In Search of Homo Sociologicus

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IN SEARCH OF

HOMO SOCIOLOGICUS

by

YUNQI XUE

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Abstract

In Search of Homo Sociologicus

by

Yunqi Xue

Adviser: Dr. Rohit Parikh

The subject of this dissertation is to build an epistemic logic system that is able to show the spreading of knowledge and beliefs in a social network that contains multiple subgroups. Epistemic logic is the study of logical systems that express mathematical properties of knowledge and belief. In recent years, there have been increasing number of new epistemic logic systems that are focused on community properties such as knowledge and belief adoption among friends.

We are interested in revisable and actionable social knowledge/belief that leads to a large group action. Instead of centralized coordination, bottom-up approach is our focus. We explore multiple methods of belief revision in social networks. Such belief revision in groups represents social influence and power to some degree. Both influence from friends and from experts are explained.

We define an intuitive concept of expected influence of a group. When different influence sources are suggesting conflicting actions, agents could make strategic decisions by analyzing expected influence of different subgroups. We then show some properties of expected influence in different network structures. We also simulate the strategic influence emerging in small-world networks which represents many real world networks.
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I thank my committee members, Dr. Liu, Dr. Jain and Dr. Yanofsky for their continuous support and guidance. I met Dr. Liu in Amsterdam during my master study. Her work had always fascinated me. Later I was fortunate to meet and work with her at Stanford University during her visits. Through many discussions, she guided me through different ideas, research topics, and papers. Dr. Jain introduced me to network theory, and showed me pioneering result in the field. Conversations with Dr. Jain were always inspiring. Dr. Yanofsky is one of my favorite professors during the first 2-year study of fundamentals in computer science. As someone who did not study computer science before, I am truly grateful for the knowledge he taught me.

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To Milinda and Neo . . .
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Chapter 1

Introduction

What is a social man (homo sociologicus)?

With this question in mind, this dissertation starts with the journey taken in the empirical fields such as experimental economics and experimental psychology. Later it extended to theoretical and philosophical areas on the related topics of *homo sociologicus*, contrasting with *homo economicus*.

The idea of *homo economicus* was originated by the prominent 19th-century scholar, John Stuart Mill. In his work in 1836, “...does not treat the whole of man’s nature as modified by the social state, nor of the whole conduct of man in society...”. Such a self-interested idea of man is also associated with the founding father of modern economics, Adam Smith, in his famous work ‘The Wealth of Nations’. He wrote: “It is not from the benevolence of the butcher, the brewer, or the baker that we expect our dinner, but from their regard to their own interest”. Later in 1881, Edgeworth even endorsed the idea as “the first principle of Economics”. The idea of homo economicus has been a major influence on economic theories and models ever since.

However, Sen points out the absurdity of such an idea in his widely cited paper, “Rational Fools: A Critique of the Behavioral Foundations of Economic Theory” in 1977. He gives an amusing example to illustrate the problem of *Homo Economicus*.

---

1Social human and Economic human respectively
assumption:

“Where is the railway station?” he asks me. “There,” I say, pointing at the post office, “and would you please post this letter for me on the way?” “Yes,” he says, determined to open the envelope and check whether it contains something valuable.

While Sen agrees with Edgeworth’s first principle in a strictly defined context, he also points out that we cannot forgo the complex psychological issues beneath choices and calls for “actually testing”, which was indeed done by experimental economists years later. With the flourishing of behavioral and experimental economics, researchers are able to bring a vast amount of data on the table to show that in many situations people do not just act on self-interest \[74\] \[25\]. The most notable publication, which is cited 1238 times to this date since 2001, is “In search of homo economicus: behavioral experiments in 15 small-scale societies” \[35\]. Their results not only show the same kind of deviation from the predications based on the homo economicus assumption, but also show that economic decisions are closely related to social/group activities.

Around the same period, a group of economists started calling for defining social preferences \[24\] \[23\].

It is clear that the homo economicus assumption works only within a tightly restricted economic context. But what about homo sociologicus? The term was introduced by German sociologist Ralf Dahrendorf in 1958 \[17\]. Here we borrow this term and examine what it means to be a social man within economic theories and framework. There is already a branch of studies in decision theory focusing on social procedures and decision making in groups \[69\] \[80\]. However, what we want to focus on is the individual level of decision making with the assumption of a social man.

Any topics related to society are almost guaranteed to be interdisciplinary studies, so is this one. We first look at some related empirical results from experimental economics, namely from Public Goods Game (PGG) and Ultimatum Game (UG).
Both games have social elements and in both empirical results deviate from the theoretical predictions. Then we review the fundamentals of classic rational choice theory, which serve as our theoretical foundation in economics. Inevitably, when we study real human decision making, we come across bounded rationality, which assumes limited computational power of real people and limited information accessibility. So what are the consequences of bounded rationality in a social man? We would like to know how rationality is bounded socially as in the results found by Bowles et al in [35]. We want to find out whether there are social signals that help decision makers make social-related decisions. Therefore we dig deeper to review the literature for signals. Another important and related area is the psychology of reasoning, which gives firm understanding of rationality. In this area, we also review the literature on one particular kind of subject, autistic people, who are considered to have relatively low social intelligence. However, some recent publications [31] [77] show that autistic people can reason about social situations, only not in the form of what normal people possess.

After reviewing in these fields, it is clear that we are homo sociologicus. Our lives are not purely driven by individual motives. We have group identity and behaviors. We also make group decisions. However what can we, computer scientists, do to further the understanding of the concept of homo sociologicus?

With these empirical studies in the background, let us get back to computational side. We review dynamic epistemic logic including its latest development on friends’ influence in a network based community. In additionally, we review some technical work from Parikh [62] that allows multiple languages in belief changes. We also review some concepts in network science.

Only after much of the reviewing and exploring, we are ready to suggest a system

\footnote{In fact, [31] is written by two highly autistic people who overcome their autism to a great extend. One of them is the famous autistic professor Temple Grandin. She described, in the book, how social rules did not come naturally to her. However, she can reason about social rules.}
which we believe can accommodate many of the issues in understanding a social person.

We are particularly interested in revisable and actionable social knowledge/belief that leads to a large group action. Instead of centralized coordination, a bottom-up approach is our focus. We explore multiple methods of belief revision in social networks. Such belief revision in groups represents social influence and power to some degree. Both influence from friends and from experts are explained.

We define an intuitive concept of expected influence of a group. When different influence sources are suggesting conflicting actions, agents could make strategic decisions by analyzing expected influence of different subgroups. We then show some properties of expected influence in different network structures. We also simulate the strategic influence emerging in small-world networks which represents many real world networks.

1.1 Organization of the Dissertation

Chapter 2 presents reviews of various theories as background, including rational choice theory, autism, signaling. Chapter 3 illustrates the contribution to the topic of social issues from computer science. We present some work in social software, dynamic epistemic logic and network science. We introduce our concept of Expert Influence in Chapter 4. Chapter 5 explains how group belief revision can lead to group actions by using a simple concept, Influence Indicator. Chapter 6 presents simulations of our system.
Chapter 2

Background

2.1 Rational Choice Theory

2.1.1 The Theory

This part of review is mostly following the related topics in Lecture Notes in Microeconomic Theory by Ariel Rubinstein [75].

Preferences

We define that a variety of options as a finite set \( X \). We then further define a binary relation \( \succsim \) that is a collection of ordered pairs of elements from \( X \). For example, when \( x, y \in X \), we can have \((x, y) \in \succsim\), which can also be denoted as \( x \succsim y \). It means option \( x \) is seen at least as good as option \( y \).

This binary relation also includes two additional definitions: one is symmetric, indifferent: \( \sim \), and the other is asymmetric strictly better: \( \succ \). For two elements \( x, y \in X \), \( x \sim y \iff [(x \succeq y) \text{ and } (y \succeq x)] \). While \( x \succ y \iff [(x \succeq y) \text{ but not } (y \succeq x)] \).

We now introduce two Axioms, Completeness and Transitivity.

\footnote{In this chapter, we use ‘options’ interchangeably with ‘alternatives’.}
Completeness  For any two options $x, y \in X$, $x \succeq y$ or $y \succeq x$.

This axiom says that a decision maker can always choose between two options. Although considering the definition of the binary relation ‘$\succeq$’ and its related definitions, it is clear that the completeness axiom allows three situations for any two options. First, when $x \succ y$, it means that the decision maker always chooses option $x$ over $y$. Similarly, the second situation is when $y \succ x$, the decision maker chooses $y$. The last situation is when $x \sim y$, which means the decision maker chooses $x$ and $y$ at random\footnote{As how Gilboa put it in his book ‘Rational Choice’.}

Transitivity  For any three options $x, y, z \in X$, if $(x \succeq y$ and $y \succeq z)$, then $x \succeq z$.

This axiom prevents cyclical preferences in both individual and group decision making situations.

Utility

When we compare two options $x, y \in X$, we often say that we prefer one of them. For example, ‘I prefer higher grades’. This can be written in following form:

$$x \succeq y \text{ if } V(x) \geq V(y)$$

Here $V : X \rightarrow \mathbb{R}$ is a function that assigns a real number to each element in $X$. In our example of grades, it has a clear numerical representation. But we also want to represent sets of options that do not have clear numerical evaluation. For example, Jill prefers Thai food over Japanese food because she likes spicy food, i.e. the spicier food has a higher value to her. Ideally we want to define a function $U : X \rightarrow \mathbb{R}$ that represents the binary relation $\succeq$ if for any two elements $x, y \in X$, $x \succeq y \iff U(x) \geq U(y)$. We call this function a utility function.

To show the existence of such utility function, we will look into two situations, namely when $X$ is finite, and when $X$ is continuous\footnote{\cite{71}}.
Finite Space  When the set $X$ is finite, a utility function that represents $\succsim$ relationship always exists. To prove that, we can first show that any subset of $X$ has a minimal element\(^3\) through induction on the size of subsets given that $X$ is complete and transitive. Then we can prove following proposition \([75]\).

**Proposition 2.1.1:** For a finite set $X$, the binary relation $\succsim \subseteq X \times X$ that is complete and transitive, has a utility representation with natural numbers.  

*Proof.* Given a finite set $X$, we can define $X_1$ as a subset of $X$ which contains all the minimal elements of $X$. Then we further define $X_2$ as a subset of $X - X_1$ with all the minimal elements in $X - X_1$. We can keep constructing such minimal subsets till we have $X = X_1 \cup X_2 \cup X_3 \ldots X_k$, and $k \leq |X|$. We then define $U(x) = k$ if $x \in X_k$. Further more, when $a \succ b$ and $a, b \in X$, we know $U(a) > U(b)$ and $a \notin X_1 \cup X_2 \cup X_3 \ldots X_{U(b)}$. When $a \succ b$, $U(a) = U(b)$.  

Continuous Space  Often in economics the set $X$ is set to be an infinite subset of a Euclidean space, $\mathbb{R}^n$. We want to show that often there is a utility representation in such a case too. Continuity requires that for two options $a, b \in X$, if $a$ is strictly preferred over $b$, then the “neighboring” elements around $a$ should be preferred over the “neighboring” elements around $b$. To formalize this, we will start with some definitions \([75]\).

**Definition 2.1.2:** Let $a$ be an element in $X$. For the set of all points that have distance less than $r$ ($r > 0$) from $a$, we call it a **ball** around $a$, and denote it as $\text{Ball}(a, r)$.  

**Definition 2.1.3 (Continuous a):** We call a preference relationship $\succsim$ on $X$ **continuous** if whenever $a \succ b$, there are balls $B_a$ and $B_b$ such that $x \succ y$ for all $x \in B_a$ and $y \in B_b$.  

\(^3\)Or minimal elements, if they are equivalent.
Definition 2.1.4 (Continuous b): We call a preference relationship $\succsim$ on $X$ continuous when the set $\{(x, y) | x \succsim y\} \subseteq X \times X$ is a closed set. In other words, for all $n$ and $a_n, b_n \in X$ such that $a_n \to a$ and $b_n \to b$, and $a_n \geq b_n$, we have $a \succsim b$.

A preference relation $\succsim$ on $X$ satisfies ‘Continuous a’ if and only if it satisfies ‘Continuous b’.

Debreu [19] proved a famous theorem that pushes a step further.

Theorem 2.1.5 (Debreu’s Theorem): For a continuous preference relationship $\succsim$ on $X$, there exists a continuous utility function $U(x) : X \to \mathbb{R}$. \qed

Choice

Both utility functions and preference relations are just the mental attitude of a person towards a set of options. However, they do not explain how that person actually makes a choice in a real life situation. A person can think of a preference ordering for different options, but it does not necessarily say that he would make a choice accordingly in a real decision problem. Therefore we need to introduce the definition of choice function. Before that we will first look at some preliminary definitions.

We will look at the set of possible alternatives $X$ again, in which any non-empty subset $A$ of $X$ could be a choice problem. Any member $x \in A$ is a choice. In some situations, the decision maker considers relevant choice problems. We pair the collection of choice problems, $D \subseteq X$, with $X$, and call $(X, D)$ a context. When a context is given, for each particular problem $A \subseteq D$, a choice function $C(A)$ outputs a unique element from $A$ that is the choice of the problem.

Since this chapter is about rational choice theory, we shall discuss what kind of behaviors are considered rational within this theory. Roughly speaking, we consider a decision maker rational when he has a preference relation $\succsim$ on the set of alternatives $X$, and facing a choice problem $A$ in context $D$, he chooses an optimal element in $A$. 
In other words, we call a choice function $C$ rationalizable when $C(A) = C_{≿}(A)$ for any $A$ in the domain of $C$.

Next, we will further review an important condition for rationalizable choice functions: condition $\alpha$ by Sen [84], which is also referred to as Chernoff’s condition [15].

**Condition $\alpha$** Given two problems $A$ and $B$, both in context $D$, we say that a choice function $C$ satisfies condition $\alpha$, if $A \supset B$ and $C(B) \in A$, then $C(A) = C(B)$. We define $C_{≿}$ as a choice function that always outputs a single most preferred element in $X$ given a preference relation $≿$. $C_{≿}$ satisfies condition $\alpha$.

Sometimes Condition $\alpha$ is also called “independence of irrelevant alternatives” and was first introduced by Arrow [6] and Nash [52]. A different way to state it would be: if an alternative $x \in B$ chosen is an element of $B$, then $x$ must be chosen from $A$.

**Dutch Book Arguments** Dutch Book Arguments state that anyone who does not try to maximize a preference relation will endure a loss. In economics, a decision maker can be Dutch-booked if he or she has intransitive preferences. For example, given three alternative values to choose from: A, B, and C, the decision maker, say Tom, has the following preference: $A \succ B$, $B \succ C$, but $C \succ A$. Then someone can take advantage of him by first selling $A$ to Tom for $B + \epsilon$; then selling $B$ to Tom for $C + \epsilon$; then selling $C$ to Tom for $A + \epsilon$. At the end, Tom has paid $3\epsilon$ with nothing in return.

**Notes on ‘Alternatives’** When we talked about Dutch Book Argument, we revealed an irrational choosing behavior that preserves intransitive preferences on a set of alternatives. In some situations, the violation of rationality is due to inaccurate or changing specification of alternatives. Now we are going to review a famous dinner example from Luce and Raiffa 1957 [48]. In a restaurant, a customer chooses chicken from the menu with only steak tartare and chicken. At the same time, he chooses
steak tartare from the menu with steak tartare, chicken and frog legs. It looks like this customer violates the condition $\alpha$, hence we could consider him irrational with his choices. However, it is possible that he realized the fact that the second menu with the frog legs indicates a high level of cooking skills. Making a steak tartare also requires high level of cooking skills. Following such reasoning, you may consider that this customer is actually not that irrational, but actually smart. Rubinstein adds this paragraph to remind us that sometimes the same set of alternatives can have a different meaning.

We also should realize the particular reasoning of this customer also implies another condition: he is a new customer. That means even though he has access to his own preferences, he does not have full information about the choices that he could make in this particular restaurant.

In Sen’s paper [86], he also discusses the problem of internal consistency.

**Choice Functions** We will continue the definition of choice functions. So far, the functions we discussed have only one solution to every choice problem. It is certainly possible that given a preference relation and a choice problem, there are more than one optimal solutions. Therefore we will explain a further fine-grained definition: *choice correspondence*.

Given a choice problem $A$, $C(A)$ is a non-empty subset of $A$. It is obvious that the decision maker has to select one element from $C(A)$. Basically $C(A)$ is set of equivalent optimal choices he could select from.

**The weak Axiom of Revealed Preference** Samuelson [87] originated the revealed preference approach in 1938 [78]. It was also proposed by Houthakker [39] and by von Neumann and Morgenstern [94]. Arrow adapted it to set-valued choice functions in [7]. Sen provides a systematic treatment of the axiomatic structure of the theory of revealed preference in [87]. Note: $xPy$ is equivalent to our earlier notation: $x \succ y$. 
Sen defines \( x \sim y \) as \( x \) is chosen while \( y \) is available but rejected.

**Definition 2.1.6:** For any \( x, y \in X \), we say \( x \) is indirectly revealed preferred to \( y \), denoted as \( x P^* y \), if only if there is a sequence \( z^i, i = 0, ..., n \), and \( z^1 = x \) and \( z^n = y \), such that for all \( i \), \( z^{i-1} P z^i \).

**Definition 2.1.7:** For any \( x, y \in X \), we say \( x \) is indirectly revealed preferred to \( y \) in the wide sense, denoted as \( x W y \), if only if there is a sequence \( z^i, i = 0, ..., n \), and \( z^1 = x \) and \( z^n = y \), such that for all \( i \), \( z^{i-1} R z^i \).

For all \( x, y \in X \), we have the following axioms \[75\]:

1. **Weak Axiom of Revealed Preference (WARP):** If \( x \sim y \), then not \( y R x \).

2. **Strong Axiom of Revealed Preference (SARP):** If \( x P^* y \), then not \( y R x \).

3. **Strong Congruence Axiom (SCA):** If \( x W y \), then for any non-empty subset \( B \) in choice problem \( A \) such that \( y \in C(B) \) and \( x \in B \), \( x \) must also belong to \( C(B) \).

4. **Weak Congruence Axiom (WCA):** If \( x R y \), then for any \( B \) in \( A \) such that \( y \in C(B) \) and \( x \in B \), \( x \) must also belong to \( C(B) \).

After showing the equivalence of all four axioms mentioned above in \[87\], Sen raises two important questions: (1) Are the rationality axioms to be used only after stipulating them to be true? (2) Are there reasons to expect that some of the rationality axioms will tend to be satisfied in choices over “budget sets” but not for other choices?

**Expected Utility**

So far in our description of rational choice theory, we assumed that the decision maker has the information about available options and outcome of the choice. However \(4x R y \) is equivalent to our earlier notation: \( x \succ y \).
in reality, we often do not have exact information about the outcome, and face risks or uncertainties. That is the relationship between actions and outcome is not deterministic.

This aspect is especially crucial for this dissertation, since ultimately we want to define choices made in a social environment that consists of many people. Then uncertainty is inevitable. Expected Utility Hypothesis, an idea, which goes as far as 1738 from Daniel Bernoulli [12], provides an important view of how to model uncertainty. In the paper, he wrote:

“Somehow a very poor fellow obtains a lottery ticket that will yield with equal probability either nothing or twenty thousand ducats. Will this man evaluate his chance of winning at ten thousand ducats? Would he not be ill-advised to sell this lottery ticket for nine thousand ducats? To me it seems that the answer is in the negative. On the other hand I am inclined to believe that a rich man would be ill-advised to refuse to buy the lottery ticket for nine thousand ducats.”

“...the determination of the value of an item must not be based on its price, but rather on the utility it yields. The price of the item is dependent only on the thing itself and is equal for everyone; the utility, however, is dependent on the particular circumstances of the person making the estimate. Thus there is no doubt that a gain of one thousand ducats is more significant to a pauper than to a rich man though both gain the same amount.”

“If the utility of each possible profit expectation is multiplied by the number of ways in which it can occur, and we then divide the sum of these products by the total number of possible cases, a mean utility [moral expectation] will be obtained, and the profit which corresponds to this

---

5 A ducat was a standard gold coin throughout Europe.
utility will equal the value of the risk in question.”

Bernoulli pointed out the problem of focusing only on monetary term and suggested to use expected utility instead. He also suggested to use logarithm to calculate the utility.

In 1944, Von Neumann and Morgenstern provide a formula in their book *Theory of Games and Economic Behavior* [94]. Later in 1954 [81], Savage introduced an alternative framework, *subjective expected utility* that was followed by the work from Aumann and Anscombe [5].

**Von Neumann-Morgenstern Utility Theorem** Before we review the Von Neumann-Morgenstern Utility Theorem (vNM Theorem), we shall explain some preliminary concepts that extend naturally to the context of uncertainty. In the earlier section, we have defined $X$ as a set of options. Here we will be more precise, and define $X$ to be a set of outcomes. The binary relation $\succeq$ on $X$ is the same as how we introduced preference. $x_1 \succeq x_2$ indicates that $x_1$ is weakly preferred to $x_2$, and $\succ$ indicates strict preference while $\sim$ indicates the indifference between the two outcome. A *lottery* on $X$ is a probability distribution: $[p_1 : x_1; p_2 : x_2; p_3 : x_3; \ldots; p_n : x_n]$. In addition, we have: $\sum_{i=1}^{n} p_i = 1$ and all $p_i \geq 0$.

In the following paragraphs, we will explain the six axioms that are stated by Von Neumann and Morgenstern [94]: completeness, transitivity, substitutability, monotonicity, continuity, and decomposability.

**Axioms** We have introduced the first two axioms (completeness and transitivity) earlier. Therefore we will just simply state them here.

**Completeness:** $\forall x_1, x_2 \in X, x_1 \succ x_2; \text{ or } x_2 \succ x_1; \text{ or } x_1 \sim x_2$

**Transitivity:** $\forall x_1, x_2, x_3 \in X, x_1 \succeq x_2 \text{ and } x_2 \succeq x_3 \Rightarrow x_1 \succeq x_3$

---

6For the convenience of readers, we will repeat some basic definition of preference that were introduced earlier.
**Substitutability:** If two outcomes are indifferent for a decision maker, then he is also indifferent between the two lotteries that contain the outcomes separately. To put it formally: Given \( x_1 \sim x_2, [p : x_1; p_3 : x_3; \ldots; p_n : x_n] \sim [p : x_2; p_3 : x_3; \ldots; p_n : x_n], \) where \( p + \sum_{i=3}^n p_i = 1. \) In other words, the outcome \( x_1 \) can be substituted with \( x_2. \)

**Monotonicity:** For all \( x_1, x_2 \in X, \) \( x_1 \succ x_2 \) and \( 1 \geq p > q \geq 0 \Rightarrow [p : x_1; 1 - p : x_2] \succ [q : x_1; 1 - q : x_2] \)

**Continuity:** For all \( x_1, x_2, x_3 \in X, \) \( x_1 \succ x_2 \) and \( x_2 \succ x_3 \Rightarrow \exists p \in [0, 1] \) such that \( x_2 \sim [p : x_1; 1 - p : x_3] \)

**Decomposability:** We denote \( P_{lj}(x_i) \) as the probability that \( x_i \) is selected by lottery \( l_j. \) The axiom states that if we have two lotteries \( l_1 \) and \( l_2 \) over \( X, \) and \( P_{l1}(x_i) = P_{l2}(x_i) \) for all \( x_i \in X, \) then \( l_1 \sim l_2. \)

An example could be following:
\[
l_1 = [0.7 : x_1; 0.3 : [0.3 : x_1; 0.7 : x_2]] \\
l_2 = [0.79 : x_1; 0.21 : x_2; 0 : x_3]
\]
According to decomposability axiom, \( l_1 \sim l_2. \)

**vNM Theorem**

**Theorem 2.1.8:** When a binary preference relation \( \succ \) on a set of outcomes \( X \) satisfies completeness, transitivity, substitutability, monotonicity, continuity and decomposability, a utility function \( u \) exists and fulfills the following two properties:

\[
u(x_1) \geq u(x_2) \text{ if only if } x_1 \succ x_2
\]
\[
u([p_1 : x_1; p_2 : x_2; p_3 : x_3; \ldots; p_n : x_n]) = \sum_{i=1}^n p_i u(x_i)
\]
2.1.2 Empirical Results

As we discussed in the introduction, the idea of homo economicus dated as far back as 1836. Sen has written a theoretical objection to the idea and argued about the exact context in which the idea might work. At the same time, the homo economicus assumption was nurturing another branch of research in economics: game theory, which emerged naturally from rational choice theory, and was heavily tested through experimental economics. Although the first reported experiment was done in 1930 by Thurstone, it was not till mid 90's that experimental economics reached its prime [73].

Through experiments, economists found many deviations in the real world results from the theory. Their initial conclusion was that real humans are not rational.

There is one particular kind of games in which subjects often behave ‘irrationally’ during experiments. Usually, this kind of game has social factors in the process. The social factors are represented in two different types of scenarios. First type is the number of players. For example, in public goods game, there are often more than 2 players. A second type is the implicit social aspect, such as in ultimatum games. In fact, the deviations of the experimental results in both games bring us back to our goal: in search of homo sociologicus. The crucial question we want to ask is: are the subjects really just being irrational or do our models need to be adapted towards the social assumption? Clearly we are in favor of the latter view. So are Bowles et al [35]. They have conducted a few games with social elements in 15 small-scale societies and show that the deviations are systematic. People are not making random social decisions. But the way they make such decisions is closely related to their day to day activities, which are shared by the social members.

Reading through such empirical results gives us both motivation and data for building new rational choice models with considerations of social aspects.

In this chapter, we will review literature in both public goods game and ultimatum game. We are going to explain the basic ideas and procedures of the two games, the
theoretical predictions, and the empirical results.

**Public Goods Game**

The theory of public goods is important for economists, policy makers and international organizations. It provides an insight into market failures, e.g. their mechanism, consumers’ and suppliers’ incentives, which are all essential for a well-functioning society. The rise of international governance, like IMF, World Bank, and UN, tells the awareness of the need of public goods. Besides governments and sociologists, economists are also interested in public goods. Do people treat public goods the same as private goods? What do they do when there is a conflict of interests between public and private goods? In 1980, Mas-Colell \[49\] published his mathematical approach to public goods theory. In the experiments, subjects have to make decisions about their public account and private account. Such experiments were conducted by different researchers but in one-shot form as Schneider and his co-authors did \[82\]. Isaac and his colleagues tried a new experiment \[40\] with the possibility of repetition. We will call their game the basic game in this dissertation. The details of the experiment are explained in the following subsections.

**The basic game** Public goods experiments have studied standard voluntary contribution mechanisms where groups have a choice to invest in a private or a common account. The private account gives the subject a return of the exact amount invested in it, while the common account gives each group member a marginal per capita return (MPCR) \[8\]. The higher the contribution into the common account, the higher the group payoff. Results have shown that contributions start very high and then decrease over time \[40\]. Also, since the decision process is repeated, participants become more experienced and free riding becomes a strategy, especially when the MPCR is low \[18\].

\[7\] Details explained in section 3.1.1.

\[8\] A basic example is explained following paragraphs: Basic Idea, Procedure, and Payoff determination.
Hence, there exists this social dilemma. To alleviate the problem of free riding and contribution decreases, studies have implemented different mechanisms to drive the contributions higher. In this section, we will look at three pairs of such variations that are particularly interesting for social concerns, namely punishment versus reward, one versus multiple punishers, and strangers versus partners.

Now let us first look at the most basic game. Isaac and his colleagues conducted 9 different experiments for the paper. We will not explain each one of them in detail, but take the essence of these experiments and tailor our ‘basic game’ for theoretical analysis.

**Basic Idea** A group of subjects who participate in this experiment have to make some decisions on their contribution to a public account, and later their total private benefit is determined by a pre-set function. No communication is allowed during the whole experiment.

**Procedure** Each participant receives an endowment $y$ at each round. He has to decide how much of $y$ he is willing to contribute to the public account. Then after the experimenter’s calculation with the payoff function, the participant is informed about his own total payoff of the round. The game is repeated for $t$ rounds.

**Payoff determination** We assume the following payoff function at round $t$ for participant $i$, who contributed $g_i^t$ to the public account, and there are $n$ participants in total:

$$\pi_i^t = y - g_i^t + a \sum_{j=1}^{n} g_j^t$$

Note: $1/n < a < 1$. The total payoff for participant $i$ is just simply: $\sum_1^t \pi_i^t$. 

Theory and Prediction In this subsection, we are mainly looking at the basic game and analyzing it from a game theoretical point of view. First of all, we have to assume that the following prediction is for the selfish and rational subjects. There are many papers written about altruistic subjects. One of the earliest such proposals was from Andreoni [4].

With that assumption, a subject would behave in such a way to maximize his payoff \( \pi_i \) in each round of the basic game. Looking at the payoff function mentioned before, we clearly see \( \frac{\delta \pi_i}{\delta g_i} = -1 + a \). Given \( 1/n < a < 1 \), we know \(-1 + a < 0\). Therefore the subject would never contribute to the public account, i.e. \( g_i = 0 \).  

Example Let us look at a experimental example of a Public Goods game. A group of 4 subjects participate in the game. At each round, everyone receives 20 units of initial endowment. They have to decide how much of the endowment, they want to contribute into private and public accounts respectively. A subject can choose to keep all the units for himself, i.e. to keep them in his private account. Then this particular round, his compensation is 20 units. If all the subjects choose to give all units to the public account, each of them will get \( \pi_i = y - g_i + a \sum_{j=1}^{n} g_j = 20 - 20 + 0.5 \times (20 \times 4) = 40 \) assuming \( a = 0.5 \).

There are 5 rounds of decisions that they have to make. They can decide to keep all the units in all 5 rounds, in which case their total compensation will be \( 20 \times 5 = 100 \) units. If all the subjects decide to contribute all the units to the public account in all 5 rounds, then they each can get \( 40 \times 5 = 200 \), which is twice as much as if they all refuse to contribute. However, subjects can also freeload on others’ contribution. For example, if all but one subject contribute all units to the public account. Then at each round, this freeloader can get \( 20 - 0 + 0.5(3 \times 20) = 50 \) units. If this happens at every round, then at the end, he gets \( 50 \times 5 = 250 \) units, which is more than if they

\(^{9}\text{Now we are just looking at the game as if it’s one-short game, because later in related paper there are analysis of repeated game.}\)
Variations  As we have mentioned before, there are many different variations of the public goods game. Here we only focus on some of the variations, which are related to our design.

Punishment vs. Reward  To many people’s surprise, when reward option is given, it actually does not work as well as punishment. In this case we are able to explain the experimental results from game theoretical perspective.

In Fehr and Gächter’s paper [26] that was influenced by [55], the game with punishment was designed in two stages. The first stage was similar to ‘the basic game’. During the second stage, the participants had opportunities to punish other participants at some cost. It turned out that with punishment, the overall contribution to the public account improved in comparison with the game without the punishment.

Sefton, Shupp and Walker wrote a paper in 2007 about the effect of rewards and sanctions[83]. The experiment with rewards has similar structure as the experiment with punishments in Fehr and Gächter’s paper. The rewards are costly just as punishment is costly, i.e. the rewards are not free. Game theoretically speaking, the punishment creates big threat to the free-riders, even without real punishment. However, in the games with rewards, it works differently. If there are expectations of rewards among the participants, then there have to be rewards, otherwise the existence of possibility of being rewarded would actually dis-encourage the high contributors. Indeed Sefton, Shupp and Walker found such a result [83].

One punisher v.s. Multiple punishers  In order to understand the effects of different punishment structures, O’Gorman et al (2009) [54] have implemented an experiment with three different conditions that have no punisher, one punisher and

\[^{10}\text{Sanctions here are technically the same as punishments in Fehr and Gächter’s paper.}\]
all punishers respectively. Their punishment system was based on a 1 : 3 ratio of the
cost of punishing for the punisher to the cost for the target. This ratio facilitated in
bringing the group with a single punisher to contribute at high levels and produce
a larger profit than the group with all being punishers. Profits were larger in the
one punisher case because punishment costs were small and punishment was more
coordinated, thus reducing inefficient loss. This shows that uncoordinated punishments
made by many people will cause inefficiency and unnecessary losses.

**Strangers vs. Partners**  It is already game theoretically surprising to find that
people punish free-riders at their own cost. Because given the fact that participants
know how many rounds there are, one could always benefit more than the others
deviating, i.e. contribute low or 0 at the last round. Applying backward induction,
a rational participant should never contribute anything from the start. There are
people who apply backward induction in a common real life situation. However, Fehr
and Gächter pushed it even further. They assigned the group members randomly for
each round so that participants are ‘strangers’ to each other. Then there is clearly no
future gain by punish someone, because one cannot even ‘educate’ the free-riders to
contribute more in the future round. However, people still punish in this setting

**Ultimatum Game**

Ultimatum game (UG) is a widely experimented and considered as ‘one of the simplest’
games in experimental game theory. Güth, Scmittberger and Schwarze first wrote an
experimental analysis of this kind of take-it-or-leave-it game in 1982.

The experimental results often deviate from the game theoretical prediction. Game
theoretically speaking, the proposer should claim as much as possible for himself, while

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11 From game theoretical point of view, it should be 0, in order to maximize the gain.
12 In next subsection more detailed the analysis of the variations is provided.
the responder should not reject any non-zero offers for him/her. However in reality, people tend to propose much more for the responders, and responders reject offers under 30% of the total share. One stream of publication, esp. in neuro-economics, tends to claim that emotional activation is the main reason for such a result \cite{79,93}. Although such a simple claim could not give detailed explanation on cross-cultural results found by Bowles et al \cite{35}. In \cite{35}, the experiments run in 15 small-scaled rather primitive societies reveal some systematic deviations from the game theoretical results across culture. It indicates a potential sophisticated mechanism beneath the behavior in the simple game.

\section{Social Outliers - Autism}

They reveal interesting facts about reasoning, epistemology, more importantly in relation with social situation.

\subsection{Introduction}

People with autism belong to a special group. They could have average intelligence or higher in some cases, but show some very specific deficits. The most obvious one is their asocial tendency. It was Kanner who started recording a special condition, autism, among children in 1938 and published it in 1943 \cite{42}. Later Rutter \cite{76} summarized three key features of this condition: (1) impaired social development; (2) delayed linguistic development; (3) insistence on sameness. All three prior features can be noticeable as early as 30 months old.

Autistic children are often inaccessible not only because of their delayed development in languages, but also because of the absence of interest in any social interaction. They often preoccupy themselves with repetitive movements, such as laying down little objects endlessly. They usually do not have pretense play \cite{44}, which is common
with normal children. Autistic children can be violent and have rages. All these traits make it very hard for their parents and people around. Some people with autism have lifelong institutionalization.

However there is another side of the autistic condition, savant talent, which is often ignored or overlooked by the parents and researchers\[13\]. Many people with autism have amazing abilities \[91\] \[92\]. Many are particularly good with jigsaw puzzles. Some have perfect pitch and can play tunes after hearing them just once. Some have perfect memory of all events as early as several months after they were born. These abilities give important cues of the autism condition as well as the deficits.

Autism is certainly an intriguing condition. Every individual with autism has an unique set of behaviors. However, the main features which we mentioned above can be reliably identified. In the 80s, it was considered a rare condition which happens 4 in every 10,000 children. However, the frequency has increased dramatically over the decades. According to Centers for Disease Control and Prevention, it is affecting 1 in every 88 children in the U.S. Understanding the matter seems urgently important.

### 2.2.2 Theory of Mind

Theory of mind, termed by Premack and Woodruff (1978), seems to be a particularly human ability. This theory describes how humans generally can think about what other people know, believe, and feel. Some researchers, such as Baron-Cohen, Leslie and Uta Frith, have linked the deficit in autism with lack of theory of mind.

Leslie \[44\] argues that pretense is part of the origins of ‘theory of mind’ and develops a meta-representational mechanism to illustrate it. In this section, we will first go through the basics of his theory and then explain its connection with autism.

Children of age two or older seem to have a more ‘sophisticated’ play: pretense play, e.g. holding a banana and pretending it is a phone. Such ability is a major development.

\[13\] A search for “autism” on Google Scholar gives 426,000 results; while for “Autism Savant” it gives only 3,560 results. There is not enough research on this particular group.
because it reveals not only children’s ability of handling distorted reality but also the beginning of a capacity for meta-representation which is seen, by Leslie, as the crucial ingredient for a theory of mind. Firstly Leslie \[44\] excludes some possibilities which appear to be pretense play, such as acting in error and functional play. Then he further defines three fundamental forms of pretense: object substitution, attribution of pretend properties, and imaginary objects. In other words, any one (or more) of the following situations \[44\] would make a play pretense play: a) object substitution: if an object is made to stand for another; b) attribution of pretend properties: a certain unreal property is used for a particular object; c) imaginary objects: there are imaginary objects. Leslie continues with definitions of primary representation, which is the first basic representational capacity for infants, and representational abuse which affects all three kinds of pretense and handles some relationships between two primary representation. In order to understand the pretense in others, a child would need a capacity for meta-representation. A major feature of the pretense theory is that it actually represents the beginning of a capacity to understand cognition itself, which is a fundamental idea in the theory of mind. Leslie also explains the isomorphism between the three types of pretense and the logical properties of sentences containing mental state terms. In the article, he gives a model for pretense as well: Decoupling Model. There are three major components in the model: 1) the perceptual processes 2) central cognitive systems 3) the decoupler. The decoupler can be further divided into three parts: the expression raiser, the manipulator and the interpreter. The decoupler is involved when a meta-representation is needed. Besides the detailed explanation of how the model works in a pretense play, Leslie also explains the gap between the two-year-old pretenders and four-year-old children who can pass the false-belief test which indicate the mastery of theory of mind.

Children with autism are impaired in pretend play (Baron-Cohen 1987; Rutter 1978; Sigman and Ungerer 1981). With the model from Leslie’s article, it is possible to
connect this symptom to the social impairment. Leslie concludes that Autistic children lack the ability in both primitive forms (pretense) and advanced forms (false-belief test) because of the impairment in their decoupling ability.

### 2.2.3 The Affective Foundation Theory

For another group of researchers\(^{14}\), the symptoms in autism are not just cognitive, but also emotional. For example, it is very difficult for autistic children to develop close emotional relationships with people, even with their parents. Hobson \(^{37}\) believes that understanding such impairment in emotional development is also crucial for understanding the autism condition as a whole. In fact, he argues further that the cognitive and language abnormalities in autism are consequences of the emotional impairment. In his opinion, ‘infants are biologically prewired to relate to people in ways that are special to people, and it is through the experience of reciprocal, affectively patterned interpersonal contact that a young child comes to apprehend and eventually conceptualize the nature of persons with mental life’ \(^{37}\) (p.104) The affect from other people directed toward objects or events is a special source of information. Through observing this type of information, young children come to understand self. Hobson believes that his process is a required foundation even for symbolic development. He applies his model to Autistic children, and explains that the difficulty in engaging affective states with others is the origin of the problem.

Then he also ran some experiments \(^{38}\) which tested their ability to recognize emotion and personal identity. The control group in these experiments are non-Autistic retarded children. In the first experiment, the subjects have to complete some tasks in which two types of faces are presented. The first type is used to observe subjects’ ability of identifying faces, while the second type is to observe their ability of recognizing emotion. Regarding the first type, there were no statistical difference between two

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\(^{14}\)Here we will just focus on one, Hobson.
groups of subjects. However, the non-Autistic retarded children are better when the emotion is involved. A second experiment was testing their ability of identifying upside-down faces. The results revealed that Autistic children were better at matching both ‘identity’ and ‘emotion’ in upside-down faces.

The experiments seem to fit with Hobson’s theory that Autistic children’s impairment is in affective connections.

2.2.4 Mirror Neurons

Starting from late 90’s, another line of research from neuroscience has been offering an alternative explanation on autism, especially on social impairment. One of the most representative work is from Gallese and his colleagues \[28\] \[27\] \[98\]. They claim that the newly discovered mirror neurons (MNs) have profound implication in understanding theory of mind, and hence in understanding of social impairment in autism.

MNs were first discovered in the macaque monkey premotor cortex. During the experiments, MNs respond in following two situations \[28\]: first, when a particular action is performed by the recorded monkey; second, when the same action performed by another monkey is observed. In \[28\], Gallese and Goldman further explain the mirror system in humans. The MNs cannot be directly studied in humans but experiments with alternative approaches have shown strong evidence for a system similar to what is discovered in monkeys. The paper \[28\] proposes that MNs are an important base for mind-reading process or at least a precursor. This claim supports a second type of model of theory of mind: a simulation based model, compared with the theory based model proposed by Baron-Cohen and his colleagues which we had looked at in an earlier section. Simulation based model suggests that when people ‘read’ others’ minds, they do not need to have a theory of all the psychological inferences, instead they just put themselves in others’ shoes to simulate the situation. Gallese and Goldman suggest
that MNs largely explain this process of mapping behavior and then understanding the others’ minds.

Williams et al continues the direction proposed by Gallese, and draw the connection among imitation, MNs and autism [98]. They argue that imitation is ‘a prime candidate for the building of a ToM (theory of mind)’, and that autistic people have deficit in imitation. The response of MNs system in human shows the ability of imitation. They conclude that such a deficit in imitation creates problems in social interaction and contribute to the lack of empathy in autistic people.

2.2.5 Discussion

Autistic people suffer a wide spectrum of problems. The deficits in social interaction and language learning are the most prominent. Both are considered to be related to the lack of ToM. As we have discussed, there are two major competing models for explaining this: Theory Theory (ToM) and Simulation Theory. A second line of problems are emotion-related, such as lack of empathy. The affective foundation theory is the first to look at these problems. Although none of the existing theory has really unveiled the root of stereo-typed behavior, such as sameness and repetition.

The newer theory, namely simulation based theory, has shown promising development by applying properties of MNs in connection with autism. It seems to explain the lack of theory of mind which affects social and language learning, and lack of empathy.

However, if we are in agreement of such deficit in MNs system in autistic people and the fact that it is the prime base for ToM and social interaction, do we get the conclusion that autistic people can never learn social interaction or theory of mind?

I believe there are deeper philosophical questions that need to be answered. What are the natural emergence of theory of mind and social meaning? Is there an un-nature process which purely rely on high-level intelligence as many autistic people have
normal to higher-than-average intelligence? These are questions beyond the scope of current study. However, I want to conclude the chapter with a quote from the chapter on the autistic professor, Temple Gardin, in “An anthropologist on Mars” by Oliver Sacks:

... What is it, then, I pressed her further, that goes on between normal people, from which she feels herself excluded? It has to do, she has inferred, with an implicit knowledge of social conventions and codes, of cultural presuppositions of every sort. This implicit knowledge, which every normal person accumulates and generates throughout life on the basis of experience and encounters with others, Temple seems to be largely devoid of. Lacking it, she has instead to ‘compute’ others’ intentions and states of mind, to try to make algorithmic, explicit, what for the rest of us is second nature. ...

The subject, Temple Grandin, shows clear self-introspection, ideas of ToM and understands that her way of perceiving social interaction is different. She actually wrote a book on social rules [31] “The Unwritten Rules of Social Relationships: Decoding Social Mysteries Through the Unique Perspectives of Autism” with another autistic adult. Who would imagine that when she was diagnosed with autism as a child, the doctor had suggested life-long institutionalization for Temple?

So maybe both Theory Theory and Simulation Theory are partially right. However, they are two different approaches to understanding of ToM and social interaction. I would call simulation-based a more natural kind. More psychological and experimental research needs to be done in this direction.

2.3 Signaling

2.3.1 Sender-Receiver Model

In this section, we will briefly review some important developments in sender-receiver configuration. I will group different theses into two types: the theory type and model
type, and disregard the chronological order. We will start with some modelers, and continue with a few theoretical developments. At the end of this section, I will summarize the key features that are important to this chapter from both groups.

Although Shannon developed *Information Theory*, I still consider him more of a modeler in the context of sender-receiver model and in comparison with other theorists whom I will discuss later. Shannon defined the early version of sender-receiver process to illustrate the information delivery from the world to a sender who signals through channels to a receiver. Lewis further developed the model in a more game theoretic setting to show that meaning is a result of sender-receiver interaction given common interest and knowledge. Skyrms pushes the model a couple of steps further in the direction of a naturalistic approach. He shows that without the assumption of common knowledge and high level of intelligence, the meaning of a signal can still evolve. In fact, his model shows a continuous application of sender-receiver configuration at all biological levels.

Both Dretske and Millikan develop detailed theoretical accounts for how information is being processed. Dretske gives enriched explanation of information in a natural and objective manner, in a sense that a signal with information is not tied to a time, a person or a history. Millikan disagrees and believes that signals are tied to a person and his/her history. In her book ‘Varieties Of Meaning’, she further defines natural signs in a local and recurrent sense. As Millikan’s theory on varieties of meaning (signs) is quite important for my discussion, we shall come back to the details in the later sections.

So which aspects of the models and theories are important for this dissertation? I will call it a quasi-Lewis-Skyrms configuration. It certainly is a sender-receiver configuration. I want to use the game theoretic aspect, in which the interaction between the sender and receiver, and re-enforcing reward, naturally stabilize a particular meaning of the signals. However, at the same time, I will incorporate Millikan’s
various refined definition of signals into the game theoretical framework. Eventually I hope such a quasi-Lewis-Skyrns configuration can be a tool to distinguish the two kinds of perceptive meaning (signals) between people. In addition, I hope we can illustrate how misinformation can happen in both cases, and why only in one case it can be corrected.

### 2.3.2 The Lewis-Skyrns Signaling Game

I will start with Lewis’ signaling game. In a typical setting, there are two agents who want to achieve a common goal, for example the sexton of the Old North Church and Paul Revere. A crucial difficulty they are facing in the Revere Story is that neither can achieve the goal alone, in this case to inform the American defenders against the British army. The sexton can observe the arrival of the army, but cannot inform; while Revere can help with defending, but cannot observe the arrival. Hence, they established a signaling game for coordination. In the game, there are three states (invasion by land, invasion by sea and no invasion), three signals accordingly (one lantern, two lantern and no lantern), and each could result in an action (preparation for invasion by land, preparation for invasion by sea and no preparation). Table 2.1 represents a normal-form game matrix for the Revere story.

More formally speaking, there are two functions ($f_c$ and $f_a$), which are used by the communicator(sender) of the signal and the audience(receiver) respectively. The

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Table 2.1: Revere 1

<table>
<thead>
<tr>
<th>States</th>
<th>Acts</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>A1</td>
</tr>
<tr>
<td>S1</td>
<td>2</td>
</tr>
<tr>
<td>S2</td>
<td>0</td>
</tr>
<tr>
<td>S3</td>
<td>0</td>
</tr>
</tbody>
</table>
sender can observe a set of states, and has a set of signals available; while the receiver can not observe the states, but has a set of actions he can use to act on each signal. \( f_c \) is a mapping from states to signals. \( f_a \) is a mapping from each signal to at most one action. Coming back to the Revere story, the sexton as the sender can observe three states \( S_1 \) (by land), \( S_2 \) (by sea), and \( S_3 \) (no invasion). He also has three possible signals to use \( M_1 \) (one lantern), \( M_2 \) (two lantern) and \( M_3 \) (no lantern). Revere as the receiver can have three different acts \( A_1 \) (preparation for land), \( A_2 \) (for sea), and \( A_3 \) (none). If \( f_c \) is a one-to-one function, it is called admissible. Similarly, \( f_a \) can be admissible. A pair of admissible functions \( < f_c, f_a > \) is a signaling system. In the game matrix, such pairs of functions give a set of equilibriums \( < (S_1, A_1), (S_2, A_2), (S_3, A_3) > \). The pair of state and act which gives both sender and receiver best payoff when the interests of the two are aligned. Hence there could be multiple signal systems. In the Revere story, we could have \( M_2 \) for \( S_1 \), \( M_1 \) for \( S_2 \), and \( M_3 \) for \( S_3 \). Receiver’s act would change accordingly too. \( A_1 \) for \( M_2 \), \( A_2 \) for \( M_1 \), and \( A_3 \) for \( M_3 \). However, the equilibrium does not change.

Now let us look at a few important assumptions in Lewis signaling games and then connect them to Skyrms’ book. The first two assumptions are common interest and common knowledge. Common interest is reflected in the payoff structure, i.e. both get maximum payoff when a signaling system is established. The fact that both parties are assumed to have a certain level of reasoning ability, which enables them to make an agreement and have higher order expectation of each other, reflects the common knowledge assumption. The third assumption is made on how a particular signal system is chosen. As we have shown above, multiple signal systems can exist in one game. Lewis assumes the players either have a prior agreement or a natural salience when choosing a particular signaling system.

Skyrms book *Signals* responds to all these three assumptions and pushes the game in a more naturalistic direction. First, common interests are not necessary, as he shows
how senders and receivers may have partial or opposite interests. He also argues that signaling games can be useful in the analysis of a wider biological spectrum without a high level reasoning capacity, thus eliminating the need for common knowledge. By using evolutionary dynamics, he shows that without any natural salience\textsuperscript{16} a signal system can emerge.

### 2.3.3 Information in ‘Signals’

Under the naturalistic assumptions, Skyrms emphasizes the importance of the information flow from the beginning of the book (page 2). He argues that the meaning of signals can be studied through the information carried by signals. Such information is further measured by its quantity and content. Skyrms derives the quantity of information from the mathematical theory of information which was originated by Claude Shannon.

As we have briefly pointed out above, Skyrms uses evolutionary dynamics to show the emergence of signaling systems. Initially, nature creates necessary states with equal probabilities. The quantity of information is how much a signal moves probabilities. If we have two states $S_1$ and $S_2$ with initial probabilities $p_{S_1} = 0.5, p_{S_2} = 0.5$, and a signal $A$ moves the probabilities of the two states to $(p_{S_1} = 0.2, p_{S_2} = 0.8)$, then the information quantity is the same as another signal $B$ which moves the probabilities to $(p_{S_2} = 0.2, p_{S_1} = 0.8)$. Skyrms defines information content as a vector which indicates not only how far a signal moves the states’ probabilities, but also in which direction. In our example, information content for signal $A$ is different from that of signal $B$, since they affect the probabilities in opposite directions.

\textsuperscript{16}Clearly, it also assumed to have no prior agreement since high level reasoning capacity is not assumed.
2.4 Social Software: From Social Procedure to Homo Sociologicus

Although we are approaching the conclusion, we are far from concluding if we do not review the seminal work on the topic from Parikh. We have purposefully excluded his work thus far hoping to explain the seemingly-unrelated past chapters through introducing a series of his past important and highly interdisciplinary works. Therefore, in this chapter we will review a few key works from Parikh related to the search of homo sociologicus. Then we will continue with a section that explains the connection between his work and the direction of this dissertation.

Parikh has had interest in social sphere for decades, such as [65] [56]. In 1995, he officially coined the term, Social Software [57] [58], and called for an interdisciplinary collaboration between computer scientists and social scientists. In [58], he points out that computer science concepts, such as algorithms and data, were already described in early philosophical works that concerns social context. At the same time, he also gives a few real life social problems, for example car parking problems, which could have been improved by using an algorithm like social procedure. He gives a famous game theory example, the Santa Fe bar problem, which was discussed first by Brian Arthur [8] in 1994, later also by Greenwald, Mishra and Parikh [32] in 1998. As Parikh explains in [58], when the demand of a public good exceeds the supply in a group of people, the conventional free market approach is not always the most desirable one socially. He states the “striking contrast between the efficiency with which purely computational processes are carried out, and the inefficiency of the social processes which are intended to be... ”. Therefore he argues that the role of algorithms and game theory in social procedures should be properly studied and positioned. [58] is an important first step that opens the research area of social software.

Like the beginning of any theory, key concepts have to be properly raised, discussed,
and defined. In the work of social software, there are a few such concepts that were elaborately explained by Parikh in [59] [60] [61]. Being an expert on the topic of knowledge, Parikh focuses on the technical aspects of knowledge in [59] based on his past work in distributed computing together with Krasucki [66] and Ramanujam [67]. He suggests that understanding the levels of knowledge under different circumstances is crucial. Later in [60], Parikh brings attention to the knowledge and logic structure of algorithms in social context. The paper displays the necessity of logical structure in the car parking problems and logical conditions in using a key.

Starting from “Logic of Society” [63], Parikh starts to list more directions that could be taking in the social software research besides the previous focuses, knowledge, logic and planning. It sheds some light on ‘softer’ issues such as rationality, incentives and preferences, culture and tradition. To be more specific, the social software project is extending to the human elements.

[69] continues this extension to game theory. In the paper, Parikh, Tasdemir and Witzel illustrate how different types of people use the same utility function differently. The knowledge of your opponent’s type in a game is a ‘game-changer’. Two relatively recent papers of Parikh, [64] and [68], are extending quest of human elements in a more philosophical direction.

The works mentioned above are just highlights of Parikh’s research. They appear to follow a direction of from system to components, from the hard issues to soft issues, and from social procedure to homo sociologicus.

2.5 The Jigsaw Puzzle: in Search of Homo Sociologicus

We first raised the need for a better theory for predicting people making social related choices through reviewing two widely experimented games: Public Goods Game and
Ultimatum Game. Then we reviewed a theoretical pillar: the rational choice theory. Parikh points out in [63], “actual behavior differs from theoretical prediction and seems to follow some pre-existing cultural pattern”. Getting back to the queuing example in [58], besides FIFO, there are other different possibilities. For example, people who come late can still go to the front of the queue. What stops him from doing that? The pure ethical concern? Or fear of others’ scolding? He suggests that the work from Lewis on convention [45] has great relevance to the subject. We, therefore, reviewed the famous signaling game from [45]. Skyrms provides a more evolutionary and biological view of the game, which could help us understand how signals and meaning form over time. Potentially, we could blend his ideas into defining social signals, which is the next step that we want to take in our research.

This dissertation is in the direction of converging with the social software project led by Parikh. The focus is how to define the sociality within each individual. More importantly how is such sociality related to our rationality and choices.

Rational choice theory serves a theoretical pillar to all concerning choices including making choices in a group, which is the focus here. However in such situation, sociality is reflected on two levels, namely the group level and individual level. Most of the work in the plethora of social software research concerns the first. We want to argue that the second level is equally important. What is sociality on an individual level? Although this is a crucial point that requires more detailed explanation in the future research, we can simply see it as a kind of social influence when individuals are making decisions. For example, in both Public Goods Game and Ultimatum Game experiments, people deviated from classical game theoretical predictions. We shall not easily conclude that they were irrational because their behaviors do not fit within conventional rational choice framework. As we have seen from the experimental results in [35] from the 15 small scale societies, such deviations are highly correlated with their social structure. Clearly, there is some kind of rationale in these people’s choice. Of course, these
subjects didn’t study different social systems and then decide one choice for a situation like Public Goods Game or Ultimatum Game. So how did they decide? What is the social influence here?
Chapter 3

Knowledge and Epistemic Logic

3.1 Knowledge

We can trace original epistemology, the study of knowledge, back to the Ancient Greek philosophy. The *Theaetetus*, which are considered Plato’s greatest work on epistemology [14], focuses on the question “What is knowledge?” In general, the study concerns the following questions: “What are the necessary and sufficient conditions of knowledge? What are its sources? What is its structure, and what are its limits?” [90].

Here in this dissertation, we are more focused on the formal aspects of the knowledge. More specifically, we will review some existing frameworks in epistemic logic and recent development in dynamic epistemic logic such as friends’ influence. At the end of this chapter, we will also discuss some technical results on language splitting which is useful for our new logic system.

3.2 Formal Frameworks

Von Wright’s work (1951), *An essay in modal logic* [95], is widely considered to be the first proper formal treatment of epistemic logic. In 1962, Jaakko Hintikka extended the work in his book *Knowledge and Belief: An Introduction to the Logic of the*
**Two Notions** Saul Kripke made a technical breakthrough in 1963 with his Kripke semantics. In the Kripke semantics, \( <W,R,|=> \) is called a Kripke-model. \( W \) represents a possible set of worlds that are accessible for the agents. \( R \) represents the relation between the worlds, which should be symmetric, reflexive, transitive, etc. \( |= \) reads as “models”. We can use this to express the Alice example again. \( m,w |= K_aP \) says that there is a model \( m \) in a world \( w \) such that the statement “Alice (\( a \)) knows statement \( P \)” is true. The following axiom \( K_a(A \rightarrow B) \rightarrow (K_aA \rightarrow K_aB) \) is called \( K \) axiom\(^1\). This axiom constitutes System \( K \), the most basic reasoning system in epistemic logic. More reasoning axioms can be added to the system. Hence we also have System \( D \), System \( T \), System \( S4 \), etc.

In the work of the Logic of Belief Revision, AGM model is the most dominant theory. A belief state includes a set of sentences (statements), which is logically closed and is called “a theory”. Let us assume \( K \) represents a belief state of an agent, i.e. \( K \) is a logically closed set of sentences. There are three types of belief changes that can happen to a belief state \( K \).

- **Expansion**: A sentence \( p \) is added to \( K \) and nothing in \( K \) is removed. The new belief set is \( K + p \).

- **Contraction**: A sentence \( p \) is removed from \( K \). The new belief set\(^2\) is \( K \div p \) with some adjustment needed.

- **Revision**: A sentence \( p \) is added to \( K \), and at the same time some sentences are removed if they contradict with \( p \). The new belief set is \( K * p \).

The last type, belief revision, can have an absurd consequence. When a new sentence that is inconsistent with \( K \) is added, all the information in \( K \) can be discarded. This is clearly unrealistic. In real life, when we learn something about the

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\(^1\) If an agent \( a \) knows the implication between the two statements \( A \) and \( B \), then if she knows \( A \), she may also know \( B \)

\(^2\) This notation differs in different literature.
weather we retain our beliefs about politics. In the last section of this chapter, we will see how this can be improved by using splitting languages.

3.3 Friends’ Influence

The most recent development of dynamic epistemic logic is to apply belief revision in social networks. We always have a group of people that are close to us and who can also influence our beliefs in many ways. More importantly understanding the belief influence in a group of people will help us understand how social belief changes happen at a local level. Liu et al \[47\] do suggest a simple yet normative model for social belief change, called ‘threshold influence’.

In their framework, they look at Friendships which is taken to be a symmetric and irreflexive relation. So an agent is a friend of any friend of hers, but she is not her own friend. In addition, they do not assume that friendship is transitive. Therefore friends of friends are not necessarily friends. Friendships among a group of people create a social network, or community. Agents start with some initial belief towards an issue \(p\). They can be \(Bp\) (believe \(p\)), \(B\neg p\) (believe the negation of \(p\)), or \(Up\) (no opinion on \(p\)). In the process of communicating with friends, this belief can be changed in two ways. When the agent is strongly influenced by friends (\(Sp\)), it leads to belief revision (\(Rp\)). Similarly, when she is weakly influenced (\(Wp\)), it leads only to belief contraction (\(Cp\)).

Symbol \(F\) means ‘all my friends’. Therefore \(FBp\) means ‘all my friends believe \(p\)’. In the framework, Liu et al further define the dual operator \(\langle F \rangle\) as ‘some of my friends’. \(\langle F \rangle Bp\) means some of my friends believe \(p\). Then they add the aspect of a threshold. For instance assuming my threshold is 100%, if I believe \(p\) but all of my friends believe \(\neg p\), then I will change my own belief to \(\neg p\). but if only some of my friends believe \(\neg p\) then I will become undecided about \(p\).

Within a group of friends, beliefs tend to be adopted from one to the other. It
creates a distribution of beliefs in the community. Liu et al [47] look at the influence one could gain from a socially connected agent.

The three possible doxastic states of a proposition $p$ can be defined through following axioms:

- Strong influence: $S\varphi \leftrightarrow (FB\varphi \land \langle F \rangle B\varphi)$

- Weak influence: $W\varphi \leftrightarrow (F\neg B\neg \varphi \land \langle F \rangle B\varphi)$

Liu et al also look at the stability of beliefs in different friendship-based networks. They defined a program, $Ip$ with following rules:

- if $Sp$ then $Rp$ else if $Wp$ then $C\neg p$

- if $S\neg p$ then $R\neg p$ else if $W\neg p$ then $Cp$

They call a community *stable* when the program $Ip$ has no effect on the belief states of the members. Some communities become stable after a few applications of $Ip$. Some can never be stable. Then these communities are said to be in flux. Figure 3.1 is one such example.

### 3.4 Language Splitting

Parikh [62] shows that a person’s beliefs can be uniquely divided into different *subject matters* and so if his beliefs are a theory $T$ in $L$ then $L$ will split naturally into
CHAPTER 3. KNOWLEDGE AND EPISTEMIC LOGIC

sublanguages $L_1 \cup L_2 \cup \ldots L_n$ and $T$ will split into $T_1 \cup T_2 \cup \ldots T_n$, each $T_i$ in $L_i$.

This idea of splitting language was naturally applied to belief revision. If I learn some political fact, it will not change my views about my teeth or the location of my children. Moreover, if have trouble with my car, I will not consult a dentist. Conversely I will not consult a garage mechanic about my teeth.

Before we look into Parikh’s axioms for language splitting, let us review the original AGM axioms which can lead to some forgetful updates.

**AGM Axioms**

1. $T \star A$ is a theory

2. $A \in T \star A$

3. If $A \iff B$, then $T \star A = T \star B$.

4. $T \star A \subseteq T + A$

5. If $A$ is consistent with $T$, i.e. it is not the case that $\neg A \in T$, then $T \star A = T + A$

6. $T \star A$ is consistent if $A$ is.

7. $T \star (A \wedge B) \subseteq (T \star A) + B$

8. If $\neg B \not\in T \star A$ then $(T \star A) + B \subseteq T \star (A \wedge B)$

*If A is consistent with T, then $T \star A = T + A$, otherwise $T \star A = Con(A)$. Given these axioms, a forgetful update is allowed by AGM, in which case when $A$ is inconsistent with $T$, all information in $T$ is discarded.*

Clearly this kind of updates are unrealistic since in real life when we learn something about weather we retain our beliefs about politics. Parikh improves this update by using Craig’s Interpolation Theorem.
Craig's Interpolation Theorem

**Theorem 3.4.1:** Let $L_1$, $L_2$ be first order languages, $L = L_1 \cap L_2$ and $T_1$, $T_2$ be theories in $L_1$, $L_2$, respectively such that $T_1 \cup T_2$ has no model (is inconsistent). Then there is some formula $\psi$ of $L$ such that $T_1 \vdash \psi$ and $T_2 \vdash \neg \psi$. In particular, if $\phi$ is an $L_1$ formula and $\xi$ is an $L_2$ formula and $T_1 \cup T_2, \phi \vdash \xi$ then there exists an $L$-formula $\psi$ such that $T_1, \phi \vdash \psi$ and $T_2, \psi \vdash \xi$. □

### 3.4.1 P Axioms

Parikh proposed following axioms to improve the belief updates, which prevent the discarding all the information in the way we mentioned in a forgetful update.

1. **Axiom P1:** If $T$ is split between $L_1$ and $L_2$, and $A$ is an $L_1$ formula, then $T \ast A$ is also split between $L_1$ and $L_2$.

2. **Axiom P2:** If $T$ is split between $L_1$ and $L_2$, $A$, $B$ are in $L_1$ and $L_2$ respectively, then $T \ast A \ast B = T \ast B \ast A$.

3. **Axiom P2g:** If $T$ split between $L_1$ and $L_2$, $A$, $B$ are in $L_1$ and $L_2$ respectively, then $T \ast A \ast B = T \ast B \ast A = T \ast (A \land B)$

4. **Axiom P3:** If $T$ is confined to $L_1$ and $A$ is in $L_1$ then $T \ast A$ is just the consequences in $L$ of $T \ast' A$ where $\ast'$ is the update of $T$ by $A$ in the sub-language $L_1$

All these axioms follow from axiom $P$ below:

**Axiom P:** If $T = Con(A, B)$ where $A$, $B$ are in $L_1$, $L_2$ respectively and $C$ is in $L_1$, then $T \ast C = Con(A) \ast' C \vdash B$, where $\ast'$ is the update operator for the sub-language $L_1$. 

Chapter 4

Network Theory

4.1 Networks

The Network Science is truly interdisciplinary. Euler’s solution of the Königsberg bridge problem in 1735 is considered the first true proof in the theory of networks [53]. The vast body of knowledge has been applied to physics, mathematics, computer science, biology, economics and sociology. With the access to cheaper computing power at the turn of this century, the science has been focusing on different problems [96], many of which are supported with empirical observations [10].

The 2003 Northeast blackout is considered a famous example of cascading failure in a network [10]. In that event, 55 million people lost their power. A more recent example (see figure 4.1) would be Google. On March 12th 2015, the search giant’s service was inaccessible for millions due to a cascading failure caused by a routing leak which originated from an Indian ISP. Both cases expose the vulnerability due to interconnectivity in a network.

For our purpose of understanding social issues, this following example which is explained by Barabási in the very beginning his book Network Science, seems more relevant.
CHAPTER 4. NETWORK THEORY

Figure 4.1: Google went down for millions of users due to a routing leak from an Indian ISP

Image 1.2b
The network of Saddam Hussein.

Figure 4.2: Image from Network Science by Albert-László Barabási
In March 2003, American forces entered Iraq. But they were unsuccessful at capturing high ranking officials including Saddam Hussein. Later, the US military reconstructed the social network of Hussein, relied on gossip and family trees instead of government documents. Using the social network diagram, there were a few successful raids including one that led to an important piece of intel: a family album. It dramatically helped with further understanding of Hussein’s trusted network. The military was eventually able to figure out the hiding place of Saddam Hussein.

Through the example, Barabási points out a few important observations of network theory:

- The predicative power: it allows even non-experts to extract useful information.
- The remarkable stability: the social network was not constructed through updated intelligence but dated information such as gossips and family photo albums.
- The choice of network can be crucial. In the case of Hussein, the military tried for months to find him through the network of the Iraqi government. But it was personal network that eventually helped.

4.2 Basic Concepts

Often ‘networks’ and ‘graphs’ are often used interchangeably by researchers. However in Network Science, we use the terms: network, node, and link, while in Graph Theory, we use the terms: graph, vertex and edge. In this chapter, we will use the terminologies from network science.

Let us review some basic concepts in network science. \( N \), Number of Nodes is the total number of components in a network. \( L \), Number of Links, is the total number of interactions between the nodes. A link can be either directed or undirected. When we
trace how a message is being sent in a network, the links are directed from senders to receivers. When we look at hand-shaking in a group of people, the links are undirected since a handshake always takes two people at the same time.

Degree, \( k \), is an important concept in network science. It is the number of links from a node to other nodes. In undirected networks, we have the following relationship between \( L \) and \( k \):

\[
L = \frac{1}{2} \sum_{i=1}^{N} k_i
\]

The average degree, \( \langle k \rangle \), of a network can be calculated in the following method:

\[
\langle k \rangle = \frac{1}{N} \sum_{i=1}^{N} k_i = \frac{2L}{N}
\]

In directed networks, we have incoming degree, \( k_{i}^{\text{in}} \) and outgoing degree, \( k_{i}^{\text{out}} \). Therefore the total degree of a node \( i \) is:

\[
k_i = k_{i}^{\text{in}} + k_{i}^{\text{out}}
\]

Following formula shows the relationship between \( L \) and \( k_{i}^{\text{in}} \) and \( k_{i}^{\text{out}} \):

\[
L = \sum_{i=1}^{N} k_{i}^{\text{in}} = \sum_{i=1}^{N} k_{i}^{\text{out}}
\]

In directed network, the average degrees of \( k_{i}^{\text{in}} \) and \( k_{i}^{\text{out}} \) are:

\[
\langle k_{\text{in}} \rangle = \frac{1}{N} \sum_{i=1}^{N} k_{i}^{\text{in}} = \langle k_{\text{out}} \rangle = \frac{1}{N} \sum_{i=1}^{N} k_{i}^{\text{out}} = \frac{L}{N}
\]

\( p_k \), the degree distribution, is the probability that a randomly selected node in the network has degree \( k \). Generally, we have \( \sum_{k=1}^{\infty} p_k = 1 \). In a fixed network that has \( N \) nodes, we have: \( p_k = \frac{N_k}{N} \), where \( N_k \) represents the number of nodes that has degree \( k \).
We can also express the average degree of a network in terms of $p_k$:

$$\langle k \rangle = \sum_{k=0}^{\infty} kp_k$$

In a network that is a complete graph, which means every node is connected to all other nodes, then we have $L_{\text{max}} = \left( \frac{N}{2} \right) = \frac{N(N-1)}{2}$. However, most of the networks observed in real life are sparse [10].

Often it is easier to talk networks in the adjacency matrix. Therefore, we will review some concepts and formula related to it. In a directed network with $N$ nodes, the adjacency matrix has $N$ rows and $N$ columns. $A_{ij} = 1$ when there is a link pointing from node $j$ to node $i$. $A_{ij} = 0$ when there is no link between node $j$ and node $i$. In an undirected network, $A_{ij} = A_{ji}$. Since $A_{ij}$ represents the links, we can get the degree $k_i$ of node $i$ from the adjacency matrix. For undirected networks:

$$k_i = \sum_{j=1}^{N} A_{ij} = \sum_{i=1}^{N} A_{ij}$$

For directed networks:

$$k_i^{\text{in}} = \sum_{j=1}^{N} A_{ij}$$

$$k_i^{\text{out}} = \sum_{i=1}^{N} A_{ij}$$

A path is a route from one node to another in a network. It can pass through the same link more than once. It can also intersect itself. A shortest path, $d_{ij}$, is a path between $i$ and $j$ with fewest number of links. In undirected networks, $d_{ij} = d_{ji}$, while in directed networks, $d_{ij} \neq d_{ji}$. A network diameter, $d_{\text{max}}$ is the maximal shortest path in a network. Average path length, $\langle d \rangle$ is $\frac{1}{N(N-1)} \sum_{i,j=1,N} d_{i,j}$ in a directed network; is $\frac{2}{N(N-1)} \sum_{i,j=1,N} d_{i,j}$ in an undirected network.

When there is a path between two nodes ($i$ and $j$), we say that they are connected,
and otherwise they are disconnected with \( d_{ij} = \infty \). A network is connected if all the pairs of nodes are connected.

In figure 4.3 (a), it is a disconnected network with two components. If we place a single link between 2 and 4, the network becomes (b) which is connected. This link is called a bridge.

In real life networks, e.g. social networks, highly density ties are often observed. Let us review a concept that is related to it. The local clustering coefficient represent the degree of how neighbors of a node are connected to each other. For a node \( i \) with degree \( k_i \), we can express the local clustering coefficient as following:

\[
C_i = \frac{2L_i}{k_i(k_i - 1)}
\]

where \( L_i \) is the total number of links between the neighbors of node \( i \).

\( C_i \) measures the local density in a network. It is between 0 and 1. When \( C_i = 1 \), the neighbors of \( i \) are fully connected and form a complete graph. When \( C_i = 0 \), none of the neighbors of \( i \) are connected.

**Figure 4.3:** Connected and Disconnected Networks [10]
4.3 The Scale-Free Property

When the WWW was first mapped out in 1999 [2], it was discovered that the network had many highly connected nodes unlike in a random network. In fact, many real networks have the same property [10]. It is called the *scale-free property*. We can represent the degree distribution of the WWW in following manner:

\[ p_k \sim k^{-\gamma} \]

We call this a *power law* distribution with a *degree exponent* \( \gamma \). After taking a logarithm of the formula above, we get: \( \log p_k \sim -\gamma \log k \).

In directed networks, we will have following:

\[ p_{k_{\text{in}}} \sim k^{-\gamma_{\text{in}}} \]

\[ p_{k_{\text{out}}} \sim k^{-\gamma_{\text{out}}} \]

When we compare a scale-free network with a random network whose distribution
Figure 4.5: A Poisson function compared with a power-law function with $\gamma = 2.1$. Both have $\langle k \rangle = 10$. \[10\]

is a Poisson distribution, we observe that the probability of high-degree nodes is much higher in a scale-free network. (See figure 7.5 and 7.6) In addition, there are many small degree nodes in scale-free networks while they are absent in random network. Figure 7.6 is an example of a scale-free network with $\langle k \rangle = 3$. “The more nodes a scale-free network has, the larger are its hubs.” \[10\].
Figure 4.6: Same two functions as above shown on a log-log plot. \[10\]

Figure 4.7: A scale-free network with \( \langle k \rangle = 3 \) \[10\]
Chapter 5

Expert Influence

5.1 Concepts

We are extending Friend Influence to show how influence on a subject travels in general. We call it *Expert Influence*. Before we discuss this type of belief updates, we are going to review some preconditions of the model.

Resolving differences of opinions among experts will bring us to the *judgment aggregation problem* which was investigated by List and Pettit [46]. Such issues are very difficult and this paper is not the right place to address it. We will start with the simple assumption of having one expert in each area in a social network at the beginning of an influence process. In addition, we look at connected social network only, i.e. there is no isolated person or groups in the network. We study how Expert Influence on a single proposition can travel through the network.

In this method, we employ all AGM axioms, P axioms from [62], and two new additional axioms that instruct agents what to do when they are influenced by an expert or a non-expert.

- **Influence from Expert to Non-expert:** Before a non-expert talks to an expert on $p$, he can believe $p$, believe $\neg p$ or have no opinion about $p$. Once the
non-expert talks to a neighboring expert about $p$, he adopts the beliefs of the expert and become an expert on $p$ himself.

- **Influence between Non-experts:** Between two non-experts, if they have different beliefs about $p$, they both contract their beliefs on $p$ since none of them are authoritative on the matter.

These conditions and axioms can be expressed formally in the following way.

1. Agents $N = \{1, 2, ...n\}$ in a social network of $G$

2. $R \subseteq N \times N$

3. $R$ is symmetric and irreflexive

4. 3 possible doxastic states:
   
   (a) believe $p$: $Bp$
   
   (b) disbelieve $p$: $B\neg p$
   
   (c) no belief about $p$: $Up = \neg Bp \land \neg B\neg p$

**Expert Influence Axioms**

- **Expert influence to a non-expert:** $R(i, j) \land E(i, p, t) \land B(i, p, t) \land (B(j, p, t) \lor B(j, \neg p, t) \lor U(j, p, t)) \land \neg E(j, p, t) \rightarrow B(j, p, t + 1) \land E(j, p, t + 1)$

  This can clearly be simplified to: $R(i, j) \land E(i, p, t) \land B(i, p, t) \land \neg E(j, p, t) \rightarrow B(j, p, t + 1) \land E(j, p, t + 1)$

- **Influence between non-experts:** $R(i, j) \land \neg E(i, p, t) \land \neg E(j, p, t) \land B(i, p, t) \land B(j, \neg p, t) \rightarrow U(i, p, t + 1) \land U(j, p, t + 1)$
**Figure 5.1:** Example 1: Single Expert - beginning

![Diagram](image1)

**Figure 5.2:** Example 1: Single Expert - during

![Diagram](image2)
5.2 Examples of Expert Influence

Let us now look at two possible scenarios of Expert Influence.

In example 1 (figure 5.1 - 5.3), 5 agents are in the network with $a$ as an expert on $p$ (marked in with a bold circle) initially. Communication can happen anytime between two agents, however, Expert Influence starts to spread only when $a$ starts communicating with her neighbors. The figures show one of the many possible influence order. Figure 2 shows that agent $a$ influenced $c$ who has already become an expert on $p$. At the same time, we can see that $b$ and $e$ also communicated. Since they had different beliefs on $p$ and none of them is an expert, they contracted their beliefs on the matter. At the end of the influence process, we can see that every agent in the network adopt the opinion of the expert and become an expert on $p$ (figure 5.3).

We have assumed that at the beginning of the influence process, there is only one expert in each area. However, it is possible to have two experts when we look at two areas of expertise. In example 2 (figure 5.4 and 5.5), the 5 agents are in same network structure as in example 1. Both $a$ and $d$ are experts, although in two areas ($p$ and $q$) which are indicated by bold and dashed circles respectively. The experts’ influence
**Figure 5.4:** Example 2 (Two Experts) - beginning

**Figure 5.5:** Example 2 - end the influence for agent b
spread out in a similar fashion as in example 1. Agent $b$ is at a special position in the network since he is connected to both expert $a$ and expert $d$. By applying the axiom P2G, we can show that $b$ will have $Ep \land Eq$. In addition, $b$ is going to be the first person in the network to become an expert on both subject matters, regardless how the two different influence processes play out.

5.3 Computation Complexity of Experts Influence

Proposition 5.3.1: The problem of influencing the whole network in one expertise is reducible to the single-source longest path problem.

The communication among all agents is non-deterministic. Any agent can be the first to talk to another and update her belief. However, once the expert starts talking to her neighbors, the expert influence does have directions. Eventually when the agent who is furthest away from the expert is influenced, then the whole network has converged to expert’s belief on $p$ in $O(N)$ time. In real networks, $\log N$ is observed.

From the complexity analysis we can see that this type of expert influence can spread the expert’s beliefs rapidly within a network of people. This may explain how hundreds of thousands people can use a one-to-one mobile app to coordinate efficiently during the Hong Kong protest in 2014.

In most of the situations in real life, we have different types of influence that sometimes conflict with one another. For example, you get an invitation to a party. Both your friends and your colleagues may received the invitation too. The two groups may have different opinions about the party. Therefore one may suggest to go and the other may suggest not to go. How do you make decisions under different influences that contradict? In the next chapter, we will suggest a simple decision making process that people could use to make strategic decisions.
Chapter 6

Coordination Under Influence

6.1 Influence and Coordination in Networks

There are many events in our daily life that require coordination with our friends and acquaintance. For example, we would like to go to a party only if enough interesting people are going. In some situations, such as a dinner party for a small group of people who we know, we can just contact our friends and ask. However, in some situations, such a gallery opening party which you have heard from some friends, you cannot contact everyone. In the second type of party, you may want to meet strangers who are interesting. On one hand, you cannot contact people you have not met. On the other hand, since your friends told you about the event, they may know some of the people who are going. Then coordination can happen through understanding influence in our social network.

In the past, we may still be able to find the information for deciding whether we would go to an event or take an action easily, since our social networks were generally smaller. In the era of infostorm, not only we are overwhelmed by information, but also having ever expending social network through all kinds of social media. How do we make good decision given different kinds of influence in a large social network?
We have introduced two different belief update methods, namely Friend Influence and Expert Influence. It is clearly that both can be separately applied in different situations. In the case of a small dinner party, we probably get influenced through friends who are invited and coordinate with them. In the case of a gallery opening party, the influence may come naturally from an artist friend who is going and is an expert of such events. It is also possible that we get influenced both from our friends and experts at the same time. How do we choose which one to follow when the two are in conflict? More importantly, in an event that requires group coordination, can we choose strategically as an individual without global planning with all the participants?

In this section, we investigate how agents can be influenced differently in one event. We will first introduce the concept of Expect Influence which indicate potential influence of an agent or a group of agents. Then we will discuss how conflicting influence may raise between two different belief-update methods. At the end, we will discuss how network structure can affect people’s choices of belief updates.

\section{6.2 Expected Influence}

During an event, such as a party, the people who are invited, are coordinating on an action which has two options either attend when there are enough interesting people going or not attend. Of course, they could contact each other and coordinate globally within a community. However, as we enter the era of infostorm, our network is growing ever larger. Such centralized coordination becomes more difficult and inefficient in some situations.

Every person in the network has her own knowledge about the local network she is in. She can then use that knowledge to decide how she wants to be influenced and take actions. In order to make such decision, she needs to have a simple indicator, \textit{Expected Influence}, which allows her to compare the influence from different groups of
people in her network. This way she can maximize her benefit from taking the action under a particular influence.

Let us look back to the example of a gallery opening. Alice received an invitation of the opening. However, she is not too familiar with the crowd. She does know that a few people in her social network are invited too. Some of them actually encouraged her to go along with them while some told her that it is going to be a boring event. Alice, being a mellow person, is happy to go if she believes that the majority of the invited guests are going. She would also be happy if she chooses to stay at home, and most people also decide not to go. Before she makes a decision, she gathers information from her social network. Then she could update her belief using the Friend Influence model which means her belief and action depend on her friends’. She could also update her belief using the Expert Influence model which means her belief and action depend on what she hears from the expert in her network, for example a seasoned artist who knows the crowd and have extensive experience with this type of gallery openings. It is possible that the two methods lead to the same action which she would take. It is also possible that two methods are in conflict. In the second case, Alice would think about her goal, being happy in this case, and strategically choose a method that maximizes the happiness. This means she needs to pick an action which is going to be taken by most people. Therefore she would assess whether her friends have more influence or the expert.

As we are influenced more and more through the social media platforms, it is harder for a community member to know how and when her acquaintances are influenced. Therefore, in this paper, we do not look at how an agent reaches to the point of being influenced and has to make a decision for her action. We are more interested in given the network, the influence she receives at a moment, and how she would strategically choose.

When we look at the influence of a person or a group of people, we do not look
at how many people they have already influenced, but how many people they can potentially influence. Therefore we define the concept in the following way.

**Definition 6.2.1:** \( I_G, G \subseteq Ag \) where \( Ag \) is the set of agents in the network, indicates the potential influence of the group \( G \). It is the number of agent in this set, \( |G| \), plus the total number of direct neighbors \((D_i)\) of each agent in group \( G \) removing the number of duplicated agents.

\[
I_G = |G| + \sum_{i=1}^{n} D_i
\]

Figure 6 is a local network that is connected to the rest of the network through E. There are 5 agents, namely A, B, C, D, and E. “Y” and “N” indicates if each agent believes that the opening will be an interesting one. “Y” means yes, and implies that the agent would probably go to the opening party. “N” means no. “u” in B means that B is undecided. The numbers in figure 6 indicates the degree of each node, i.e. the number of neighbors of each agent. E is the expert in this local network, who has a reputation of being the expert. Therefore the number of her neighbors is known to everyone in the local network. Some agents’ neighbors are completely observable in the local network, such as A, B, C, and D. Some agent’s neighbors, such as the 10
neighbors of E, are not all on this local graph. Here we also assume that the rest of 9 neighbors of E are not in this local graph.

The influence of agent A is \( I_{\{A\}} = 3 \). The influence of agent B and C, \( I_{\{B,C\}} \), is 4. Influence of agent E and C, \( I_{\{E,C\}} \), is 13.

### 6.3 Belief Conflict and Coordination Equilibria

Now we continue to look at the gallery opening example in figure 6 and find out how an agent can choose between the two methods of belief update. Since agent B is undecided, we will focus on her and see how she can update her belief and choose the better update that maximize her happiness.

Just as we mentioned earlier, there is only one expert in the local network. In this example, it is E. Furthermore we will assume that the expert is always more actively talking about her expertise to the people in her network. In the example here, it means agent B get influenced by E first. The other agents, A and D are friends\(^1\) of B. We can see that all of B’s friends think it will be a great party and probably going, while the expert E believes that the party will not be so great. So how is B going to be influenced and how will she choose what to do?

B receives the information from E first and updates her belief to E’s according to the expert method that was defined earlier in the paper. Then B communicates with her friends and finds out that her friends think otherwise. Now she has a choice between being influenced by her friends and being influenced by the expert.

B has a coordination problem to solve. She wants to make a choice that majority of the global network would choose. However, she does not have much knowledge about the actual global network. She does not know how many are invited. She certainly does not know who they are and how they are connected to each other. She knows her neighbors and the number of neighbors they each have. What she could do

\(^1\)Experts are not friends.
Table 6.1: Influence Matrix

<table>
<thead>
<tr>
<th></th>
<th>Friends</th>
<th>Expert</th>
</tr>
</thead>
<tbody>
<tr>
<td>Agent B</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>Agent D</td>
<td>0</td>
<td>13</td>
</tr>
</tbody>
</table>

is to choose an update that gives her maximized influence in her local network. We further assume that the expert cannot be influenced by others since she believes that she has the authority on the matter. B can consider the pair-wise coordination with each of her friends.

Let us look at the coordination between B and D. Once they communicate their beliefs with each other, they both know that the friends belief differs from the expert’s belief. Since they each represent a different opinion, they are vague about the actions they take, going to the party or not, so that they will not hurt each others’ feeling. They ponder what to do given the Influence Matrix from table 1. When they choose different beliefs to act upon (\{Friends, Expert\} and \{Expert, Friends\}), they are both not happy. When they are choosing the same belief to act upon, then their happiness depends on the expected influence.

\[
I_{\text{Friends}} = I_{B,D} = |\mathcal{G}_{\{B,D\}} + \mathcal{D}_B + \mathcal{D}_D| = 4
\]

\[
I_{\text{Expert}} = I_{B,E} = I_{D,B,E} = |\mathcal{G}_{\{B,E\}} + \mathcal{D}_B + \mathcal{D}_E| = 13
\]

Because agent B is the undecided one, therefore when we estimate the influence we have B’s degree in both methods. When if they choose expert method, it is \(I\{B,E\}\) for B, and \(I\{D, B, E\}\) for D.
In this payoff matrix, we have two equilibria, with \{Friends, Friends\} as a risk-dominant equilibrium and \{Expert, Expert\} as a payoff-dominant equilibrium \[34\] \[13\]. It is reasonable to assume that they will pick the payoff-dominant equilibrium as they both know the influence of the expert is higher.

Gintis \[30\] suggests that a payoff-dominant equilibrium is selected when players believe that the Principle of Honest Communication, according to which players keep their promises unless they can benefit by violating these promises and being believed.

We further make the natural assumption that regardless of the influence method, it is always better to pick the group that gives more expected influence. This can explains some social hypes \[16, p. 37\] through a perspective of social networks, such as the famous El Farol Bar Problem \[9\]. This also shows that people tend to believe and act on views that give them highest potential influence and ignore the source of the information.

### 6.4 Different Types of Networks

With this simple influence indicator, we can investigate the relationship between a belief update method and the structure of the network. We will show a few properties in some basic network structures.

**Proposition 6.4.1:** In a social network that is a complete graph (fully connected), two methods are the same. In fact, any group \( G \in \wp(A) \) has the same influence \( I^* = |A| \).

**Proof.** By induction on \( G \). ■

When a social network is so tightly connected, e.g. figure 7, each individual can potential influence everyone, which also means each of them has little to none influence. It is also natural to assume that such networks have few conflicts in general.
Figure 6.2: Influence in a complete graph

![Complete Graph](image)

Figure 6.3: Influence in a ring graph with 4 agents

![Ring Graph with 4 Agents](image)

Figure 6.4: Influence in a ring graph with 5 agents

![Ring Graph with 5 Agents](image)
Figure 6.5: Influence in a star graph

**Proposition 6.4.2:** In a social network of $|Ag|$ number of agents, if the structure is a *ring*, friends $I_{\{i,j\}} = 4$ always have equal influence as the expert $I_{\{i,e\}} = 4$. □

*Proof.* Any of the two neighboring agents have influence of 4. Induction on $|Ag|$. ■

**Proposition 6.4.3:** In a social network that is a structure of a *star* with $|Ag| \geq 4$, $I_{\{i,e\}}$ always has the most influence if $e$ is at the center. Otherwise friend influence is always stronger. □

*Proof.* (1) When the expert is at the center

$$I_{\{i,e\}} = |Ag|$$

since $e$ is connected to everyone. $I_{\{i,j\}} = 2$ for all $i \neq j$ and $i, j \neq e$, since none of the two none-experts are neighboring each other.

(2) When a non-expert is at the center

$$I_{\{c,i\}} = |Ag| = I_{\{c,e\}}$$

with $c$ as the agent who is at the center. $I_{\{i,e\}} = 2$ for the influence of any non-expert and expert (except the influence of the center agent and expert).

We can see that the network structure has an effect on how agents would choose which type of belief updates as well. In a social network that is fully connected or
has a ring structure, the agents are likely to treat the two methods equally if they are in conflict. However in a network that has a structure of a star, the choice depends on the position of the agents and the expert. An expert in the center knows that he has enormous power that can influence the whole network. It also shows that whoever has absolute control of the information flows has the control of behavior of the whole network. We should note that, comparing with the game theoretical approach we discussed earlier, these three propositions requires additional assumption that everyone knows the network structure.
Chapter 7

Simulating Social Influence

After studying some of the properties of influence indicator, we are interested in seeing how it could work on bigger and more complex networks. Therefore in this chapter, we are going to introduce two different types of network: Watts-Strogatz [97] and Barabási-Albert [1]. We will then show some simulation results in these networks.

We discover that the expert\(^1\) has different degrees of social power in different network structures. In particular, a well-connected expert has a lot of power in the Barabasi-Albert network, which is more similar to real world social network.

The random graphs from Paul Erdős and Alfréd Rényi [22] (ER graphs) is a network model used in many applications. The ER graphs have two major issues in comparison with real-world networks. First of all, these graphs do not have local clustering which often observed in real life. Secondly, the degree distribution of ER graphs is basically a Poisson distribution. However in the actually networks observed, the degree distribution is a power law [72].

The Watts-Strogatz model and Barabási-Albert models are two different approaches trying to improve the original ER graph to make the representation closer to real life.

In the first section, we will start with introducing the structure of Watts-Strogatz

\(^1\)We have always assumed 1 expert in the network. In the simulation we also set the number of expert to 1. This can be easily changed and extended in the future work.
network. Then we will show some simulation results in this type of network. In the second section, Barabási-Albert network will be explained. We will also see how the simulation results in this type of networks are quite different from Watts-Strogatz network.

7.1 Watts-Strogatz Networks

The Watts-Strogatz (WS) network is proposed by Watts and Strogatz in *Nature* [97]. It is a model that generate a particular kind of graphs with short average path lengths and high clustering. WS network is a step closer to the real-life network. In particular, it creates local clustering which is not in ER graphs.

A graph, $G$, has $N$ nodes with a mean degree of $K$. $\beta$ is a parameter that is in $[0, 1]$. A WS graph with $N$ nodes and $\frac{NK}{2}$ edges is constructed in following two steps [97]:

- Construct a ring lattice that is a graph with $N$ nodes. Each node has $K$ neighbors with $\frac{K}{2}$ on each side.
- For each nodes $n_i$ in $n_0, ..., n_{N-1}$, we connect the edge $(n_i, n_j)$ ($i < j$) with probability of $\beta$.

As $\beta \to 1$, the WS graph converges to a ER graph with the clustering coefficient of $\frac{K}{N}$. And its degree distribution becomes a Poisson distribution as in a usual ER graph.

A WS graph shows local clustering without having hubs and a scale-free distribution of degrees. Therefore when we simulate the social influence, both friend and expert influence, we expect that power of influence is not as evenly distributed as in a ER graph, but will have some degrees of segregation.

In each of the following 4 simulations, there are three figures. We start with the initial network of agents with blue represents “Yes”, red represents “No”, and yellow
represents “Uncertain” to an event. Second figure is the final situation after 6 rounds of interactions. The last figure exhibits the history of changes in terms of the number of nodes in blue, red and yellow respectively.

As we assumed in the earlier chapter, there is only one expert in the group of 30 agents. The expert is always in blue.\textsuperscript{2}

In this first example of a WS network, red is the largest group initially with 17 agents, then blue with 8 agents and yellow with 5. Over 3 rounds of influence, the situation seems to be stabilized into two groups: 18 red agents and 12 blue agents.

In this first example of a WS network, red is again the largest group initially with 15 agents, then blue with 5 agents and yellow with 10. Over 3 rounds of influence, the situation seems to be stabilized into three groups: 21 red agents, 2 yellow agents and 7 blue agents.

From these two example, we can find some commonalities about influence in WS\textsuperscript{2}.

\textsuperscript{2}This can be easily changed into red or yellow. The general result will not change.
**Figure 7.2:** WS Network - 1 - final

**Figure 7.3:** WS Network - 1 - history
Figure 7.4: WS Network - 2 - initial

Figure 7.5: WS Network - 2 - final
networks. First of all, when there are different opinions about an event and with two types of influence methods, the agents did not converge into one color. Secondly, it took relatively short time for the network to stabilize. Thirdly, the yellow agents may not disappear completely, but tend to converge into blue or red. For example, in the second simulation, there are a third of agents in yellow initially. At the end, there was only two left.

We conclude that when there is no major hubs in the network, agents’ opinion tend to stabilize among the local clusters.

### 7.2 Barabási-Albert Networks

The Barabási-Albert (BA) model \[1\] is an algorithm to generate random scale-free networks that resemble networks observed in real-life more closely.

A BA network generation starts with a group of connected \(n_0\) nodes. Then each new node is connected to \(n\) nodes \((n \leq n_0)\) with probability \(p_i\).

\[
p_i = \frac{k_i}{\sum_j k_j}
\]
Figure 7.7: BA Network - 1 - initial

$k_i$ is the degree of node $i$, $\sum_j k_j$ is the sum of degrees of all pre-existing nodes $j$. Nodes with many links tend to have even more new links. This enables the scale-free (power-law) degree distributions.

This type of degree distribution is observed in many real social networks [11]. So let us simulate the types of influence in this framework.

All the setups are similar to WS network, except that expert is the hub of the network, i.e. with the most connection.

In the first simulation, it starts with 15 red agents, 8 yellow agents, and 7 blue agents. Although blue color has the least number of agents initially, it becomes the only color very quickly after only two rounds of interactions.

The second simulation shows similar ending that the whole network converges to blue. It starts out with 15 red agents, and many more yellow agents (13 of them). However after 3 rounds, 3 blue agents influence the whole network.

From these 4 examples of both WS and BA network simulations, we can see that
Figure 7.8: BA Network - 1 - final

Figure 7.9: BA Network - 1 - history
Figure 7.10: BA Network - 2 - initial

Figure 7.11: BA Network - 2 - final
hubs in the networks are quite influential. However if there is no hubs, the power of influence is diminished.

We have ran a dozen more simulations for both networks. The patterns are very similar. Most of the time, WS networks segregate into two groups (red and blue) with a few yellow agents left occasionally. BA networks always converge into blue. Many more data points are needed for more definitive conclusion. That will be in our future work.
Chapter 8

Summary

We started this research with a strong believe in the power of groups that people are not only selfish like homo economicus but rather more pro-social like homo sociologicus. In order to deeply understand human behavior, we explored in many fields such as rational choice theory, cognitive science (autism specifically), social software. With these field as background, we eventually settled our research focus on group coordination that is achieved through bottom-up communication methods.

8.1 Conclusion

In this dissertation, we are primarily interested decentralized social coordination. When people make decisions in participating social events, they often get influenced by different people in their social networks. We explained two possible belief update methods, namely Friend Influence and Expert Influence. Instead of assuming large amount of common knowledge, we assume limited local knowledge. We introduced a simple indicator for influence in a network. Sometimes two belief updates lead to two different suggestions. When this happens, each individual can use the influence indicator to assess the potential influence of various groups, e.g. her friends or the expert. Given the two options, each agent can make a strategic decision that maximizes
her benefits. We also discussed how different network structures may affect agents’
decisions. In some structures, the two methods always have the same influence. In
some structures, such as a star, we can predicate the choice of agents by looking at
their positions in the network.

Our simulation reflects our theoretical finding, which shows the importance of the
network structure in spreading social powers.

8.2 Future Directions

It is clear that this is just a start of truly understanding human as homo sociologicus
as in how we coordinate and achieve wonders in large groups. There are multiple
directions that we can pursue.

Firstly, a more properly defined language of influence can be developed. So far,
we have set inference rules, discussed examples of influence and simulated it. A
well-defined language can give us a deeper understanding. It will probably also allow
us to have more experts in one field.

Secondly, there are many network properties that we can explore further in terms
of social influence. It would be interesting to explore the relationship between the
network positions of the expert and spreading of the influence.

Last but not least important is to study the relationship between self utility and
group utility \cite{41}. Although there is strong evidence of social conformity \cite{99}, we, as
intelligent beings, have our own minds. For certain issues, we may follow the crowd.
But there are definitely issues that are so important to us, we are not willing to change
our mind even if people around us hold different opinions.


