying a negative relationship between $[ln (\gamma_{T}/\gamma_{to})]$ and $ln \gamma_{to}$ (i.e., convergence in levels.)

If $\beta > 1$, the function explodes.
NEOCLASSICAL SOLOW-SWAN MODEL EXTENSION & CONVERGENCE

In the neoclassical framework of growth, the condition of a closed economy implies that the convergence of output and income must coincide. As stated previously, this assumes that net foreign assets are equal to zero. However, a distinction can be made between domestic product and income or domestic capital stock and assets.

For an open economy, even if we assume that the economies share similar technologies, the per capita capital stock and output will converge to the prevailing economies, but the per capita incomes will not converge. This is because each economy faces constant returns in the (global) capital market.

If we think of each state in the U.S. as a distinct economy, then each state can be described as an open economy. This is because its residents can migrate, exchange goods and technologies and borrowing can occur across borders. However, Barro & Sala-i-Martin (1992) find that the per capita domestic product and incomes, contrary to open-economy theory, do converge.

To offer an explanation to this contradiction of findings for the U.S. states, they postulate that domestic product behaves like national incomes and thus will converge if we allow capital to include human capital and place a ceiling on the external debt/capital stock ratio. This means that the government of each state cannot finance, by borrowing nationally, limitless amounts of their expenditures in their investment of human capital.
(i.e., education.) Stated differently, if we add a borrowing constraint on human capital, which capital has been expanded to include both physical and human capital, then convergence of domestic product/output and income will coincide for an open-economy model with similar technology economies.

If states do not share similar technologies, can convergence of output and income occur?

Even though the states may not share similar technologies at the start but are open economies where technology is allowed to diffuse across states, then there is a potential to imitate so that poor states can grow faster than rich states. In this case, convergence of this type is possible.

If, on the other hand, the states do not share similar technologies and the economies are open in the sense that capital is free to migrate, then capital mobility can create a divergence of per capita output and capital stocks. This may occur because higher capital/labor ratios are usually associated with higher values of technologies and the higher the value of technology, the more it offsets the diminishing returns on the marginal product of capital. This may cause both physical and human capital to move from poor to richer economies rendering the convergence of product and income ambiguous.
Another factor that upholds convergence between per capita product and income is the free flow of labor. Nonetheless, even though Barro & Sala-i-Martin (1992) find that net in-migration is positively correlated with the initial per capita income (similar results I found for the pre-unification period in Section II, Chapter 2.2.A2) the results show that the estimated convergence coefficients, the Betas, are only marginally affected when the net in-migration is inserted as an explanatory variable in the neoclassical growth rate equation regression model.
From [Equation 15],

\[ \ln \left( \frac{Y_t}{L_t} \right) - \ln \left( \frac{Y_{t0}}{L_{t0}} \right) = \alpha - \beta_2 \ln \left( \frac{Y_{t0}}{L_{t0}} \right) - \beta_3 \ln \left( \frac{Z_{t0}}{L_{t0}} \right) \]

let \( \frac{Y}{L} = \gamma \) and \( \frac{Z}{L} = z \) so that,

\[ \ln \left( \frac{\gamma_t}{\gamma_{t0}} \right) = \alpha - \beta_2 \ln \gamma_{t0} - \beta_3 \ln z_{t0} \]

When applying this equation to many economies (but eliminating the \( i \) subscript representing each economy, for simplicity) in the limit, if the economies have converged then the difference between the two periods’ per capita level of income goes to zero, thus \( \gamma_t = \gamma_{t0} \), \( \frac{\gamma_t}{\gamma_{t0}} = 1 \) and \( \ln \left( \frac{\gamma_t}{\gamma_{t0}} \right) = 0 \). The above equation is reduced to:

\[ 0 = \alpha - \beta_2 \ln \gamma_{t0} - \beta_3 \ln z_{t0} \]

If we assume a linear equality restriction for the coefficients \( \beta_2 \) and \( \beta_3 \)—an equiproportional change in the growth rate for an equiproportional change in the \( \ln \gamma_{t0} \) and \( \ln z_{t0} \), then:

\[ \beta_2 + \beta_3 = 1 \quad \text{or} \quad \beta_3 = 1 - \beta_2 \]

inserting this into the above equation:

\[ 0 = \alpha - \beta_2 \ln \gamma_{t0} - \beta_2 \ln z_{t0} \]

distributing the parenthesis,

\[ 0 = \alpha - \beta_2 \ln \gamma_{t0} - \beta_2 \ln z_{t0} + \beta_2 \ln z_{t0} \]

rearranging,

\[ 0 = \alpha - \ln z_{t0} - \beta_2 \ln \gamma_{t0} + \beta_2 \ln z_{t0} \]
combining the betas,

\[ 0 = \alpha - \ln z_{10} - \beta_2 (\ln \gamma_{10} - \ln z_{10}) \quad \text{or,} \quad 0 = \alpha - \ln z_{10} - \beta_2 \ln (\gamma_{10} / z_{10}) \]

From the above equation, if we apply the restriction that \( \gamma_{10} \neq z_{10} \) \(^{172}\), then we can summarize:

If \( \beta_2 = 0 \), then \( \beta_3 = 1 \). The above equation reduces to \( 0 = \alpha - \ln z_{10} \).

Thus \( \alpha = \ln z_{10} \) implying that the differences between the initial and ending per worker capita income is completely explained by the initial level of per worker per capita \( Z \) variable and that the initial per capita level of income in the base year has no effect on the differences between the initial and ending per worker capita income. Thus, there would be no convergence in the level of per capita incomes but the initial level of the 'conditional' variable \( Z \), would alone explain the rate of growth. This is different than the result under ABC. The relationship in CBC is no longer undefined as in ABC.

If \( \beta_2 = 1 \), then \( \beta_3 = 0 \) then \( \alpha - \ln \gamma_{10} = 0 \) and \( \alpha = \ln \gamma_{10} \). This implies that the initial level of per worker per capita completely explains the difference between the initial and ending per worker capita income but that the 'conditional' \( Z \) variable has no effect. This yields the same results obtained in ABC.

\(^{172}\) This is a plausible restriction as it would not make sense to have a second explanatory variable that is equal to the first explanatory variable; else it would have no additional explanatory power.
If $\beta_2$ is between 0 and 1, then $\beta_3$ is also between 0 and 1, exclusive. This occurs only when the $\ln \gamma_{t0} > \ln z_{t0}$. Thus, the slope is negative and convergence is evident.

However, two main differences in comparing CBC to ABC arise. The slope (CBC) < slope (ABC) in absolute value and $\alpha$ (CBC) < $\alpha$ (ABC); both diminished by the effect of the 'conditional' variable $Z$.

If $\beta_2 < 0$, then $\beta_3 > 1$. This would imply that the absolute value of $\ln z_{t0} > \ln \gamma_{t0}$, making the slope positive, signifying divergence in levels. However in comparing this divergence to that of ABC, this (CBC) divergence is reduced by the effect of the 'conditional' variable $Z$.

The addition of more than one structural variable ($Z$) to explain the steady-state differences of each country, would have similar effects on the concept of convergence as described above.
### HIGH-OECD-INCOME (HOIC) ECONOMIES

<table>
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* Members of the European Union.

*Source: 2001 World Development Indicators on CD-ROM*
HIGH-INCOME NON-OECD (HNOIC) ECONOMIES

* ** *** Andorra
* ** *** Aruba
* ** *** Bahamas, The
* ** *** Bermuda
* ** *** Brunei
* ** *** Cayman Islands
* ** *** Channel Islands
** *** Cyprus
* ** *** Faeroe Islands
* ** *** French Polynesia
* ** *** Greenland
** *** Guam
 Hong Kong, China
 Israel
 Kuwait
* ** *** Liechtenstein
 Macao, China
* ** *** Monaco
** ** Netherlands Antilles
 New Caledonia
* ** *** Northern Mariana Islands
** *** Qatar
* ** *** San Marino
 Singapore
 Slovenia
 United Arab Emirates
** *** Virgin Islands (U.S.)

* excluded from average population growth rate due to missing data
** excluded from average per capita GDP due to missing data
*** excluded from average savings due to missing data

Source: 2001 World Development Indicators on CD-ROM
**UPPER-MIDDLE INCOME (UMC) ECONOMIES**

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* excluded from average population growth rate due to missing data.
** excluded from average per capita GDP due to missing data.
*** excluded from average savings due to missing data.

*Source: 2001 World Development Indicators on CD-ROM*
## LOWER-MIDDLE INCOME (LMC) ECONOMIES

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* excluded from average population growth rate due to missing data.
** excluded from average per capita GDP due to missing data.
*** excluded from average savings due to missing data.

Source: 2001 World Development Indicators on CD-ROM
** Appendix A9**

**LOW-INCOME (LIC) ECONOMIES**

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** excluded from average per capita GDP due to missing data.
*** excluded from average savings due to missing data.

*Source: 2001 World Development Indicators on CD-ROM*
DEFINITIONS AND SOURCES OF VARIABLES
USED IN SECTION II.

DEATHS:
Death is the permanent disappearance of all evidence of life at any time after live birth has taken place. It is defined as the postnatal cessation of vital functions without capability of resuscitation. This definition therefore excludes foetal deaths. Data were extracted in actual year-end totals and converted to natural logs. The series are available from 1950 to 1992 for both East and West Germany.

Source: These data were extracted from the Demographic Yearbook of the UN, German Tables. 1950-1992. Issues 10-52.

*DIVORCE:
Divorce is a final legal dissolution of a marriage; that a separation from husband and wife which confers on the parties the right to remarriage under civil, religious and/or other provisions, according to the laws of each country. Data were extracted in actual year-end totals and converted to natural logs. The series are available from 1950 to 1989 for East Germany and from 1950 to 1990 for West Germany.

Source: These data were extracted from the Demographic Yearbook of the UN, German Tables. 1950-1990. Issues 10-50.

* Data is speculative for East Germany.
EMPLOY:

Employment in manufacturing, Index: 1975=100.

The index is calculated as:

\[1-\left\{\frac{\text{base-year value} - \text{current-year value}}{\text{base-year value}}\right\} \times 100.\]

Data were then converted to natural logs and the series are available from 1950 to 1996 for both East and West Germany.

Source: These data were extracted from the Groningen Growth and Development Centre, October 1999, reproduced in Van Ark, page 38.

F-MORT:

Foetal mortalities is defined as death prior to the complete expulsion or extraction from its mother from conception with duration of pregnancy of 28 or more completed weeks of gestation. The death is indicated by the fact that after such separation, the fetus does not breath or shows any other evidence of life—such as beating of the heart, pulsation of the umbilical cord, or definite movement of voluntary muscles. These are synonymous with the events reported under the pre-1950 terms stillborn. Data were extracted in actual year-end totals and converted to natural logs. The series are available from 1950 to 1989 for East Germany and from 1950 to 1990 for West Germany.

Source: These data were extracted from the Demographic Yearbook of the UN, German Tables. 1950-1990. Issues 10-50.
**GDP PP:**

Output per person employed, Index: 1975=100.

The index is calculated as:

\[
1 - \frac{(\text{base-year value} - \text{current-year value})}{\text{base-year value}} \times 100.
\]

Data were then converted to natural logs and the series are available from 1950 to 1996 for both East and West Germany.

*Source: These data were extracted from the Groningen Growth and Development Centre, October 1999, reproduced in Van Ark, page 38.*

**I-MORT:**

Infant mortalities is the permanent disappearance of all evidence of life after live birth has taken place. It is defined as the postnatal cessation of vital functions without capability of resuscitation for an infant child. Data were extracted in actual year-end totals and converted to natural logs. The series are available from 1950 to 1989 for East Germany and from 1950 to 1990 for West Germany.

*Source: These data were extracted from the Demographic Yearbook of the UN, German Tables. 1950-1990. Issues 10-30.*

* Data is speculative for East Germany.
L-BIRTHS:
Live birth is the complete expulsion or extraction of a living being from its mother, irrespective of the duration of pregnancy. Living is defined such that after such separation from its mother, it breathes or shows any other evidence of life—beating of the heart, pulsation of the umbilical cord, or definite movement of voluntary muscles.
Data were extracted in actual year-end totals and converted to natural logs. The series are available from 1950 to 1989 for East Germany and from 1950 to 1992 for West Germany.

Source: These data were extracted from the Demographic Yearbook of the UN, German Tables. 1950-1992. Issues 10-52.

*MARRIAGE:
Marriage is the act, ceremony or process by which the legal relationship of husband and wife is constituted. The legality of the union may be established by civil, religious, or other means of as recognized by the laws of each country. Data were extracted in actual year-end totals and converted to natural logs. The series are available from 1950 to 1989 for East Germany and from 1950 to 1991 for West Germany.

Source: These data were extracted from the Demographic Yearbook of the UN, German Tables. 1950-1991. Issues 10-51.

* Data is speculative for East Germany.


**MIGR:**

Migration Rate is a computed variable, using the variables listed in this appendix, defined as:

\[
MIGR_t = POPUL_t - (L-BIRTHS_t - DEATHS_t + POPUL_{t-1})
\]

Data once computed from actual totals were then converted to natural logs. The series are available from 1951 to 1989 for East Germany and from 1951 to 1992 for West Germany.

*Source: Computed from the data extracted from the Demographic Yearbook of the UN, German Tables. 1951-1992. Issues 11-52.*

**POPUL:**

All population figures are mid-year estimates of the actual total population. Data were then converted to natural logs. The series are available from 1950 to 1994 for both East and West Germany.

*Source: These data were extracted from the Demographic Yearbook of the UN, German Tables. 1950-1994. Issues 10-54.*

**PROD:**

Labor productivity in manufacturing, Index: 1975=100. The index is calculated as:

\[
[1-\{(\text{base-year value} - \text{current-year value}) / \text{base-year value}\} * 100.]
\]

Data were then converted to natural logs and the series are available from 1950 to 1996 for both East and West Germany.

*Source: These data were extracted from the Groningen Growth and Development Centre, October 1999, reproduced in Van Ark, page 38.*
Graph G1

Sigma Convergence
HOIC Economies
(West Germany)

Note: Standard Deviation of ln GDP divided by mid-year population expressed in constant 1995 US$.
Graph G2

Sigma Convergence
UMC Economies
(East Germany)

Standard Deviation of In GDP divided by mid-year estimates of the actual total population.

Note: See Appendix A1.

Graphical Measurement of Dispersion

Graph G3
Graphical Measurement of Dispersion
Sigma Convergence
Migration

Note: Standard Deviation of ln Migration defined as: current year's (population - live births + deaths) - previous year's population.
Source: See Appendix A1.
Note: Standard Deviation of In year-end actual number of live births.

Source: See Appendix A1.

Graphical Measurement of Dispersion

Sigma Convergence

Graph G5
Note: Standard deviation of in-year-end actual number of marriages.

Source: See Appendix A1.
Graph G7

Graphical Measurement of Dispersion
Sigma Convergence
Divorces

Note: Standard Deviation of In year-end actual number of divorces.
Source: See Appendix A1.
Graphical Measurement of Dispersion
Sigma Convergence
Deaths

Note: Standard Deviation of In year-end actual number of deaths.
Source: See Appendix A1.
Note: Standard Deviation of in-year-end actual number of foetal mortalities.

Source: See Appendix A1.

Graph G9: Standard Deviation of the GDR & FRG Regions
Standard Deviation of the \( \ln \) Infant Mortality for the GDR & FRG Regions

Graphical Measurement of Dispersion
Sigma Convergence
Infant Mortalities
Graph G11

Graphical Measurement of Dispersion
Sigma Convergence
Gross Domestic Product Per Person Employed

Note: Standard Deviation of ln GDP (output) per person employed index where 1975 = 100.
Source: See Appendix A1.
Graphical Measurement of Dispersion
Sigma Convergence
Employment in Manufacturing

Note: Standard Deviation of ln employment in manufacturing index where 1975 = 100.
Source: See Appendix A1.
Graph G13

Graphical Measurement of Dispersion
Sigma Convergence
Labor Productivity in Manufacturing

Note: Standard Deviation of ln labor productivity in manufacturing index where 1975 = 100.
Source: See Appendix A1.
Graphical Measurement of Dispersion
Sigma Convergence
Employment in Manufacturing

Note: Standard Deviation of \( \ln \) employment in manufacturing index where \( 1975 = 100 \).
Source: See Appendix A1.
Graph G15

Graphical Measurement of Dispersion
Sigma Convergence
Gross Domestic Product Per Person Employed

Note: Standard Deviation of $\ln$ GDP (output) per person employed index where 1975 = 100.
Source: See Appendix A1.
Graphical Measurement of Dispersion
Sigma Convergence
Population

Note: Standard Deviation of \( \ln \) mid-year estimates of the actual total population.
Source: See Appendix A1.
Graphical Measurement of Dispersion
Sigma Convergence
Labor Productivity in Manufacturing

Note: Standard Deviation of $\ln$ labor productivity in manufacturing index where $1975 = 100$.
Source: See Appendix A1.
BIBLIOGRAPHY


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