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Essays on the Transmission of Risk and Volatility Across International Financial Markets

Evan Warshaw

The Graduate Center, City University of New York

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ESSAYS ON THE TRANSMISSION OF RISK AND VOLATILITY ACROSS INTERNATIONAL FINANCIAL MARKETS

by

EVAN WARSHAW

A dissertation submitted to the Graduate Faculty in Economics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York

2018
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This manuscript has been read and accepted by the Graduate Faculty in Economics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

ESSAYS ON THE TRANSMISSION OF RISK AND VOLATILITY ACROSS INTERNATIONAL FINANCIAL MARKETS

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Advisor: Professor Merih Uctum

This dissertation consists of three distinct but related chapters studying the behavior of international financial markets. Chapter 1 explores volatility spillovers between European equity and foreign exchange markets from 2009 until 2016. In contrast to traditional empirical methods, this study uses realized volatility estimates to analyze causal linkages across the frequency domain. Volatility spillovers are tested using the CAC 40, DAX, and FTSE 100 equity indices and the USD/GBP, USD/EUR, and GBP/EUR exchange rates. Results show that volatility spillovers are bidirectional and asymmetric across the frequency domain. Daily volatility spillover from equity to foreign exchange markets is significant at high, mid-range, and low frequencies, whereas foreign exchange to equity market spillover is significant only at lower frequencies. Weekly analysis reinforces these results. However, volatility spillover from equity to foreign exchange markets is less persistent and insignificant from the CAC 40 and DAX to the USD/EUR. These findings highlight the need to consider a wide-range of frequencies when testing for spillover effects.

Chapter 2 analyzes risk spillovers across North American equity markets from 1995 until 2016. Downside and upside Conditional Value-at-Risk (CoVaR) are estimated after modeling the dynamic dependence structure for each equity market pair using generalized autoregressive score (GAS) copulas. US-CAN and CAN-MX dynamic correlations trend upwards over the sample period while the US-CAN correlation fluctuates around a higher long-run average. Conditional tail
dependence is symmetric in all cases and increases substantially following the Global Financial Crisis, reflecting greater co-movement under extreme economic conditions. Downside and upside risk spillovers are significant and asymmetric for each equity market pair, where downside risk spillovers are more severe. Risk spillover magnitude varies significantly with conditioning direction. Asymmetric behavior is observed across high/low risk and high/low risk spillover periods.

The third and final chapter investigates the dynamic dependence structure between U.S. equity market returns and fluctuations in dollar strength using generalized autoregressive score (GAS) copulas. Analysis is conducted for both large and small equity market sizes, as well on a sector-specific basis to tease out differences attributed to international exposure. Significant variation is observed in the dynamic correlations over time, where they exhibit extreme persistence but with a tendency for rapid trend reversals. Dynamic correlations are negative on average with brief positive episodes. Symmetric tail dependence is significant for ten of the twelve equity market measures considered, peaking during periods of exuberance and financial stress when correlations are positive. Sub-sector tail dependence tends to follow the broader market behavior but with large deviations in overall magnitude across time.
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Chapter 1

Asymmetric Volatility Spillover Between European Equity and Foreign Exchange Markets: Evidence From The Frequency Domain

1.1 Introduction

Europe has experienced economic and socio-political turmoil in the years following the Global Financial Crisis. From 2010 until 2013, many European countries experienced sovereign debt crises. More recently, there are concerns with regards to a fragile Italian banking sector with high levels of non-performing loans and socio-political upheaval (e.g., Brexit). These factors, as well others, influence the behavior of European equity and foreign exchange markets. Such circumstances highlight the need for a better understanding of the transmission of volatility across these markets. In particular, both individual and institutional investors may gain from such an analysis by improving the assessment and management of risk. We approach the problem of untangling the
nature of volatility spillovers between European equity and foreign exchange markets from a new perspective within the context of the frequency domain, revealing further complexities in volatility spillovers.

What does the existence of volatility spillover between financial markets imply? This question has been widely addressed by the literature. The transmission of volatility is closely linked to the flow of information (Ross 1989). Volatility spillover is the market response to the flow of information. Engle et al. (1990) attribute the delay between the arrival and processing of information as one of the main contributing factors to the spread of volatility across markets. In addition, there may be financial market friction related delays. For example, trading costs and low asset liquidity may prohibit rapid actionable responses to news shocks. Furthermore, not all investors hold the same beliefs and thus may interpret and respond to market developments differently (Shalen 1993). Ultimately, volatility spillover across markets is a complex process with many factors affecting initiation, strength, and duration. Consequently, focusing on volatility spillover in terms of causality over varying frequencies is worthwhile.

We focus on the three largest equity markets in Europe (France, Germany, and the U.K.), proxied by the CAC 40, DAX, and FTSE 100, and their main underlying exchange rates (USD/EUR, GBP/EUR, and USD/GBP). We adopt a novel approach to analyze the nature of volatility spillovers between these markets. First, we use high frequency intra-day data to generate realized volatility measures at both the daily and weekly observational frequency. Utilizing realized volatility provides a “model free” means of capturing the integrated, or latent, volatility processes (Andersen et al. 2001, 2002, Barndorff-Nielsen and Shephard 2001). The most widely used alternative approaches are to estimate either generalized autoregressive conditional heteroskedastic (GARCH) or stochastic volatility models. However, these models are restrictive in their assumptions, provide a potentially over-smoothed approximation of volatility, and are potentially misspecified. Second, we use the Breitung and Candelon (2006) approach to test for causality across the frequency domain. Frequency domain analysis captures both directional and frequency-specific asymmetries,
allowing for richer analysis. To the best of our knowledge, this is the first work to analyze volatility spillovers across financial markets using realized volatility and frequency domain causal analysis.

Our findings show that volatility spillover between European equity and foreign exchange markets is bidirectional and asymmetric across the frequency domain in nearly all cases. Daily volatility spillover from equity prices to exchange rates is significant at high, mid-range, and low frequencies for all six equity indices and exchange rate pairs. Evidence of daily volatility spillover originating from foreign exchange markets is generally weaker but significant at low frequencies. Weekly analysis confirms these results for volatility spillover from exchange rates to equity prices. Some minor differences are observed for the equity to exchange rate direction when using weekly volatilities. However, overall results are consistent with the daily analysis. The result of bidirectional spillover is in contrast to most empirical studies that consider the similar variables and time periods. Furthermore, the analysis highlights the need to consider a wide range of frequencies (e.g., extremely low) when testing for the presence of volatility spillover across financial markets.

The remainder of the paper is organized as follows. Section 2 provides an overview of the relevant literature on volatility spillovers between equity and foreign exchange markets. Section 3 presents and discusses the frequency domain causality methodology. Section 4 presents the data. Section 5 discusses the empirical findings. Finally, Section 6 gives concluding remarks.

1.2 Literature Review

1.2.1 Theoretical Foundations

There are two main frameworks used to link equity and foreign exchange markets. The first, developed by Dornbusch and Fischer (1980), emphasizes the role of exchange rate competitiveness in driving trade flows which affect firm profits. Consider a domestic exporter and assume the firm’s stock price is the present discounted value of future cash flows. An increase (decrease) in exchange rate volatility increases (decreases) the level of uncertainty surrounding their real
income and expected future cash flows. Such a development should be reflected in the stock price’s behavior. A similar argument can be made for domestic importers. Here, exchange volatility affects expectations of future import costs and the production costs associated with using imported intermediate goods. While Jorion (1990) and Sercu and Vanhulle (1992) find some firm-level evidence supporting this theory, the aggregate impact of this causal channel is somewhat unclear. Ultimately, the degree to which exchange rate fluctuations affect the broader equity market depends on the degree of trade openness, relative mix of importers and exporters, and ability of domestic firms to hedge exchange rate risk through the use of derivatives contracts.

The second framework, referred to as the “portfolio balance model”, focuses the demand for financial assets and capital flows. Early work by Frankel (1983) and Branson and Henderson (1985) suggests that shifting asset demand drives capital flows which in turn impacts domestic currency demand. Assuming that there are no capital restrictions, a country with positive expectations of future growth (e.g., an economic boom) will likely experience a capital inflow. As a consequence, the domestic currency will appreciate. However, this approach, as well as the trade-flow model, are both partial equilibrium models and impose directionality.

In a general equilibrium framework, Hau and Rey (2006) build an international portfolio diversification model where investors cannot completely hedge exchange rate risk due to the presence of incomplete financial markets. Equilibrium equity prices, exchange rates, and capital flows are all determined jointly. Portfolio re-balancing is conducted in response to both equity price and exchange rate fluctuations. Suppose that a portfolio’s foreign equities rapidly appreciate in value. In the context of this model, re-balancing implies that the investor sells some of the foreign assets and buy domestic ones. Any foreign profits, assuming that there are for this example, must be converted to the domestic currency in order to purchase domestic equities. As such, the increase in the demand for domestic currency affects the exchange rate. Suppose instead that there is an exchange rate shock such as an unanticipated depreciation. The investor will buy and sell domestic and international equities to offset any changes to their portfolio’s risk profile. Therefore, the
initial exchange rate shock is reflected to some extent in the behavior of the respective equity price movements.

1.2.2 Characteristics of Volatility Spillovers

Volatility spillover between equity and foreign exchange markets is a complex process with empirical findings varying based on factors such as sample periods, financial development, and economic state. One question addressed in the literature is whether or not volatility spillovers are asymmetric. Here, asymmetry refers to the differences in the processing of positive and negative cross-market news shocks (e.g., leverage effects). Kanas (2000, 2002) find that volatility spillovers are positive and symmetric. More recent evidence suggests the prevalence of positive, asymmetric volatility spillover (Aloui, 2007; Morales, 2008; Walid et al., 2011). Leverage effects are generally strongest in the equity to exchange rate direction. Multivariate exponential GARCH (MV-EGARCH) models are estimated to capture asymmetric behavior, combining crossplacements of standardized residuals amongst variance equations with leveraged effects.

In addition to asymmetry in terms of positive versus negative shocks there is also evidence of asymmetry in spillover direction. Equity market volatility tends to more strongly influence foreign exchange market volatility than vice versa (Kanas, 2000, 2002; Apergis and Rezitis, 2001; Caporale et al., 2002; Aloui, 2007; Morales, 2008; Choi et al., 2009). Dominance of the equity market to foreign exchange rate path is observed for both developed and developing markets, as well as for commodity and non-commodity currencies.[1]

Studies by Aloui (2007), Choi et al. (2009), and Walid et al. (2011) suggest a time varying component to the volatility spillover relationship. Focusing on the New Zealand equity market and several NZD cross-rates, Choi et al. (2009) find that volatility spillover is weaker following the Asian Financial Crisis of 1997-1998. Aloui (2007) examines European equity and foreign

---

[1] Commodity currency behavior is closely tied to price fluctuations of goods such as precious metals, crude oil, natural gas, industrial metals, and ores. The three most liquid commodity currencies are the Australian Dollar (AUD), Canadian Dollar (CAD), and New Zealand Dollar (NZD).
exchange markets pre- and post-Euro adoption. Volatility spillover from equity to foreign exchange markets is stronger in both the pre- and post-Euro periods, exhibiting significant leverage effects. Spillover persistence is higher in the pre-Euro period for both transmission directions.

As opposed to performing sub-sample analysis, Walid et al. (2011) directly model time variation using a Markov-Switching (MS) EGARCH framework. They classify two regimes to capture general economic states: “calm” (high mean, low variance) and “turbulent” (low mean, high variance). Interestingly, they find a significant difference in the spillover sign across regimes. Volatility spillover is positive during calm regimes and negative during turbulent periods. News signals tend to be noisier during turbulent regimes, which may partially explain the sign change.

While the aforementioned empirical approaches to testing for volatility spillover can reveal direction, sign, magnitude, and leverage, they are limited by their parameterization. Volatility in a particular equity market tends to be highly persistent (Corsi, 2009). Introducing the necessary additional lags to capture long-memory volatility processes in a MV-EGARCH or BEKK-GARCH (Engle and Kroner, 1995) model increases the parameter space and further complicates estimation. As such, casual analysis provides an attractive alternative empirical approach.

1.2.3 Causal Linkages

Empirical research focusing on causal linkages between equity and foreign exchange market volatility employs econometric methods centered around either univariate GARCH (Cheung and Ng, 1996; Hong, 2001) or BEKK-GARCH models (Caporale et al., 2002). For the former, cross-correlation function (CCF) tests are performed at various lags and leads using the standardized squared residuals from a first-stage GARCH model. The BEKK-GARCH causality-in-variance (CIV) approach entails estimating a bivariate GARCH system and then separately performing upper and lower diagonal Wald tests on the covariance structure.

Using the CIV approach, Caporale et al. (2002) study the causal relationship between equity and foreign exchange volatility for several East Asian countries before and after the Asian Financial
Crisis of 1997-1998. Results depend on the level of financial development and sub-period with evidence in favor of unidirectional CIV from equity to foreign exchange markets. Similarly, Aloui (2007) find evidence in support of unidirectional CIV for French and German equity markets. Using the CCF methodology, Koseoglu and Cevik (2013) also report unidirectional causality in the equity to exchange rate direction for several Central and Eastern European countries.

There are limitations associated with the aforementioned econometric methodologies. While the CCF approach can test for causality at various lags and leads, it is hindered by its reliance on first-stage univariate GARCH estimates. Unaccounted for serial correlation in the conditional mean and variance equation increases the size of the test. Furthermore, the test’s power is reduced when considering very large lags and leads. Comparing the properties of CCF and CIV tests, Hafner and Herwartz (2008) find that the CCF test exhibits lower power and is more sensitive to misspecification than CIV. That said, the CIV test suffers from size and power distortions in the presence of extreme volatility clustering such as around financial crises (Javed and Mantalos, 2015). Furthermore, the CIV methodology only recovers spillover direction and the Wald restrictions are only tested at one lag in the covariance process.

Taking an alternative approach, Leung et al. (2017) estimate a two-stage regression model. First, they estimate univariate GARCH models. Next, regressions are performed on the recovered conditional volatilities. Control variables are included to account for financial stress, contagion, and crises. Consistent with past studies, volatility spillover is significant and positive. Volatility spillover strength between equity and foreign exchange markets is lower during financial crisis, reflecting the state-dependent nature explored by Walid et al. (2011). However, there are two serious econometric concerns which may adversely impact second-stage estimate reliability. Leung et al. (2017) use a highly restrictive GARCH model with no conditional mean structure, further assuming disturbances are normally distributed. Financial asset returns often exhibit highly non-normal behavior with fat-tails and skewness. Finally, their second-stage regression analysis implicitly assumes directional exogeneity from equity to foreign exchange markets. While instruments are
used to control for potential control variable endogeneity, no such effort is done to account for endogeneity in volatilities.

This study adopts a more flexible approach, providing richer causal analysis. Taking advantage of the information conveyed by high frequency data, intra-day returns are used to generate more accurate “model free” realized volatility measures. In doing so, we by-pass the typical first-stage GARCH model and lessen the chance of misspecification. Focusing on frequency domain, we address not only the existence of causality but can pinpoint the significant frequencies in each direction. Consider two daily asset returns A and B. Suppose there exists bidirectional causality in the volatilities of A and B. Further assume that causality is significant from A to B at high frequencies (< 5 days) but only at low frequencies (> 30 days) from B to A. Such a relationship can be recovered using frequency domain approach without leading to power distortions as with the CCF test. Finally, we can observe causal patterns at non-integer intervals.

1.3 Testing For Causality In The Frequency Domain

Following Breitung and Candelon (2006), we test for volatility spillovers between equity and foreign exchange markets using a frequency domain framework. Focusing on the frequency domain facilitates causality testing over a wide-range of frequencies (high-to-low) without requiring differing lag structures. This approach can reveal both directional and frequency-specific asymmetries.

Let \( z_t = [x_t, y_t]' \) be a two-dimensional vector for \( t = \{1, ..., T\} \). Assume \( z_t \) has a finite-order vector autoregressive (VAR) functional form given by

\[
\Theta(L)z_t = \epsilon_t, \tag{1.1}
\]

where \( \Theta(L) = I - \Theta_1 L - \Theta_2 L^2 - \cdots - \Theta_p L^p \) is a \((2 \times 2)\) polynomial lag operator such that \( L^k z_t = z_{t-k} \). Further assume that \( \epsilon_t \overset{i.i.d.}{\sim} (0, \Sigma) \) and \( \Sigma \) is positive definite. For simplicity, the following derivations abstract from the inclusion of deterministic terms such as a constant, trend,
or dummy variables in Eq.(1.1).

Let $G$ be the lower triangular matrix of the Cholesky decomposition $G'G = \Sigma^{-1}$ such that $E[\eta_t\eta_t'] = I$ and $\eta_t = G\epsilon_t$. If the system is stationary, then the moving average representation takes the form

$$z_t = \Phi(L)\epsilon_t = \begin{bmatrix} \Phi_{11}(L) & \Phi_{12}(L) \\ \Phi_{21}(L) & \Phi_{22}(L) \end{bmatrix} \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix},$$
$$= \Psi(L)\eta_t = \begin{bmatrix} \Psi_{11}(L) & \Psi_{12}(L) \\ \Psi_{21}(L) & \Psi_{22}(L) \end{bmatrix} \begin{bmatrix} \eta_{1,t} \\ \eta_{2,t} \end{bmatrix},$$

(1.2) (1.3)

where $\Phi(L) = \Theta(L)^{-1}$ and $\Psi(L) = \Phi(L)G^{-1}$.

From Eq.(1.3), the vector moving average spectral density of $x_t$ is expressed as

$$f_x(\omega) = \frac{1}{2\pi} \left\{ |\Psi_{11}(e^{-i\omega})|^2 + |\Psi_{12}(e^{-i\omega})|^2 \right\},$$

(1.4)

where $\omega \in (0, \pi)$ is the angular frequency\(^2\) Using $f_x(\omega)$, Geweke (1982) and Hosoya (1991) suggest following the measure of causality:

$$M_{y\rightarrow x}(\omega) = \log \left[ \frac{2\pi f_x(\omega)}{|\Psi_{11}(e^{-i\omega})|^2} \right],$$

(1.5)

$$= \log \left[ 1 + \frac{|\Psi_{12}(e^{-i\omega})|^2}{|\Psi_{11}(e^{-i\omega})|^2} \right].$$

(1.6)

$M_{y\rightarrow x}(\omega) = 0$ only if $|\Psi_{12}(e^{-i\omega})| = 0$. If this is true, then $y$ does not cause $x$ at frequency $\omega$.

\(^2\)Let $\tau$ be the observational frequency of the underlying data (e.g., daily, weekly, monthly, etc...). The relationship between the observational and angular frequencies takes the form $\tau = 2\pi/\omega \in (2, \infty)$. For example, if the underlying data is daily and $\omega = 0.5$, then $\tau \approx 12.6$ days.
the components of $\Psi_{12}(L)$,
\[ \Psi_{12}(L) = \frac{-g^{22}\Theta_{12}(L)}{|\Theta(L)|}, \tag{1.7} \]
where $g^{22}$ is the lower diagonal element of $G^{-1}$ and $|\Theta(L)|$ is the determinant of $\Theta(L)$. It follows that $y$ does not cause $x$ at frequency $\omega$ if
\[ |\Theta_{12}(e^{-i\omega})| = \left| \sum_{k=1}^{p} \theta_{12,k}\cos(k\omega) - \sum_{k=1}^{p} \theta_{12,k}\sin(k\omega) \right| = 0. \tag{1.8} \]
$\Theta_{11,j}$ and $\Theta_{12,j}$ are the coefficients of the lag polynomials $\Theta_{11}(L)$ and $\Theta_{12}(L)$.

To simplify the notation, re-parameterize $\alpha_j = \theta_{11,j}$ and $\beta_j = \theta_{12,j}$. The VAR equation for $x_t$ is then written as
\[ x_t = \alpha_1 x_{t-1} + \ldots + \alpha_p x_{t-p} + \beta_1 y_{t-1} + \ldots + \beta_p y_{t-p} + \epsilon_{1,t}. \tag{1.9} \]
The null hypothesis $M_y \to x(\omega) = 0$ is equivalent to the linear restriction
\[ H_0 : R(\omega)\beta = 0 \tag{1.10} \]
where $\beta = [\beta_1, \ldots, \beta_p]'$ and
\[ R(\omega) = \begin{bmatrix} \cos(\omega) & \cos(2\omega) & \cdots & \cos(p\omega) \\ \sin(\omega) & \sin(2\omega) & \cdots & \sin(p\omega) \end{bmatrix} \]
Eq.(1.10) is tested using an ordinary F test where $F \sim F_{2,T-2p}$ for $\omega \in (0, \pi)$. 
CHAPTER 1. ASYMMETRIC VOLATILITY SPILLOVER

1.4 Data

1.4.1 Realized Volatility Construction and Behavior

Collected from Bloomberg, the data consist of five-minute intra-day spot equity and exchange rate prices from January 2, 2009 through December 30, 2016. The CAC 40, DAX, and FTSE 100 indices proxy for French, German, and British equity markets, respectively. Equity indices are denominated in local currencies. We focus specifically on the USD/EUR, USD/GBP, and GBP/EUR exchange rates. Only intra-day observations that occur during regular trading hours are used. British, French, and German equity markets are open from 9:00AM to 5:30PM Central European Time (CET), leading to a total of 103 intra-day observations on complete trading days.\footnote{Incomplete trading days are a consequence of holiday related early closings. On such days, French and German equity markets close at 2:00PM CET while the British equity market closes at 1:30PM CET.} Intra-day returns are defined as \( r_{i,t} = 100 \times [\ln(P_{i,t}) - \ln(P_{i-1,t})] \) for \( i = \{1, \ldots, N\} \) returns and \( t = \{1, \ldots, T\} \) days.

Daily realized variance for each series calculated using the sum of squared intra-day returns\footnote{Restricting the analysis to regular trading hours removes the need to perform scaling adjustments such as those proposed by Martens (2002) and Hansen and Lunde (2005).}

\[
RV^d_t = \sum_{i=1}^{N} r_{i,t}^2. \tag{1.11}
\]

It follows that realized volatility, \( RVOL^d_t = \sqrt{RV^d_t} \). We do not make any explicit corrections for potential market micro-structure noise. Liu et al. (2015) show that 5-minute RV as calculated by Eq.(1.11) outperforms most other RV estimators in terms of in- and out-of-sample accuracy even in the presence of micro-structure noise. Moreover, their results are robust to asset class.

We proceed using the log of realized volatility, \( \log(RVOL^d_t) \), so that the data more closely resembles a Normal distribution and to avoid any potential non-negativity issues. Exploring differences that may arise at varying observational frequencies, weekly log-realized volatility is cal-
culated using an intra-week average of the daily estimates:

\[
\log(RVOL^w_s) = \frac{1}{K_s} \sum_{k=1}^{K_s} \log(RVOL_{k,s}).
\]  (1.12)

\(K_s\) is the number of daily observations in a given week \(s = \{1, ..., S\}\). This approach is similar to the traditional heterogeneous autoregressive (HAR) empirical methodology, which uses a five day trailing average instead of intra-week average.\(^5\)

Table 1.1 presents descriptive statistics for each series. Consistent with Corsi et al. (2008), each series exhibits non-normal distributional behavior, significant ARCH effects, and serial correlation. Bodart and Candelon (2009) show that neither non-normality nor conditional heteroskedasticity significantly impact the power or size of the frequency domain causality test. However, outliers are problematic, leading to size distortions. Meaningful differences between the traditional skewness and the outlier robust (MC-Skew) estimates suggests the presence of outliers.\(^6\)

For illustrative purposes we plot daily realized volatility (in levels) in Figure 1.1. Several outliers are clearly visible. Notable spikes in equity and exchange rate volatility are present towards the end of 2011 (European sovereign debt crisis), mid-2015 (negative oil shock and Chinese stock market crash), and June 2016 (Brexit). Bodart and Candelon (2009) note that even a small number of outliers can affect the power of the frequency domain causality test, ultimately leading to over-rejection of the null hypothesis of no causality for a given \(\omega\). As such, we need to control for outliers before proceeding with any causal analysis.

\(^5\)See Corsi et al. (2012) for further details.

\(^6\)Introduced by Brys et al. (2004), the medcouple approach estimates an outlier robust skewness measure.
Table 1.1: Descriptive Statistics For Log Realized Volatility

<table>
<thead>
<tr>
<th></th>
<th>Daily</th>
<th>Weekly</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>CAC40</td>
<td>DAX</td>
</tr>
<tr>
<td>Mean</td>
<td>-0.071</td>
<td>-0.086</td>
</tr>
<tr>
<td>Median</td>
<td>-0.101</td>
<td>-0.106</td>
</tr>
<tr>
<td>Max</td>
<td>1.563</td>
<td>1.651</td>
</tr>
<tr>
<td>Min</td>
<td>-1.600</td>
<td>-1.652</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>0.419</td>
<td>0.436</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.214</td>
<td>0.183</td>
</tr>
<tr>
<td>MC-Skew.</td>
<td>0.089</td>
<td>0.050</td>
</tr>
<tr>
<td>S.W.</td>
<td>0.995***</td>
<td>0.997***</td>
</tr>
<tr>
<td>LBQ(5)</td>
<td>4554***</td>
<td>4763***</td>
</tr>
<tr>
<td>ARCH(5)</td>
<td>625***</td>
<td>858***</td>
</tr>
<tr>
<td>Obs.</td>
<td>2050</td>
<td>2033</td>
</tr>
</tbody>
</table>

Notes: Daily US$ / EUR, GBP / EUR, and USD / GBP statistics are presented for each series paired against the CAC 40 but do not differ substantially for the other sample sizes (available upon request); MC-Skew. is the outlier robust medcouple skewness measure; S.W. is the Shapiro-Wilks statistic; Augmented Dickey-Fuller (ADF) unit root tests are performed including a constant term with optimal lag length determined by AIC; * p-val < 0.01, ** p-val < 0.05, *** p-val < 0.01.
1.4.2 Outlier Adjustments

We adopt the adjusted outlier (AO) methodology developed by Hubert and Vandervieren (2008) to isolate outliers in the data. This particular approach is attractive since it accounts for outliers in a manner robust to skewness and does not assume any particular underlying distribution such as when using the Median Absolute Deviation (MAD) methodology.
Let $X$ be a $(T \times 1)$ vector of data. Define $AO_t$ such that

$$AO_t = AO(x_t, X) = \begin{cases} \frac{x_t - \text{med}(X)}{w_2 - \text{med}(X)}, & \text{if } x_t > \text{med}(X) \\ \frac{\text{med}(X) - x_t}{\text{med}(X) - w_1}, & \text{if } x_t < \text{med}(X) \end{cases}$$

where $AO_t = 0$ only if $x_t = \text{med}(X)$ where $\text{med}(\cdot)$ is the median function and

$$w_1 = \begin{cases} Q_{0.25} - 1.5e^{-4MC} IQR, & \text{if } MC > 0 \\ Q_{0.25} - 1.5e^{-3MC} IQR, & \text{if } MC < 0 \end{cases}$$

$$w_2 = \begin{cases} Q_{0.75} + 1.5e^{3MC} IQR, & \text{if } MC > 0 \\ Q_{0.75} + 1.5e^{4MC} IQR, & \text{if } MC < 0 \end{cases}$$

$MC$ is the outlier robust skewness measure of $X$, $Q$ is the quantile function, and $IQR = Q_{0.75} - Q_{0.25}$. Note that if $MC = 0$ then $w_1$ and $w_2$ collapse to standard Tukey box plot whiskers.

$AO_t$ is calculated for all $t \in T$ yielding a $(T \times 1)$ vector $AO$. Define the cutoff statistic, $k$, calculated from the distribution of $AO$ as

$$k = Q_3(AO) + 1.5e^{3MC(AO)} IQR(AO).$$

If a given $AO_t > k$ then the underlying $x_t$ is marked as an outlier.

Table 1.2 displays the corresponding percentage of observations marked as outliers for each $\log(RVOL)$ series. There are a small but non-zero percentage of outliers for each of the daily series. Outliers are less problematic at the weekly frequency. Constructing weekly log-realized volatility as an intra-week average smooths over short-lived spikes in volatility.
Table 1.2: Percentage of Outliers

<table>
<thead>
<tr>
<th></th>
<th>CAC 40</th>
<th>DAX</th>
<th>FTSE 100</th>
<th>USD/EUR</th>
<th>GBP/EUR</th>
<th>USD/GBP</th>
</tr>
</thead>
<tbody>
<tr>
<td>Daily</td>
<td>0.59%</td>
<td>0.25%</td>
<td>0.10%</td>
<td>0.10%</td>
<td>0.05%</td>
<td>0.78%</td>
</tr>
<tr>
<td>Weekly</td>
<td>0.00%</td>
<td>0.24%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.96%</td>
</tr>
</tbody>
</table>

Notes: Daily USD/EUR, GBP/EUR, and USD/GBP outlier percentages correspond to each series paired against the CAC 40 but do differ substantially for the other sample sizes (available upon request).

Outliers are replaced with their $m$-observation centered average,

$$x_{m,j} = \left(\frac{1}{m-1}\right) \sum_{k=j-m+2}^{j} x_k,$$

where $j = \{t, s\}$ for daily or weekly frequencies. The range of observations ($m$) is set equal to 10 if the underlying data are daily and 6 if weekly. A 10-day centered average is consistent with Bodart and Candelon (2009) for daily data. Furthermore, a wide range is necessary to minimize the potential issue of clustered outliers due to volatility persistence.

1.5 Empirical Results

Each daily VAR model is estimated including a constant term and set of day of the week dummies, $D = \{D_{TU}, D_W, D_{TH}, D_F\}$. Daily dummy variables are included to control for potential intra-week seasonality as observed in international equity (Berument and Kiymaz 2001, Kiymaz and Berument 2003, Scharth and Medeiros 2009) and to a lesser extent for foreign exchange markets.
Each weekly VAR model includes only a constant term.

Tables 1.3 presents the optimal VAR lag lengths by group and frequency. Optimal lag length is determined by AIC. The large number of lags for each daily model reflects volatility persistence in both equity and foreign exchange markets. The weekly models, on the other hand, are much smaller systems with either three or four lags.

<table>
<thead>
<tr>
<th>Series</th>
<th>USD/EUR</th>
<th>GBP/EUR</th>
<th>USD/GBP</th>
</tr>
</thead>
<tbody>
<tr>
<td>CAC 40</td>
<td>Daily</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Weekly</td>
<td>4</td>
<td>4</td>
</tr>
<tr>
<td>DAX</td>
<td>Daily</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Weekly</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>FTSE 100</td>
<td>Daily</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td>Weekly</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>3</td>
<td>3</td>
</tr>
</tbody>
</table>

Notes: Daily VARs include a constant term and set of exogenous day of the week dummy variables while the weekly VARs include only a constant term; Lag length determined by AIC.

Frequency domain causality test results are presented by plotting the p-values corresponding to each $F$ test for Eq.(1.10) over the range of $\omega \in (0, \pi)$. $\omega$ are plotted on the x-axis and p-values on the y-axis. P-values below 0.10 imply rejection of the null hypothesis of no causality at a given $\omega$. Evidence of causality for $\omega$ near $\pi$ indicates short-run (high frequency) volatility spillover, whereas causality at $\omega$ near 0 are reflective of co-movement in the long-run (low frequency) or permanent components of market volatility (Bodart and Candelon 2009).

\footnote{Full results are available upon request.}
1.5.1 Daily Volatility Spillover

Figure 1.2 presents the frequency domain causality test results at the daily level. For the CAC 40, volatility spillover is stronger in the equity to exchange rate direction than vice versa for both exchange rate pairs. More specifically, CAC 40 volatility spillover is significant across most tested frequencies (high, mid-range, and low). In contrast, USD/EUR and GBP/EUR volatility only cause CAC 40 volatility at mid-to-low frequencies. For the USD/EUR, we observe marginal rejection for $\omega \in [1.908, 2.042]$ but strong evidence of causality for $\omega \in [0.686, 1.218]$ and $\omega \leq 0.360$. Disregarding the former, this translates to significant volatility spillover between approximately 5 to 9 days and beyond 17 days. Similarly, GBP/EUR volatility only causes CAC 40 volatility for low frequencies $\omega \leq 0.578$ ($\geq 11$ days). These findings are contrary to the evidence presented by [Aloui (2007)] in the post-Euro period, who only finds evidence of causality using the cross-correlation function (CCF) test from the CAC 40 to the USD/EUR. Focusing on the pre-Euro period, [Kanas (2000)] and [Yang et al. (2004)] similarly only find significant volatility spillover from the CAC 40 to USD/EUR.

As with the CAC 40, volatility spillover from the DAX to the USD/EUR and GBP/EUR exchange rates is significant at high, mid-range, and low frequencies. Volatility spillover from the USD/EUR and GBP/EUR to the DAX is primarily significant at low frequencies. For the USD/EUR, strong evidence of spillover is present for $\omega \in [0.252, 0.630]$ (10 to 25 days). Focusing on the GBP/EUR, we reject the null of no causality for $\omega \in [0.195, 0.549]$. Here, this translates to roughly 11 to 32 days. As before, our finding of bidirectional causality between the DAX and USD/EUR deviates from [Aloui (2007)] post-Euro analysis. In his study, he only finds significant spillover from the USD/EUR to the DAX.
Volatility spillover is again stronger in the equity to foreign exchange market direction for the FTSE 100 against the GBP/EUR and USD/GBP. However, as compared to the CAC 40 and DAX analysis, we see weaker evidence of causality at mid-range frequencies between 3 and 10.
days. This behavior is more prominent for volatility spillover from the FTSE 100 to the USD/GBP exchange rate. GBP/EUR volatility significantly causes FTSE 100 volatility for $\omega \leq 1.260$ ($\geq 5$ days), whereas USD/GBP volatility spillover is weakly significant for $\omega \in [0.943, 1.220]$ and strongly significant for $\omega \leq 0.609$ ($\geq 10$ days). [Leung et al. (2017)] also find evidence of significant volatility spillover from the USD/GBP and GBP/EUR to the FTSE 100. However, their results suggest a weaker relationship when considering the GBP/EUR than we find here. In their analysis, they do not consider spillover in the equity to exchange rate direction. Focusing on the pre-Euro period, [Kanas (2000, 2002)] only find significant volatility spillover from the broader FT All Share index to the USD/GBP.

A clear pattern emerges across markets when focusing solely on the equity to exchange rate direction. Outside of the (FTSE 100 USD/GBP) model, spillover is insignificant around $\omega \approx 1.9$ to $\omega \approx 2.3$; this translates to roughly 2.7 to 3.3 days. Given that spillover is significant in the $\omega \approx [2.3, \pi)$ range, this pattern of behavior potentially suggests overshooting in the foreign exchange market. More specifically, equity market volatility causes an initial over-reaction in exchange rate volatility followed by an under-reaction.

Overall, these results suggest bidirectional volatility spillover that is asymmetric across the frequency domain. Moreover, our findings are consistent with the more recent portfolio balance framework [Hau and Rey 2006]. There are substantial differences in the significant causal frequencies by direction, an important quality of the relationship between equity and foreign exchange markets overlooked by past empirical work. Equity market volatility causes exchange rate volatility at high, mid-range, and low frequencies for all six equity index and exchange rate pairs. Volatility spillover in the other direction tends to be weaker but still significant. Exchange rate volatility primarily causes exchange market volatility at lower frequencies; here, volatility spillover is highly persistent except for in the (DAX, USD/EUR) and (DAX, GBP/EUR) cases. That said, volatility spillover from the USD/EUR and GBP/EUR is still significant up to approximately 25 and 32 days, respectively.
1.5.2 Weekly Volatility Spillover

We now focus on volatility spillover occurring at a weekly observational frequency to see if market behavior deviates from what is observed at the daily frequency. In part, this approach is motivated by Corsi (2009), who remarks that traders react differently to volatility measured over various horizon lengths. More specifically, long-term volatility may matter more to short-term traders than short-term volatility. At the same time, the processing speed of information conveyed over long versus short periods of time may vary. Portfolio managers, or individual and institutional investors, may react differently to slower moving volatility signals.

Figure 1.3 presents the weekly volatility spillover results for each equity and exchange rate pair. We find evidence of bidirectional causality for four of the six equity market and exchange rate pairs, where volatility spillover is significant in the exchange rate to equity direction for each pair. Moreover, we observe the same pattern in this direction as compared with the daily analysis. On average, exchange rate volatility causes equity price volatility beyond four weeks at the high end (FTSE 100, GBP/EUR) and seven weeks on the low end (DAX, USD/EUR).

The results somewhat differ when focusing on the equity to exchange rate direction. We fail to reject the null hypothesis of no causality at any $\omega \in (0, \pi)$ for the CAC 40 and DAX to the USD/EUR exchange rate, implying that volatility spillover is insignificant. However, there is evidence of significant spillover for the CAC 40 and DAX towards the GBP/EUR exchange rate. For the former this occurs between $\omega \in [0.411, 1.491]$ (4 to 15 weeks) and the latter over $\omega \in [0.669, \pi]$ (2 to 9 weeks). Volatility spillover from the FTSE 100 to the GBP/EUR and USD/GBP exhibits a similar behavior as the DAX and GBP/EUR where spillover is significant up to about 7 and 11 weeks, respectively.
These findings reinforce our daily analysis, suggesting bidirectional volatility spillover between equity and foreign exchange markets which is asymmetric in the frequency domain. The exceptions being the unidirectional results for the (CAC 40, USD/EUR) and (DAX, USD/EUR) pairs
which reflect the trade-flow narrative (Dornbusch and Fischer, 1980). While the weekly volatility spillover from equity to foreign exchange markets do not exhibit the same degree of persistence as with the daily analysis, causality remains significant for on average up to two months.

1.6 Concluding Remarks

This study analyzes volatility spillovers between the CAC 40, DAX, and FTSE 100, and the USD/EUR, GBP/EUR, and USD/GBP exchange rates using a frequency domain approach over 2009 through 2016. Starting from high-frequency intra-day data, we construct realized volatility estimates for each series to avoid potential complications associated with GARCH and stochastic volatility models. The main contribution is to focus on volatility spillover between these markets through the frequency domain and in the process highlight overlooked extreme persistence. We adopt the Breitung and Candelon (2006) VAR-based frequency domain framework to conduct our analysis.

Our empirical findings are summarized as follows. Volatility spillovers between European equity and foreign exchange markets are bidirectional and asymmetric across the frequency domain. At the daily observational frequency, we find a stronger causal relationship moving from equity to foreign exchange markets than vice versa. Volatility spillover in this direction is significant at high, mid-range, and low frequencies for all six equity indices and exchange rate pairs. On the other hand, volatility spillover from the foreign exchange to equity markets is significant only at lower frequencies. Bidirectional causality is consistent with the portfolio balance approach to modeling equity prices and exchange rates. Mid-to-low frequency spillover is suggestive of portfolio re-balancing effects, whereas high-frequency spillover is more likely to be attributable to market entry/exit and contagion effects. Overall, our results suggest a stronger relationship in the exchange rate to equity direction than the existing literature, reflecting the importance of considering especially low frequencies.
Focusing on a weekly volatility, we observe the same relationship in the foreign exchange to equity market direction. However, volatility spillover is less persistent in the equity to foreign exchange market direction. Only at the weekly level do we find any evidence of unidirectional causality, occurring for the (CAC 40, USD/EUR) and (DAX, USD/EUR) pairs. Overall, the weekly analysis re-affirms the findings of daily volatility spillover.

In summary, our results reveal that volatility transmission between European equity and foreign exchange markets is asymmetric across the frequency domain. Ignoring low frequencies may lead to inaccurate assessments of the strength and degree of volatility spillover persistence. As such, considering volatility spillover across the frequency domain is particularly important for risk management. One of the challenges that a portfolio manager faces is to both quantify their portfolio’s current level of risk and anticipate future risk. The analysis presented in this study is useful with regards to the latter. Furthermore, the rise of algorithm based high-frequency trading places a greater emphasis on predicting market volatility. Such an investment strategies tends to perform best when markets are volatile and pricing discrepancies arise.

Further analysis may explore what differences emerge when focusing on volatility spillover in the frequency domain between emerging equity and foreign exchange markets. Another potential avenue is to extend the Breitung and Candelon (2006) framework to directly account for heterogeneous autoregressive (HAR) effects.
Chapter 2

Extreme Dependence and Risk Spillovers In North American Equity Markets

2.1 Introduction

The North American economic region has become increasingly integrated following the adoption of NAFTA and floating of the Mexican Peso in 1994. The removal of trade barriers and capital controls led to a substantial increase in cross-border trade in goods and services and foreign direct investment (FDI). Over 1995 through 2016, regional FDI increased by over 470% (US BEA; Parliament of Canada); the largest increase stems from cross-border FDI between Canada and Mexico. Total trade in goods and services for the US with Canada and Mexico increased nearly one- and three-fold, respectively. Similarly, trade between Canada and Mexico increased by 485% (Parliament of Canada). At the same time, there has been a marked increase in cross-border buying and selling of equities. Gross portfolio flows between the US and Canada (Mexico) increased by 281% (502%) (US TIC). More widespread investing across North American equity markets leaves portfolio managers, as well as individual investors, with greater exposure to regional shocks. As such, understanding the transmission of risk across US, Canadian, and Mexican equity markets is
CHAPTER 2. EQUITY MARKET RISK SPILLOVERS

essential for the execution of effective diversification strategies, as well as risk analysis.

We explore the impact of increased regional integration by addressing two specific questions. First, how do North American equity markets move together over time and is there evidence of significant tail behavior (joint co-movement at the extremes)? We capture this behavior by estimating pair-wise dynamic dependence structures using the generalized autoregressive score (GAS) copula framework of Creal et al. (2013). Second, what can be inferred about the nature of downside and upside risk spillovers across these equity markets given our understanding of their dependence structures? Risk spillovers are calculated using recent advances in the estimation of downside and upside Conditional Value-at-Risk (CoVaR) by means of dynamic copulas (Mainik and Schanning, 2014; Reboredo and Ugolini, 2015); downside (upside) risk reflects potential extreme long (short) position losses. Risk spillover significance is tested using the bootstrapped two-sample Kolmogorov-Smirnov (KS) test developed by Abadie (2002). To the best of the author’s knowledge, this is the first study to both rigorously examine the dependence structures and explore the transmission of risk across US, Canadian, and Mexican equity markets using dynamic copulas.

Our findings show that the relationship amongst US, Canadian, and Mexican equity markets strengthens over time. Dynamic correlations for the US-MX and CAN-MX equity market pairs trend upwards over the sample period. The US-CAN correlation does not exhibit any substantial trend but is somewhat stronger in the latter half of the sample. Conditional tail dependence, a measure of extreme co-movement, increases substantially during and after the Global Financial Crisis. Ultimately, North American equity markets exhibit greater co-movement during both calm and extreme periods. Rising correlations are consistent with the existing literature and real economic data trends; however, few, if any, studies consider the evolution of tail dependence amongst North American equity markets.

Drawing from the dynamic dependence structure estimates, we find downside and upside risk spillovers are significant for each equity market pair and spillover direction. Moreover, risk spillovers are asymmetric along two dimensions. First, downside risk spillovers are larger in mag-
CHAPTER 2. EQUITY MARKET RISK SPILLOVERS

magnitude than upside risk spillovers in all cases. Potential long-position losses conditional on an extreme decline in the designated secondary equity market value are greater than short-position losses conditional on an extreme increase in secondary market value. Second, downside and upside risk spillover severity varies significantly in spillover direction. The relative size, development, and importance of each equity market in a given pair greatly influences the degree of risk spillover. These results hold when focusing on both periods of high/low risk and high/low risk spillover.

The rest of the paper is as follows. Section 2 surveys recent empirical literature on the behavior of North American equity markets. In Section 3, we discuss all of the employed econometric methodologies. Section 4 details the data. Section 5 presents the results with accompanying analysis. Concluding remarks are given in Section 6.

2.2 Literature Review

Most empirical analysis on the relationship between US, Canadian, and Mexican equity markets searches for evidence of co-movement and long-run behavior as indicated by cointegration. The motivation for this approach stems from the passage and later implementation of NAFTA on January 1, 1994. Typically, these studies take three approaches: i) pre- and post-NAFTA cointegration tests; ii) cointegration tests in the presence of structural breaks; and/or iii) dynamic correlation modeling.


1The Mexican Peso crisis is another but less emphasized potential source of structural change.
time-variation and instability in the long-run relationships amongst each of the equity market pairs. Stable relationships are only found during periods of greater financial stress, reinforcing the notion that international equity markets exhibit significant tail dependence.

Lahrech and Sylwester (2013) estimate dynamic correlations for US, Canadian, and Mexican equity market returns using a tri-variate extension of the ADCC-GARCH model (Caporale et al., 2006). While the US-CAN dynamic correlation is relatively stable, the pair-wise time varying correlations for US-MX and CAN-MX display upward trends in the post-NAFTA period. The analysis presented by Lahrech and Sylwester (2013) suffers from a flaw. ADCC-GARCH, and DCC-GARCH models in general, fail to take into account the possibility of tail dependence. In utilizing dynamic copulas, we provide a richer analysis that examines not only the time-varying behavior of cross-market correlations but also the dependence across markets during periods of extreme tail behavior.

Few, if any, studies rigorously study the underlying dependence structure between all three equity markets. Ning (2009) employs a dynamic copula analysis in the spirit of Patton (2006b). However, Ning (2009) only considers the US-CAN relationship and neither US-MX nor CAN-MX. Furthermore, the utilized methodology has several shortcomings. Most notably, the dynamic structures are independent of the underlying copula density function; this issue potentially explains their strangely behaving lower and upper US-CAN tail dependence estimates. We directly link model dynamics to the underlying copula density using a generalized autoregressive score (GAS) copula approach of Creal et al. (2013).

More closely related to the methodological approach of this paper, Ji et al. (2017) analyze equity market risk spillovers from the US to the other G7 countries using Markov Switching copulas over January 1915 through February 2017. Risk spillovers are found to be significant and asymmetric. Downside risk spillover is stronger than upside risk spillover, and spillover originating

\footnotetext[2]{Using several rolling windows, Phengpis and Swanson (2006) also find correlations trending upwards for the US-MX and CAN-MX equity market pairs.}
from the US is stronger than from the rest of the G7. However, their analysis suffers from several
drawbacks. First, they use an extremely long one-hundred year sample but only allow for two
potential regimes. The past century is marked by significant structural change in capital markets,
foreign exchange markets, and evolving monetary policy. Moreover, Ji et al. (2017) use monthly
data, which yields over-smoothed estimates of dynamic correlations, tail dependence, and thus
downside and upside risk spillovers. Finally, as with Ning (2009), copula dynamics are ad-hoc and
independent of the underlying copula density function.

2.3 Methodology

2.3.1 Copula Functions and Dependence

We employ the conditional copula model proposed by Patton (2006b), which builds upon the foun-
dational work of Sklar (1959). Define $Y_t = [Y_{1,t}, ..., Y_{N,t}]'$. If $Y_t$ has the conditional joint distribu-
tion $F_t$ and conditional marginal distributions $\{F_{1,t}, ..., F_{N,t}\}$, then

$$ Y_t | F_{t-1} \sim F_t = C_t(F_{1,t}, ..., F_{N,t}). \quad (2.1) $$

$C_t$ is the conditional copula of $Y_t$ and $F_{t-1} = g(Y_{t-1}, Y_{t-2}, ...)$. Define $U_t = [u_{1,t}, ..., u_{N,t}]'$ where $u_{i,t} = F_{i,t}(Y_{i,t})$ for $i = \{1, ..., N\}$. It follows that,

$$ U_t | F_{t-1} \sim C_t. \quad (2.2) $$

Therefore, we are able to recover the conditional joint distribution, $F_t$, by combining estimates of
the conditional marginal distributions together with some specified conditional copula function;
this approach is preferable to directly estimating high-dimensional multivariate distributions. In
this study, we consider the bivariate case where $N = 2$.

In addition to estimating traditional measures of correspondence such as linear and rank order
correlation coefficients, copulas can also be used to examine tail behavior. Define the lower and upper tail dependence coefficients $\lambda_L$ and $\lambda_U$ as

$$
\begin{align*}
\lambda_L &= \lim_{u \to 0} P[F_1(Y_1) \leq u | F_2(Y_2) \leq u] = \lim_{u \to 0} \frac{C(u, u)}{u} \\
\lambda_U &= \lim_{u \to 1} P[F_1(Y_1) \geq u | F_2(Y_2) \geq u] = \lim_{u \to 1} \frac{1 - 2u + C(u, u)}{1 - u},
\end{align*}
$$

where $\lambda_L, \lambda_U \in [0, 1]$. Significant lower (upper) tail dependence reflects a tendency for $Y_1$ and $Y_2$ to jointly crash (boom).

Table 2.1 presents the functional forms with corresponding tail dependence measures for several copulas commonly used in financial economics. The Normal copula traditionally serves as a benchmark model as it does not allow for either upper or lower tail dependence. Clayton and Gumbel copulas are asymmetric copulas that only capture lower and upper tail dependence, respectively. Rotated, or survival, forms of each copula can be attained by the 180-degree data transformation $(u_{1,t}, u_{2,t}) \rightarrow (1 - u_{1,t}, 1 - u_{2,t})$. Finally, the Student’s t-copula allows for symmetric upper and lower tail dependence and converges to the Normal copula as $\nu \to \infty$.

<table>
<thead>
<tr>
<th>Name</th>
<th>Copula Function</th>
<th>Parameter(s)</th>
<th>Tail Dependence</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td>$C_N(u_1, u_2; \rho) = \phi^{-1}(u_1) \phi^{-1}(u_2)$</td>
<td>$\rho \in (-1, 1)$</td>
<td>$\lambda_L = \lambda_U = 0$</td>
</tr>
<tr>
<td>Clayton</td>
<td>$C_C(u_1, u_2; \delta) = \left(u_1^{-\delta} + u_2^{-\delta}\right)^{-1/\delta}$</td>
<td>$\delta \in (0, \infty)$</td>
<td>$\lambda_L = 2^{-1/\delta}, \lambda_U = 0$</td>
</tr>
<tr>
<td>Rotated Clayton</td>
<td>$C_{RC}(u_1, u_2; \delta) = C_C(1 - u_1, 1 - u_2; \delta) + u_1 + u_2 - 1$</td>
<td>$\delta \in (0, \infty)$</td>
<td>$\lambda_L = 0, \lambda_U = 2^{-1/\delta}$</td>
</tr>
<tr>
<td>Gumbel</td>
<td>$C_G(u_1, u_2; \delta) = \exp \left(-\left(\log(u_1)\right)^\delta + \left(\log(u_2)\right)^\delta\right)^{1/\delta}$</td>
<td>$\delta \in [1, \infty)$</td>
<td>$\lambda_L = 0, \lambda_U = 2 - 2^{1/\delta}$</td>
</tr>
<tr>
<td>Rotated Gumbel</td>
<td>$C_{RG}(u_1, u_2; \delta) = C_G(1 - u_1, 1 - u_2; \delta) + u_1 + u_2 - 1$</td>
<td>$\delta \in [1, \infty)$</td>
<td>$\lambda_L = 2 - 2^{1/\delta}, \lambda_U = 0$</td>
</tr>
<tr>
<td>Student’s t</td>
<td>$C_T(u_1, u_2; \rho, \nu) = T_{\rho, \nu}\left(t_1^{-1}(u_1), t_2^{-1}(u_2)\right)$</td>
<td>$\rho \in (-1, 1), \nu \in (2, \infty)$</td>
<td>$\lambda_L = \lambda_U = h(\rho, \nu)$</td>
</tr>
</tbody>
</table>

Notes: $\phi$, $t$, and $T_{\rho, \nu}$ are the bivariate standard normal, univariate $t$, and bivariate $t$-distribution cumulative distribution functions, respectively; $\rho$ and $\nu$ are the Pearson correlation coefficient and $t$ degree-of-freedom parameter; $h(\rho, \nu) = 2t_{\nu+1} \left(-\sqrt{\nu} + \sqrt{1 - \rho}/\sqrt{1 + \rho}\right)$.  

Finally, the Student’s t-copula allows for symmetric upper and lower tail dependence and converges to the Normal copula as $\nu \to \infty$.  


2.3.2 GAS Dynamics

The conditional copula can be extended in numerous ways to allow for dynamic behavior. In this study, we adopt the Creal et al. (2013) generalized autoregressive score (GAS) methodology. The GAS model presents a simple and intuitive observation driven approach where dynamics are tied to the score of the copula density function with respect to the time-varying parameter(s) at each point in time.

For simplicity, assume that the conditional copula function \( C_t(\cdot) \) possesses only one time-varying parameter, \( \delta_t \). As shown in Table 2.1, \( \delta_t \) is often restricted to a certain parameter space. Following Creal et al. (2013), we model the transformed parameter,

\[
f_t = h(\delta_t) \iff \delta_t = h^{-1}(f_t).
\]  

(2.5)

The GAS model is then described by the following three equations:

\[
f_{t+1} = \omega + \alpha \frac{1}{2} I_t^{-1} s_t + \beta f_t
\]

(2.6)

\[
s_t = \frac{\partial}{\partial \delta_t} \log c_t(u_{1,t}, u_{2,t}; \delta_t)
\]

(2.7)

\[
I_t \equiv E_{t-1}[s_t s_t'] = I(\delta_t),
\]

(2.8)

where \( \beta \in (0, 1) \). The “forcing variable”, \( s_t \), is determined by the shape of the copula density function at each point in time as captured by the score with respect to \( \delta_t \). \( s_t > 0 \) \( (< 0) \) implies a strengthening (weakening) relationship.

---

\[\footnote{Manner and Reznikova (2012) provide a detailed summary on recent advances in the estimation of time-varying copulas.}

\[\footnote{Creal et al. (2013) highlight the GAS framework’s flexibility by showing how it can be used to recover more traditional GARCH and multiplicative error models (MEM) such as ACD and ACI.}\]
CHAPTER 2. EQUITY MARKET RISK SPILLOVERS

2.3.3 Marginal Distribution Models

We estimate the following conditional mean and variance models to filter the data and recover the marginal distributions:

\[ Y_{i,t} = \mu_i(Z_{t-1}; \theta_i) + \sigma_i(Z_{t-1}; \theta_i)\epsilon_{i,t}, \quad i = 1, 2, 3 \] \hspace{1cm} (2.9)

\[ \epsilon_{i,t} \mid \mathcal{F}_{t-1} \sim i.i.d \ F_{skew-t}(0, 1; \theta_i), \forall \ t \] \hspace{1cm} (2.10)

where \( Z_{t-1} \in \mathcal{F}_{t-1} \). The conditional mean equation is estimated using an autoregressive model. For the variance equation, we use the GJR-GARCH specification of Glosten et al. (1993). Optimal conditional mean and variance equation variable orders are determined by comparing AIC. We allow for up to five lags in the mean equation and consider variance orders of (1,0,1) through (2,2,2). \( F_{skew-t} \) represents the Hansen (1994) skewed-t distribution, which nests the symmetric Normal and Student’s t-distributions as special cases.

The following probability integral transformation is performed after estimating each optimal model, \( \hat{u}_{i,t} = F_{skew-t}(\hat{\epsilon}_{i,t}; \hat{\theta}_i) \), where \( \hat{\epsilon}_{i,t} \) are the estimated standardized residuals from Eqs.(2.9)-(2.10). If the optimal marginal model is correctly specified then \( \hat{u}_{i,t} \sim i.i.d \ U[0, 1] \). We employ several goodness-of-fit tests to ensure that this condition is satisfied.

2.3.4 Copula Estimation and Inference

Conditional copulas can be estimated using traditional maximum likelihood methods. Differentiating Eq.(1) for \( N = 2 \) yields,

\[ f_t(Y_t; \theta) = f_{1,t}(Y_{1,t}; \theta_1) \cdot f_{2,t}(Y_{2,t}; \theta_2) \cdot c_t(F_{1,t}(Y_{1,t}; \theta_1), F_{2,t}(Y_{2,t}; \theta_2); \theta_c), \] \hspace{1cm} (2.11)

where

\[ c_t(F_{1,t}(Y_{1,t}; \theta_1), F_{2,t}(Y_{2,t}; \theta_2); \theta_c) = \frac{\partial^2 C_t(F_{1,t}(Y_{1,t}; \theta_1), F_{2,t}(Y_{2,t}; \theta_2); \theta_c)}{\partial F_{1,t}(Y_{1,t}; \theta_1) \partial F_{2,t}(Y_{2,t}; \theta_2)} \]
and \( f_{i,t}(Y_{i,t}; \theta_i) = \partial F_{i,t}(Y_{i,t}; \theta_i)/\partial Y_{i,t} \). Taking the natural log of Eq.(2.11) and summing over \( t = \{1, \ldots, T\} \), we can define the joint log-likelihood function,

\[
L(\theta) = \sum_{t=1}^{T} \log f_{1,t}(Y_{1,t}; \theta_1) + \sum_{t=1}^{T} \log f_{2,t}(Y_{2,t}; \theta_2) \\
+ \sum_{t=1}^{T} \log c_t(F_{1,t}(Y_{1,t}; \theta_1), F_{2,t}(Y_{2,t}; \theta_2); \theta_c),
\]

where \( \theta = \theta(\theta_1, \theta_2, \theta_c) \).

Equation (2.12) is estimated using the two-stage ‘Inference Functions for the Margins’ (IFM) procedure [Joe and Xu, 1996]. First, \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \) are each separately estimated by maximum likelihood. \( \hat{\theta}_c \) is then estimated by maximum likelihood conditional on \( \hat{\theta}_1 \) and \( \hat{\theta}_2 \). As Patton (2013) notes, second-stage copula standard errors are too small in absence of controlling for first-stage parameter estimation error. We employ the stationary block-bootstrap of Politis and Romano (1994) using \( N = 100 \) replications to calculate robust second-stage standard errors. Average block length is determined by applying the Politis and White (2004) test to the estimated dynamic copula parameter.

### 2.3.5 Copulas and Conditional Value-at-Risk Measures

Let \( Y_t = [Y_{1,t}, Y_{2,t}]' \) be some financial asset returns for \( t = \{1, \ldots, T\} \). Downside and upside Value-at-Risk (VaR) for \( Y_{i,t} \) at confidence level \( (1 - \alpha) \) and time-horizon \( t \) are defined as \( P(Y_{i,t} \leq VaR_{\alpha,t}) = \alpha \) and \( P(Y_{i,t} \geq VaR_{1-\alpha,t}) = \alpha \), respectively. We can compute these values directly using the estimated marginal models from Eqs.(2.9)-(2.10). \( VaR_{\alpha,t}(Y_{i,t}) = \hat{\mu}_{i,t} + F_{skew-t}^{-1}(\alpha; \hat{\theta}_i)\hat{\sigma}_{i,t} \) and \( VaR_{1-\alpha,t}(Y_{i,t}) = \hat{\mu}_{i,t} + F_{skew-t}^{-1}(1 - \alpha; \hat{\theta}_i)\hat{\sigma}_{i,t} \), where \( \hat{\mu}_{i,t} \) and \( \hat{\sigma}_{i,t} \) are the estimated conditional mean and volatilities. \( F_{skew-t}^{-1}(\alpha; \hat{\theta}_i) \) and \( F_{skew-t}^{-1}(1 - \alpha; \hat{\theta}_i) \) represent the \( \alpha \) and \( (1 - \alpha) \) quantiles of the corresponding estimated skewed t-distribution.

Joe (1997) shows the IFM procedure is consistent, while Joe (2005) and Patton (2006a) show the efficiency loss as compared to a one-stage estimation is relatively small.
In practice, \(VaR_{\alpha,t}\) and \(VaR_{1-\alpha,t}\) are used to capture potential long (short) position losses due to an extreme decrease (increase) in the value of \(Y_{i,t}\). However, these measures only reflect isolated risk and ignore the current behavior of \(Y_{j,t}\) for \(i \neq j\). Within the context of North American equity markets, any measure of downside and upside risk should also take into consideration the behavior of one or more of the other equity markets; this is particularly important given the relative size and importance of the different equity markets (e.g., US versus Mexico).

Following [Girardi and Ergün (2013)], downside and upside Conditional Value-at-Risk (CoVaR) at confidence levels \((1 - \alpha)\) and \((1 - \beta)\) and time-horizon \(t\) are defined as

\[
P(Y_{i,t} \leq CoVaR_{\beta,t}(Y_{i,t})|Y_{j,t} \leq VaR_{\alpha,t}(Y_{j,t})) = \beta, \quad i \neq j
\]  

\[
P(Y_{i,t} \geq CoVaR_{\beta,t}(Y_{i,t})|Y_{j,t} \geq VaR_{1-\alpha,t}(Y_{j,t})) = \beta, \quad i \neq j.
\]  

[Mainik and Schaanning (2014)] show that if the conditional joint distribution of \(Y_{1,t}\) and \(Y_{2,t}\) can be represented by \(F_t = C_t(F_{1,t}, F_{2,t})\), then \(CoVaR_{\beta,t}\) can be represented in terms of the conditional copula. Without loss of generality, consider \(CoVaR_{\beta,t}(Y_{i,t})\). Suppressing variable \(Y_{i,t}\) and parameter \((\theta)\) notation for brevity and re-writing Eqs.(2.13)-(2.14) yields

\[
C_t(F_1(CoVaR_{\beta,t}), F_2(VaR_{\alpha,t})) = \alpha \beta
\]  

\[
1 - F_1(CoVaR_{\beta,t}) - F_2(VaR_{1-\alpha,t}) + C_t(F_1(CoVaR_{\beta,t}), F_2(VaR_{1-\alpha,t})) = \alpha \beta,
\]  

where \(F_i(\cdot) = F_{\text{skew-}i}(\cdot; \hat{\theta}_i)\) for \(i = 1, 2\).

Let \(w_t = F_1(CoVaR_{\beta,t})\) and note that \(F_i(VaR_{\alpha,t}) = \alpha\). Simplifying Eqs.(2.15)-(2.16),

\[
C_t(w_t, \alpha) = \alpha \beta
\]  

\[
\alpha - w_t + C_t(w_t, 1 - \alpha) = \alpha \beta.
\]
Define \( w^*_t = w_t(\alpha, \beta, C_t) \in [0, 1] \) as the largest solution to either Eq.(2.17) or (2.18). After solving for \( w^*_t \), CoVaR can be calculated as \( \text{VaR}_{w^*_t,t} \) (Mainik and Schaanning, 2014). We adopt the two-step method of Reboredo and Ugolini (2015) to estimate downside and upside CoVaR for all \( t \in T \) and \( i = \{1, 2\} \). First, we solve either Eq.(2.17) or (2.18) for \( w^*_t \) given \( \alpha, \beta, C_t \in (0, 1) \) and \( C_t(\cdot; \hat{\theta}_c) \). Next, we calculate \( \text{CoVaR}_{\beta,t}(Y_{i,t}) = \text{VaR}_{w^*_t,t}(Y_{i,t}) = \hat{\mu}_{i,t} + F^{-1}_{\text{skew-t}}(w^*_t; \hat{\theta}_i)\hat{\sigma}_{i,t} \).

Proposed by Abadie (2002), we test for the significance of downside and upside equity market risk spillovers, as well as for the presence of asymmetric spillover, using two- and one-sided versions of the bootstrapped two-sample Kolmogorov-Smirnov (KS) test.\(^6\) First, define the two-sided equal-distribution KS statistic for sample sizes \( m \) and \( n \) as

\[
\text{KS}^{\text{EQ}} = \left( \frac{mn}{m+n} \right)^{\frac{1}{2}} \sup_x |G_m(x) - H_n(x)|,
\]

where \( G_m \) and \( H_n \) are the cumulative CoVaR, VaR, or normalized CoVaR/VaR empirical distribution functions. The two-sided test is used to assess \( H_0: G_m = H_n \) versus \( H_1: G_m \neq H_n \). Similarly, the one-sided test of first order stochastic dominance takes the form

\[
\text{KS}^{\text{FSD}} = \left( \frac{mn}{m+n} \right)^{\frac{1}{2}} \sup_x (G_m(x) - H_n(x)).
\]

Here, \( H_0: G_m \leq H_n \) and \( H_1: G_m > H_n \) or \( H_0: G_m \geq H_n \) and \( H_1: G_m < H_n \). Whether testing for equality in distribution or for first order stochastic dominance, p-values need to be bootstrapped because the distribution under the null is generally unknown.

\(^6\)The two-sample KS test has been similarly used to assess the significance of risk spillovers across financial sectors (Bernal et al., 2014), sovereign debt markets (Reboredo and Ugolini, 2015), equity and foreign exchange markets (Reboredo et al., 2016), and oil and equity markets (Mensi et al., 2017).
2.4 Data

End-of-week closing prices for the S&P500, S&P/TSX Composite (henceforth referred to as TSX), and Mexbol’s IPC equity indices are collected from Bloomberg over January 3, 1995 through December 30, 2016. Each index is denominated in USD to avoid currency-related valuation effects. We focus on the post-1995 period as the Bank of Mexico maintained a fixed USD/Peso exchange rate until December 22, 1994. Weekly rather than daily frequency data are utilized to avoid potential distortions due to varying holiday schedule closures. Weekly returns are defined as \( r_t = 100 \times \ln(P_t/P_{t-1}) \) and plotted below in Figure 2.1.

Figure 2.1: Weekly Equity Returns

\[ \text{Jondeau and Rockinger (2006)} \text{ remark that removing daily observations alters the temporal structure, leading to potential marginal model and copula estimate distortions.} \]
Table 2.2 presents descriptive statistics for equity returns series. Ljung-Box-Q (LBQ) tests reveal significant serial correlation up to four lags (one trading month) for the S&P500 and IPC. Conditional heteroskedasticity is present for each series with ARCH effects significant at the 1% level (see Figure 2.1 for further evidence). Each returns series deviates significantly from the Normal distribution with substantial skewness and kurtosis. Finally, each series is stationary over the sample period.

Table 2.2: Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P500</th>
<th>TSX</th>
<th>IPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.138</td>
<td>0.117</td>
<td>0.151</td>
</tr>
<tr>
<td>Median</td>
<td>0.253</td>
<td>0.345</td>
<td>0.342</td>
</tr>
<tr>
<td>Max</td>
<td>11.356</td>
<td>14.960</td>
<td>22.850</td>
</tr>
<tr>
<td>Min</td>
<td>-20.084</td>
<td>-25.553</td>
<td>-29.528</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.412</td>
<td>3.009</td>
<td>4.274</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.774</td>
<td>-1.050</td>
<td>-0.404</td>
</tr>
<tr>
<td>Kurtosis</td>
<td>9.474</td>
<td>10.798</td>
<td>7.911</td>
</tr>
<tr>
<td>S.W.</td>
<td>0.941***</td>
<td>0.918***</td>
<td>0.947***</td>
</tr>
<tr>
<td>LBQ(4)</td>
<td>16.064***</td>
<td>4.547</td>
<td>10.229**</td>
</tr>
<tr>
<td>ARCH(4)</td>
<td>123.385***</td>
<td>168.085***</td>
<td>159.797***</td>
</tr>
<tr>
<td>ADF</td>
<td>-13.374***</td>
<td>-10.338***</td>
<td>-12.961***</td>
</tr>
</tbody>
</table>

Notes: The sample size is 1147 observations for each series; S.W. is the Shapiro-Wilks normality test statistic; Augmented Dickey-Fuller (ADF) unit root test are performed including a constant term and with optimal lag length determined by AIC; * p-val < 0.10; ** p-val < 0.05; *** p-val < 0.01.
2.5 Empirical Results and Analysis

2.5.1 Marginal Model Results

Table 2.3 presents the marginal model estimates. Optimal AR orders are 2, 0, and 4, respectively. A GJR-GARCH(1,1,1) structure is specified for each conditional variance equation. Allowing for leverage, symmetric ARCH effects are only significant (weakly) for TSX returns. Conditional heteroskedasticity is primarily driven by negative news shocks for the S&P500 and IPC. GARCH effects are significant at the 1% level for each model, reflecting strong serial correlation in the variance processes. The Hansen (1994) degree-of-freedom (t-DoF) and skewness (t-Skew) parameters are both significant at the 1% level for each marginal model. As such, there is sufficient evidence to suggest the presence of left-skewed fat-tailed distributions.

Given the requirements of Sklar (1959), we must check whether \( \hat{u}_{i,t} \overset{i.i.d.}{\sim} U[0, 1] \) before proceeding further. First, we test whether or not \( \hat{u}_{i,t} \overset{i.i.d.}{\sim} U[0, 1] \). Define \( \bar{\hat{u}}_i = \frac{1}{T-1} \sum_{t=1}^{T} \hat{u}_{i,t} \). We regress \( (\hat{u}_{i,t} - \bar{\hat{u}}_i)^k \) on \( L = 12 \) own lags for \( k = \{1, 2, 3, 4\} \) and then perform an LM test to look for remaining serial correlation across each distribution’s moments. Columns 2-4 of Table 2.4 provide the respective simulated p-values. We fail to reject the null hypothesis of no serial correlation up to 12 lags for each test with the exception of the fourth moment of the IPC marginal model. However, rejection only occurs at the 10% significance level and can be thought of as relatively minor. Next, we address whether \( \hat{u}_{i,t} \sim U[0, 1] \) using the Cramer-von Mises (CvM) distributional test. Column 6 of Table 2.4 displays the corresponding simulated p-values. We fail to reject the null hypothesis that \( \hat{u}_{i,t} \sim U[0, 1] \) for each model. Overall, these findings suggest that each marginal model is correctly specified.
Table 2.3: Marginal Model Estimates

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P500</th>
<th>TSX</th>
<th>IPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>M-Constant</td>
<td>0.138**</td>
<td>0.117</td>
<td>0.151</td>
</tr>
<tr>
<td></td>
<td>(0.071)</td>
<td>(0.093)</td>
<td>(0.122)</td>
</tr>
<tr>
<td>AR(1)</td>
<td>-0.075**</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(2)</td>
<td>0.053</td>
<td>0.085**</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(3)</td>
<td>-0.046</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AR(4)</td>
<td>0.027</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>V-Constant</td>
<td>0.317***</td>
<td>0.301***</td>
<td>0.822**</td>
</tr>
<tr>
<td></td>
<td>(0.108)</td>
<td>(0.102)</td>
<td>(0.361)</td>
</tr>
<tr>
<td>ARCH</td>
<td>0.009</td>
<td>0.042*</td>
<td>0.021</td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.025)</td>
<td>(0.019)</td>
</tr>
<tr>
<td>Asym. ARCH</td>
<td>0.303***</td>
<td>0.172***</td>
<td>0.202***</td>
</tr>
<tr>
<td></td>
<td>(0.085)</td>
<td>(0.055)</td>
<td>(0.078)</td>
</tr>
<tr>
<td>GARCH</td>
<td>0.783***</td>
<td>0.833***</td>
<td>0.829***</td>
</tr>
<tr>
<td></td>
<td>(0.049)</td>
<td>(0.030)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>t-DoF</td>
<td>12.599***</td>
<td>9.245***</td>
<td>9.384***</td>
</tr>
<tr>
<td></td>
<td>(4.278)</td>
<td>(2.649)</td>
<td>(2.249)</td>
</tr>
<tr>
<td>t-Skew</td>
<td>-0.229***</td>
<td>-0.258***</td>
<td>-0.152***</td>
</tr>
<tr>
<td></td>
<td>(0.044)</td>
<td>(0.044)</td>
<td>(0.044)</td>
</tr>
<tr>
<td>LL</td>
<td>2461</td>
<td>2694</td>
<td>3137</td>
</tr>
</tbody>
</table>

Notes: Robust standard errors in parentheses; * p-val < 0.10; ** p-val < 0.05; *** p-val < 0.01.
Table 2.4: Marginal Model Goodness-of-Fit

<table>
<thead>
<tr>
<th></th>
<th>1$^{st}$ Moment</th>
<th>2$^{nd}$ Moment</th>
<th>3$^{rd}$ Moment</th>
<th>4$^{th}$ Moment</th>
<th>CvM</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P500</td>
<td>0.588</td>
<td>0.859</td>
<td>0.428</td>
<td>0.403</td>
<td>0.859</td>
</tr>
<tr>
<td>TSX</td>
<td>0.636</td>
<td>0.104</td>
<td>0.799</td>
<td>0.157</td>
<td>0.763</td>
</tr>
<tr>
<td>IPC</td>
<td>0.223</td>
<td>0.121</td>
<td>0.105</td>
<td>0.080</td>
<td>0.745</td>
</tr>
</tbody>
</table>

Notes: Simulated moment test p-values correspond to LM tests performed on regressions of $(\hat{u}_{i,t} - \bar{\hat{u}}_{i})^k$ on $L = 12$ own lags for $k = \{1, 2, 3, 4\}$ where $\hat{u} = \sum_{t} \bar{\hat{u}}_t$, $N = 1,000$ simulations; Simulated Cramer-von Mises (CvM) test p-values are presented, $N = 1,000$ simulations.

Figure 2.2 displays equity market pair-wise scatter plots of $(\hat{u}_{i,t}, \hat{u}_{j,t})$ for $i \neq j$ to explore the behavior of the joint marginal distributions. Each pair exhibits a positive relationship with most observations falling in the (0,0) to (1,1) direction. Deviations from this pattern suggest fluctuations in the relationship strength over time. Furthermore, clustering around (0,0) and (1,1) is indicative of potential lower and upper tail dependence, respectively.

Figure 2.2: Probability Integral Transformed Standardized Residuals

2.5.2 Copula Estimates

We estimate static representations of the copulas listed in Table 2.1 to help guide our choice of dynamic models. Table 2.5 displays the AIC corresponding to each copula and equity market pair.
The optimal model in each case is highlighted in bold. Observe that the Normal, Rotated Gumbel, and Student’s t-copulas are the top three models in terms of AIC regardless of equity pair. In each case, the Student’s t-copula is optimal. As such, we proceed by estimating GAS representations of these three particular copula functions.

### Table 2.5: Static Copula Information Criteria

<table>
<thead>
<tr>
<th></th>
<th>Normal</th>
<th>Clayton</th>
<th>R. Clayton</th>
<th>Gumbel</th>
<th>R. Gumbel</th>
<th>Student’s t</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P500 and TSX</td>
<td>-755</td>
<td>-664</td>
<td>-543</td>
<td>-699</td>
<td>-767</td>
<td><strong>-790</strong></td>
</tr>
<tr>
<td>S&amp;P500 and IPC</td>
<td>-558</td>
<td>-516</td>
<td>-370</td>
<td>-493</td>
<td>-583</td>
<td><strong>-592</strong></td>
</tr>
<tr>
<td>TSX and IPC</td>
<td>-469</td>
<td>-429</td>
<td>-334</td>
<td>-432</td>
<td>-494</td>
<td><strong>-505</strong></td>
</tr>
</tbody>
</table>

Notes: AIC statistics are presented for each estimated copula; Optimal models are in bold.

Parameter transformations must be specified prior to estimating each copula. We model \( \delta_t = 1 + \exp(f_t) \) for the Rotated Gumbel GAS copula, restricting \( \delta_t \in [1, \infty) \). For the Normal and Student’s t-GAS copulas, we estimate the dynamic correlation parameter using the transformation, \( \rho_t = (1 - \exp(-f_t))/(1 + \exp(-f_t)) \), restricting \( \rho_t \in (-1, 1) \). Furthermore, for the Student’s t-GAS model, we estimate the inverse t-degree of freedom parameter, \( \nu^{-1} \), and hold it fixed over time to decrease the computational burden.

Table 2.6 presents the estimates for each copula grouped by equity market pair. The coefficient on the forcing variable, \( \hat{\alpha} \), is significant for seven of the nine estimated models. The underlying dynamic processes are highly persistent with \( \hat{\beta} \) significant at the 1% level and close to one for each model. \( \hat{\beta} \) is extremely high for the S&P500 and IPC and TSX and IPC Normal GAS copulas, a potential sign of model misspecification. Finally, \( \hat{\nu}^{-1} \) is significantly greater than zero for all three Student’s t-GAS copulas, implying the presence of tail dependence for each equity market pair.

\[ \text{Testing whether } \hat{\nu}^{-1} > 0 \text{ amounts to a nested test of the Normal versus Student’s t-copula functional form. Rejection of } \hat{\nu}^{-1} = 0 \text{ implies rejection of the Normal copula relative to the Student’s t.} \]
### Table 2.6: GAS Copula Estimates

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P500 and TSX</th>
<th>S&amp;P500 and IPC</th>
<th>TSX and IPC</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>R. Gumbel</td>
<td>Student's t</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.077 (0.179)</td>
<td>-0.010 (0.033)</td>
<td>0.132 (0.136)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.045** (0.19)</td>
<td>0.156*** (0.047)</td>
<td>0.160*** (0.055)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.955*** (0.105)</td>
<td>0.921*** (0.159)</td>
<td>0.924*** (0.080)</td>
</tr>
<tr>
<td>( \nu^{-1} )</td>
<td>0.170*** (0.039)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>-782 (0.970)</td>
<td>-795 (0.970)</td>
<td>-827 (0.970)</td>
</tr>
<tr>
<td>CvM</td>
<td>0.260</td>
<td>0.410</td>
<td>0.970</td>
</tr>
</tbody>
</table>

Notes: Block-bootstrap standard errors in parentheses, \( N = 100 \) repetitions with an average block length of 60 weeks; Simulated Cramer-von Mises (CvM) test p-values are presented with rejections in bold, \( N = 100 \) simulations; * p-val < 0.10; ** p-val < 0.05; *** p-val < 0.01.

We use a modified version of the CvM test to assess the goodness-of-fit of each copula density to the underlying data. The modification consists of performing a multivariate probability integral transform, or “Rosenblatt transform”, prior to employing the test; this approach is necessary due to the dynamic copula structure\(^9\). The second panel of Table 2.6 shows that each Rotated Gumbel and Student’s t-GAS copula is reasonably specified. As previously suggested, misspecification is an issue for the S&P500 and IPC and TSX and IPC Normal GAS copulas.

Next, we perform the Rivers and Vuong (2002) non-nested model selection test to more rigorously determine the optimal GAS copula model. Table 2.7 presents the t-static for each pair-wise test. Positive (negative) and significant t-statistics imply dominance of the first (second) compared model. The Rotated Gumbel GAS copula is preferred to the Normal GAS copula for two of three equity market pairs. The only non-rejection is in the S&P500 and TSX case. Observe that the Stu-

\(^9\)See Diebold et al. (1999) and Remillard (2017) for further details.
dent’s t-GAS copula is superior to both the Normal and Rotated Gumbel GAS copulas for all three equity market pairs. As such, we proceed using the Student’s t-GAS copulas to generate dynamic correlations and conditional tail dependence estimates.

Table 2.7: GAS Copula Model Selection

<table>
<thead>
<tr>
<th></th>
<th>S&amp;P500 and TSX</th>
<th>S&amp;P500 and IPC</th>
<th>TSX and IPC</th>
</tr>
</thead>
<tbody>
<tr>
<td>R. Gumbel vs. Normal</td>
<td>-0.215</td>
<td>1.717</td>
<td>6.897</td>
</tr>
<tr>
<td>Student’s t vs. Normal*</td>
<td>4.633</td>
<td>6.345</td>
<td>9.966</td>
</tr>
<tr>
<td>Students t vs. R. Gumbel</td>
<td>5.891</td>
<td>5.546</td>
<td>6.872</td>
</tr>
</tbody>
</table>

Notes: Rivers and Vuong (2002) non-nested model selection test t-statistics are presented; A positive (negative) significant t-statistic implies dominance of the first (second) model considered; * This is a test of nested models.

Figure 2.3 plots the Student’s t-GAS estimated dynamic correlations and conditional tail dependence. The correlation between S&P500 and TSX returns is generally stronger than the other market pairs yet exhibits more volatility and fluctuates around its long-run average of 0.692. S&P500 and IPC and TSX and IPC returns correlations are increasing over most of the sample, suggesting a strengthening relationship across equity markets. These findings are consistent with Aggarwal and Kyaw (2005), Phengpis and Swanson (2006), and Lahrech and Sylwester (2013); however, their studies either employ rolling correlation calculations or the DCC-GARCH model and consider earlier sample periods (pre-Global Financial Crisis). Observe that the S&P500 and IPC correlation is stronger than for the TSX and IPC for the majority of the sample period; this is likely due to the dominant role of the U.S. equity market in the greater North American region.

As shown in panel (b) of Figure 2.3, conditional tail dependence between S&P500 and TSX returns is substantially higher than for the other equity market pairs; S&P500 and TSX returns are more likely to co-move during extreme economic events. While the correlation between the S&P500 and IPC is stronger than for the TSX and IPC on average, tail dependence is stronger for
CHAPTER 2. EQUITY MARKET RISK SPILLOVERS

Figure 2.3: Student's-t-GAS Copula Correlation and Tail Dependence

(a) Dynamic Correlation

(b) Conditional Tail Dependence
the TSX and IPC pair relative to the S&P500 and IPC. There appears to be a fundamental shift in tail behavior starting from the peak of the pre-crisis boom in 2006. Compared to the 1995 to 2006 period, average tail dependence over 2007 to 2016 increases by 20.0%, 89.3%, and 82.6%, respectively. Elevated tail dependence in the wake of the Global Financial Crisis implies that North American equity markets are more likely to jointly boom and crash.

Overall, these results indicate that the degree of co-movement in North American equity markets increased over time both in terms of general correspondence (correlation) and during periods of extreme market behavior (booms and busts). Portfolio managers and investors need to consider behavior at the extremes when accounting for risk, as relying solely on correlations is potentially misleading and may induce a false sense of protection through regional diversification This is particularly important given the prolific rise in the use of exchange traded funds (ETFs) as investment vehicles whose function is to track underlying indices such as the S&P500.

2.5.3 Downside and Upside Risk Spillover

Downside and upside CoVaR estimates are calculated for all three equity market pairs and conditioning directions using the two-step method described in Section 3.5. We also calculate each equity return’s isolated downside and upside VaR using their respective fitted marginal distributions. In all cases, we assume that $\alpha = \beta = 0.05$, focusing specifically on the 95% confidence levels.

Figure 2.4 plots the downside and upside VaR and CoVaR estimates for each equity market pair. The caption above each sub-figure denotes the CoVaR conditioning direction. For example, the sub-figure in the first column of the first row displays downside and upside CoVaR for S&P500 returns conditional on the VaR for TSX returns. Downside and upside VaR in this sub-figure

---

10 Considering international portfolio diversification problems, Ang and Bekaert (2002) and Ait-Sahalia and Hurd (2016) explore the ramifications of failing to account for time-varying economic states and extreme market behavior.

11 See Krause and Tse (2013) for a discussion on the relationship between U.S. and Canadian equity index ETFs.

12 We also consider $\alpha = \beta = 0.01$. Results at the 99% confidence interval do not differ substantially and are available upon request.
correspond to the isolated estimates for S&P500 returns. Risk spillovers generally move together over time regardless of equity market pair and are greatest during three distinct periods: i) the Asian Financial Crisis in 1997-1999; ii) the end of the Dot-Com bubble in 2000-2001; and iii) the Global Financial Crisis in 2008-2009. Downside and upside CoVaR for the IPC also exceed ±20% conditional on S&P500 and TSX VaR at the tail end of the Mexican Peso Crisis in early 1995. The largest single week CoVaR occurs for the IPC conditional on S&P500 VaR during the Global Financial Crisis; here, downside and upside CoVaR peak at -42.35% and 38.21%, respectively.

Table 2.8 presents the two-sample $K S^{FSD}$ statistics with bootstrapped p-values for the test of $H_0$: $CoVaR(D) \geq VaR(D)$ versus $H_1$: $CoVaR(D) < VaR(D)$ and $H_0$: $CoVaR(U) \leq VaR(U)$ versus $H_1$: $CoVaR(U) > VaR(U)$. Rejection of the null hypothesis implies that risk spillover is significant such that downside or upside CoVaR are larger in magnitude than downside or upside VaR. We conduct the test for each equity market pair and spillover direction. As shown in columns 3 and 6, which considers the entire cumulative distributions (designated ‘All’), downside and upside risk spillovers are significant at the 1% level in all cases. For example, the $K S^{FSD}$ statistic for the test of downside CoVaR for S&P500 returns conditional on the downside VaR for TSX returns versus isolated downside VaR for S&P500 returns is 0.494 with a p-value of 0.000; this implies that potential S&P500 losses are greater when the TSX has already experienced an extreme decline in value. Overall, we find that the secondary equity market status significantly impacts the potential primary market long and short position losses.
Figure 2.4: Downside and Upside Value-at-Risk (VaR) and Conditional Value-at-Risk (CoVaR)
### Table 2.8: Downside and Upside Risk Spillovers

<table>
<thead>
<tr>
<th>CoVaR</th>
<th>VaR</th>
<th>Downside</th>
<th></th>
<th></th>
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<th></th>
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<td></td>
<td></td>
<td>All</td>
<td>Lower 10(^{th})</td>
<td>Upper 90(^{th})</td>
<td>All</td>
<td>Lower 10(^{th})</td>
<td>Upper 90(^{th})</td>
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<tr>
<td>S&amp;P500</td>
<td>TSX</td>
<td>S&amp;P500</td>
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<td>0.719</td>
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<td>S&amp;P500</td>
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<td>S&amp;P500</td>
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<tr>
<td>IPC</td>
<td>S&amp;P500</td>
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<td>0.555</td>
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<td>IPC</td>
<td>TSX</td>
<td>IPC</td>
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<td>0.542</td>
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</tbody>
</table>

Notes: Two-sample Kolmogorov-Smirnov (KS) test statistics are presented with bootstrapped p-values below in brackets, N = 1000 replications; The tests consist of \( H_0: \text{CoVaR}(D) \geq \text{VaR}(D) \) versus \( H_1: \text{CoVaR}(D) < \text{VaR}(D) \) and \( H_0: \text{CoVaR}(U) \leq \text{VaR}(U) \) versus \( H_1: \text{CoVaR}(U) > \text{VaR}(U) \); 'All' implies testing over the entirety of the cumulative distributions; Lower 10\(^{th}\) and Upper 90\(^{th}\) refer to below the 10\(^{th}\) and above the 90\(^{th}\) quantiles of each cumulative distribution, respectively.

In addition to testing over the entire cumulative distributions, we also test for significant risk spillover below the lower 10\(^{th}\) and above the upper 90\(^{th}\) quantiles. For downside risk, below the lower 10\(^{th}\) (above the 90\(^{th}\)) quantile implies that the degree of risk in financial markets is currently high (low). For upside risk, the correspondence is reversed. Columns 4-5 and 7-8 of Table 2.8 display the downside and upside risk spillover test results by quantile, respectively. As before, risk spillover is significant for all considered CoVaR and VaR combinations and at both ends of the cumulative distributions. Moreover, the \( \hat{K}^{FS} \) statistics are substantially larger at the ends of the
distribution than when considering the entire distribution; this is particularly true when downside and upside risk is low. These findings suggest that downside and upside CoVaR is greater than VaR even when potential long and short position losses are low. Ignoring these facts can lead portfolio managers to under-estimate potential downside and upside risk. Furthermore, individual investors who choose to invest primarily in broad-market equity ETFs would be exposed as well.\(^1\)

2.5.4 Asymmetric Risk Spillover

We explore potential risk spillover asymmetries along two dimensions. First, we consider whether downside risk spillover is significantly larger than upside risk spillover. Following Reboredo et al. (2016), downside and upside CoVaR are normalized by the corresponding VaR. More formally, we test \(H_0: \text{CoVaR}/\text{VaR}(D) \leq \text{CoVaR}/\text{VaR}(U)\) versus \(H_1: \text{CoVaR}/\text{VaR}(D) > \text{CoVaR}/\text{VaR}(U)\). Next, we test whether downside and upside risk spillovers are asymmetric by spillover direction. For example, does downside risk spillover from the TSX to the S&P500 differ from the S&P500 to the TSX? In focusing on this relationship, we highlight the importance of market specific factors such as the dominant role that the U.S. equity market plays within the greater North American region. This test takes the form \(H_0: \text{CoVaR}(j)/\text{VaR}(j)(Y_1|Y_2) = \text{CoVaR}(j)/\text{VaR}(j)(Y_2|Y_1)\) versus \(H_1: \text{CoVaR}(j)/\text{VaR}(j)(Y_1|Y_2) \neq \text{CoVaR}(j)/\text{VaR}(j)(Y_2|Y_1),\) where \(j = \{\text{Down, Up}\}\).

As shown in Table 2.9, we find that downside risk spillover is considerably larger than upside risk spillover regardless of equity market pair and spillover direction.\(^2\) On average, downside risk spillover is 21.3% larger than upside risk spillover; on a model pair basis, this figure ranges from 14.6% to 28.5%. The largest degrees of asymmetry, 28.2% and 28.5% on average, are observed for spillover from the S&P500 to the TSX and from the IPC to the TSX. Similarly, Ji et al. (2017) find asymmetric risk spillover between U.S. to Canadian equity markets. Furthermore, our find-

\(^{13}\)ETF holdings now amount to over US$2.9 trillion with approximately 81% allocated to corporate equities (Federal Reserve Financial Accounts, 2017Q2).

\(^{14}\)Given that each equity market pair is modeled with a Student’s t-GAS copula, asymmetric risk spillover is driven primarily by differences in the marginal distributions.
ings are consistent below (above) the 10\textsuperscript{th} (90\textsuperscript{th}) quantiles. As opposed to capturing the general level of risk, normalized values in the lower (upper) tail of the distribution imply that the severity of downside and upside risk spillover are low (high).

Table 2.9: Asymmetric Risk Spillover: Downside Versus Upside

<table>
<thead>
<tr>
<th></th>
<th>CoVaR</th>
<th>VaR</th>
<th>All</th>
<th>Lower 10\textsuperscript{th}</th>
<th>Upper 90\textsuperscript{th}</th>
</tr>
</thead>
<tbody>
<tr>
<td>S&amp;P500</td>
<td>TSX</td>
<td>S&amp;P500</td>
<td>0.989</td>
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<td>S&amp;P500</td>
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<td>0.899</td>
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</table>

Notes: Two-sample Kolmogorov-Smirnov (KS) test statistics are presented with bootstrapped p-values below in brackets, N = 1000 replications; The test consists of $H_0: \text{CoVaR/VaR}(D) \leq \text{CoVaR/VaR}(U)$ versus $H_1: \text{CoVaR/VaR}(D) > \text{CoVaR/VaR}(U)$; ‘All’ implies testing over the entirety of the cumulative distributions; Lower 10\textsuperscript{th} and Upper 90\textsuperscript{th} refer to below the 10\textsuperscript{th} and above the 90\textsuperscript{th} quantiles of each cumulative distribution, respectively.
Spillover direction plays an important role in determining the magnitude of risk spillover. Table 2.10 presents the results for each directional test. Consider the first entry in column 2 to the right of ‘S&P500 and TSX’. In this case, we are testing whether downside risk spillover from the TSX to S&P500 is equivalent to downside risk spillover from the S&P500 to the TSX. Here, we reject this hypothesis given the bootstrapped p-value of 0.000. Over 1995 to 2016, downside risk spillover from the TSX to the S&P500 is 19.1% lower than vice versa. Similarly, upside risk is 14.0% lower. These estimates are relatively small compared to the other two equity market pairs. On average, downside and upside risk spillovers are 25.6% and 34.1% lower when focusing on the IPC to TSX direction as compared to TSX to IPC. The S&P500 and IPC pair exhibits the greatest degree of asymmetry with differences of 41.5% (downside) and 44.8% (upside). As such, risk spillover from the IPC to S&P500 is nearly half as severe as risk spillover from the S&P500 to IPC.

Table 2.10: Asymmetric Risk Spillover By Conditioning Direction

<table>
<thead>
<tr>
<th></th>
<th>Downside</th>
<th></th>
<th></th>
<th>Upside</th>
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<td>Lower 10&lt;sup&gt;th&lt;/sup&gt;</td>
<td>Upper 90&lt;sup&gt;th&lt;/sup&gt;</td>
<td>All</td>
<td>Lower 10&lt;sup&gt;th&lt;/sup&gt;</td>
<td>Upper 90&lt;sup&gt;th&lt;/sup&gt;</td>
</tr>
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<td>S&amp;P500 and TSX</td>
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<td>0.974</td>
<td>0.737</td>
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<td>0.921</td>
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<tr>
<td>S&amp;P500 and IPC</td>
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<td>0.868</td>
<td>0.630</td>
<td>0.746</td>
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<td>[0.000]</td>
<td>[0.000]</td>
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<tr>
<td>TSX and IPC</td>
<td>0.340</td>
<td>0.991</td>
<td>0.851</td>
<td>0.673</td>
<td>0.754</td>
<td>1.000</td>
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</table>

Notes: Two-sample Kolmogorov-Smirnov (KS) test statistics are presented with bootstrapped p-values below in brackets, N = 1000 replications; The test consists of \( H_0: \text{CoVaR/VaR}(Y_1|Y_2) = \text{CoVaR/VaR}(Y_2|Y_1) \) versus \( H_1: \text{CoVaR/VaR}(Y_1|Y_2) \neq \text{CoVaR/VaR}(Y_2|Y_1) \); ‘All’ implies testing over the entirety of the cumulative distributions; Lower 10<sup>th</sup> and Upper 90<sup>th</sup> refer to below the 10<sup>th</sup> and above the 90<sup>th</sup> quantiles of each cumulative distribution, respectively.
In summary, we find that risk spillovers are highly asymmetric across North American equity markets. Downside risk spillovers are more severe and the spillover direction is important as well. Understanding and accounting for these asymmetries is key for portfolio managers given the greater inter-dependence observed in North American equity markets over the past two decades.

### 2.6 Concluding Remarks

In this study, we analyzed the dynamic dependence structure between US, Canadian, and Mexican equity market returns and explored the resulting implications for cross-market downside and upside risk spillovers over 1995 to 2016. S&P500, TSX, and IPC equity indices are used proxy for each market, respectively. Dynamic dependence structures were estimated using the two-stage ‘Inference Functions from the Margins’ (IFM) procedure. First, marginal distributions were estimated by fitting optimally specified AR-GJR-GARCH models with skewed-t disturbances on each equity returns series. Conditional on these results, we then estimated several generalized autoregressive score (GAS) copulas and using the optimal models calculated dynamic correlations and conditional tail dependence. Given the first-stage marginal distribution and second-stage copula estimates, we adopted the two-step approach of Reboredo and Ugolini (2015) to calculate downside and upside Conditional Value-at-Risk (CoVaR). Finally, we employed bootstrapped two-sample Kolmogorov-Smirnov (KS) tests to assess the significance of downside and upside risk spillover and whether any asymmetries persisted.

The dynamic dependence structure for each equity market pair is characterized by a Student’s t-GAS copula which allows for a time-varying correlation process and symmetric conditional tail dependence. US-MX and CAN-MX correlations are increasing over the sample period. While the time-varying US-CAN correlation does not exhibit any defined trend, estimates are somewhat higher in the later half of the sample on average. Rising correlations reflect more substantial co-movement. Similarly, we also find that conditional tail dependence increases over time. Most of
the change in tail dependence follows the Global Financial Crisis, suggesting that the crisis left North American equity markets more sensitive to positive and negative regional shocks.

Downside and upside risk spillovers are significant and asymmetric in all cases. Two-sample KS tests reveal that downside and upside CoVaR is significantly larger than isolated downside and upside VaR for all considered equity market pairs and spillover directions. On average, downside risk spillover is 21.3% larger than upside risk spillover. Again, two-sample KS test are performed and show that downside risk spillover is significantly larger than upside risk spillover in all cases. Finally, we find that risk spillover direction significantly influences the strength of the spillover regardless of whether considering downside or upside risk. All of these results hold when testing over the lower 10\textsuperscript{th} and upper 90\textsuperscript{th} quantiles of the CoVaR, VaR and normalized CoVaR/VaR cumulative distributions.

Ultimately, the analysis presented in this study provides insight in to the transmission of risk across North American equity markets. Our findings have important implications for the management of risk, both at the individual and institutional levels, highlighting the prevalence of tail behavior and asymmetric characteristics of risk spillovers.
Chapter 3

The Dynamic Dependence Structure Between U.S Equity Market Performance and Dollar Strength

3.1 Introduction

Recent developments in the global economy have generated a renewed interest, academic and otherwise, in the relationship between U.S. equity market performance and fluctuations in the value of the dollar. Understanding this relationship is particularly important given the size and prominence of U.S. equity markets in relation to the greater international financial system, as well as the dollar’s global reserve currency status. This is especially true in the post-Global Financial Crisis economic climate where financial markets demonstrate a somewhat fickle nature with heightened sensitivity to socio-political and economic shocks. In this study, we recover the dynamic dependence structure between U.S. equity returns and fluctuations in the value of the dollar using recent advances in the estimation of dynamic copulas.

An extensive amount of literature is devoted to understanding the relationship between equity
prices and exchange rates. Early theoretical models emphasized the roles of either the current or capital accounts in a partial-equilibrium setting [Dornbusch and Fischer, 1980; Frankel, 1983; Branson and Henderson, 1985], focusing on the importance of trade balances and asset demand driven capital flows. More recent endeavors favor a general equilibrium international portfolio balancing approach [Hau and Rey, 2004, 2006]. Here, investors seek to solve an international asset allocation problem in the presence of both equity and foreign exchange rate risk. Foreign exchange rate risk cannot be completely hedged away and investors must re-balance their portfolios through the purchase or sale of domestic and foreign equities.

Most empirical research on U.S. equity markets tends to investigate the nature of exchange rate risk premia [Ding and Hu, 2012; Chakraborty et al., 2015; Hughen, 2015]. Alternatively, Cho et al. (2016) explores the role of flight-to-quality induced capital flows in determining the correlation across markets. However, both approaches are somewhat limited in capturing the joint-relationship between U.S. equity and foreign exchange markets. More closely related to the methodology of this study, Patton (2006b) suggest using time varying copulas to recover the dynamic dependence structure between two asset price returns. Copula oriented analysis is well suited to estimate dependence structures, whether static or dynamic; the methodology is flexible enough to recover measures of both correlation and extreme tail dependence while also allowing for asymmetric behavior. Other models such as the multivariate dynamic conditional correlation (DCC) GARCH specification developed by Engle (2002) and Tse and Tsui (2002) are unable to adequately capture tail behavior. As such, copulas are widely used in financial economics to explore the co-movement of financial markets and asset prices with implications for risk management and portfolio diversification [Jondeau and Rockinger, 2006; Creal et al., 2013; Oh and Patton, 2015; Salvatierra and Patton, 2015; Reboredo et al., 2016].

We study the dynamic dependence structure between U.S. equity returns and fluctuations in the strength of the dollar using a class of recently developed generalized autoregressive score (GAS) copulas. Proposed by Creal et al. (2013), the GAS framework is a flexible observation driven dy-
namic model. GAS dynamics are directly tied to the behavior of the underlying copula probability density function at each point in time, whereas the dynamic structure introduced by Patton (2006b) and latter adopted by Michelis and Ning (2010), Ning (2010), Dajcman (2013), and Reboredo et al. (2016) to study the joint behavior of equity and foreign exchange markets is somewhat arbitrary. First, however, we estimate conditional mean and variance AR-t-GARCH models to filter each returns series and capture the behavior of the marginal distributions. Conditional on the first-stage analysis, we then conduct the dynamic copula estimations.

The analysis is directed along two dimensions. First, we consider differences in the dynamic dependence structure attributable to equity market size. S&P 500 and S&P 600 index returns are each paired against broad trade-weighted dollar index returns, designated as large and small market capitalization (cap) models. To further explore the behavior of U.S. equity markets, we then consider differences attributable to market sector. Here, we utilize S&P 500 GICS Level I sub-sector index returns.

Our findings suggest a generally negative correlation for both the large and small market cap models, where the large cap correlation is more likely to be positive for brief periods of time. Contrary to prevailing sentiment, the small cap correlation series is not always the weaker of the two. We find symmetric tail dependence present for both equity market sizes, where the small cap tail dependence is the stronger of the two for the majority of the sample. Tail dependence is strongest when the correlations are positive, reflecting greater risk present in the markets when the relationship between equity and dollar returns deviates from its long-run negative relationship. Sub-sector dynamic correlations are negative for each sector over the majority of the sample. While most sub-specific correlations follow a similar path the energy, telecommunications, and utilities sectors deviate somewhat substantially. The greatest difference in sub-sector behavior is reflected in conditional tail dependence estimates. The more internationally exposed sectors such as consumer discretionary, industrials, and financials exhibit stronger and more volatile symmetric tail dependence. To the best of the author’s knowledge this is the first study to rigorously model the dynamic
dependence structure of both overall U.S. equity market performance, as well as sector-specific, against changes in the value of the dollar.

The remainder of the paper is organized as follows. Section 2 provides a brief introduction to copula theory and and tail dependence. Section 3 explores the properties of the data, guiding first-stage marginal model selection. Section 4 details all estimation methodologies and presents the resulting empirical findings accompanied by analysis. Finally, concluding remarks are given in Section 5.

3.2 A Primer on Copulas

3.2.1 Copula Functions and Estimation Approaches

The copula is a multivariate cumulative distribution function (CDF) whose marginal distributions are \( F_i \overset{i.i.d.}{\sim} U[0,1] \). Consider a bivariate case of Sklar’s Theorem [Sklar, 1959]: let \( F_{XY} \) be the joint distribution function for two variables with corresponding marginal distributions \( F_X \) and \( F_Y \). There exists a copula \( C(\cdot) \) such that for all \( x, y \) in \( \mathcal{R} \),

\[
F_{XY}(x, y; \theta_x, \theta_y, \theta_c) = C(F_X(x; \theta_x), F_Y(y; \theta_y); \theta_c). \tag{3.1}
\]

If \( F_X \) and \( F_Y \) are continuous, then \( C(\cdot) \) is unique. Conversely, if \( C(\cdot) \) is a copula function and \( F_X \) and \( F_Y \) are marginal CDFs, then \( F_{XY} \) is a joint CDF with margins \( F_X \) and \( F_Y \). Sklar’s theorem implies that we can first separately model each marginal distribution and then use \( C(\cdot) \) to estimate the dependence structure between \( X \) and \( Y \). The transformed problem is substantially simpler than directly estimating the joint distribution.

Copulas are conveniently estimated using maximum likelihood methods. Differentiating Eq.(3.1),

\[
f_{XY}(x, y; \theta_x, \theta_y, \theta_c) = f_X(x; \theta_x) \cdot f_Y(y; \theta_y) \cdot c(u, v; \theta_x, \theta_y, \theta_c), \tag{3.2}
\]
where \( f_{XY}, f_X, f_Y, \) and \( c \) are density functions. Define \( u = F_X(x; \theta_x) \) and \( v = F_Y(y; \theta_y) \). Next, taking the natural log of Eq.(3.2) yields

\[
L_{XY}(x, y; \theta_x, \theta_y, \theta_c) = L_X(x; \theta_x) + L_Y(y; \theta_y) + L_C(u, v; \theta_x, \theta_y, \theta_c),
\]

(3.3)

where \( L_{XY} = \log(f_{XY}), L_X = \log(f_X), L_Y = \log(f_Y), \) and \( L_C = \log(c(u, v)) \). Consistent, efficient, and asymptotically normal one-step full information maximum likelihood (FIML) estimates are obtainable under standard regularity conditions by maximizing the right hand side of Eq.(3.3) with respect to \( \theta = (\theta_x, \theta_y, \theta_c) \). However, the FIML procedure can become unwieldy as the parameter space increases in \( \theta_x, \theta_y, \) and \( \theta_c \).

Proposed by Joe and Xu (1996), the Inference Functions for the Margins (IFM) procedure provides a more desirable two-step maximum likelihood methodology to estimating copula functions. First, \( L_X(x; \theta_x) \) and \( L_Y(y; \theta_y) \) are separately estimated by maximum likelihood to retrieve estimates of \( \hat{\theta}_x \) and \( \hat{\theta}_y \). Conditional on the first-step, \( L_C(u, v; \hat{\theta}_x, \hat{\theta}_y, \theta_c) \) is then estimated by maximum likelihood. Joe (1997) shows that the IFM procedure produces consistent and asymptotically normal estimates under standard regularity conditions. Some efficiency loss is incurred using the two-step approach; however, Joe (2005) and Patton (2006a) show the loss is minimal and the two-step methodology doesn’t rely on numerically estimating complex and computationally demanding Hessian matrices. There is one further drawback to using a two-step estimation. Naive second-stage standard errors are biased downwards due to inherited parameter estimation error. Simulated second-stage standard errors are calculated to correct for this issue.\footnote{See Patton (2013) for further details on correcting for parameter estimation error in copula estimation.}
3.2.2 Relationship To Tail Dependence

Certain copulas allow for the possibility of tail dependence, or joint behavior at the extremes. Define the *lower and upper tail dependence coefficients* $\lambda_L$ and $\lambda_U$,

\[
\lambda_L = \lim_{u \to 0} \frac{C(u, u)}{u},
\]

\[
\lambda_U = \lim_{u \to 1} \frac{1 - 2u + C(u, u)}{1 - u},
\]

where $\lambda_L, \lambda_U \in [0, 1]$. The lower and upper tail dependence coefficients describe the behavior of the joint distribution as the probability transformed data converges to either (0,0) (lower tail) or (1,1) (upper tail). In a financial market context, lower tail dependence would reflect joint crashes and upper tail dependence joint booms.

The Normal (Gaussian) copula is often considered a “benchmark” as it exhibits zero tail dependence ($\lambda_L = \lambda_U = 0$). Let $u, v \overset{i.i.d.}{\sim} U[0, 1]$. The Normal copula is given by,

\[
C_N(u, v; \rho) = \Phi^{-1}(\rho \Phi^{-1}(u), \Phi^{-1}(v)),
\]

where $\Phi^{-1}$ is the inverse standard normal CDF and $\rho$ is the Pearson correlation coefficient. Closely related, the Student’s t-copula allows for symmetric tail dependence, nesting the Normal copula as a special case. Here,

\[
C_t(u, v; \rho, \nu) = t_{\rho, \nu}^{-1}(t_{\nu}^{-1}(u), t_{\nu}^{-1}(v)),
\]

where $\nu$ is the t-degree of freedom parameter. Substituting Eq.(3.7) in to Eqs.(3.4)-(3.5) and evaluating the respective limits, $\lambda_L = \lambda_U = 2 \cdot t_{\nu+1}(-\sqrt{\nu + 1 - \rho/\sqrt{1+\rho}})$. While the Normal and Student’s t-copulas are attractive due to their simplicity, there are instances where other copula density functions are required. In particular, there may be cases where *only* upper or lower tail
dependence is significant or both upper and lower tail dependence exist but are asymmetric.

### 3.3 Data

End-of-week closing prices for the S&P 500 (large cap), S&P 600 (small cap), and ten main S&P 500 GICS Level I sub-sector indices are collected from Bloomberg spanning June 28, 1996 until June 30, 2017. The Federal Reserve’s trade-weighted broad currencies (TWDBC) index is selected to proxy for dollar strength. While ICE’s Dollar Index (ticker: DXY) is more widely recognized and quoted in financial news, it provides a poor approximation of dollar strength. The DXY is limited to only six major industrialized nations and maintains fixed weights as opposed to more realistic annually adjusted weights. More recent attempts to accurately value the U.S. dollar explore alternative weighting schemes but tend to be more limited in availability and sample length.

Returns are calculated as $r_t = 100 \times \left[ \ln(P_t) - \ln(P_{t-1}) \right]$, resulting in a total of 1096 observations. Table 3.1 presents summary statistics characterizing the distributions of each series. Small cap returns are approximately 19.4% more volatile than their large cap counterpart. The Energy, Financials, and Information Technology (Info Tech) sectors are most volatile on both a sectors and compared to the broader S&P 500. Moreover, all equity returns series are an order of magnitude more volatile than the broad currencies index. Serial correlation is present for some but not all series. ARCH effects are found highly significant when allowing for up to four lags (one trading month). The Shapiro-Wilks (S.W.) distribution test is employed to explore whether each series deviates significantly from the normal distribution. As expected, each returns series exhibits highly non-normal behavior. Finally, each series is stationary over the sample period.

---

1 Manner and Reznikova (2012) and Patton (2012) provide detailed surveys on the estimation and applications of copulas in financial economics.
2 June 28, 1996 coincides with the launch date of the S&P 500 GICS Level I sub-sector indices.
3 The Wall Street Journal (BUXX) and Bloomberg (BBDXY) measures attempt to directly control for liquidity using triennial BIS foreign exchange turnover data. However, both series are limited by their sample length.
### Table 3.1: Descriptive Statistics

<table>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.117</td>
<td>0.169</td>
<td>0.154</td>
<td>0.119</td>
<td>0.156</td>
<td>0.123</td>
<td>0.116</td>
<td>0.091</td>
<td>0.013</td>
<td>0.068</td>
<td>0.013</td>
<td>0.068</td>
</tr>
<tr>
<td>Median</td>
<td>0.207</td>
<td>0.379</td>
<td>0.222</td>
<td>0.189</td>
<td>0.236</td>
<td>0.187</td>
<td>0.329</td>
<td>0.293</td>
<td>0.031</td>
<td>0.183</td>
<td>0.031</td>
<td>0.183</td>
</tr>
<tr>
<td>Std. Dev.</td>
<td>2.447</td>
<td>2.921</td>
<td>2.895</td>
<td>2.895</td>
<td>2.473</td>
<td>2.346</td>
<td>2.817</td>
<td>2.858</td>
<td>2.466</td>
<td>0.689</td>
<td>0.689</td>
<td>0.689</td>
</tr>
<tr>
<td>Skewness</td>
<td>-0.753</td>
<td>-0.523</td>
<td>-0.424</td>
<td>-0.424</td>
<td>-0.424</td>
<td>-0.424</td>
<td>-0.424</td>
<td>-0.424</td>
<td>-0.424</td>
<td>-0.424</td>
<td>-0.424</td>
<td>-0.424</td>
</tr>
</tbody>
</table>

| S.W. | 0.942*** | 0.953*** | 0.942*** | 0.928*** | 0.950*** | 0.879*** | 0.948*** | 0.952*** | 0.958*** | 0.960*** | 0.953*** | 0.936*** | 0.971*** |

Notes: S.W. stands for the Shapiro-Wilks normality test; Augmented Dickey-Fuller (ADF) tests are conducted including a constant term with optimal lag length determined by AIC; * p-val < 0.10; ** p-val < 0.05; *** p-val < 0.01.
3.4 Empirical Methodology, Results, and Analysis

Following the IFM procedure, we first estimate each marginal model via maximum likelihood. In Section 3.4.1 we detail each model specification, present the empirical findings, and evaluate fit and appropriateness. The GAS copula framework is presented in Section 3.4.2, with estimates and analysis discussed in Sections 3.4.3 and 3.4.4.

3.4.1 Marginal Model Specifications and Results

Let \( r_t \) be a \((T \times 1)\) vector of returns. We estimate AR(1)-t-GARCH conditional mean and variance marginal models of the following form:

\[
\begin{align*}
    r_t &= \phi_0 + \phi_1 r_{t-1} + \epsilon_t \quad (3.8) \\
    \sigma^2_t &= \omega + \sum_{i=1}^{p} \alpha_i \epsilon^2_{t-i} + \sum_{i=1}^{q} \beta_i \sigma^2_{t-i} \quad (3.9) \\
    z_t &= \frac{\epsilon_t}{\sigma_t}, \quad z_t \sim i.i.d. \ t\nu \quad (3.10)
\end{align*}
\]

Optimal GARCH specifications for each model are determined by comparing BIC.\(^5\)

Next, the following probability integral transformation is performed on the standardized residuals:

\[
\hat{u}_t = t\nu \left( \frac{r_t - \hat{\phi}_0 - \hat{\phi}_1 r_{t-1}}{\hat{\sigma}_t} \right). \quad (3.11)
\]

If Eqs.(3.8)-(3.10) are correctly specified then \( \hat{u}_t \sim i.i.d. U[0, 1] \).

Table 3.2 presents all marginal model estimates. The initial findings for each model are supportive. The AR(1) term is significant for five of thirteen models, suggesting only mild evidence of serial correlation in equity returns at a weekly frequency. Furthermore, each conditional variance specification follows a GARCH(1,1) process. All series exhibit significant and strong ARCH

---

\(^5\)We consider GARCH orders of (1,1), (2,1), (1,2), and (2,2).
effects and volatility clustering. \( \hat{\nu} \) is significant at the 1% level for each model, reflecting the presence of fat tails. We employ the Engle and Ng (1993) joint bias test on the standardized residuals of each model to explore whether the GARCH(1,1) specification fails to account for any asymmetries in underlying the volatility processes. The corresponding F-statistics are presented in the bottom panel of Table 3.2. We fail to reject the null hypothesis of no sign bias or asymmetric leverage effects for each model and proceed using the AR(1)-t-GARCH(1,1) specification for each marginal model.

Table 3.3 presents the results of two further diagnostic tests performed on each marginal model. Here, we are checking to see whether each \( \hat{\nu}_t \overset{i.i.d}{\sim} U[0,1] \). Simulated p-values are calculated for each test to account for parameter estimation error associated with the probability integral transformation. First, we perform a LBQ test for serial correlation up to twelve lags on \((\hat{\nu}_t - \bar{\hat{\nu}})_k\) for \( k = \{1, 2, 3, 4\} \) where \( \bar{\hat{\nu}} = T^{-1} \sum_t \hat{\nu}_t \). Rejection across several moments of the distribution indicates marginal model misspecification. We fail to reject the null hypothesis of no serial correlation up to twelve lags for each case with the exception of the third moment for the dollar index returns model. Next, we use the Cramer-von Mises (CvM) distribution test to determine if \( \hat{\nu}_t \sim U[0,1] \). As shown, we fail to reject the null hypothesis of \( \hat{\nu}_t \sim U[0,1] \) for each model. Overall, these findings are supportive of each marginal model specification.

Finally, we explore the joint behavior of the probability transformed residuals to motivate our selection of copulas. Figure 3.1 presents scatter plots of each equity model \( \hat{\nu}_t \) paired against \( \hat{\nu}_{t\text{wdtc},t} \). The plots suggest a generally negative relationship for each pair, with clustering along the (0,1) to (1,0) direction. That said, relationship strength varies substantially on a case-by-case basis. For each pair there exists a non-insignificant number of observations in the positive direction, implying that the correlation varies both in magnitude and direction over time. Furthermore, there is only weak evidence of tail dependence. However, this does not imply that tail dependence is weak or insignificant at each point in time.
Table 3.2: Marginal Model Estimates

<table>
<thead>
<tr>
<th></th>
<th>Broad Market</th>
<th>S&amp;P 500 Sub-Sectors</th>
<th>U.S. Dollar</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_0 )</td>
<td>0.117</td>
<td>0.169*</td>
<td>0.154*</td>
</tr>
<tr>
<td></td>
<td>(0.074)</td>
<td>(0.089)</td>
<td>(0.087)</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>-0.079**</td>
<td>-0.043</td>
<td>-0.056</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.040)</td>
<td>(0.038)</td>
</tr>
<tr>
<td>( \omega )</td>
<td>0.221**</td>
<td>0.705***</td>
<td>0.411***</td>
</tr>
<tr>
<td></td>
<td>(0.111)</td>
<td>(0.203)</td>
<td>(0.110)</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>0.145***</td>
<td>0.168***</td>
<td>0.193***</td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.031)</td>
<td>(0.032)</td>
</tr>
<tr>
<td>( \beta )</td>
<td>0.820***</td>
<td>0.747***</td>
<td>0.761***</td>
</tr>
<tr>
<td></td>
<td>(0.051)</td>
<td>(0.043)</td>
<td>(0.033)</td>
</tr>
<tr>
<td></td>
<td>(1.644)</td>
<td>(1.829)</td>
<td>(1.927)</td>
</tr>
</tbody>
</table>

| LL            | 2380         | 2602                | 2544        | 2178        | 2741   | 2735       | 2418   | 2546        | 2797      | 2708       | 2576   | 2405      | 1054   |
| Joint Bias    | 1.227        | 0.058               | 0.180       | 0.255       | 0.265  | 0.275      | 1.733  | 0.024       | 0.339     | 0.171      | 0.538  | 1.355     | 0.146  |

Notes: Robust standard errors in parentheses; F-statistics are presented for each Engle and Ng [1993] joint bias test; * p-val < 0.10; ** p-val < 0.05; *** p-val < 0.01.
### Table 3.3: Marginal Model Goodness-of-Fit

<table>
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<tbody>
<tr>
<td>First Moment</td>
<td>0.598</td>
<td>0.674</td>
<td>0.940</td>
<td>0.826</td>
<td>0.958</td>
<td>0.736</td>
<td>0.614</td>
<td>0.850</td>
<td>0.728</td>
<td>0.650</td>
<td>0.624</td>
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<tr>
<td>Second Moment</td>
<td>0.894</td>
<td>0.496</td>
<td>0.930</td>
<td>0.342</td>
<td>0.982</td>
<td>0.926</td>
<td>0.414</td>
<td>0.986</td>
<td>1.000</td>
<td>0.504</td>
<td>0.370</td>
<td>0.598</td>
<td>0.686</td>
</tr>
<tr>
<td>Third Moment</td>
<td>0.398</td>
<td>0.824</td>
<td>0.740</td>
<td>0.536</td>
<td>0.902</td>
<td>0.904</td>
<td>0.914</td>
<td>0.240</td>
<td>0.960</td>
<td>0.896</td>
<td>0.578</td>
<td>0.506</td>
<td><strong>0.036</strong></td>
</tr>
<tr>
<td>Fourth Moment</td>
<td>0.890</td>
<td>0.184</td>
<td>0.808</td>
<td>0.682</td>
<td>0.994</td>
<td>0.964</td>
<td>0.656</td>
<td>0.990</td>
<td>1.000</td>
<td>0.630</td>
<td>0.898</td>
<td>0.348</td>
<td>0.570</td>
</tr>
<tr>
<td>CvM</td>
<td>0.482</td>
<td>0.328</td>
<td>0.510</td>
<td>0.686</td>
<td>0.552</td>
<td>0.668</td>
<td>0.698</td>
<td>0.556</td>
<td>0.686</td>
<td>0.308</td>
<td>0.830</td>
<td>0.436</td>
<td>0.500</td>
</tr>
</tbody>
</table>

Notes: Simulated p-values are presented for each test, N = 500 simulations; The moment test consists of performing an LBQ(12) test on \( (\hat{u}_t - \bar{\hat{u}})^k \) for \( k = \{1, 2, 3, 4\} \) where \( \bar{\hat{u}} = T^{-1} \sum_t \hat{u}_t \); CvM is Cramer-von Mises distribution test.
CHAPTER 3. U.S. EQUITIES AND DOLLAR STRENGTH

Figure 3.1: Probability Integral Transformed Standardized Residuals
3.4.2 The GAS Copula Model

Creal et al. (2013) propose a flexible and general observation driven dynamic estimation methodology motivated by the behavior of a probability density’s score function over time. For simplicity, assume that copula $C_t(\cdot)$ possess only one time-varying parameter, $\delta_t$. In practice, $\delta_t$ is often restricted to a certain parameter space dependent on the copula function. To deal with this issue, we transform the restricted parameter $\delta_t$ into another parameter $f_t$ that is more flexibly modeled:

$$f_t = h(\delta_t) \iff \delta_t = h^{-1}(f_t).$$

(3.12)

The GAS model is then described by the following three equations:

$$f_{t+1} = \omega + \alpha I_t^{-1/2} s_t + \beta f_t$$

(3.13)

$$s_t = \frac{\partial}{\partial \delta_t} \log c_t(u_t, v_t; \delta_t)$$

(3.14)

$$I_t \equiv E_{t-1}[s_t s_t'] = I(\delta_t),$$

(3.15)

where $\beta \in (0, 1)$. The “forcing variable”, $s_t$, is determined by the shape of the copula density function at each point in time as captured by the score with respect to $\delta_t$.

Normal and Student’s t copulas are selected for the underlying densities given the weak evidence of tail dependence and no clear visual support for asymmetry seen in Figure 3.1. For each copula model we estimate the time varying correlation using the transformation, $\delta_t = (1 - e^{-f_t})/(1 + e^{-f_t})$, restricting $\delta_t \in (-1, 1)$. Finally, we model the inverse t-degree of freedom parameter, $\nu^{-1}$, for the Student’s t-GAS copula and hold it fixed over time to decrease the computational burden.
3.4.3 Dynamic Dependence By Equity Market Size

Table 3.4 presents the model estimates grouped by market capitalization. The coefficients for the forcing variable ($\hat{\alpha}$) and autoregressive term ($\hat{\beta}$) are significant across copulas and market size. $\hat{\beta}$ are close to one in all cases, suggesting each dynamic process is highly persistent. Testing whether $\hat{\nu}^{-1}$ is significantly greater than zero amounts to a nested test comparing the Normal and Student’s t-GAS copulas. As $\hat{\nu}^{-1}$ is significant at the 1% level for both the large and small cap models, we reject the Normal in favor of the Student’s t-GAS copula.

Table 3.4: Large and Small Market Cap GAS Copula Estimates

<table>
<thead>
<tr>
<th></th>
<th>Large Cap</th>
<th></th>
<th></th>
<th>Small Cap</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>Student’s t</td>
<td>Normal</td>
<td>Student’s t</td>
<td>Normal</td>
<td>Student’s t</td>
</tr>
<tr>
<td>$\omega$</td>
<td>-0.006</td>
<td>-0.007</td>
<td>-0.009</td>
<td>-0.005</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.014)</td>
<td>(0.053)</td>
<td>(0.044)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.056***</td>
<td>0.111***</td>
<td>0.043**</td>
<td>0.065*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.034)</td>
<td>(0.019)</td>
<td>(0.038)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.987***</td>
<td>0.987***</td>
<td>0.982***</td>
<td>0.989***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td>(0.016)</td>
<td>(0.069)</td>
<td>(0.075)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\nu^{-1}$</td>
<td>0.207***</td>
<td></td>
<td>0.232***</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td></td>
<td>(0.061)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LL</td>
<td>73.548</td>
<td>85.511</td>
<td>51.562</td>
<td>67.801</td>
<td></td>
<td></td>
</tr>
<tr>
<td>BIC</td>
<td>-126.097</td>
<td>-143.024</td>
<td>-82.126</td>
<td>-107.604</td>
<td></td>
<td></td>
</tr>
<tr>
<td>CvM</td>
<td>0.622</td>
<td>0.628</td>
<td>0.502</td>
<td>0.480</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Notes: Simulated standard errors in parentheses, N = 500 simulations; Simulated Cramér-von Mises (CvM) p-values are presented, N = 500 simulations; * p-val < 0.10; ** p-val < 0.05; *** p-val < 0.01.
A modified Cramer-von Mises test is used to more rigorously assess the appropriateness of each copula density to the underlying data. Here, a multivariate probability integral transform, or “Rosenblatt transform”, is conducted prior to employing the test. The probability integral transform is necessary due to the dynamic copula structure\(^6\). The bottom panel of Table 3.4 shows a failure to reject the null hypothesis that each model is reasonably specified. However, in light of the significant \(\hat{\nu}^{-1}\), we proceed using Student’s t-GAS copula models to generate dynamic correlation and conditional tail dependence estimates.

The top panel of Figure 3.2 presents the estimated dynamic correlations and conditional tail dependence by market size. Correlations between equity and dollar index returns are negative for the majority of the sample, aligning with the predictions of Hau and Rey (2006); dollar appreciations (depreciations) are associated with negative (positive) equity market returns. Except for the late 1990s, large and small Cap correlations track each other closely over the sample period. Week-over-week changes in the large cap correlation are roughly 69\% more volatile than that of the small cap series, demonstrating the greater degree of exchange rate sensitivity faced by larger firms. Contrary to prevailing sentiment, there are many periods in which small cap equities’ correlation strength is stronger. More specifically, the small cap correlation is greater in magnitude 35.1\% of the time and at a rate of 6.0\% while in the opposite direction. Ignoring these facts, investors may overestimate potential diversification protection against dollar value fluctuations across equity market size.

Prolonged positive correlation periods tend to end violently with rapid trend/sign reversals, the largest of which occurs in the early 2000s for the large cap correlation series. During the late 1990s, tight U.S. monetary policy and capital inflows strengthened the U.S. dollar. Rising interest rates should typically lead to a declining correlation, where equity markets fall as the dollar appreciates. However, overly optimistic market sentiment instead drove U.S. equity markets to new highs. These forces, amongst other factors, resulted in stronger co-movement. As the Dot-Com

\(^6\)See Diebold et al. (1999) and Rémillard (2017) for further details.
bubble burst, investors’ expectations adjusted and abrupt changes in monetary and fiscal policy sent the correlations crashing. The large cap correlation fell from a high of 0.339 on 12/24/1999 to a low of -0.423 on 12/01/2000; this amounts to more than a full reversal within a twelve-month period. The small cap correlation fell as well, but to a lesser extent and over a shorter period.

Figure 3.2: Large and Small Market Cap Model Dynamics
A similar pattern of behavior persists around the Global Financial Crisis. The largest *combined* drop in correlation magnitudes occurs shortly following BNP Paribas’ decision to freeze fund redemptions on 8/09/2008. Overall, large and small Cap correlations fell from highs of 0.191 and 0.049 on 8/29/2008 to -0.465 and -0.391 by the end of 2008; they recovered slightly before proceeding to fall even further in to the second half of 2009 as the crisis gave way to recession.

Conditional tail dependence for each model is calculated using the dynamic correlation series and corresponding inverse t-degree of freedom estimates from Table 3.4. As seen in the bottom of Figure 3.2, there are four distinct periods with substantial increases in tail dependence: i) following the 1997-1998 Asian Financial Crisis and through the Dot-Com bubble collapse in 2000; ii) mid-2002 until 2003 as the U.S. prepared for and initiated the second Iraq war; iii) during the Global Financial Crisis; and iv) following the 2016 U.S. presidential election. Large cap tail dependence exceeded 0.10 in the first two cases, suggesting a greater than 10% likelihood of extreme co-movement between changes in the value of the dollar and S&P 500 returns. For the latter two periods, this percentage is smaller but not insignificant. These economic periods reflect times when the underlying dynamic correlation is positive, suggesting investors should be wary when equity and foreign exchange markets move in tandem for prolonged periods of time. While simple rolling or DCC-GARCH generated dynamic correlations can recover the correlation behavior, neither methodology is able to capture the underlying tail dependence.

Conditional tail dependence fundamentally changes in latter half of the sample. Overall tail dependence levels are lower for both the large and small cap models following the Global Financial Crisis. Splitting the sample in to 1996-2006 and 2007-2017, average weekly tail dependence decreases by 55.6% and 48.3%, respectively. Elevated tail dependence in the first half of the sample may be attributed to the higher frequency of currency-related economic events experienced by the basket of countries included in the broad currencies dollar index (e.g., the Mexican Peso crisis in 1995, the Russian default in 1998, the Euro introduction in 1999, and the Argentinian default in 2001), as well as more substantial U.S. interest rate volatility. In contrast to the 1990s
and early 2000s, a near zero Effective Federal Funds Rate persisted from the end of 2008 through late 2015.

### 3.4.4 Dynamic Dependence By Equity Market Sector

Moving beyond broad market analysis, we now focus on the dynamic relationship between each individual S&P 500 GICS Level I sub-sector against the dollar value. This analysis provides an additional means by which to explore the role that international exposure affects the dynamic dependence structure.

Tables 3.5 and 3.6 present the GAS copula estimates by sector. \( \hat{\alpha} \) and \( \hat{\beta} \) are significant across sectors and copula models with the exception of \( \hat{\alpha} \) for the Telecommunications (Telecom) sector. In this case, \( \hat{\beta} \approx 1 \) regardless of copula density, suggesting a near integrated process. The estimated inverse t-degree of freedom parameter, \( \hat{\nu}^{-1} \), is significant for all sectors except for Telecom and Utilities. Finally, simulated CvM p-values suggest either copula structure is reasonably specified for each market sector. As before, copula appropriateness rests on the significance of \( \hat{\nu}^{-1} \). We use the Normal GAS specification for the telecommunications and utilities models and Student’s t-GAS for the remaining sectors to generate dynamic correlations and conditional tail dependence estimates.
<table>
<thead>
<tr>
<th></th>
<th>Cons. D.</th>
<th>Cons. S.</th>
<th>Energy</th>
<th>Financials</th>
<th>Health</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>Student’s t</td>
<td>Normal</td>
<td>Student’s t</td>
<td>Normal</td>
</tr>
<tr>
<td>$\omega$</td>
<td>-0.007</td>
<td>-0.008</td>
<td>-0.007</td>
<td>-0.008</td>
<td>-0.003</td>
</tr>
<tr>
<td></td>
<td>(0.035)</td>
<td>(0.015)</td>
<td>(0.014)</td>
<td>(0.012)</td>
<td>(0.092)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.049**</td>
<td>0.103***</td>
<td>0.056***</td>
<td>0.125***</td>
<td>0.032**</td>
</tr>
<tr>
<td></td>
<td>(0.021)</td>
<td>(0.036)</td>
<td>(0.021)</td>
<td>(0.039)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.982***</td>
<td>0.981***</td>
<td>0.983***</td>
<td>0.981***</td>
<td>0.995***</td>
</tr>
<tr>
<td></td>
<td>(0.055)</td>
<td>(0.024)</td>
<td>(0.021)</td>
<td>(0.016)</td>
<td>(0.082)</td>
</tr>
<tr>
<td>$\nu^{-1}$</td>
<td>0.173***</td>
<td>0.153***</td>
<td>0.127***</td>
<td>0.195***</td>
<td>0.155***</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.056)</td>
<td>(0.056)</td>
<td>(0.062)</td>
<td>(0.057)</td>
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<tr>
<td>LL</td>
<td>43.884</td>
<td>53.303</td>
<td>41.842</td>
<td>50.102</td>
<td>107.916</td>
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<tr>
<td>CvM</td>
<td>0.606</td>
<td>0.606</td>
<td>0.684</td>
<td>0.656</td>
<td>0.814</td>
</tr>
</tbody>
</table>

Notes: Simulated standard errors in parentheses, N = 500 simulations; Simulated Cramér-von Mises (CvM) p-values are presented, N = 500 simulations;

* p-val < 0.10; ** p-val < 0.05; *** p-val < 0.01.
Table 3.6: S&P 500 Sub-Sector GAS Copula Estimates (continued)

<table>
<thead>
<tr>
<th></th>
<th>Industrials</th>
<th>Info Tech</th>
<th>Materials</th>
<th>Telecom</th>
<th>Utilities</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
<td>Student’s t</td>
<td>Normal</td>
<td>Student’s t</td>
<td>Normal</td>
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<tr>
<td>$\omega$</td>
<td>-0.009</td>
<td>-0.011</td>
<td>-0.009</td>
<td>-0.010</td>
<td>-0.001</td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.021)</td>
<td>(0.038)</td>
<td>(0.025)</td>
<td>(0.176)</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.053***</td>
<td>0.114***</td>
<td>0.049**</td>
<td>0.108**</td>
<td>0.052***</td>
</tr>
<tr>
<td></td>
<td>(0.019)</td>
<td>(0.100)</td>
<td>(0.021)</td>
<td>(0.043)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.983***</td>
<td>0.981***</td>
<td>0.986***</td>
<td>0.979***</td>
<td>0.987***</td>
</tr>
<tr>
<td></td>
<td>(0.017)</td>
<td>(0.026)</td>
<td>(0.096)</td>
<td>(0.064)</td>
<td>(0.018)</td>
</tr>
<tr>
<td>$\nu^{-1}$</td>
<td>0.217***</td>
<td>0.219***</td>
<td>0.106**</td>
<td>0.049</td>
<td>0.049</td>
</tr>
<tr>
<td></td>
<td>(0.061)</td>
<td>(0.063)</td>
<td>(0.049)</td>
<td>(0.048)</td>
<td>(0.048)</td>
</tr>
<tr>
<td>LL</td>
<td>55.447</td>
<td>70.517</td>
<td>43.522</td>
<td>55.462</td>
<td>101.590</td>
</tr>
<tr>
<td>CvM</td>
<td>0.472</td>
<td>0.470</td>
<td>0.790</td>
<td>0.832</td>
<td>0.464</td>
</tr>
</tbody>
</table>

Notes: Simulated standard errors in parentheses, N = 500 simulations; Simulated Cramer-von Mises (CvM) p-values are presented, N = 500 simulations;
* p-val < 0.10; ** p-val < 0.05; *** p-val < 0.01.
Figure 3.3 plots the dynamic correlations for each sector. As suggested by Figure 3.1 correlations are fairly volatile over time with both positive and negative periods. Similar to the broad market correlations, the sub-sector series are negative for most of the sample. Furthermore, the sector-specific correlations tend to more closely track the greater equity market when there are major economic developments (e.g., the Global Financial Crisis); this is likely attributable to the existence of tail dependence between the S&P 500 and its sub-sector components. The Financials and Telecom sectors yield the most and least volatile dynamic correlations, respectively. While most sectors follow a similar time path, the Telecom and Utilities sector dynamic correlations deviate substantially from the rest. Diversification by investing in equities across sectors may provide some hedge against dollar fluctuations when sector-specific correlations diverge from one another.

Figure 3.4 presents the conditional tail dependence estimates calculated from each t-GAS model. As before, tail dependence is greatest when the corresponding dynamic correlations are positive or strengthening in the positive direction. This particular feature is common across all models despite varying magnitudes over time. The Consumer Staples (Cons. S), Financials, Industrials, and Info Tech sectors consistently display the highest degree of tail dependence. From these, Info Tech tail dependence is highest on average. However, most of the extreme values occurs around the Dot-Com bubble. Financials sector tail dependence is greatest following the 2016 U.S. presidential election, reflecting the impact of high growth expectations coupled with announced monetary policy tightening. While most sectors’ dynamic correlations (except for Utilities) move in the positive direction during this period, a large increase in tail dependence is only seen for the Financials sector.
Figure 3.3: S&P 500 Sub-Sector Dynamic Correlations
Figure 3.4: S&P 500 Sub-Sector Conditional Tail Dependence
Simply focusing on the correlation between sector-specific equity and dollar index returns is clearly misleading. While correlations provide some insight into the relationship over time, they fail to account for changes in the dependence structure under extreme conditions. In particular, it is when U.S. equity markets co-move with changes in the value of the dollar that investors should be most concerned about tail dependence. This is especially important given the rapid adoption of passive investment strategies by both individual and institutional investors. There are a plethora of exchange traded funds (ETFs) that investors may purchase to tracking entire market segments, as opposed to individual firms with a sector.

3.5 Concluding Remarks

This study examined the time varying dependence structure between U.S. equity market performance and fluctuations in dollar strength using a generalized autoregressive score (GAS) copula framework. Equity market size and sector representation were considered, exploring the extent to which dynamic dependence structures vary by international exposure within and across market size. U.S. dollar strength was measured against a broad, annually adjusted trade-weighted basket of currencies. Motivated by the underlying data properties, conditional mean and variance models were estimated to filter each series and to generate the residuals necessary for copula estimation. Normal and Student’s t-GAS copulas were estimated conditional on the first-stage marginal model estimates.

We find highly persistent dynamic correlation processes and weak but significant symmetric tail dependence for both large and small size U.S. equity market measures. Dynamic correlations are negative on average, suggesting that dollar appreciation (depreciation) corresponds with negative (positive) equity returns. Positive correlation periods tend to reverse themselves in a rapid and

\[7\] In real terms, U.S. end of quarter exchange traded fund (ETF) holdings grew from $2.1B in 1996Q1 to $2.4T by 2016Q3, a staggering 113,048% increase. The share of holdings allocated to corporate equities, such as the SPDR SPY, fluctuates between 70-80% on average (Federal Reserve Financial Accounts).
dramatic fashion, where these episodes exhibit the strongest symmetric tail dependence between markets; these episodes tend to occur during times of market exuberance. Contrary to prevailing wisdom, small market cap correlations are stronger and opposite in sign than large cap for a non-insignificant number of observations. In light of these results, investing across equity market size may prove to be insufficient as a currency hedge.

While sub-sector correlations follow similar paths over time, there are large differences in the magnitude responses to various economic events. The Telecommunication and Utilities sector correlations deviate most dramatically from the other sub-sectors and broader market as a whole. Each sub-sector correlation series is negative over most of the sample, with scattered and brief positive periods. Tail dependence is symmetric but generally weak for eight of the ten sectors; the greatest strength is observed for sectors most exposed to international trade, financial, and commodity markets (e.g., Financials and Industrials).

Our findings have practical implications for portfolio diversification and risk management, both from the perspective of individual and institutional investors. Traditional analysis of the relationship between U.S. equity and dollar returns fails to account for tail dependence. In doing so, the risks associated with co-movement under extreme economic conditions are often ignored. While the relationships at both the aggregate and sub-sector level are mostly negative, there are clearly periods in which tail dependence is present. This analysis is particular important given the rise of passive investing strategies where investors focus solely on index tracking.

Beyond the scope of this study, further analysis may consider a higher dimension problem where individual bilateral exchange rates are used instead of a dollar index. Modeling dynamic high order multivariate distributions limits the choice of copulas applicable. Oh and Patton (2015) suggest a stochastic factor-driven framework to capture the information content of large groups of variables. However, there remains the issue of how to determine the appropriate number of factors and then provide reasonable interpretation for each factor. Data structure provides further complication, consisting of one equity and \( N \) exchange rate returns series, as opposed to \( N \) series of
the same asset class. As an alternative methodology, Almeida et al. (2015) propose using D-Vine organization to handle high-dimensions where dynamics are captured either through a stochastic auto-regressive copula (SCAR) (Hafner and Manner, 2012) or GAS processes (Creal et al., 2013). Here, the challenge lies in how to organize the tree structure.
Bibliography


