Abrupt Climate Transitions

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Abrupt Climate Transitions

by

Christine Ramadhin

This manuscript has been read and accepted for the Graduate Faculty in Earth and Environmental Sciences in satisfaction of the dissertation requirements for the degree of Doctor of Philosophy.

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Abstract

Abrupt Climate Transitions

By Christine Ramadhin

Advisor: Dr. Chuixiang Yi

The Earth’s climate system displays a long history of nonlinear abrupt transitions which have resulted in significant ecosystem disruption and are recorded in the geologic data. Today significant anthropogenic changes are occurring in many Earth systems that seem to be pushing these toward critical thresholds. Thus, increasing the possibility of a transition to alternative states which can have unfavorable consequences. Therefore, it becomes compelling to forecast when and how these transitions will occur so that decision-makers can devise appropriate strategies to avoid or cope with the effects of a changeover to a new alternative state. However, due to the highly nonlinear nature of critical transitions, current understanding of knowing when thresholds have been crossed, or predictions of the rate, extent and nature of a climate transition have remained a challenge with considerable uncertainty. This dissertation aims to improve understanding of abrupt climate transitions by examining published literature and paleoclimate data of the nonlinear entrances and exits to and from interglacial conditions.

In the first part of this study, a conceptual model of two negative sea ice feedbacks that may have caused slower glaciations is proposed to explain the asymmetrical shape observed in the glacial-interglacial cycles. This proposal implies that negative feedbacks play an important role in moderating rate of climate transitions and should be considered in future studies estimating the speed of critical transitions.
Second, this dissertation examines multiple paleotemperature datasets, for changes in temporal variance before an entrance or exit from an interglacial. Increased variance has been shown to announce the imminence of critical thresholds in earlier theoretical and modeling studies of dynamic systems with different underlying gradually changing forcing. Here, using empirical data and two statistical methods, the moving variance and the Ansari-Bradley tests, the results indicate, increases in paleotemperature variance announces a glacial termination. Additionally, this study found evidence suggesting that the size of the variance increase forecasting an interglacial may predict its maximum. Thus, implying that the magnitude of the preceding variance increase has the potential of being developed as a novel tool to estimate the interglacial maximum following it.

Finally, this study examined $\delta^{13}C$ isotopic records covering the last five termination events from both shallow and deep Atlantic Ocean sources and found a decrease in values coinciding with glacial terminations. This provides an added line of evidence supporting the hypothesis of a previous study that suggested thermal destabilization of methane clathrates occurred during the last glacial termination. Additionally, the results show that the decrease in $\delta^{13}C$ observed following the start of a warming event, implies the temperature increase might have caused the methane clathrate dissociation. The main implication of this finding is that current global warming can potentially destabilize the existing large methane clathrate reservoir buried under permafrost as sea ice continues to melt and resulting in amplified warming due to the climate-carbon feedback. For future work, a simple climate-carbon model is assembled along with detailed derivation of the equations and is presented here with the intent of further analyzing the mathematical phenomenon of bifurcation and critical thresholds in the climate system.
Acknowledgements

First, I would like to thank my advisor, Dr. Chuixiang Yi for giving me the opportunity to work on this project dedicating his time and effort to countless scientific discussions, reports and presentations over the years which have all challenged me to develop multidisciplinary skills over the years. His unwavering dedication to his research has motivated and inspired me and I hope to take at least a fraction of that passion to my future endeavors. I would also like to express my gratitude to Dr. George Hendrey who encouraged me to think critically with his insightful questions and comments enabling me to grow both professionally and personally. He has also invested many hours in reading and improving my manuscripts over the years. Sincere thanks to my other committee members, Dr. Cecilia McHugh, Dr. Gary Hemming and Dr. Nir Krakauer who have taken time to read, comment, encourage and offer insightful advice on my research.

Special thanks to the Earth and Environmental Sciences Departments at the Graduate Center and Queens College, City University of New York (CUNY) for providing a supportive environment and financial assistance especially to the Graduate Center for the Quantitative Reasoning Fellowship and Queens College for the Gural Awards and other fellowships. I am grateful to the department chairs, Dr. Greg O’Mullan, Dr. Jeffrey Bird and Executive Officer, Dr. Monica Varsanyi for their thoughtful support and invaluable guidance in this process. I am thankful to my friend and lab mate, Eric Kutter for patiently reading my manuscripts, offering a positive voice and being an excellent sounding board. I would also like to thank my other lab and office mates Dr. Xiyan Xu, Anand Dookie, Dr. Hongbin Chen, Derek Mu, Dr. Suhua Wei, Dr. Wei Fang, Dr. Shan Gao, Dr. Peipei Xu, Dr. Yukikazu Murakami, Ayo Deas and many others who have shared their friendships and enthusiasm for science with me.
Finally, I would like to thank my wonderful mom, Rampyearie Ramadhin and brother David Ramadhin for their unconditional love and support without which this project would not have been possible. In this journey, I have also experienced the help of many wonderful friends, Celeste Ramnarain, Nerissa Mohammed, the Singh family, Gladys Sapiago, Mustafa Kamal and others. And lastly, special thanks to Mufasa who has loved, comforted me and encouraged me to think more creatively.
Common Abbreviations and Symbols used in this Dissertation

AMOC - Atlantic Meridional Overturning Circulation

EBM – Energy Balance Model

EPICA - European Project for Ice Coring in Antarctica

Ka – Kilo annum

LGM – Last Glacial Maximum

Ma – Mega annum

MIS - Marine Isotope Stages

NADW – North Atlantic Deep Water

NH – Northern Hemisphere

NCDC - National Climatic Data Center

NOAA – National Oceanic and Atmospheric Administration

ODP - Ocean Drilling Program

PETM - Paleocene-Eocene Thermal Maximum

ppbv – parts per billion volume

ppmv – parts per million volume

PW - Petawatts

SST - Sea Surface Temperature

TI – Termination 1

$\delta^{13}C$ - ratio of stable isotopes $^{13}C:^{12}C$ (delta C 13)

$\delta^{18}O$ - ratio of stable isotopes $^{18}C:^{16}C$ (delta O 18)

‰ – per mil
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Chapter 1 Introduction

1.1 Motivation

According to the National Research Council (NRC) 2002 report, *Abrupt Climate Change: Inevitable Surprises*, “an abrupt climate change is when the climate system is forced to cross some threshold, triggering a transition to a new state at a rate determined by the climate itself and faster than the cause.” In this work, the glacial-interglacial cycles are studied as one example of these types of abrupt climate transitions from the past. Paleoclimate data shows that abrupt climate transitions are common throughout the Earth’s history (Hays et al., 1976; Imbrie et al., 1993; Raymo et al., 1998). These climate change events are usually followed by significant ecosystem disruption (Scheffer et al., 2001). Today extensive anthropogenic changes in various Earth systems are occurring and may be pushing the climate system toward tipping points and critical transitions (Alley et al., 2003). The paleoclimate data cover a multitude of abrupt climate changes. Therefore, investigating the mechanisms and behavior of Earth system variables from the past may help in developing insights on how the system will behave when exposed to changing conditions in the future. The goal of this dissertation is improving understanding of abrupt climate transitions by analyzing changes in the behavior of paleoclimate variables around the vicinity of the nonlinear transitions of glacial terminations and glacial inceptions.

1.2 Background

1.2.1 Glacial-Interglacial Cycles

An interglacial is considered a warm period with conditions such as present day with little ice cover and may last between 10-30 Ka while glaciations are cold periods with extensive glaciers lasting for about 100 Ka as observed during the latter Pleistocene (Imbrie et al., 1992, Lang and
Wolff, 2011, Berger et al., 2016). Even though proxy records show that the glacial-interglacial cycles were the most salient pattern of recent geological climate, there is still no consensus on the underlying causal mechanism and sequence of events for these climate shifts (Hays et al., 1976; Imbrie and Imbrie, 1980; Imbrie et al., 1993; Saltzman, 2002; de Boer et al., 2010). These cycles are thought to be triggered by changes in solar insolation that reach the surface of the Earth and is termed the Milankovitch or astronomical theory of climate (Berger et al., 1999). In the 1930s, Milankovitch proposed that the main initiator of these climate transitions is due to orbital changes, identifying three major orbital variations: obliquity, precession, and eccentricity and is so far the best accepted theory (Berger et al. 2007). Geologic data record glacial and interglacial cycles, based on variations of the oxygen isotope ratio ($\delta^{18}$O) as and are known as Marine Isotope Stages (MIS) where odd numbers denote interglacials while even numbers identify glaciations (Berger et al., 2016). Following this nomenclature, this manuscript will use the term MIS followed by a number to identify the specific interglacial being addressed as shown in Figure 1.1.

Proxy data obtained from oxygen isotopes of sediments collected from the deep sea showed good agreement between the orbital cycles and provided supporting evidence for the Milankovitch theory (Hays et. al., 1976). However, these changes in insolation values are too small to account for the large changes in temperature observed between the glaciations and interglacials (Wunsch, 2004; Kohler et al., 2005).
Previous studies have considered internal processes in the climate system that can change the impact of forcings and enable these long-term climate changes observed in the paleoclimate data (Raymo, 1997; Banderas et al., 2012). For example, two pioneer studies have long shown that changes in the ice-albedo feedback (Appendix B, Figure B.2) of the Earth may be a major player in abrupt climate shifts by regulating the amount of insolation the Earth receives (Budyko, 1969; Sellers, 1969). A later study using an Energy Balance Model (EBM) with clouds added to this, not only considering the global ice cover but included cloud cover, as clouds can also have an impact on the albedo of the Earth (Källén et al., 1979). More recent studies would confirm that the ice – albedo feedback plays an important role in enabling glacial-interglacial transitions due to changes in ice cover (Paillard, 1998; Parrenin and Paillard, 2003; Calov et al., 2005a). A
comprehensive list and description of some of these feedbacks at work in the climate system during the transitions into and out of an interglacial are given in Appendix B.

1.2.2 Characteristics of Glacial-Interglacial Cycles

Since the goal of this work includes improving knowledge of how internal climate variables work together to enable transitions into and out of an interglacial, it is important to define the steady state values of these variables for glaciations and interglacials. Glaciations and interglacial events can each be characterized by similar features such as temperature, atmospheric carbon, ice volume, sea levels, precipitation and atmospheric dust concentration. Glacial-interglacial temperature differences vary spatially with larger differences seen in higher latitudes where significantly lower temperatures characterize glaciations (Shakun et al., 2012). Based on a compilation of 104 high-resolution records the average global temperature difference between the last glacial maximum and peak interglacial conditions is -4.9°C (Shakun et al., 2010). Ice core data shows that glacial-interglacial temperatures have been strongly correlated with the concentration of the greenhouse gases; atmospheric CO₂ and CH₄ concentrations (Petit et al., 1999). The concentration of these gases are derived from air bubbles in the ice cores from Antarctica. Glacial periods had an average of 180 ppmv CO₂ and CH₄ concentrations of 320 – 350 ppmv while interglacials record an average of 280-300 ppmv CO₂ and CH₄ values at 650 – 770 ppbv (Petit et al., 1999; Lüthi et al., 2008).

Using pollen grains preserved in lakes in France and Russia, glaciations are found to be characterized by lower precipitation values than interglacials at 400-480 mm yr⁻¹ for the glaciations and 800 mm yr⁻¹ (Cheddadi et al., 2005; Tarasov et al., 2007). Wu et al., (2007) used an inverse vegetation model to show precipitation in the tropics were more variable during the last glacial maximum (LGM) with a range of values of 200 - 1000 mm yr⁻¹ lower than the current interglacial.
Global sea levels have been found to inversely mirror global ice volume as these variations are the result of the exchange of water between ice and ocean (Lambeck et al., 2014). Estimates showed land-based ice sheets during the last glacial maximum exceeded current volume by about $52.5 \times 10^6 \text{ km}^3$ while sea levels were about -120 m lower than present day as shown in MIS 2 (Yokoyama et al., 2000). Glacial cycles are characterized by a lower aeolian dust volume in comparison with interglacials with high-low concentrations of 800 - 15 $\mu$g kg$^{-1}$ (Lambert et al., 2008).

1.2.3 Mechanisms of Glacial-Interglacial Transitions

Numerous mechanisms to explain the shifts from a cold glaciation to a warm interglacial period and the changeover back to a glacial state have been proposed. This chapter introduces a few of these here focusing on those most relevant to this dissertation.

1.2.4. i Terrestrial Mechanisms

It has been suggested that the rapid glacial termination is due to the terrestrial biosphere acting as a large carbon reservoir due to an enhanced CO$_2$ fertilization-Biosphere feedback (Appendix B, Figure B.4) during an interglacial (Zeng, 2003). This carbon gets buried under the massive ice sheets during the long glaciation but is rapidly released increasing atmospheric CO$_2$ by about 30 ppmv upon warming and melting of the ice accelerating the warming through the positive climate-carbon feedback (Appendix B, Figure B.1) (Zeng, 2003). Later model studies of the terrestrial biosphere showed the advancing of the boreal forest farther North during an interglacial and therefore greater carbon storage by the biosphere in this period (Brücher and Brovkin, 2013). A reexamination of West Antarctic ice core data found that the atmospheric CO$_2$ increased much faster than the response time of the deep oceans which is on the order of 1000s of years (Marcott et al., 2014). There were three major pulses during the last deglaciation of 10-15 ppmv in less than
two centuries suggesting that some other component of the Earth system other than the ocean must be involved (Marcott et al., 2014).

1.2.4. ii Ocean Circulation Mechanisms

The climate-carbon feedback shown in Appendix B, Figure B.1, is also influenced by changes in ocean circulation and CO₂ solubility which can lead to significant changes in atmospheric CO₂ concentration (Ewen et al., 2004). Ocean circulation changes are an essential variable in facilitating glacial-interglacial cycle transitions (Imbrie et al., 1993; Cortijo et al., 1994; Khodri et al., 2001; Rahmstorf, 2006). During glacial terminations, a positive feedback mechanism is initiated with increased warming resulting in shifting of the Southern Hemisphere Westerlies poleward causing increased upwelling of deep nutrient-rich waters and outgassing of CO₂ to the atmosphere leading to greater warming (Toggweiler et al., 2006). Anderson et al., (2009) added support when they studied sediment cores taken from the Southern Hemisphere looking at changes in burial rate of opal and discovered that during terminations there is increased upwelling of deep carbon-rich water and outgassing of CO₂ to the atmosphere. During glacial inceptions the Southern Westerlies shift north with global cooling, sea ice formation is enhanced around Antarctica, ocean stratification increases, mixing and upwelling are reduced, decreased outgassing of CO₂ from the Antarctic deep-water dropping its atmospheric concentration and further cooling (Watson and Garabato, 2006).

Variations in the Atlantic Meridional Overturning Circulation (AMOC) is also shown to have an impact on climate. According to Rahmstorf, (2002 and 2006) the AMOC has three circulation modes based on global climate. He showed that there is a warm mode with North Atlantic Deep Water (NADW) formation south of Greenland, a cold mode with NADW forming North East of Greenland and a switched off mode where vertical stratification of the North Atlantic is strong.
These are all linked to climate fluctuations between warm and cold periods. Observation data confirms the existence of varying modes of operation and the significance of the AMOC in determining climate by following the circulation through its three modes as various climate events progressed including a cold Heinrich event, the Younger Dryas and Bolling warming events (McManus et al., 2004).

1.3 Dynamic Systems Theory

The theory of dynamical systems is used to study interactions among system components and recognize the current state as the result of a balance of these interactions where the state is known as an attractor represented by stability landscape diagrams as shown in Appendix A (Scheffer and Carpenter, 2003). Each stable state is represented by a well also known as a basin of attraction and the separating hill symbolizes an unstable state, the slope corresponds to the rate of change thus equilibrium is found where the line is horizontal, the slope is zero (Scheffer, 2009). The upper part of Figure A.1 in Appendix A shows the transition from stable conditions during an interglacial stage, considered one stable state. Then as conditions change, multiple stable and unstable equilibrium states develop as the the bifurcation point (red arrow) is approached, beyond which any small change in conditions can tip the system over to another stable state (a glacial state in this case). The bottom of Figure A.1 represents a system of three stable states, interglacial at higher temperatures, an unstable state (dashed part of the curve) and glaciation at the cooler end of the curve. Any perturbation of the system while on the unstable part of the curve will cause it to move to one of the stable attractors (Scheffer and Carpenter, 2003). A critical transition occurs when the system shifts from one equilibrium state to another (Scheffer et al., 2009).
1.3.1 Bifurcations and Threshold Points

As conditions change a system may respond smoothly or may be insensitive to changes over a certain range of parameter values and suddenly respond strongly after a threshold point has been surpassed resulting in large unexpected changes, usually to another stable state (Scheffer et al., 2012). This threshold point is known as a bifurcation point; it is the point at which the system’s qualitative behavior changes (Scheffer et al., 2009). In Figure A.1 shown in Appendix A, fold bifurcation points F₂ and F₁ are shown. When the system is on the upper part of the curve, close to the bifurcation point, F₂ any perturbation allowing the system to surpass this bifurcation may induce a critical transition or catastrophe shift to the lower part of the curve (Scheffer et al., 2012). At that point even if conditions revert to those of F₂, a changeover to the first stable state will not occur until conditions reverse to F₁, the second bifurcation point, this behavior is known as hysteresis in the system (Scheffer, 2009).

1.3.2 Nonlinear Transitions: Glacial Terminations and Inceptions

The entrance and exits of interglacial periods are considered nonlinear since this is a response to small gradual changes in solar radiation amplified by instabilities in the ice sheets and the carbon cycle (Berger et al., 2016). Therefore, from a dynamical systems perspective, the glaciations and interglacials can be thought of as stable attractors and glaciations and terminations as critical transitions between these two stable equilibriums (glaciations and interglacials). For example, during the equilibrium state of an interglacial small astronomical forcing can move past the bifurcation point initiating a glacial inception transition. The development of an intermediate stage can be thought of as an unstable equilibrium where continued changes allow for a full transition to the alternative stable state of full glacial conditions.
1.4 Research Objectives and Outline of Dissertation

This chapter provided a bit of background on the characteristics of the latter Pleistocene glacial-interglacial cycles and the nonlinear mechanisms of the transitions between these climate variations. It introduced some basic concepts from dynamical system theory that are relevant in helping to decipher the behavior of the entrance and exits to an interglacial. The primary goal of this dissertation is to explore the paleoclimate data for information in behavior changes of climate variables during the glacial -interglacial transitions that may shed light on the underlying mechanisms of nonlinear climate transitions in general. It is hoped that the results from this research can be applied to improving understanding of how anthropogenic activities today may be causing the tipping points to be surpassed and induce a nonlinear climate transition in the near future.

To meet the goal of this dissertation the following main questions about glacial-interglacial transitions are addressed in chapters 2, 3 and 4.

- Why are glacial inceptions slower than terminations?
- Are there characteristic changes in the variability of paleoclimate temperature as critical tipping points are surpassed?
- Can $\delta^{13}$C isotopic records determine if methane clathrates were destabilized during glacial terminations? If so, did these occur before or after the onset of warming?

Chapter 2 of this dissertation discusses the asymmetry of the glacial-interglacial cycles and earlier explanations for the fast terminations and slow glaciations. It then considers the nonlinearity of the transitions between periods and the importance of feedback mechanisms. Then two negative sea ice feedbacks are proposed as being responsible for the development of
an intermediate stage during glacial inceptions prolonging the changeover process and creating the asymmetry observed.

**Chapter 3** explores a concept from dynamical systems theory. A system can become unstable after crossing some critical threshold and abruptly transition from one stable state to another even though the changes in forcing may be gradual and therefore the prediction of abrupt transition is difficult to predict (Scheffer et al., 2001). Here paleotemperature data from nine sites are analyzed for changes in variability preceding glacial terminations of the last 450 Ka. This is done to determine the feasibility of using temperature variability as an early warning signal of a climate transition even though the cause of the shift is still being debated.

**Chapter 4** evaluates δ¹³C isotopic records for evidence of thermal destabilization of methane clathrates during past warm climate transitions and hence the possibility of destabilization of existing reservoirs due to current global warming. Finally, **Chapter 5** presents a summary of the findings for this study and the development of a simple climate-carbon model. It ends with recommendations for the future direction of using stability analysis to determine critical thresholds for the ice-albedo and climate-carbon feedback.
Chapter 2 Why are Glaciations slower than Terminations?

2.1 Introduction

Paleoclimate data analyses reveal that Earth’s climate could rapidly switch from one stable state to another in just a few decades. While external forcings such as changes in insolation can determine the onset of abrupt climate shifts, the underlying mechanisms of these transitions are influenced by increased instability of internal Earth system dynamics. This is particularly true of feedback mechanisms that cause non-linear responses of the entire system. Our understanding of nonlinear regime shifts is limited, but climate transitions can cause severe ecosystem disruption. Consequently, a comprehensive understanding of factors controlling past climate regime shifts should prove useful in discerning how current anthropogenic changes may be modifying Earth system dynamics today and could affect the pace of future transitions. This study aims to improve understanding of the mechanisms of abrupt climate change by highlighting the important role of negative feedbacks in regulating the pace of a climate transition. Specific to the glacial-interglacial cycles of the Quaternary period, two negative sea ice feedback mechanisms with the potential to become dominant during the glacial inception and decrease the pace of the process are identified and described in this chapter.

2.1.1 Motivation for this Study

Current anthropogenic changes may be moving the climate system toward threshold points at which the climate will shift with consequent ecosystem disruption, thus adding urgency for comprehensive knowledge of climate transitions (Alley et al., 2003; Alley et al., 2004; Lenton et al., 2008; Lenton, 2011). However, predictions of the rate, extent and endpoint of a climate transition are difficult because of limited understanding of the nonlinearity of the climate system (Robcock, 1985; Lockwood, 2001; Rial et al., 2003; Alley et al., 2004; Dakos et al., 2008; Soden
and Held, 2006). For these reasons, to be able to anticipate the course of a potential climate transition, research that aims to decrease our limitations in predicting the timing and process of nonlinear climate transitions is indispensable (Alley et al., 2003; Alley, 2004; Alley et al., 2005; Lenton et al., 2008; Dakos et al., 2008). Since paleoclimate data show the existence of multiple steady states such as the recent glacial-interglacial cycles observed during the Quaternary (Imbrie et al., 1993b; Lisiecki and Raymo, 2005), studies of paleoclimate data may help elucidate key processes at work in the Earth system dynamics as demonstrated by Fischer et al., (2018). Hence, in this chapter, the key mechanisms of glacial terminations and inceptions are studied.

### 2.1.2 Glacial-Interglacial Cycle Asymmetry

During the latter part of the Pleistocene, Earth’s climate were dominated by the glacial-interglacial cycles where an interglacial is a warm period with little global ice cover lasting 10-30 Ka, while glaciations are cold periods with extensive glaciers existing about 100 Ka (Imbrie et al., 1992; Lang and Wolff, 2011; Berger et al., 2016). In the 1930s, Milankovitch proposed that the main initiators of these climate transitions are orbital changes with consequential deviations in the Northern Hemisphere (NH) summer insolation (Berger et al., 1999; Hansen et al., 2008; Berger et al., 2010; Berger et al., 2016). Proxy data obtained from oxygen isotopes of ocean sediments later confirmed this relationship (Hays et al., 1976).

The glacial-interglacial cycles are shown in Figure 2.1, where temperature change data from the European Project for Ice Coring in Antarctica (EPICA) Dome C ice core (Jouzel et al., 2007), atmospheric CO$_2$ (Lüthi et al., 2008), surface temperature relative to present in the Arctic (Bintanja and van de Wal, 2008) and insolation at 60°N (Berger et al., 1999) are plotted.
Figure 2.1 Glacial-Interglacial Cycle Asymmetry. The dashed blue curve shows atmospheric CO₂ concentrations (Lüthi et al., 2008). The solid red curve shows proxy temperature data from EPICA Dome C (Jouzel et al., 2007) relative to present day and the orange curve shows reconstructed atmospheric surface air temperature north of ~45°N (Bintanja and van de Wal, 2008) for the last 400 Ka. The dark blue dotted curve shows insolation values at 60°N in June (Bintanja and Selten, 2014) overlaid on the temperature and CO₂ concentrations. Glacial terminations are highlighted with a gray box and inceptions by a blue arrow. Both temperature and atmospheric CO₂ concentration display an asymmetrical curve not observed in the insolation values.

Data from NOAA National Climatic Data Center (NCDC) [https://www.ncdc.noaa.gov/data-access/paleoclimatology-data/datasets/ice-core].

The data was assembled and resampled on an evenly spaced time axis of 1 to 400 Ka with 3 Ka intervals using MATLAB linear interpolation. The termination of a glacial period takes approximately 10 Ka (Berger et al., 2016), as shown in Figure 2.1, (gray box) and occurs rapidly after a threshold in Northern Hemisphere (NH) summer insolation is crossed (Kawamura et al., 2007). Glacial inceptions (depicted in Figure 2.1 by the gray arrow) are similarly triggered by
astronomical forcing when insolation in June at 62.5°N reaches minimum threshold values of about 440 Wm$^{-2}$ (Royer et al., 1983; Kawamura et al., 2007; Müller and Pross, 2007; Mysak, 2008). However, unlike terminations, which take thousands of years, the changeover to a glacial period takes tens of thousands of years to be completed (Tziperman and Gildor, 2003; Peacock et al., 2006; Lisiecki and Raymo, 2007). This results in an interesting asymmetrical shape as observed in Figure 2.1 or the well-known ‘sawtooth’ shape of the glacial cycles (Lisiecki and Raymo, 2007). This asymmetry is also observed using oxygen isotope data from benthic foraminifera (forams) and is not just a regional phenomenon (Raymo, 1994) as has been noted in several other studies (Pollard, 1983; Bond and Lotti, 1995; Tziperman and Gildor, 2003; Lisiecki and Raymo, 2007). Although insolation changes trigger these cycles, a similar asymmetry is not observed in insolation changes (Berger et al., 2016) and there is no consensus on the mechanism for its formation (Raymo, 1998).

2.2 Role of Feedbacks in Nonlinear Climate Transitions

Feedback mechanisms increase the climate system’s sensitivity to variation of any forcing (e.g., insolation) by amplifying or diminishing the effect, thereby regulating transitions between multiple equilibrium states (Cess, 1976; Rial et al., 2003; Roe and Baker, 2007). These mechanisms are essential to understanding the stability of the climate system: positive feedback mechanisms work to destabilize, while negative feedbacks stabilize the system (Cess, 1976; Rial et al., 2003; Roe and Baker, 2007).

While it is mostly agreed that insolation changes are the trigger for glacial-interglacial transitions (Le Treut and Ghil, 1983; Royer et al., 1983; Mysak, 2008), to explain the pace of the transitions researchers have focused on internal mechanisms. This is because a similar asymmetry is not observed in the orbital forcing (Figure 2.1) suggesting a nonlinear response by the climate.
system (Lisiecki and Raymo, 2007). The influence of the climate-carbon feedback plays a significant role since the concentration of atmospheric carbon and temperature changes show a positive correlation (Petit et al., 1999). Other projections on the mechanism of the glacial-interglacial cycles have identified the ice-albedo feedback as being another significant player in regulating the glaciations and terminations (Budyko, 1969; Sellers, 1969).

2.2.1 Proposed Mechanisms for Asymmetry

The ideas used to explain the glacial-interglacial asymmetry identified several mechanisms such as changes in ocean circulation patterns (Toggweiler et al., 2006, Anderson et al., 2009) and the terrestrial biosphere (Zeng, 2003; Brovkin et al., 2012). These mechanisms control atmospheric carbon concentration and the climate-carbon feedback. Also prevalent is the modification of conditions that affect the strength of the ice-albedo feedback and the amount of insolation absorbed. These two feedbacks are illustrated in Appendix B, Figures 7.2.1 and 7.2.2 respectively. These include varying ice sheet mass (Le Treut and Ghil, 1983), ice volume thresholds (Paillard, 1998) ice altitude (Kageyama et al., 2004), ice calving instabilities (Weertman, 1974; Pollard, 1983; Watts and Hayder, 1983), accumulation of dust (Galleé et al., 1992; Peltier and Marshal, 1995), isostasy (Crucifix et al., 2001; Abe-Ouchi et al., 2013), sea ice extent (Jochum et al., 2011) and oceanic heat transport (Royer et al., 1983, Cortijo et al., 1994, Khodri et al., 2001, Sirocki et al., 2005, Calov et al., 2005b, Jochum et al., 2011).

2.2.2 Climate-Carbon Feedback Mechanisms

Atmospheric CO$_2$ and temperature changes show good correlation during the glacial-interglacial transitions where increased warming during an insolation maximum leads to higher atmospheric CO$_2$ concentrations, enhanced greenhouse effect and more warming (Petit et al., 1999; Watson and Garabato, 2006). For the ocean carbon feedback, CO$_2$ stored in the deep ocean and
circulation changes have been shown to be influential in increasing atmospheric carbon and amplifying the initial warming caused by an insolation maximum during terminations (Toggweiler, 1999; Sigman and Boyle, 2000; Toggweiler et al., 2006).

Looking at changes in ocean circulation, models show that increased warming can result in shifting of the Southern Hemisphere Westerlies poleward, causing increased upwelling of deep nutrient-rich waters and outgassing of CO$_2$ to the atmosphere, leading to greater warming (Toggweiler et al., 2006). Other model studies also support this response of the deep ocean to warming, increasing atmospheric CO$_2$ and enhancing the climate-carbon feedback thereby increasing the rapidity of glacial terminations (Hogg, 2008). This was confirmed by data from sediment cores taken from the Southern Ocean looking at changes in burial rate of opal during terminations revealing increased upwelling of deep carbon-rich water and outgassing of CO$_2$ to the atmosphere (Anderson et al., 2009). This phenomenon can be reversed during glacial inceptions, where the Southern Westerlies shift north with global cooling, sea ice formation is enhanced around Antarctica, stratification increases between the surface and deep ocean, reducing upwelling and outgassing of carbon from the Antarctic deep water thus dropping its atmospheric concentration and further cooling the climate (Watson and Garabato, 2006).

Carbon is also stored on land although the effect on the climate-carbon feedback is smaller than ocean storage (Sigman and Boyle, 2000). During glacial terminations, increasing atmospheric CO$_2$ can be a negative feedback, through the CO$_2$ fertilization feedback by boosting the rate of photosynthesis which in turn lowers atmospheric CO$_2$ (Bauska et al., 2016). This feedback allows greater terrestrial carbon storage by as much as 288 GtC during interglacials (Brovkin et al., 2012) and facilitates improved terrestrial carbon storage during interglacials, which can then become frozen in permafrost or buried under glaciers during glaciations (Zeng, 2003). This terrestrial
reservoir can release carbon rapidly during deglaciations as the ice-sheets retreat and respiration increases, further hastening the deglaciation process (Zeng, 2003). During the last termination, atmospheric CO$_2$ increased in pulses of 10-15ppm in less than two centuries, suggesting a source other than the ocean (Marcott et al., 2014).

2.2.3 Ice-Albedo Feedback Mechanisms

The ice-albedo feedback plays a crucial role in glacial-interglacial cycles (Budyko, 1969; Sellers, 1969). Model studies find that as an ice-sheet ages, dust accumulation increases. This dust loading enhances ice-sheet ablation during terminations due to decreased ice-albedo (Galleé et al., 1992; Peltier and Marshall, 1995; Bar-Or et al., 2008). Ice-sheet instabilities and calving mechanisms are positive feedbacks that can enhance ablation, reducing albedo and increasing the rate of glacial terminations (Källén et al., 1979; Imbrie and Imbrie, 1980; Pollard, 1983; Watts and Hayder, 1984; Paillard, 1998; Lisiecki and Raymo, 2005). An ice calving mechanism occurs where a small increase in temperature results in melting ice, higher sea levels, water seeping under the outer layers of ice-sheets, then further melting, eventual ice-sheet collapse, decreasing albedo and further warming (Weertman, 1976; Pollard, 1983). This feedback is especially significant in speeding up terminations because the ice calving results in cold oceans, decreased evaporation and precipitation followed by reduced ice accumulation (Watts and Hayder, 1984).

The load-accumulation feedback is another feedback crucial in facilitating the asymmetry observed in the glacial-interglacial cycles by enhancing the rate of terminations (Le Treut and Ghil, 1983). This is where a large continental-size ice-sheet leads to depression of the underlying bedrock, increasing temperature and basal melting thereby decreasing ice-sheet height and consequently less snowfall and ice accumulation, enhancing ice-sheet break up during terminations (Crucifix et al., 2001; Le Treut and Ghil, 1983). Additionally, the type of surface on
which the ice-sheet rests influences the rate of ablation such that soft beds can melt as weight and friction increase, hastening the ablation rate and encouraging basal sliding and melting, increasing the rate of glacial terminations (Clark et al., 1999).

The Temperature-Precipitation feedback during terminations is a negative feedback where a small temperature increase results in greater evaporation and increased precipitation followed by more ice accumulation and enhanced albedo with consequent cooling (Källén et al., 1979; Ghil et al., 1987; Tziperman and Gildor, 2003). Observation data of ice accumulation at the Greenland Ice Sheet Project (GISP2) site shows an increase during warm periods due to increased precipitation (Cuffey and Clow, 1997). However, when temperatures increase past a certain threshold, ablation becomes greater than ice accumulation so that the negative feedback is no longer dominant (Huybrechts and Oerlemans, 1990; Tziperman and Gildor, 2003). This effect is also seen in model studies of the Antarctic ice-sheet where 1°C uniform warming can cause an increase of 1.43x10^{11} m^3 of ice (Huybrechts and Oerlemans, 1990).

During the glacial inception process, model studies show an increase in ice sheet build up, as the height of the ice sheet increases so does the elevation which leads to cooler temperatures favoring ice volume increase growth (Kageyama et al., 1999; Wang and Mysak 2002). Sea ice-Clouds feedback is a negative feedback during glacial inceptions (Jochum et al., 2012). As the sea ice extends due to cooler temperatures caused by reduced summer insolation, the extent of cloud cover over high latitude regions is reduced as evaporation is hindered, leading to decreased albedo and increasing temperature (Jochum et al., 2012).

Variation in the Atmospheric Meridional heat transport serves as a negative feedback during glacial inceptions because a larger meridional temperature gradient develops as NH temperature cools resulting in increased atmospheric heat transport to the Arctic from 2.88 PW to 3.00 PW
resulting in warming (Jochum et al., 2012). However, this is countered by a positive feedback as this enhanced pole-equator temperature gradient also causes increased moisture transport poleward, increasing precipitation on the now cooler land surface facilitating ice-sheet growth and glaciation (Royer et al., 1983). This enhanced precipitation is concentrated in the Northern high latitudes since sediment data from Central Europe found the decreased insolation caused dry conditions (Sirocko et al., 2005). As persistent annual increases in snowfall continue, moisture supplied by the enhanced meridional gradient leads to the buildup of ice-sheets over North America (Cortijo et al., 1994, Khodri et al., 2001, Riesebrokken et al., 2007). This increases the strength of the ice-albedo feedback, leading to further cooling, amplifying the effects of insolation changes (Calov and Ganopolski, 2005b).

One explanation of the asymmetrical glacial-interglacial cycles is the introduction of an intermediate stage during the glacial inception transition and the presence of an ice volume threshold that must be crossed to continue the descent toward glacial temperatures (Paillard, 1998). However, looking at the temperature changes over time during the last three glacial inceptions (indicated by the gray arrows) in Figure 2.1, temperature does not merely descend to the intermediate stage. Instead, it is observed to rapidly decrease after the transition is initiated before rising again to form an intermediary stage, suggesting an additional mechanism at work. This chapter builds on this idea of an intermediate stage with the goal of improving understanding of the importance of negative feedbacks in climate transitions.

2.2.4 Millennial Scale cycles

Within the glacial states are shorter term, millennial-scale cycles, known as the Dansgaard–Oeschger and Heinrich events, which display a pattern similar to glacial terminations and inceptions, characterized by a rapid rise in temperature followed by a slow return to initial
conditions (MacAyeal, 1993; Schulz et al., 1998; Ganopolski and Rahmstorf, 2001). The Dansgaard–Oeschger (D/O) cycles are marked by a sudden increase in Greenland’s temperature (5-10°C) in as little as a decade followed by gradual cooling back to glacial conditions with a recurrence time of about 1,470 years (Ganopolski and Rahmstorf, 2001). Heinrich events are characterized by colder temperatures possibly caused by the release of icebergs into the North Atlantic, occurring every 7 Ka to 10 Ka during glaciations (Bond and Lotti, 1995).

One explanation for Heinrich events is the ‘binge/purge’ theory where the ice-sheet grows until the mass reaches a critical point beyond which accumulated geothermal energy results in melting of the frozen basal sediment (MacAyeal, 1993). The thawed sediment creates a slippery foundation for the ice-sheet, enhancing sliding and break up, releasing numerous icebergs into the Hudson Strait and the North Atlantic (MacAyeal, 1993). This explanation of the Heinrich event introduces the idea that accumulated geothermal energy may help break an ice sheet. Can this explanation be used to provide insights into the mechanisms at work during the longer term glacial inception process? Here it is proposed that it may have played a role in delaying the glaciation transition.

2.3 Proposed Negative Feedbacks Responsible for Slower Inception Process

In this chapter, the focus is on the Northern Hemisphere as previous studies indicate this region is critical in glacial cycle transitions (Imbrie et al., 1992, Berger et al., 1999). While positive feedback amplifies a perturbation and increases the pace of transitions, negative feedbacks do the opposite (Cess, 1976; Rial et al., 2003; Roe and Baker, 2007). Based on this, it is suggested that the dominance of two negative feedback mechanisms during the glacial inception process as environmental conditions become favorable to facilitate their dominance. These are shown in Figure 2.2 and illustrated in Figure 2.3 as being responsible for slowing the glacial transition process.
2.3.1 Ocean-Atmosphere Mass Exchange

*Sea ice - Precipitation* is one of the first negative feedback mechanisms that becomes dominant during glaciations as illustrated by the upper part of the schematic in Figure 2.2. After the insolation minimum is crossed, temperature decreases, sea ice extends rapidly, albedo increases, further decreasing temperatures and sea ice growth, air-sea moisture exchange decreases and so does precipitation, starving the newly formed sea ice. This immature sea ice becomes vulnerable to ablation, and sudden enhanced rapid ocean turnover can break up the sea ice, decreasing albedo, and temperature increases again resulting in the intermediate stage or mild glacial conditions as described by Paillard (1998).

The cartoon in Figure 2.3 attempts to paint the scenario of how the *Sea ice - Precipitation* mechanism works to slow glaciation. The insets of Figure 2.3 for each of the four panels show the stage of the glacial inception process with A starting at interglacial conditions and D at full glacial conditions. Figure 2.3A shows the Arctic region during interglacial conditions, with little sea ice formation but a strong energy and mass ocean-atmosphere exchange. The glaciation process is initiated after the insolation minimum is crossed (Royer et al., 1983; Wang and Mysak, 2002; Müller and Pross, 2007; Mysak, 2008). Literature does indicate a NH temperature drop upon initiation of glaciation (Royer et al., 1983; Jochum et al., 2012). The lower temperatures allowing sea ice to extend rapidly, increasing albedo, further decreasing temperatures. This helps to explain the initial drop in temperature in response to decreased NH summer insolation as observed in Figure 2.1 and Figure 2.3B inset (blue arrow). The larger extent of the air-sea ice barrier results in less moisture loss to the atmosphere and decreased precipitation.

Based on the temperature-precipitation feedback, with less precipitation in the Arctic region,
ice accumulation diminishes and sea ice growth is hindered. Based on this concept, it is suggested that the large sea ice pack formed upon the initiation of the glaciation process becomes starved as precipitation and accumulation becomes limited so that the newly formed sea ice thins and becomes more vulnerable to ablation. As the sea ice resilience to ablation is smaller, enhanced ocean movements result rapid break up, shown in Figure 2.3C and inset (blue arrow), indicates this is the intermediate stage being created. This stage remains dominant allowing increased evaporation due to the newly uncovered ocean, and increased precipitation. As overall cooler than interglacial temperatures prevail with low insolation values, the broken sea ice reforms into a sturdier sheet that is more resistant to ocean turnover (Figure 2.3D).

![Figure 2.2](image)

**Figure 2.2** Schematic diagram of proposed operation of the dominant negative sea ice feedback mechanisms in the North Atlantic and Arctic Ocean where an insolation minimum triggering glacial onset results in increased sea ice extent.
This initial sea ice growth increases albedo, leading to further cooling and sea ice growth which eventually leads to conditions that favor the proposed negative feedbacks. The upper path shows the effect of increased sea ice on ocean-atmosphere mass exchange or *Sea ice - Precipitation* feedback by decreased evaporation, decreased moisture content and precipitation, which starves the sea ice, followed by thinning sea ice and vulnerability to ocean turnover. The lower part describes the *Sea ice - Insulation* feedback where the energy barrier that sea ice creates between the ocean and atmosphere decreasing heat exchange, elevating deep ocean temperatures, increasing buoyancy in the deep ocean, causing eventual ocean turnover and sea ice break up. The sea ice break up decreases albedo and leads to increased warming. These inhibit the glaciation transition process resulting in the formation of the intermediate stage observed during the transition.

### 2.3.2 Ocean-Atmosphere Energy Exchange

The second dominant feedback suggested is *Sea ice - Insulation*. This feedback is also responsible for the slower glaciation process through control of air-sea energy exchange, as portrayed by the lower part of the schematic in Figure 2.2. The insolation minimum triggers the glaciation process; NH temperature drops. The sea ice extent expands, reducing air-sea energy exchange, geothermal energy builds up in the deep ocean increasing its buoyancy, and eventually causing rapid ocean turn-over and sea ice ablation, returning to higher temperatures of the intermediate stage. The first proposed feedback assists this sea ice disintegration: *Sea ice - Precipitation* that produced starved sea ice suffering from enhanced vulnerability to ablation.

The cartoon shown in Figure 2.3 attempts to depict the scenario of how the dominance of the negative feedback sea ice-insulation can help create the intermediate stage and slow the pace of glacial inception. The first panel shows interglacial conditions Figure 2.3A and inset (blue arrow)
shows the stage on the temperature curve with good air-sea energy exchange. After the glaciation process begins and temperature decreases, the sea ice increases over the Arctic Ocean reducing energy loss to the atmosphere (Figure 2.3B). With reduced heat loss, there is geothermal energy build up in the deep ocean increasing seawater buoyancy and at some critical point leading to an ocean turn over and sea ice ablation, illustrated in Figure 2.3C. The first proposed feedback likely enhances the sea ice disintegration; Sea ice - Precipitation exchange described above where decreased precipitation results in thinning sea ice and increased susceptibility to perturbations such as turbulent ocean mixing.

Sea ice disintegration decreases the albedo resulting in higher temperatures and the development of the intermediary stage depicted in Figure 2.3C inset and in the temperature data of Figure 2.1 where there is a return to almost interglacial conditions. This stage remains dominant temporarily with increased evaporation allowed by the newly uncovered ocean, and increased precipitation. Perhaps the sea ice formation and break up process repeats itself a few times until the ice is sturdy enough to resist rapid ocean turnover leading to changes in local North Atlantic convection sites instead such as the moving of North Atlantic Deep Water (NADW) formation further south and descent to full glacial conditions as depicted in Figure 2.3D and inset (blue arrow).
**Figure 2.3** Illustration showing how the proposed two dominant negative feedbacks operate to slow down the glaciation process with an inset showing a conceptual model of the glaciation process. The blue arrow shows the glacial transition zone being described. **A.** Interglacial conditions exist, there is strong exchange of energy (dashed orange arrow) and evaporation of moisture (blue arrow) from the ocean. **B.** Initiation of glacial inception as insolation minimum is crossed and sea ice sheet extends and begins insulating the ocean while energy accumulates, increasing deep ocean temperatures and buoyancy, thereby creating an unstable top heavy water column. **C.** At some critical point, when the water column is sufficiently unstable resulting in overturning and breakup of the young sea ice, albedo decreases and temperatures increases, creating the observed intermediary stage. **D.** Sea ice has reformed from the broken pieces of ice due to continued low summer insolation. This reformed ice is thicker and sturdier making it better able to resist ocean turnover, leading to changes in the ocean circulation patterns instead. The inset in each panel shows a conceptual model based on **Figure 2.1** and shows the general pattern of
The glacial cycle transition with the higher peak representing maximum interglacial while the lower peak represents the intermediate stage. The blue arrow shows the stage of the transition being addressed in each panel.

2.4 Discussion

Published literature lends support to the idea presented here by indicating sea ice plays a significant role as a control in climate fluctuations by regulating albedo, air-sea exchange of gases and energy flux. In this section, the literature is explored to investigate the feasibility of the dominance of the sea ice-precipitation feedback during glaciation.

2.4.1 Evidence to support Sea ice - Precipitation Feedback

Sea ice plays a significant role in climate fluctuations (Stein et al., 2017) and the extremely cold air temperatures of Greenland during glaciations suggesting extensive sea ice cover in the North Atlantic (Broecker, 2000). Both proxy and modeling data indicate significant ice cover over the Arctic Ocean during past glacial maximums (Colleoni et al., 2009; Jakobsson et al., 2016). This evidence of large sea ice cover in the Arctic Ocean supports the suggestion that the lower sea surface temperature (SST) facilitates the continued expansion of sea ice, rapidly increasing coverage of the Arctic Ocean. Previous studies show sea ice influence climate by regulating albedo and air-sea exchange of both energy, moisture (Weyl, 1968; Gildor and Tziperman, 2000; Thomas et al., 2016) and gas (CO₂) exchange (Budikova, 2009; Rysgaard et al., 2011) specifically Arctic sea ice (Landais et al., 2006; Bauska et al., 2016). Recent observational data from the Arctic confirm that sea ice extent does influence atmospheric moisture and hence precipitation (Stroeve et al., 2011). Hydrogen isotope ratios (d-excess values) of the last 8 Ka helps to establish that sea ice has a stronger control on regional Arctic precipitation by indicating the existence of drier conditions on Greenland as sea ice extent increases (Kopec et al., 2016). For this reason, the
suggestion of increased sea ice causing decreased air-sea exchange and cold, dry atmospheric conditions to develop in the Arctic, reducing precipitation as depicted in Figure 2.3B, is reasonable.

2.4.2 Evidence to support Sea ice - Insulation Feedback

There is literature support for the Sea ice - Insulation feedback becoming dominant during glacial inception. This response of increased deep ocean temperature as the sea ice extent increases has been confirmed by model results (Rial and Saha, 2011). In a study of Dansgaard-Oeschger events, the ECBilt-Clio model demonstrates the effect of sea ice on deep ocean temperature where a 13-15 x 10^6 km^2 sea ice extent led to deep ocean temperature increase of 2-4°C at 1.5-3.5 km depth, and 2-5°C increase at 0.5-1.5 km depth (Rial and Saha, 2011). The model indicates lower temperatures at the top of the water column and warmer at the bottom so that it becomes convectively unstable (Rial and Saha, 2011). It is suggested here that, this instability is also experienced during glaciation when the sea ice extent increases leading to a top-heavy water column and eventually turbulent vertical mixing, which encourages in sea ice ablation. There are model studies supporting sea ice thickness as a control of air-sea energy flux. For example, the atmospheric HIRHAM regional climate model showed sea ice acts to regulate air-sea heat flux by indicating stronger heat flux when sea ice thickness is reduced (Curry et al., 1995).

The presence of a durable sea ice cover with resilience to ocean turnover which changes the local North Atlantic convection sites (Rinke et al., 2006) helps confirm the idea of the effective role negative sea ice feedbacks in the Arctic Ocean played as a control of the glaciation pace. Ocean circulation changes are considered a fundamental variable in facilitating glacial-interglacial cycle transitions (Imbrie et al., 1993b; Cortijo et al., 1994; Rahmstorf, 2002; McManus et al., 2004). Shifting position of NADW formation is demonstrated by Rahmstorf, (2006) who showed
the Atlantic Meridional Overturning Circulation (AMOC) has three circulation modes. There is a shifting in convection location geographically in response to changing temperatures, influencing climate through events including cold Heinrich events, and the Younger Dryas and Bolling warming events (Rahmstorf, 2006).

2.5 Conclusions

Paillard, (1998) showed the existence of an intermediate stage during glacial inception during which an ice volume threshold must be crossed before the transition to full glacial conditions can occur and help stimulate the asymmetry of the glacial cycles. Negative feedbacks play a critical role in maintaining the stability of an equilibrium stage and accordingly work against instability and climate transitions (Cess, 1976; Rial et al., 2003; Roe and Baker, 2007). Based on published literature, the dominance of two negative feedbacks are suggested, sea ice-precipitation and sea ice-insulation that provide a physical cause of the intermediary stage during glaciations that makes the process much slower than terminations.

The dominance of Sea ice - Precipitation and Sea ice - Insulation feedbacks as proposed here have implications for models that will replicate the climate dynamics of the glacial-interglacial cycle transitions and other critical transitions by emphasizing the role of negative feedbacks. Understanding the behavior of this aspect of the climate system is useful in predicting how it will respond to ongoing anthropogenic changes. Given the unusually fast rate of anthropogenic changes the Earth system is currently undergoing the risk of crossing thresholds and transitioning to another climate state becomes greater. Undoubtedly, improving our understanding of how negative feedbacks work will help in estimating the speed of transitions.
Chapter 3  Temperature Variability as an Indicator of Impending Abrupt Climate Shifts

3. 1 Introduction

Abrupt transitions though common in Earth systems are challenging to predict and can result in significant ecosystem disruption (Scheffer et al., 2001). Past studies have identified increased variance of a dynamic system as a predictor of abrupt changes using modeling studies (van Ness and Scheffer, 2003; Kleinen et al., 2003; Carpenter and Brock 2006). Here using published proxy temperature data from nine sites, the Ansari-Bradley test is used to identify changes in temperature variability to elucidate the significance of temperature variance with increasing instability in announcing an approaching climate regime shift. Additionally, the simple moving variance is determined to allow easy comparison with the Ansari-Bradley test and visualization of changes in temporal temperature variability. The results show an increase in temperature variability is discernible before a glacial termination providing empirical evidence for this early warning signal in the paleotemperature data. The Ansari-Bradley test compared well with the moving variance and may be a useful tool to quickly identify deviations in the variance of a temperature time series. Further, it supports the idea that increasing temperature variability or increases in extremes of a system with multiple steady states can be a useful warning signal of impending abrupt change. These results also imply that peak variance can be a viable metric to estimate the magnitude of a temperature transition.

3.1.1 Motivation for this Study

Previous studies show that astronomical forcings can pace the timing of transitions into and out of ice ages as observed in the glacial-interglacial cycles (Hays et al., 1976; Raymo, 1997). These cycles are evident globally both in the ice cores and sediment records (Hays et al., 1976, Petit et al., 1999). For the last 800 Ka, glaciations usually lasted around 100 Ka while interglacial
conditions persisted for about 30 Ka (Berger et al., 2016). **Chapter 1** provides a more detailed description of glacial-interglacial cycles and their characteristics. Today’s global climate is currently undergoing changes, apparently as a consequence of the rapid rate of increase in the concentration of atmospheric CO₂ (Solomon et al., 2009). The rate of change is unprecedented over the Holocene and may be pushing the climate system toward a tipping point, increasing the probability of an abrupt climate shift similar to those observed in the paleoclimate data (Alley et al., 2003, Solomon et al., 2009). Abrupt transitions might result in significant ecosystem and societal disruptions because of the difficulty of adapting to the fast-paced changes (Alley et al., 2003). Climate regime changes can have severe catastrophic consequences in various types of ecosystems (Scheffer et al., 2001). Therefore, for most natural and manmade systems, it is crucial to predict abrupt transitions between alternative stable states to prevent or adapt to new conditions (Liu et al., 2015). Despite the evidence of past abrupt climate shifts and the ramifications of such transitions for society today, predicting these are difficult due to limited understanding of uncertainties in the climate system, forcings and feedback mechanisms (Alley et al., 2003, Dakos et al., 2008, McNeall et al., 2011). Since the paleoclimate data shows the existence of past abrupt transitions such as the prominent ancient long-term global climate switches from cold glaciations to warm interglacials (Hays et al., 1976, Petit et al., 1999, EPICA Community Members, 2004), does it also hold evidence of key behavioral changes that signal an abrupt change is looming?

### 3.1.2 Insights from Paleoclimate Transitions

Analysis of paleoclimate data of past climate transitions can improve our understanding of how the climate system responds to small changes in forcings (Hansen et al., 2013). For example, Rohling et al. (2012) demonstrate this by using paleoclimate data to narrow the range of climate sensitivity to 0.3–1.9K W⁻¹ m² at 95% probability. In addition to providing insights on the
mechanisms of climate dynamics, paleoclimate data can provide a good way to test model output (Taylor et al., 2012). In this study, paleoclimate data are used to improve understanding of the generic changes the climate system may experience before a transition.

3.1.3 Examples of Past Climate Transitions

Past abrupt climate changes have occurred after critical thresholds have been crossed with continued change in the climate driver enabling rapid climate transitions much faster than the rate of change in the underlying forcing, establishing that the climate system is nonlinear (Rial et al., 2003). This section reviews two major past climate transitions: the recent transformation of a lush, moist Sahara to the desert conditions of today shown in Figure 3.1; and the ancient Greenhouse to Icehouse transition shown in Figure 3.2. This is done to illustrate the nonlinearity of the climate system by showing how the gradual changes in forcings can trigger an abrupt regime change after critical thresholds are surpassed. This nonlinearity complicates forecasting abrupt climate changes because it is difficult to know when critical thresholds are being approached or even crossed.

Around 5.5 Ka, the Saharan desert abruptly changed from being moist and vegetated to the unvegetated, desert conditions seen today, and this is documented by a change in aeolian dust sedimentation off the coast of West Africa (deMenocal, 2000). deMenocal (2000) compared the gradual decrease in Northern Hemisphere solar radiation which is the trigger for the desertification observed in the dust record with the abrupt increase observed in the dust deposition, demonstrating the effect of positive feedbacks and thresholds in the climate system. Desert and moist conditions represent two alternative stable states in the system and the sudden crossover at that critical tipping point from one stable state to another is unimaginable, since a similar abrupt change is not observed in the forcing (Scheffer et al., 2001).
Figure 3.1 Insolation values at 60° N June (Bintanja and Selten, 2014) for the last 9 Ka (top panel) and terrigenous dust (%) of West Africa to oceanic sediments from ODP Site 658C (deMenocal et al., 2000) (bottom panel). Dry conditions result in higher dust flux from the land surface to the ocean sediments. Although the insolation decrease responsible for the change in precipitation over the Saharan desert is gradual, the transition from moist humid to desert conditions is abrupt demonstrating the nonlinearity of the climate system.

Figure is modified from deMenocal et al., (2000).
Figure 3.2 The transition of Earth’s climate from extremely hot to the current glaciated state occurred abruptly around 33.7 million years (Ma), a decrease in atmospheric CO₂ is thought to be responsible for this transition. $p$CO₂ record spanning 32 to 35 Ma from an Ocean Drilling Program (ODP) Site 925 located in the western equatorial Atlantic Ocean shows a slow decrease in atmospheric CO₂ (Zhang et al., 2013) (top panel). The CO₂ decrease led to colder conditions; this temperature decrease is reflected by changes in stable carbon isotope records (Tripati et al., 2005) (bottom panel).

At the Eocene-Oligocene boundary, the change from a greenhouse to an icehouse climate state occurred 33.7 Ma resulting in the development of ice sheets in Antarctica and was thought to be driven by long-term decreasing CO₂ concentrations (Tripati et al., 2005). The carbonate data shown in Figure 3.2 lower panel shows the Greenhouse-Icehouse climate transition (Tripati et al., 2005). CO₂ concentrations dipped through some tipping point, allowing the buildup of a large Antarctic ice sheet in as little as 200 Ka creating the icehouse world of today (Kump, 2009).
Looking at the data, the $p\text{CO}_2$ record presented by Zhang et al., (2013) shown in Figure 3.2 (upper panel) shows a slow stepwise decrease until some threshold is crossed and causing a drastic drop in temperature as seen in Figure 3.2 lower panel. Abrupt state shifts are largely determined by gradually changing variables when tipping points have been exceeded though not necessarily observed in the system variables themselves (Scheffer et al., 2001). These examples outline how gradual changes in solar radiation and $\text{CO}_2$ concentrations resulted in large scale climate regime shifts while not displaying a similar abrupt changeover itself. If there was a way to signal when these thresholds have been surpassed, and an imminent transition will occur, society will benefit greatly by being given time to develop adaptive techniques (Lenton et al., 2012).

3.2 What happens before Critical Transitions

When conditions change gradually until a critical threshold is reached, this point is called a bifurcation, any further change in conditions beyond the critical threshold will now result in a transition to an alternative stable state fueled by the effect of positive feedbacks (Scheffer et al., 2009; Dakos et al., 2012). Theoretical studies show that nonlinear systems experience a characteristic increase in their return times to equilibrium after a threshold is crossed (Wissel et al., 1984). This slowdown in recovery rates of a dynamical system approaching a critical transition is also known as decreased resilience and can be used as a signal of an impending changeover (Strogatz, 1994; Scheffer et al., 2009). This sluggishness can be detected as an increase in the memory of the system after a small perturbation and is termed ‘critical slowdown’ (van Nes and Scheffer 2007; Dakos et al., 2012). Previous studies did find longer recovery times of a dynamical system after it is perturbed using it as a signal of an impending climate shift to entirely different conditions (Dakos et al., 2008, Lenton et al., 2012, Veraart et al., 2012). Stability landscape diagrams (Appendix A) represents multiple steady state dynamical systems where each stable
state of the system is shown as well. Systems further away from a bifurcation are shown to have higher resilience with deeper wells (Scheffer et al., 2009). For these dynamical systems, the stability landscape may change before a major transition either by becoming more sluggish or by “flickering” so that an alternative state emerges and allows occasional flips to the other state (Scheffer et al., 2012). There are a number of ways to detect the subtle changes in a dynamical system after thresholds have been crossed; for example autocorrelation and increased variance (Lenton et al., 2012).

3.2.1 Increased Autocorrelation

One way this phenomenon of ‘critical slowdown’ can be detected in a time series is by increased autocorrelation (Scheffer et al., 2009; Dakos et al., 2012). Dakos et al., (2008) analyzed eight past abrupt climate change time series, for example: the end of the Greenhouse Earth; the end of earlier glaciations; the end of the Younger Dryas; the Bolling-Allerod transition; and desertification of North Africa, to find evidence of these recovery slowdowns prior to a transition. They looked for changes in autocorrelation in the time series for each of the datasets just prior to a transition, finding an increase before a shift occurred, indicating a slowdown in the rate of return in the systems in all the cases examined. Veraart et al., (2012) using experimental data obtained by exposing cyanobacteria to increasing amounts of sunlight until the population collapsed, observed increased recovery times or decreased system resilience as the distance to the critical point got smaller.

3.2.2 Increased Variance

Another tool researchers used to determine proximity to environmental tipping points is increased variance in a key system driver (Dakos et al., 2012; Lenton et al., 2012). Carpenter and Brock (2006) observed greater variability in a lake’s phosphorous concentration prior to a shift
from an oligotrophic to a eutrophic state. They interpreted this increased variability as an early warning signal of an imminent change in system states. Drake and Griffin (2010) studying population dynamic changes in deteriorating environments found increased variability of the zooplankton Daphnia Magna prior to extinction. van Nes and Scheffer (2003), using an individual-based simulation model of aquatic macrophyte growth, demonstrated increased oscillation amplitudes when close to a catastrophic bifurcation. Kleinen et al., (2003) using the Stommel model (Stommel 1961) to investigate the stability of the ocean’s thermohaline circulation, also found increased variability in overturning as the temporal distance to bifurcation decreased.

Following from the examples above, the hypothesis of this study states, that temperature variability in a time series will increase so that there are more extremes observed in the record, just before a transition to a new, stable state occurs. This will be tested in this study by scrutinizing the changes in paleo-temperature variance in published paleotemperature time series data obtained from nine globally distributed sites to detect signals of forthcoming nonlinear glacial terminations and inceptions. Here, similarly to Dakos et al., (2008), past abrupt climate transitions are also examined to understand the feasibility of using increased variance as a metric for paleoclimatology datasets. This study differs from Dakos et al., (2008), by employing a different statistical metric to identify early warning signals of abrupt climate transitions and explores the possibility of a novel method to estimate the magnitude of a transition. Instead of increased autocorrelation as an early warning signal, increased variance is used as a predictor of imminent transition and two methods to detect this are compared – the Ansari-Bradley test and moving variance.
### 3.3 Data and Methods

The reconstructed temperature time series data used in this study is available for download from NOAA National Centers for Environmental Information formerly known as the National Climatic Data Center (NCDC) database website. The data are proxy temperature reconstructions from ice core and marine sediments from sites around the globe (Table 3.1). These datasets provide a globally comprehensive picture with sites located in both polar and equatorial regions (Figure 3.1). The interglacial stages are delineated in Figures 3.4-3.12 based on sea level highs as given in Berger et al., (2016). The recent interglacials, those younger than 450 Ka, were found to be the most intense (Lang and Wolff, 2011). Since this study is focused on the entrances (glacial terminations) and exits (glacial inceptions) from interglacials, the period with the strongest interglacials, i.e., from 0 to 460 Ka covering MIS1 through MIS 11 was selected. Two testing methods are used to allow a comparison of the results between the two and provide insight on their robustness in detecting increased variance as an early warning signal for time series data.
<table>
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<th>No.</th>
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<th>Location</th>
<th>Paleo-temperature Proxy</th>
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<td>Jouzel et al., 2007</td>
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<td>Planktonic foraminifera (Mg/Ca)</td>
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<td>North Atlantic</td>
<td>Alkenone, SST</td>
<td>Lawrence et al., 2009</td>
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</tbody>
</table>
3.3.1 Paleotemperature Data

The LR04 data stack used in this analysis is part of a time series of the mean land surface air temperature relative to the present day with an average spacing of 0.1 Ka and expressed in °C. This was reconstructed from the LR04 stack marine sediment δ¹⁸O for the region between 40-80°N running from 0 to 3 Ma using a comprehensive ice-sheet and ocean temperature model (Bintanja and van de Wal, 2008). Derivation of temperature from marine oxygen isotope records contains uncertainty because isotope storage in ice-sheet and the temperature of deep water affects the recorded δ¹⁸O (Bintanja et al., 2005). In order to extract temperature from this data and account for the limiting factors of ice-sheet volume influence and deep ocean temperature on oxygen isotope ratios, the development of the model based approach was necessary (Bintanja et al., 2005). To analyze the effectiveness of their approach, Bintanja et al., (2005) compared their reconstructed mid latitude Northern Hemisphere temperature records with δD (deuterium) record from Dome C, Antarctica of the last 400 Ka obtaining a very small error curve which suggests good coherence.

The European Project for Ice Coring in Antarctica (EPICA) 800 Ka Deuterium Data and Temperature Estimates temperature record is based on high resolution deuterium measurements, δ¹⁸D_{ice} taken from a deep ice core drilled in East Antarctica (75° 06' S, 123° 21' E) (Jouzel et al., 2007). The EPICA Dome C record runs from 0.0384 to 801.662 Ka containing 5788 data points and therefore has a mean spacing of 0.14 Ka. Uncertainty in ice core measurements are due to the mismatch between air bubbles trapped in the ice and the surrounding ice with ice-gas age differences ranging from 100-1000 years depending on accumulation rates of the region (Schwander et al., 1997). To improve accuracy of the data set, experiments were performed with the European Centre/Hamburg Model General Circulation Mode that incorporated that
incorporated water isotopes which supports the temperature records obtained from the ice core data (Jouzel et al., 2007).

Most of the marine sediment sites shown in Figure 3.3 relied on alkenone unsaturation $U_{37}^K$ to estimate sea surface temperature (SST) such as the Ocean Drilling Program (ODP) sites 1090, 882, 846 and 982. Martínez-Garcia et al., (2009) modified ODP Site 1090 SST data age-model using graphic correlation to the EPICA Dome C record and the software Analyseries to enable direct comparison of the marine records to the continental one. For ODP Site 1090, SSTs are obtained from two marine sediment cores PS2489-2 (42°52.4'S, 8°58.4'E) sampled at 1 to 5 Ka and ODP Site 1090 (42°54.8'S, 8°53.9'E) sampled at 2 Ka, in the Sub-Antarctic Atlantic Ocean with PS2489-2 covering the period 0 to 500 Ka (Martínez-Garcia et al., 2009). The temperature estimates are based on the behavior of the alkenone paleo-temperature where analytical errors were below 0.5°C and within the method error range (Martínez-Garcia et al., 2009). The ODP Site 1090 record goes from 0 to 1085.94 Ka with 472 data points and therefore a mean spacing of 2.3 Ka. SST was estimated for ODP Site 882 (50°21'N, 167°35'E) in sub-Arctic Pacific using alkenone unsaturation indices (Martínez-Garcia et al., 2010). This record spans from 4.27 to 3638.88 Ka with a mean temporal spacing of 2.4ka.

ODP Site 846 (3°S, 91°W) is a high resolution alkenone reconstructed SST in the low latitude region of the Eastern equatorial Pacific, and it is 5 Ma long with a resolution of 3 Ka (Lawrence et al., 2006). Alkenones are compounds produced by some marine surface algae and the degree of saturation observed in the sample, depends on the growth temperature so that alkenone unsaturation index can be used reliably to estimate SST (Conte et al., 2006). ODP Site 982 (57.31°N, 16.52°W) is also alkenone derived high resolution SST but of the North Atlantic
Ocean and is 4 Ma long sampled at an average of 3 Ka resolution but the time period 1200-1750 Ka and 1250-400 Ka were sampled at 10 Ka resolution (Lawrence et al., 2009). ODP Site 722 (16°37.308’N, 59°47.718’E) gives SST of the Arabian Sea for the last 3330.48 Ka and is based on stable isotope data published by Clemens et al., (1996) and has a low resolution of 7 Ka and therefore larger uncertainties in the data (Herbert et al., 2010).

SST data was reconstructed from the Mg/Ca ratios of surface-dwelling foraminifera Core MD06-3018 (23°00’S, 166°09’E) and subject to seasonal bias, this is on the eastern side of the New Caledonia Trough (Russon et al., 2010). They did repeated measurements of samples from this core and comparison with similar datasets yield values nearly within the sample reproducibility and calibration errors. This dataset goes from 0 to 1543.5 Ka and contains 260 data points, therefore has a sampling resolution of 5.9 Ka. Core MD97-2040 (2°02’N, 141° 46’E) also used Mg/Ca measurements in planktonic foraminifera to derive SST of the center of the western equatorial Pacific warm pool spanning the last 1.75 Ma with a resolution of 5 Ka (de Garidel-Thoron et al., 2005).
Figure 3.3 Location of the paleo-temperature data sites used in this study.

Note: Since LR04 stack is data-averaged over a spatial range and is not one data site, it was not included here.

3.3.2 Ansari-Bradley Test

The Ansari-Bradley test is used to identify changes in the paleo-temperature variability. While the classic t and F-tests can be used to identify changes in the means and variances of data sets, these assume that the data comes from a Gaussian distribution; since the Ansari-Bradley is a nonparametric test, it is considered a better solution (Trauth et al., 2009). The Ansari-Bradley test performs a two-sided rank-sum test of the null hypothesis that assumes both samples are from a distribution with similar shape and median but different dispersions (Trauth, 2015). Trauth et al., (2009) demonstrated the effectiveness of the Ansari-Bradley test by detecting significant changes in the variability of paleo-climate dust flux data. In this chapter, this statistic is employed to detect significant shifts in the variability of temperature in the paleo records to detect the changes in
temperature variability preceding the transition to glacial conditions (glacial inceptions) and interglacial conditions (terminations). The length of the average glacial cycle is approximately 100 Ka (Imbrie et al., 1993). Based on this the running Ansari-Bradley tests are computed using a paired sliding window size of 30 data points (L = 30) each of which is equivalent to 100ka. The Ansari-Bradley test results are shown in panel B of Figures 3.4-3.12.

### 3.3.3 Moving Variance Test

Additionally, the moving variance with a moving window of 31 Ka was computed to compare with the Ansari-Bradley test to get a better understanding of the soundness of the Ansari-Bradley test in estimating the change in paleo-temperature variability over time. At each site and for each interglacial during the last 460 Ka, the peak temperature and peak variance are determined and given in Tables 3.2-3.10 in Appendix 7.1 b. The peak temperature is the maximum value identified for the interglacial following the termination analyzed. The peak variance for each interglacial is then identified as the maximum point of variance that precedes the transition to that interglacial. Using the data from Tables 3.2-3.10, the peak variance versus the peak temperature is plotted and shown in panel D of Figures 3.4-3.12 along with the r-squared values of these correlations.

### 3.4 Results and Discussion

This work analyzes paleo-temperature variance change before known ancient climate transitions to see if there are significant deviations in behavior that can help us determine the possibility of an upcoming climate shift. Two methods are used and nine sites located around the globe to help improve the robustness of the results. As with most analyses, there are assumptions and limitations to consider when interpreting the results. Here, one limitation of using changing variability as a signal of an impending regime shift is that it may be caused by
other factors unrelated to a forthcoming transition such as noise (Carpenter and Brock, 2006). Another limitation is that low resolution data can result in underestimation of variance change so it is possible that a change of state may occur without noticeable preceding variance change in the data record (Dakos et al., 2008, Dakos et al., 2012).

Additionally, analyses of the robustness of statistical tests in detecting signals forecasting an abrupt climate shift found that noise-induced transitions are not predictable using statistical methods (Ditlevsen and Johnson, 2010; Livina et al., 2012). By using more than one method to detect changes in variance and a variety of data sites for the same time period, an attempt is made here to address these shortcomings of using the indicator “increased variance”. This will allow comparison between sites and events to shed light on the robustness of increased variability as an early warning signal of abrupt climate changes.

The results from the Ansari-Bradley and moving variance tests are shown in **Figures 3.4-3.12** where the temperature record resampled on an evenly-spaced time scale running from 1 to 460 Ka in 3 Ka intervals are shown in panel A, the deviations in mean temperature variance from the Ansari-Bradley test is shown in B. Panel C shows the temperature (solid line) and moving variance (dotted line) for the last 460 Ka; D, shows the peak variance preceding glacial termination and the peak temperature of the interglacial that followed that variance increase. This is repeated for all nine data sites analyzed.
From the results, the Ansari-Bradley test detected significant transitions in the temperature variability for both glacial terminations and inceptions for all the interglacials covered by this time period for the polar sites - LR04 Data stack and the EPICA Dome C data. However, for the other sites, this was not always the case. The moving variance test results compared well with the Ansari-Bradley test identifying similar variance change but is unable to detect signs of an upcoming glacial inception.

**Figure 3.4.A** presents the temperature change relative to present in °C for the region between 40-80°N and covers the last five interglacial stages identified as MIS 1 through MIS 11 and depicts the glacial terminations with their assigned numbers. The running Ansari-Bradley test identified both major and minor changes in temperature variability in the LR04 stack data site shown in **Figure 3.4.B**. The Ansari-Bradley test revealed significant changes in temperature variability before abrupt changes. For example, looking at the entrance to MIS 5e (TII) around 145 Ka in **Figure 3.4, A** where there is a sudden increase in temperature, the Ansari-Bradley test in **B**
finds significant deviations in temperature variability just prior to it. The exit from MIS 5e (Figure 3.4.A) at around 130 Ka in Figure 3.4.B, again the Ansari-Bradley test shows significant changes in the variability. Going over to MIS 7e, TIII occurring around 250 Ka (Figure 3.4.A), the Ansari-Bradley test (Figure 3.4.B) shows a small deviation in variability occurring just prior and similarly for the exit from MIS 7e, although here those deviations are smaller than those for MIS 5e. For the glacial termination to MIS 9, TIV (Figure 3.4.A), there is a minor deviation in variability while for the exit of this interglacial there is a larger deviation detected by the Ansari-Bradley test (Figure 3.4.B). For MIS 11 starting around 430 Ka, TV (Figure 3.4.A), there is a significant change in temperature variability (Figure 3.4.B) and similarly for the exit.

From the temperature and moving variance (Figure 3.4.C), an increase in variance is discernable before all the glacial terminations covered by this time. However, unlike the Ansari-Bradley findings (Figure 3.4.B), there is no visible change in variance detected by the moving variance (Figure 3.4.C) for an exit from an interglacial or glacial inception. Since there is a distinct increase in variance (Figure 3.4.C) before all the glacial terminations, it seems intuitive that there is a relationship between the peak variance and the interglacial maximum temperature that follows. Figure 3.4.D shows there is a strong positive correlation between the peak variance and maximum interglacial temperatures for the LR04 stack data.
Figure 3.5 EPICA Dome C temperature data set

Figure 3.5A presents the temperature change relative to present in °C from the EPICA Dome C ice core and covers the last five interglacial stages identified as MIS 1 through MIS 11 and depicts their entrances and exits. Similarly to the LR04 Stack data shown in Figure 3.4, the running Ansari-Bradley test (Figure 3.5B) identified both major and minor changes in temperature variability. Going through each interglacial stage, it is seen that the Ansari-Bradley test revealed significant changes in temperature variability before abrupt changes. For the entrance to MIS 5e (Figure 3.5.A) at Termination I (TI) marked by a sudden increase in temperature, the Ansari-Bradley test (Figure 3.5.B) finds moderate deviations in temperature variability just prior to the event. The exit from MIS 5e (Figure 3.5.A) at around 130 Ka in Figure 3.5.B, here the Ansari-Bradley test shows significant changes in the variability much larger than that detected for the entrance to this interglacial. The entrances and exits to MIS 7e and 9, (Figure 3.5. A), are predicted by the Ansari-Bradley test (Figure 3.5.B) but with smaller deviations in variability similar to that for the LR04 data above (Figure 3.4.B). The interglacial MIS 11 is also announced starting around
430 Ka (Figure 3.5.B), with a significant change in temperature variability for its entrance and exit.

From the temperature and moving variance curves presented in Figure 3.5.C, there is an observable increase in variance for all the glacial terminations covered by this time period. However, similar to the LR04 data (Figure 3.4.C) there is no significant change in variance announcing the glacial inceptions as is seen with the Ansari-Bradley test. The peak variance versus the peak temperature of the interglacial following that peak shows a good correlation (Figure 3.5.D) although not as strong as LR04 data since the r-squared value is lower at 0.47.

The moving variance results for ODP1090 Site in Figure 3.6.C shows an increase before the glacial termination events except for the entrance to MIS 11 where the increase in variance seems to come a bit after the event started. However, this site shows a very good relationship between the peak variance and the peak temperature in Figure 3.6.D of the interglacial predicted with an r-squared value of 0.83 giving this relationship good prospects as a possible predictor of the maximum temperature of the interglacial that follows.
Figure 3.6 ODP Site 1090

Figure 3.6.A shows the temperature over time from the sub-Antarctic site (ODP1090) and covers the last five interglacial stages identified as MIS 1 through MIS 11 although here these are not as distinctly defined as in the LR04 (Figure 3.4.A) and EPICA Dome C (Figure 3.5.A) data. For ODP Site 1090 (Figure 3.6.B) the running Ansari-Bradley test results display signals very much like LR04 stack and EPICA Dome C above where increases in variability are detected just before an abrupt transition is observed. MIS 7e continues to be detected by a smaller deviation in variability than the other interglacials.
Figure 3.7 ODP Site 882

Figure 3.7.A shows the temperature over time from the sub-Arctic site, ODP 882 Site and covers the last five interglacial stages identified as MIS 1 through MIS 11. Similar to ODP Site 1090 (Figure 3.6.A) above these are not as distinctly defined as in the LR04 (Figure 3.4.A) and EPICA Dome C (Figure 3.5.A) data. The running Ansari-Bradley test for ODP Site 882 is shown in Figure 3.7.B and has detected both major and minor transitions in temperature variability. Unlike LR04 stack, EPICA Dome C and ODP Site 1090, most of the detected signals announcing approaching glacial inceptions and terminations were not found by Ansari-Bradley test at this site. Although some are detected such as Termination II (TII), this may be due to a lower resolution data set, in the ODP Site 882, the average spacing of the sampled data set before interpolation is about 2.4 Ka. The moving variance in Figure 3.7.C suffered a similar fate and did not display a discernable increase in variance forecasting a termination as expected. However, the regression plot of peak variance and peak temperature in Figure 3.7.D showed moderate promise with an r-squared value of 0.43.
Figure 3.8 ODP Site 846

**Figure 3.8.A** shows the temperature over time from the Eastern Equatorial Pacific, ODP846 Site spanning the last 460 Ka covering the interglacials MIS 1 through MIS 11. The interglacials for this site are better defined than the ODP Site 882 although not as well as the LR04 and the EPICA Dome C datasets shown above. The running Ansari-Bradley test for ODP Site 846 (Figure 3.8.B) has detected both major and minor transitions in temperature variability and similar to LR04 stack, EPICA Dome C and ODP Site 1090, most of the detected signals announced the approaching temperature transitions such as glacial inceptions and terminations. MIS 7 which the Ansari-Bradley test predicted with small deviations in the LR04 and EPICA Dome C data set shows up here with moderate deviations in temperature variability.

The moving variance test results for ODP846 Site in **Figure 3.8.C** does show an increase before each of the glacial terminations but do not announce the glaciation events. Interestingly for this site, MIS 7 is predicted with a large variance increase while the other sites analyzed here tend to show a smaller change for this termination (TIII). The regression plot of peak variance and the
peak temperature of the interglacial given in Figure 3.8.D, does not have a good correlation implying that perhaps this relationship is not as strong for equatorial regions.

**Figure 3.9 Core MD06-3018 Site**

**Figure 3.9.A** shows the temperature of the tropical western Pacific (Core MD06-3018 dataset) spanning the last 460 Ka covering the interglacials MIS 1 through MIS 11. The interglacials are well defined in comparison with that of ODP Site 882. The running Ansari-Bradley results are shown in **Figure 3.9.B** and have detected both glacial terminations and inceptions by showing a change in variability forecasting each event. In keeping with LR04 and the EPICA Come C sites, the Ansari-Bradley predicts the termination before MIS 7 (TIII) with small deviations in temperature variability.

The moving variance test results for Core MD06-3018 data in **Figure 3.9.C**, does not show an increase before each of the glacial terminations and did not announce any of the major transitions in temperature observed. This is similar to ODP Site 882 which may have been due to
low resolution data and may the issue at this site also since it has a low sampling resolution of 5.9 Ka. Accordingly, the straight line plot of peak variance and temperature in Figure 3.9.D shows little to no correlation.

**Figure 3.10 ODP Site 722**

The temperature of the Arabian Sea, spanning the last 460 Ka and covering interglacials MIS 1 through MIS 11 is shown in Figure 3.10.A. The running Ansari-Bradley results are shown in Figure 3.9.B and display some major and minor deviations in the temperature variability, but it does not distinctly forecast glacial terminations and inceptions at this site. The moving variance test in Figure 3.9.C has results akin to the Ansari-Bradley test for this site, and no relationship exists for the regression plot of peak variance and temperature in Figure 3.9.D. As is the case with sites ODP Site 882 and Core MD06-3018 shown above, this one has a low resolution of 7 Ka and may be the culprit behind the inability to predict the major temperature transitions.
Figure 3.11 Core MD97-2140 temperature dataset

Core MD97-2140 temperature dataset of the Western Pacific warm pool is shown in Figure 3.10.A. It spans the last 460 Ka and includes interglacials MIS 1 through MIS 11. The running Ansari-Bradley results are shown in Figure 3.11.B and show some major and minor deviations in the temperature variability announcing the entrance and exits from interglacials except for MIS 9 which is unclear. The moving variance test in Figure 3.11.C shows an increase in variance before the terminations and the regression plot of peak variance before a termination and peak interglacial temperature in Figure 3.11.D has a reasonably good correlation with an r-squared value of 0.598.
Figure 3.12 ODP Site 982

Figure 3.12.A shows the temperature of the North Atlantic for the last 460 Ka and includes interglacials MIS 1 through MIS 11. The running Ansari-Bradley results are shown in Figure 3.12.B and show some major and minor deviations in the temperature variability that precedes the glacial terminations and inceptions and following the majority of the sites studied here predicted the entrance to MIS 7e with a minor deviation. The moving variance test in Figure 3.12.C shows an increase in variance before the terminations except for MIS 11 which is not announced at all. The regression plot of peak variance before a termination and peak interglacial temperature shown by Figure 3.12.D has a reasonably poor correlation albeit it is better than the low latitude sites; Core MD06-3018, ODP Sites 722 and 846.

An interesting trend emerged from both the Ansari-Bradley tests and the moving variance, where MIS 7e is announced by a small change in variance; this is consistent across the sites. Previous research has found MIS 7e to be a cooler, less intense interglacial in comparison with those found in the last 450 Ka (Lang and Wolff, 2011). It is quite intriguing that this is reflected
in the quantity of variance change and shows there is promise in developing the use of peak variance forecasting a transition to estimate the intensity of that change. Here the use of this metric is explored with favorable results for the high latitude sites such as the LR04, the EPICA Dome C data and ODP Site 1090, more research in this direction may help develop its use to a more robust metric.

There has been an abundance of research focused on understanding and predicting when and how climate transitions occur but this remains challenging (Alley et al., 2003). The results from the Ansari-Bradley tests shown in Figures 3.4-3.12 were able to predict both the glacial terminations and inceptions except for sites ODP 882 and 722; both sites have low sample resolution. The moving variance test predicted all the glacial terminations except for sites ODP 882, 722 and Core MD06-3018. One of the challenges of using empirical methods of abrupt regime shift is the need for high resolution data because low resolution may result in underestimation of variance due to low frequencies (Dakos et al., 2008; Dakos et al., 2012). Low resolution data may explain why some sites such as ODP 882 and 722 did not display a noticeable change; this test may be used along with an additional test to improve accuracy. Additionally, a comparison of data from sites around the globe shows glacial termination patterns vary spatially (Shakun et al., 2012) and may explain why the results may vary across sites. Overall, these results suggest that there is an increased temperature variance before an abrupt climate change and that changes in variance can be used as an early warning signal of impending transition regardless of the underlying mechanism causing the shift. They also demonstrate the effectiveness of the Ansari-Bradley test in detecting changes in variance in a time series as this test was able to forecast both glacial terminations and inceptions while the moving variance only predicted the terminations.
Today’s increasing atmospheric anthropogenic CO$_2$ is causing significant changes to the climate system with potentially dangerous consequences (Alley et al., 2003, Solomon et al., 2009). One of the new temperature trends already being observed today is an increase in daily extremes with much colder nights and hotter days (Alexander et al., 2006; Tebaldi et al., 2006). Past studies (van Ness and Scheffer, 2003; Kleinen et al., 2003; Carpenter and Brock, 2006) and this present investigation of the paleotemperatures show that an increased variability can be one way to predict the imminence of a system shift. Is this new temperature trend of increased extremes in night and day temperatures portending an abrupt climate shift? Is it already underway with flickering to the new state occurring? Perhaps this method of looking at increased temperature variance may be useful in indicating current proximity to transition to another alternative climate state.

3.5 Conclusions

These findings have important implications in predicting nonlinear transitions in Earth systems. Past studies looked at increased variability as an indication of imminent transition in a system using model studies. For example, van Ness and Scheffer, (2003) used the macrophyte model Charisma, Kleinen et al (2003) used the Stommel model of ocean circulation changes, Carpenter and Brock (2006) used a system of differential equations representing phosphorus, lake water and surface sediments to look for predictors of regime change. This paper builds on these by using proxy temperature reconstructions to demonstrate this concept of increased variance as an abrupt climate change signal.

The Ansari-Bradley test found increased temperature variability preceding glacial termination and inceptions and suggests this is a reasonably robust tool to quickly detect an imminent climate transition. The Ansari-Bradley results compared well with that of the moving variance test but both methods experienced problems with low resolution datasets and therefore
should be used along with another early warning signal method, for example, autocorrelation. Additionally, the Ansari-Bradley method was able to announce a glacial inception which was not seen by the moving variance. Altogether, these findings demonstrate the usefulness of both the Ansari-Bradley method and moving variance as alternative methods in detecting changes in variability for future time series analyses. Additionally, this chapter explores the potential of the novel tool of using the preceding peak variance to predict the intensity of interglacials, implying the magnitude of transitions can be estimated. Variance change should prove an essential metric in monitoring Earth systems amidst ongoing anthropogenic changes to determine stable state boundaries of a system.
Chapter 4 Evidence of Thermal Destabilization of Methane Clathrates during Glacial Terminations

4.1 Introduction

Paleoclimate data show atmospheric carbon changing along with temperature during the glacial-interglacial transitions suggesting that the climate-carbon feedback plays an essential role in the transitions (Petitt et al., 1999). It has been suggested that once atmospheric levels reach a certain critical threshold, the climate-carbon feedback can result in a rapid transition to an alternative stable state (Paillard, 2001). The geological record shows that around 55.7 Ma, an abrupt increase of global temperatures of 5-8°C occurred and lasted about 200 Ka, now known as the Paleocene-Eocene Thermal Maximum (PETM) (McInerney et al., 2011). It was found that at about the same time a massive amount of methane was released into the atmosphere (Kennett and Stott, 1991). This is recorded by a large negative $\delta^{13}$C anomaly in the paleoclimate data and based on this evidence it was proposed the temperature spike is related to the methane pulse (Kennett and Stott, 1991). Instability of methane clathrates has also been proposed as a major source of increased atmospheric carbon during the last glacial termination based on evidence provided from dating methane craters in the Arctic (Andreassen et al., 2017). If methane clathrates are indeed destabilized during glacial terminations, then these should be recorded by a negative carbon excursion during glacial terminations as in the PETM.

This chapter builds on this concept by analyzing $\delta^{13}$C isotopic records from shallow and deep Atlantic Ocean sources during glacial terminations. The hypothesis is that there should be a negative carbon excursion, though not as large as seen during the PETM, coinciding with or lagging the temperature increase during terminations signaling thermal methane clathrate destabilization. The results reported here indicate a negative carbon-13 excursion averaging 0.41
for shallow Atlantic Ocean data during glacial terminations. and provide an additional line of
evidence demonstrating the instability of methane clathrates during warming. The consequent
release of methane significantly increases the pace of the transition by enhancing the positive
climate-carbon feedback mechanism. These results imply 1) methane clathrates are a major carbon
source during warmings, indicating a strong indicator of an active climate-clathrate feedback and
2) destabilization of current reservoirs has the potential to inject large amounts of carbon abruptly
into the atmosphere. These reservoirs should be considered a significant player in model studies
of projected warming in today’s ongoing anthropogenic climate change. Additionally, these
findings suggest the importance of understanding how destabilizing the methane clathrate reservoir
might have impacted past climate change and may do so again in the near future.

4.1.1 Motivation for this Study

Ice cores contained tiny bubbles of air that allowed the determination of atmospheric
concentrations of CO₂ and CH₄; these varied in step with temperature during the glacial-
interglacial cycles (Petit et al., 1999; EPICA Community Members, 2004). The climate-carbon
feedback illustrated in Appendix B, it is a positive feedback and thought to play a large role in
amplifying the magnitude of the astronomical forcing during glacial terminations (Toggweiler,
1999). Even though numerous studies have attempted to explain the mechanisms behind the
change in atmospheric carbon concentration during the glacial-interglacial cycles, there is still no
consensus on a mechanism (Zeng, 2003). Today atmospheric concentration of CO₂ is rapidly
rising and consequently, there have been recent increased global temperatures (Solomon et al.,
2009). Atmospheric CO₂ levels are currently at 411ppm based on measurements taken from the
Mauna Loa Observatory, Hawaii (www.co2.earth). Destabilization of methane clathrates can
occur due to rising temperatures (McDonald, 1990). Methane clathrates are considered a possible source of the previous carbon injection into the atmosphere during the PETM (Gu et al., 2011).

Given that methane clathrates can become destabilized with rising temperatures and have been associated with past abrupt temperature increase, it is important to understand the role of methane during warmings. Investigating its behavior during glacial terminations may provide some clues. The methane clathrate dynamics will be an important consideration when estimating the potential effect of modern day atmospheric carbon concentrations and global temperature rise. The climate-clathrate feedback occurs when rising temperatures destabilize the clathrate reservoir. This causes methane to be released into the ocean and atmosphere enhancing the climate-carbon feedback and global temperatures increase further (Harvey and Huang, 1995). Previous studies, for example, MacDonald, (1990) considering only permafrost regions estimated a release of about 3 GtC over a 200-year period. Recent observations indicate increased methane emissions from the East Siberian Arctic Shelf due to subsea permafrost degradation (Shaklova et al., 2015). Here δ¹³C isotopic records are explored for a negative excursion to help constrain the source of the carbon increase during glacial terminations and provide some insight on the mechanism responsible for the glacial-interglacial carbon variation.

4.2 Methane Variation during Glacial-Interglacial Cycles

Glacial to interglacial atmospheric variation of carbon were consistent with CO₂ rising from 180 to 280 – 300 ppmv (Figure 2.1) and CH₄ increased from 320 – 350 to 650 – 770 ppbv (Figure 4.1) (Petit et al., 1999). There was a high correlation between these greenhouse gases and temperature with r-squared values of 0.71 and 0.73 for CO₂ and CH₄ respectively (Petit et al., 1999). The good correlation between temperature, atmospheric CO₂ and CH₄ suggests that these gases are important in amplifying the orbital forcing and facilitating the glacial-interglacial
transitions (Raynard et al., 1993). A glacial to interglacial CO$_2$ change from 180-280 ppmv to 300 ppmv represent a 42 to 56% increase while CH$_4$ increase from 350 to 770 ppbv is a large increase of 120% suggesting a higher variability for methane during terminations.

**Figure 4.1** Temperature over time from the EPICA Dome C ice core (Jouzel et al., 2007) in the top panel while the bottom panel gives methane concentrations from EPICA Dome C (Loulergue et al., 2008) shown by the solid red line and the dashed blue curve shows methane concentrations from the Greenland Ice Sheet Project 2 (GISP) (Brook et al., 1996). The grey bar highlights the last glacial termination.

### 4.2.1 Sources of Methane during Terminations

Increases in variability of methane above normal fluctuations indicates changes in sources, and these include wetlands, burning biomass, oceans, lakes and methane clathrates buried under permafrost (Raynard et al., 1993). Nisbet, (1992) proposed that a significant source of this methane came from degassing of methane clathrates during glacial termination warming. Chappellaz et al.
(1997) using model studies attempted to identify the source of the methane increase during the last glacial termination and found emissions of 53 teragram per year for tropical sources and 40 teragrams per year for boreal sources but could not confirm that the boreal source stems from destabilization of CH₄ clathrates. Another study showed that increased precipitation during warming might have led to increased tropical wetlands and greater emission of CH₄. (Severinghaus and Brook, 1999). Yet another study using palynological records from central Brazil argued that there is evidence of severe water stress being experienced by some tropical regions, limiting or preventing the development of wetlands (Salgado-Labouriau et al., 1998). However, the source of the methane increase remains controversial. A recent reexamination of CH₄ reconstruction from ice cores suggests that tropical regions are a major source of this methane increase during terminations (Bock et al., 2017).

4.2.2 Antarctic Methane lagged Arctic Records

Direct comparison between Greenland and Antarctic methane records revealed an average interpolar difference of 44 ± 7 ppbv during the last glacial termination (Chappellaz et al., 1997). This is observed in Figure 4.2 in the lower panel, during the last glacial termination the EPICA Dome C record is shown to lag its GISP 2 counterpart by a few thousand years. This may be an indication of destabilization of methane clathrates in the shallow Arctic region during past warmings such as during terminations due to warming and larger emissions of methane into the ocean and atmosphere. And hence the Arctic region’s significance when considering thermal destabilization of methane clathrates.

4.3 Methane Clathrates: Formation and Stability

Methane is a potent greenhouse gas and variation of its concentration in the atmosphere can cause shifts in the Earth’s radiative balance (McDonald, 1990). Methane clathrates are solids
formed in sediments under high pressure and cold temperatures (Kenneth et al., 2000). These are cube lattices formed when water freezes in which gases (methane) can become trapped inside and remain stable over a narrow range of temperature and pressure (McDonald, 1990). Generally, this means at depths below 300m and temperatures below 5°C (Andreassen et al., 2017). The methane gas can form from the decomposition of organic material in sediments (Dickens, 2003). During photosynthesis plants preferentially take up $^{12}$C so that organic material when considered in a ratio of stable carbon isotopes of $^{13}$C: $^{12}$C (known as $\delta^{13}$C) would have low $\delta^{13}$C values (Libes, 1992). Therefore, a negative excursion in $\delta^{13}$C can possibly signal an addition of $^{13}$C carbon to the atmosphere (Dickens, 2003). One direct consequence of warming bottom water in the past has been thermal methane clathrate dissociation along continental slopes and the consequent release of massive amounts of methane into the ocean and atmosphere recorded by a negative excursion in the $\delta^{13}$C (Dickens et al., 1995).

4.3.1 Role of Methane Clathrates during the Paleocene–Eocene Thermal Maximum (PETM)

Evidence of the PETM is shown in Figure 4.2, this is an ancient climate event that occurred about 56 Ma where global ocean temperatures rose about 5-8°C, coinciding with a rapid release of carbon into the atmosphere continuing for about 20 Ka (McInerney et al., 2011). This event lasted about 200 Ka and is marked by a significant negative carbon isotopic excursion at the same time as a decrease in $\delta^{18}$O values. The decreased $\delta^{18}$O values reflect a temperature increase (Cramer et al., 2009). This decrease in $\delta^{13}$C suggests the addition of a large quantity of $^{13}$C depleted carbon to the ocean and atmosphere resulting in a climate-carbon positive feedback enhanced warming (Dickens et al., 1995; Slujis et al., 2007).
Figure 4.2 Pacific δ¹⁸O and Pacific δ¹³Cmax data compilation for the tropical Pacific for the last 80Ma (Cramer et al., 2009). The grey bar marks the timing of the PETM event about 56 Ma. Decreasing δ¹⁸O suggests warmer temperatures and this is indicated by the orange arrow.

Prior explanations of the cause for dissociation of methane clathrates include erosion or seismic activity making the continental margin steeper allowing methane gas to escape and result in warming (Katz et al., 2001). However, Thomas et al., (2002) argue that this would mean the onset of the negative carbon isotope excursion would have preceded the decrease in δ¹⁸O or temperature increase. Using high resolution stable isotope records Thomas et al., (2002) affirm thermal destabilization of methane clathrates occurred by showing temperature increase preceded the decrease in δ¹³C confirming the plausibility of thermal destabilization of methane clathrates and the abrupt increase in temperature seen during the PETM.

4.3.2 Role of Methane Clathrates during Glacial Terminations

Destabilization of methane clathrates is not a rare phenomenon that only occurred during the PETM. It is ongoing and seems to occur whenever ocean temperature increases above some critical point, for example during glacial terminations. Andreassen et al., (2017) found evidence of recent eruptions when studying about 100 large craters found on the Barents Sea floor. These
were formed by explosive gas eruption that had been covered by sea ice during the last glacial maximum. They suggest as the sea ice retreated during termination, it relieved pressure over the sea floor beneath which methane clathrates had accumulated over the glaciation period, forming pressurized mounds. The sudden relief of pressure allowed methane clathrate dissociation and abrupt release of large amounts of gases into the water column and collapse of the mounds creating the craters observed on the sea floor. This massive release of methane into the water column differs from slow seepage where most of the gases may become oxidized or trapped in sediments; the greater quantity of these emissions increased the potential of the gas escaping to the atmosphere (Andreassen et al., 2017).

Figure 4.3 from the United States Geological Survey (USGS) gives the location of sites where methane clathrates are known to occur based on sampling or have been inferred using indirect methods such as seismic reflectors and pore-water freshening in core samples (Hester and Brewer, 2009). Methane clathrates today can be found mainly in permafrost regions and in continental slope sediments around the globe (Harvey and Huang, 1995). The global methane hydrate estimates vary widely. Using the volume of clathrate stability zone and the volume filled with clathrates, MacDonald (1990) estimated about 400 GtC methane hydrates are currently stored in permafrost regions and 11,000 GtC are in ocean sediments. Harvey and Huang, (1995) used a combination of findings from previous studies and a hydrate model to estimate a global inventory of methane clathrates where 800 GtC are in terrestrial sediments and 24,000 GtC are buried in marine sediments. A more recent estimate based on a hydrate model that considered sediment type, geothermal gradient and sea floor depth predicted there should be about 74,400 Gt of methane in ocean hydrates (Klauda and Sandler, 2005). A more recent study adopted a global estimate of value of 1800 GtC and 2400 Gt of buried methane (Ruppel and Kessler, 2016).
Figure 4.3 Map showing the location of known and inferred locations of gas hydrate occurrence as compiled by the USGS (Hester and Brewer, 2009).


4.4 Data and Methods

In this analysis, the goal is to investigate the paleoclimate records of temperature, methane and the stable carbon isotopes to help determine the relationship between glacial termination warming and stability of methane clathrates buried in the ocean floor. All the datasets in this evaluation are listed in Table 4.1. These were downloaded from NOAA National Centers for Environmental Information formerly known as the National Climatic Data Center (NCDC) database website [https://www.ncdc.noaa.gov/data-access/paleoclimatology-data/datasets]. Two datasets from East Antarctica were used, the European Project for Ice Coring in Antarctica (EPICA) 800 Ka Temperature Estimates (Jouzel et al., 2007) and the EPICA Dome C Ice Core
800KYr Methane Data (Loulergue et al., 2008). These were compared with the Atlantic Regional 450 KYr Benthic $\delta^{13}$C Stacks (Lisiecki et al., 2008). A simple comparison of the timing of the temperature rise at each glacial termination and the corresponding decrease in the stable carbon isotope ratio was used to identify whether the changes coincided.

4.4.1 Paleoclimate Data

The EPICA 800 Ka Temperature Estimates is based on high resolution deuterium measurements, $\delta^{18}$D$_{\text{ice}}$ taken from a deep ice core drilled in East Antarctica (75º 06' S, 123º 21' E) (Jouzel et al., 2007). A more detailed description can be found in Chapter 3 in Paleotemperature section. The EPICA Dome C Ice Core 800 Ka Methane Dataset comes from the same ice core as the EPICA temperature (Loulergue et al., 2008) described above and both of these datasets show clearly defined interglacial maxima in Figure 4.4. The atmospheric methane concentration for the last 450 Ka varied widely with a maximum of 907 ppbv and a minimum of 342 ppbv as shown in Figure 4.4.b. The methane data were obtained from air bubbles in the ice core using gas chromatography, at a high resolution of 0.38 Ka and mean analytical uncertainty of 10 ppbv (Loulergue et al., 2008).

The benthic $\delta^{13}$C Stacks used were assembled from previous studies and put on a common age model by aligning each dataset $\delta^{18}$O with the $\delta^{18}$O of the LRO4 records (Lisiecko and Raymo, 2005). A more detailed description of the LRO4 records can be found in Chapter 3 in the Paleotemperature records. Two $\delta^{13}$C datasets were used here, the shallow North Atlantic sites and the Deep Atlantic Sites. These datasets are from 0 to 450 Ka with 226 data points, resulting in a temporal resolution of 1.99 Ka. The shallow North Atlantic site stack is a compilation of data from four sites: DSDP 552, ODP 980, ODP 982 and ODP 983 while the deep Atlantic Sites are from ODP 929, ODP 1089, GeoB 1041 and GeoB 1211 (Lisiecki et al., 2008).
Table 4.1 List of paleo-climate records used in this evaluation

<table>
<thead>
<tr>
<th>Name of Dataset</th>
<th>Location</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>EPICA Dome C Temperature Estimates</td>
<td>East Antarctica</td>
<td>Jouzel et al., 2007</td>
</tr>
<tr>
<td>EPICA Dome C Methane</td>
<td>East Antarctica</td>
<td>Loulergue et al., 2008</td>
</tr>
<tr>
<td>Deep and Shallow Atlantic Benthic $\delta^{13}$C</td>
<td>Atlantic Ocean</td>
<td>Lisiecki et al., 2008</td>
</tr>
</tbody>
</table>

4.4.2 Method

To determine the temporal lag between the temperature rise and the $\delta^{13}$C negative excursion, the point at which temperature begins to increase during warming and the minimum point in the stable carbon isotope during the warming period are identified. The timing of the glacial termination was determined from previously published literature. The time difference between the start of warming and the time of largest $\delta^{13}$C negative excursion was found to give the lag time as shown in Table 4.2. In this investigation, the following timings of glacial terminations were used. Based on ice sheet data, the onset of the last glacial termination I was determined to around 18 Ka (Denton et al., 2010). The age of termination II was determined to be about 142 Ka using U/Th dating of a core from a calcite vein at Devils Hole (Winograd et al., 1992). The age of terminations III was determined to be 278 Ka based on Ar-40/Ar-39 geochronology of Roman volcanic province tephra (Karner and Renne, 1998). Marra et al., (2008) constrained the age of glacial terminations IV and V as 366 ± 10 and 437 ± 7 Ka respectively using $^{40}$Ar/$^{39}$Ar dating of volcanic layers.

4.5 Results and Discussion

The main results of this study are presented in Figure 4.4 and Table 4.2. The temperature data clearly show the timing of the last five glacial terminations and these are highlighted by the
grey bars in Figure 4.4.a. Atmospheric methane variations for these glacial terminations are shown in Figure 4.4.b. For each of the last five terminations shown in Figure 4.4, there is an atmospheric methane concentration increase coinciding with a decrease in the δ^{13}C value for both the deep Atlantic and shallow North Atlantic sites. The stable carbon isotope changes spanning the last 450 Ka are shown in Figure 4.4.c and the points of maximum decrease coinciding with glacial termination warming events are highlighted along with their δ^{13}C values. These show all five glacial terminations were accompanied by corresponding carbon-13 depletions both for the deep and shallow Atlantic.

For the most recent glacial termination, deep Atlantic Ocean carbon-13 ratios dropped to a low of -0.49 from a high of -0.09 ‰ while the shallow North Atlantic experienced a low value of 0.76‰, moving from a high value of 1.15‰. There is no temporal lag time between the onset of termination I and the carbon-13 ratio decrease for the deep Atlantic and 6 Ka for the shallow Atlantic data as shown in Table 4.2. During termination II, the benthic δ^{13}C for the deep ocean has a low value of -0.71‰ while the shallow Atlantic follows up with its own low value of 0.63‰. The δ^{13}C negative excursions for termination II has a lag time of 12 Ka for the shallow Atlantic, but in contrast to the other data points, the deep Atlantic slightly led the warming. Ice core data for this glacial–interglacial transition reveal a strong negative δ^{13}CO₂ excursion of -7.5 ‰ that lagged the warming of termination II (Lourantou et al., 2010).
Figure 4.4 Comparison of paleoclimate records spanning the last 450 Ka covering five glacial terminations marked by the grey bar. **a.** Temperature over time from the EPICA Dome C ice core (Jouzel et al., 2007). **b.** Methane concentrations from EPICA Dome C (Loulergue et al., 2008). **c.** Compilation of benthic δ¹³C from deep North Atlantic site (black line) and shallow North Atlantic sites (blue line) (Lisiecki et al., 2008). The square markers on the curve mark the points of maximum δ¹³C excursions during the termination event highlighted.

For termination III the deep Atlantic shows carbon-13 ratio depletion with a low value of 0.44‰ while the shallow Atlantic had a low value of 0.56. Both the deep and shallow Atlantic exhibited a long lag phase of 20 and 32 Ka. Termination IV has δ¹³C minimum values of -0.80 and 0.58‰ for deep and shallow Atlantic respectively are much lower than terminations I and II and positive lag times of 3.43 and 9.43 Ka. Similar to termination IV, termination V had an extended lag phase for both deep and shallow Atlantic with 26 and 32 Ka respectively. The oldest termination analyzed here, V has lower δ¹³C values than the younger four with -1 for the deep
Atlantic and 0.26 for the shallow Atlantic. The lag time between the onset of termination and the carbon-13 ratio minimum is comparable with I and II at 3 and 9 Ka.

**Table 4.2 Temporal lag time in (Ka) between warming and maximum benthic δ¹³C excursion**

<table>
<thead>
<tr>
<th>Termination Event</th>
<th>Deep Atlantic</th>
<th>Shallow Atlantic δ</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0</td>
<td>6</td>
</tr>
<tr>
<td>II</td>
<td>-4</td>
<td>12</td>
</tr>
<tr>
<td>III</td>
<td>20</td>
<td>32</td>
</tr>
<tr>
<td>IV</td>
<td>26</td>
<td>32</td>
</tr>
<tr>
<td>V</td>
<td>3</td>
<td>9</td>
</tr>
</tbody>
</table>

From Table 4.2, deep and shallow Atlantic δ¹³C experienced a carbon-13 ratio depletion after warming for all the events studied here. This supports the hypothesis that warming transition may have destabilized methane clathrates and hastened the pace of the transition due to the positive climate-carbon feedback during recent glacial terminations. An exception is termination II in the deep Atlantic where maximum carbon-13 ratio depletion led the onset of warming. Previous studies suggest that there may have been small warming events or interstadials just prior to the onset of glacial termination II (Moseley et al., 2015). This may be a possible explanation for the carbon-13 ratio depletion lead on the onset of termination for this event. Additionally, the deep Atlantic experienced a δ¹³C ratio depletion before the shallow Atlantic Ocean sites. A warming climate can result in heat diffusing to the sea floor raising the temperature and enabling methane clathrate dissociation from the bottom up (Harvey and Huang, 1995).
Figure 4.5 Conceptual diagram illustrating glacial to interglacial changes in methane clathrates stability. **A.** During glaciations, the region is covered by a marine ice-sheet forming a sort of cap increasing pressure on the sea floor. The red arrows show movement of gases from hydrocarbon pools moving upwards but are trapped forming methane clathrates (blue triangles). Blue dotted line represents $\delta^{13}$C of benthic foraminifera record of changing methane flux rates from glacial to interglacial conditions. **B.** During termination events, the warming causes the sea ice to retreat decreasing pressure on the sea floor and decomposition of methane clathrates allowing a mound of built up of gases to form known as gas hydrate pingos. **C.** Eventually the gas hydrate pingo bursts due to dissociation of the gas hydrates forming the crater seen today. This event abruptly ejects large amounts of methane into the water where it has a high potential of getting into the atmosphere and contribute to increased temperature through the climate-carbon feedback. This increase in methane is recorded by decreased $\delta^{13}$C averaging 0.41 for the shallow Atlantic Ocean data of benthic foraminifera for the last five glacial terminations.


4.6 Conclusion

**Figure 4.5** depicts how warming may have led to destabilization of methane clathrates in past deglaciation events which are recorded by the craters and decreased $\delta^{13}$C of benthic foraminifera.
The findings of this chapter suggest stable carbon isotope records from Atlantic Ocean sites provide an added line of evidence for the thermal destabilization of methane clathrates during glacial termination warmings and hence evidence for the climate-clathrate feedback. It lends support to the hypothesis of massive expulsion of methane from hydrate reservoirs in the Arctic sea forming craters during the last glacial termination (Andreassen et al., 2017). For each glacial termination studied here, the warming shift is accompanied by a decrease in the benthic $\delta^{13}$C values that lags the onset of glacial termination by an average of 9 Ka for the deep Atlantic and 18.2 Ka for the shallow Atlantic sites. This can significantly improve our understanding of the thermal stability of methane clathrates buried under permafrost. Given the large amount of methane clathrate currently buried under the ocean around the globe (McDonald, 1990) and ongoing rising global temperatures (Solomon et al., 2009), methane clathrate dynamics of the past may offer good clues on potential and future destabilization of current reservoirs.
Chapter 5  Discussion and Conclusions

5.1 Introduction

Previous chapters of this dissertation discussed the idea that glacial-interglacial transitions cannot be explained by the astronomical forcing alone and that internal feedback mechanisms are required to amplify the orbital forcing. Also illustrated are some of the challenges that this nonlinearity of abrupt climate transitions causes in predicting the timing of past abrupt climate transitions since these sudden jumps between climate states shifts are not always reflected in the forcing. Given these overarching concepts of abrupt climate transitions, this dissertation sought to improve understanding of the climate parameters influencing the dynamics of these shifts. This work examines an example of past abrupt climate transition – glacial-interglacial cycles. This chapter briefly summarizes the main findings of each of the chapters in the context of the questions outlined in the introduction and provides an outline for future studies building on what has been learned here.

From Chapter 2, two negative sea-ice feedbacks are proposed to explain the development of an intermediate state during the glacial inception process and attempt to provide an account for the asymmetrical shape of the glacial-interglacial cycles. These proposed feedbacks are based on a review of published works on the feedback mechanisms dominant during the entrance and exits of interglacials. Chapter 3 found changes in temperature variance before known past abrupt climate transitions – glacial terminations, suggesting this can be an added tool for predicting an abrupt climate transition or any critical transition in a dynamical system. The Ansari-Bradley test proved useful as a quick detection method to predict both glacial terminations and inceptions, but effectiveness decreased with low-resolution data. Additionally, this chapter explores the use of a novel metric where the magnitude of the preceding peak variance of temperature can be used as
an early signal of the next interglacial maximum. In Chapter 4, evaluated benthic $\delta^{13}$C values and found evidence of possible thermal destabilization of methane clathrates during the warming transition of glacial terminations. Decreased values for the benthic foraminiferal carbon isotope ratios are observed for both the deep and shallow Atlantic Ocean just following the onset of the warming event – glacial termination.

Today humans are changing some aspects of the climate system, for example, the composition of the atmosphere by increasing its carbon content. It is important to better constrain the behavior of the critical feedbacks with gradual changes in forcing to understand their range of safe operation and tipping points. Here a simple climate-carbon model that incorporates the two principal feedbacks discussed in earlier chapters and illustrated in Appendix 7.2; the climate-carbon and the ice-albedo feedback mechanisms is assembled. Then an outline of how a future study may use this model to assess how changes in these feedback parameters may affect equilibria and stability of the climate system.

5.2 Synthesis of Chapters and Limitations of Paleoclimate Data

The main aim of this work is to examine the paleoclimate data and published paleoclimate works to extract information on how climate variables behaved during past abrupt climate transitions such as the glacial-interglacial cycles. This section synthesizes the findings of the three major issues examined here such as the glacial-interglacial cycle asymmetry, identifying characteristic changes that may be indicative of critical thresholds being surpassed and evidence of thermal methane clathrate destabilization during the last five glacial terminations.

The use of paleoclimate proxy data is important to derive lessons from the past, however, there are limitations that should be kept in mind when making inferences. The NRC (1995) book
titled, *Natural Climate Variability on Decades to Century Timescales* shed light on the main limitations which include:

i. The age model used may vary across datasets and makes using the absolute age of samples problematic.

ii. The accuracy, resolution and the distribution of the proxy indicator used may vary globally or seasonally and introduce bias.

iii. The records may not be continuous due to disturbance and some records are a combination of cores in an effort to produce a continuous time series.

Accurate dating is a significant problem since data of the climate variable from different locations may not be available for the time period or the presence of annual varves may be missing.

5.2.1 Why are Glacial Inceptions slower than Termination?

Chapter 2 of this dissertation seeks to shed light on the question, “Why are glacial inceptions slower than terminations?”. The relatively fast glacial terminations and the slower glacial inceptions are illustrated using paleoclimate data in Figure 2.1. Although there are proposed mechanisms for the existence of this phenomenon, there is no current consensus on one. Prior proposals for this asymmetry include changes in ocean circulation, for instance, Toggweiler et al., (2006) and Anderson et al., (2009) that vary the atmospheric concentration of CO₂ and increase the dominance of the strong climate-carbon feedback. Others focused on another powerful positive feedback, the ice-albedo feedback as becoming dominant due to changes in global ice volume or its dust content, for example, Weertman, (1974), Le Treut and Ghil, (1983) and Peltier and Marshal, (1995). These explanations focused on the dominance of positive feedback mechanisms
(illustrated in Appendix B) during glacial terminations, these internal mechanisms are thought to amplify the astronomical forcing and speed up transition to the warm conditions of an interglacial.

Here it is proposed that negative feedbacks becoming dominant are responsible for the development of an intermediary stage observed during the glaciation process and are specified as the critical feedbacks slowing the transition. The proposed feedbacks illustrated in Figures 2.2 and 2.3 suggest that at the onset of glacial inception, sea ice expands rapidly as the temperature in the NH drops but as the ice extent increases it serves to insulate the ocean decreasing air-sea exchange. The first negative feedback proposed as becoming dominant is Sea ice – Precipitation feedback. This is where the inhibited air-sea exchange eventually results in decreased evaporation from the ocean in the Arctic reducing the atmospheric moisture and therefore precipitation. With less precipitation in the Arctic region, there is less ice accumulation, and sea ice growth is hindered, rendering it more vulnerable to ablation. The second proposed feedback reinforces this by resulting in greater ocean turbulence and sea ice ablation.

The second proposed feedback is the Sea ice - Insulation feedback portrayed in Figures 2.2 and 2.3 which becomes dominant with enhanced insulation of the Arctic Ocean reducing heat loss and increasing geothermal energy build up in the deep ocean. This leads to increasing deep water buoyancy and at some critical point, to vertical ocean turbulence and sea ice ablation. Sea ice disintegration decreases albedo resulting in higher temperatures and the development of a temporary intermediary stage observed during glaciation. As cooler than interglacial temperatures persist, the broken sea ice reforms solidifying the broken pieces and most likely sturdier and more resilient to ocean turbulence so that eventually there is a shift in the location of convection sites allowing the sea ice to flourish, increased ice-albedo feedback and continued transition to full glacial conditions.
The dominance of these two feedbacks have implications for models that will replicate the climate dynamics of the glacial-interglacial cycle transitions and similar critical transitions of dynamical systems by emphasizing the role of negative feedbacks. Given the unusually fast rate of anthropogenic changes the Earth system is currently undergoing, the risk of crossing thresholds and transitioning to another climate state becomes greater and will likely be a phenomenon of interest. Therefore, improving understanding of how negative feedbacks facilitate climate regime change will help in estimating the speed of transitions of the future.

5.2.2 Temperature Variability as an Indicator of Impending Abrupt Climate Shifts

This chapter (3) observes that abrupt climate transitions were prevalent in the past citing not only the glacial-interglacial transitions but also the Greenhouse to Icehouse shift and the sudden changeover from wet humid to desert conditions in the Sahara. The Greenhouse to Icehouse occurred about 33.7 Ma ago possibly due to long term decreasing atmospheric CO$_2$ concentrations (Tripati et al., 2005). The switch from a moist to dry climate in the Sahara is thought to be caused by gradually decreasing NH insolation (deMenocal, 2000). The underlying cause of these changes may not display a similar shift making it difficult to predict but theoretical and modeling studies confirm that there are generic changes in a dynamic system when tipping points are crossed and these can instead be utilized as an early warning system (Scheffer et al., 2001).

Chapter 3 found there is increased variance in published proxy temperature data that can be used to forecast impending climate transitions of the past using both moving variance and Ansari-Bradley tests. The Ansari-Bradley test was also able to predict glacial inceptions in addition to terminations while the moving variance was limited to terminations. However, the results are not consistent for all nine temperature datasets, the effectiveness of variance change in forecasting a transition decreased with lower resolution data and latitudes. This may be due to differences in
spatial variability of glacial terminations as Lang and Wolff (2011) found significant variability among interglacials and sites in their comparison of interglacial variability paper. This suggests that the use of variance change as an early warning signal of abrupt transitions should be done alongside another method such as changes in autocorrelation to verify the robustness of the results. Another fascinating finding of this study is that both the mobbing variance test and the Ansari-Bradley test found a smaller increase in variance announcing MIS 7e which had previously been found to be a cooler interglacial (Lang and Wolff, 2011). This implied that there is a relationship between the size of the variance change before an abrupt transition and the transition maximum. An exploratory study done here shows good correlation between the magnitude of the preceding peak variance of temperature and the interglacial maximum it predicted. This is especially true for datasets ODP Site 1090, LR04 data stack, EPICA Dome C and ODP Site 882 with r-squared values of this relationship being 0.826, 0.687, 0.465 and 0.433 respectively.

Since this method of variance change relies on the characteristic changes in a dynamical system and not necessarily the mechanism causing the shift, it can be applied across a variety of disciplines such ecology, paleoceanography, societal shifts or man-made systems (Scheffer et al., 2009). Here the simple tools of moving variance and the Ansari-Bradley test have been demonstrated as quick detection tool in a dynamical system. Future analyses may include applying this method to tree ring data to detect changes in resilience and possible impending transitions in a forest, first by applying it to paleo-data and then to current growing forest to detect changes and guide management practices. Today numerous major changes are occurring in various essential Earth systems that may be putting these closer to abrupt transitions with consequences (Rockström et al., 2009). Having a fast and easy way to forecast a possible impending transition may help
decision makers in planning to manage adaptation or other issues for future abrupt transitions in Earth Systems.

5.2.3 Evidence of a Possible Climate-Clathrate Feedback during Glacial Terminations

A large negative carbon isotopic excursion is observed around 56 Ma ago coinciding with an abrupt temperature increased as evidenced by the $\delta^{18}$O drop (McInerney et al., 2011). Chapter 4 examines the paleoclimate data for evidence of a similar negative carbon isotopic excursion during the warming periods of glacial terminations. Methane clathrates are found buried under permafrost and along continental margins and are stable within a small range usually below 300m and at temperatures below 5°C (Andreassen et al., 2017). Estimates of the existing methane clathrate reservoirs vary a bit, but a recent estimate puts the value at around 74,400 Gt (Klauda and Sandler, 2005). Today’s global temperature is currently increasing and will likely continue to increase (IPCC SR15, 2018). It is important to understand the role of temperature increase in methane clathrate destabilization and the addition of methane to the atmosphere. This chapter turns to the ocean $\delta^{13}$C isotopic records for a sign of destabilization during glacial terminations. It is also important to determine the timing of the destabilization, whether this occurred before the warming event or after.

An earlier study dated methane craters found in the Arctic Ocean and determined the timing of these coincided with the last glacial termination (Andreassen et al., 2017). Chapter 4 lends support to this hypothesis of destabilization of methane clathrates during the last glacial termination by showing a decrease in $\delta^{13}$C isotopic records for both the shallow and deep Atlantic Ocean sources during glacial terminations for the last five termination events. Additionally, the data (Table 4.2) shows that the warming preceded the decrease in $\delta^{13}$C isotopic records for both the shallow and deep Atlantic Ocean for all the glacial terminations covered by the records.
examined. All termination except for Termination II (TII), the deep Atlantic Ocean shows the carbon-13 depletion occurring just prior to the onset of the termination event. This may have been caused by a prior small warming event that took place just before TII as Moseley et al., (2015) showed an interstadial might have occurred immediately before the termination.

The results suggest that methane clathrates have been destabilized with warming in recent past events not just supplying an added line of evidence demonstrating methane clathrates were destabilized during the last termination but also the last five glacial terminations. Given the large reservoir of methane clathrates today, the major implication is that current global warming can destabilize these reservoirs leading to a large amplification of global warming through the climate-clathrate feedback.

5.3 Future work

This work has established that the climate system is nonlinear due to the operation of internal feedbacks specifically the climate-carbon and ice-albedo feedbacks illustrated in Appendix 7.2 and the challenge this creates in predicting a climate transition. A further step would be to understand how small changes in these feedback parameter values in a simple coupled climate-carbon model would change the stability and perhaps identify the bifurcation points of these feedbacks given climate conditions of glacial terminations and inceptions. Today’s climate conditions are already being changed drastically (Solomon et al., 2009; IPCC SR15, 2018). The insights gained by analyzing the bifurcation and critical threshold points of the climate-carbon and ice-albedo feedback parameter values can be applied to improving understanding of present-day conditions and potential future climate shifts. Here a climate-carbon model is assembled, and an outline of a future stability analysis of this model is presented.
5.3.1 Development of a Simple Climate-Carbon Model

Here a simple Climate-Carbon model of two governing equations are presented in equations (1) and (2) where T represents global mean temperature and \( C_a \) global atmospheric \( CO_2 \) and are a function of parameters. Energy balance model (EBM) modified from Yi et al., (2001) by adding the greenhouse effect parameter from Cox et al., (2006).

\[
C_a \frac{dT}{dt} = \frac{l_0}{4} (1 - \alpha(T) - \varepsilon_a (1 - \varepsilon_{a1})(a_1 + a_2 T) + \frac{\Delta T_{2xCO_2}}{\log 2} \log \left( \frac{C_a}{C_a(0)} \right) = f_T(T, C_a), \quad (1)
\]

Where \( \varepsilon_{a1} = \varepsilon_0 + \varepsilon_T T \).

Details of the modification of the original EBM from Yi et al., (2001) is given in Appendix D.

Where \( \lambda = \Delta T_{2xCO_2} \)

\( \lambda \) is the climate sensitivity to \( CO_2 \), which is the temperature change with a doubling of atmospheric \( CO_2 \), this is estimated to be 2.6–4.1°C (Rahmstorf, 2008) or 6°C when slower feedbacks such as ice-albedo are included (Hansen, 2008).

All parameter and their values are listed in Table 5.1.

Parameterization of albedo based on temperature from McGuffie and Henderson-Sellers (2005)

\[
\begin{align*}
\propto (T) &= \propto_g, & &\text{for } T \leq T_g \text{ (glaciations)} \\
\propto (T) &= \propto_i, & &\text{for } T \geq T_i \text{ (interglacials)} \\
\propto (T) &= \propto_g + b(T_g - T), & &\text{for } T_g < T < T_i \text{ (transition)}
\end{align*}
\]

\( b = \text{rate temperature change during transitions} \)

\( T_i = 2.3 - 6^\circ \text{C- glacial temperatures} \)

\( T_g = 13-16^\circ \text{C - interglacial temperatures} \)
And a global carbon model is developed where \( K_o C_a \) represents carbon input to the atmosphere from the ocean and \( R_o + R_1 (T - T_g) \) represents input from the biosphere from respiration while the output is from photosynthesis as \( \sigma \frac{I_0}{4} [1 - \alpha(T)] \).

\[
\frac{dC_a}{dt} = K_o C_a + R_o + R_1 (T - T_g) - \sigma \frac{I_0}{4} [1 - \alpha(T)] = f_{Ca}(T, C_a).
\] (2)
Table 5.1 List of model parameters and estimated global values.

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Values</th>
<th>Units</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solar constant</td>
<td>(I_0 )</td>
<td>1370</td>
<td>Wm(^{-2})</td>
<td>McGuffie and Henderson-Sellers, 2005</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1360</td>
<td>Wm(^{-2})</td>
<td>Yi et al., 2001</td>
</tr>
<tr>
<td>Albedo - glaciation</td>
<td>(\alpha_g)</td>
<td>0.62</td>
<td></td>
<td>Budyko, 1969</td>
</tr>
<tr>
<td>Albedo - interglacial</td>
<td>(\alpha_i)</td>
<td>0.32</td>
<td></td>
<td>Budyko, 1969</td>
</tr>
<tr>
<td>Surface emissivity</td>
<td>(\varepsilon_s)</td>
<td>0.98</td>
<td></td>
<td>Yi et al., 2001</td>
</tr>
<tr>
<td>Emissivity coefficient</td>
<td>(\varepsilon_o)</td>
<td>0.1553</td>
<td></td>
<td>Yi et al 2001</td>
</tr>
<tr>
<td>Water vapor coefficient</td>
<td>(\varepsilon_T)</td>
<td>0.00212</td>
<td>°C(^{-1})</td>
<td>Yi et al., 2001</td>
</tr>
<tr>
<td>Linear Coefficient of OLW</td>
<td>(a_1)</td>
<td>199</td>
<td>Wm(^{-2})</td>
<td>Budyko, 1969</td>
</tr>
<tr>
<td></td>
<td></td>
<td>212</td>
<td>Wm(^{-2})</td>
<td>Cess, 1976</td>
</tr>
<tr>
<td>Linear Coefficient of OLW</td>
<td>(a_2)</td>
<td>1.45</td>
<td>Wm(^{2/3})C</td>
<td>Budyko, 1969</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.6</td>
<td>Wm(^{2/3})C</td>
<td>Cess, 1976</td>
</tr>
<tr>
<td>Glacial atmospheric CO(_2)</td>
<td>(C_{ao_g})</td>
<td>381.6</td>
<td>GtC</td>
<td>Petit et al., 1999</td>
</tr>
<tr>
<td></td>
<td></td>
<td>424</td>
<td>GtC</td>
<td>Petit et al., 1999</td>
</tr>
<tr>
<td>Glacial atmospheric CO(_2)</td>
<td>(C_{ao_i})</td>
<td>593.6</td>
<td>GtC</td>
<td>Petit et al., 1999</td>
</tr>
<tr>
<td></td>
<td></td>
<td>636</td>
<td>GtC</td>
<td>Petit et al., 1999</td>
</tr>
<tr>
<td>Glacial Temp</td>
<td>(T_g)</td>
<td>6</td>
<td>°C</td>
<td>Holden et al., 2009</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.89</td>
<td>°C</td>
<td>Crowley, 2000</td>
</tr>
<tr>
<td>Interglacial Temp</td>
<td>(T_i)</td>
<td>16</td>
<td>°C</td>
<td>Data from NCDC NOAA website</td>
</tr>
<tr>
<td></td>
<td></td>
<td>13.7</td>
<td>°C</td>
<td></td>
</tr>
<tr>
<td>Air-sea carbon flux coefficient</td>
<td>(K_0)</td>
<td>-0.1996</td>
<td>yr(^{-1})</td>
<td></td>
</tr>
<tr>
<td>Annual respiration</td>
<td>(R_0)</td>
<td>7.65E+01</td>
<td>GtC yr(^{-1})</td>
<td>Raich and Potter, 1995</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6.80E+01</td>
<td>GtC yr(^{-1})</td>
<td>Raich and Potter, 1995</td>
</tr>
<tr>
<td>Temperature sensitivity of soil respiration, Q10</td>
<td>(R_1)</td>
<td>2.26</td>
<td>GtC yr(^{-1}) °C(^{-1})</td>
<td>Zhou et al., 2009</td>
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<tr>
<td></td>
<td></td>
<td>1.43</td>
<td>GtC yr(^{-1}) °C(^{-1})</td>
<td>Zhou et al., 2009</td>
</tr>
<tr>
<td>Light use efficiency</td>
<td>(\sigma)</td>
<td>3.97E-14</td>
<td>GtC m(^2) yr(^{-1}) W(^{-1})PAR</td>
<td>Wei et al., 2017</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.78E-14</td>
<td>GtC m(^2) yr(^{-1}) W(^{-1})PAR</td>
<td>Wei et al., 2017</td>
</tr>
</tbody>
</table>

5.3.2 Stability Analysis Research Outline

First the equilibrium or steady state can be determined by setting (1) and (2) to 0, and solving simultaneously for steady state temperature (\(T_s\)) and steady state atmospheric CO\(_2\) (\(C_{as}\)) so that they become (3) and (4) in the transition case.

\[
\frac{dT}{dt} = \frac{C_{ao}}{4} \left[ 1 - \alpha_g + b(T_g - T) \right] - \varepsilon_s \left( 1 - \varepsilon_0 - \varepsilon_T T - \varepsilon_c \ln \left( \frac{C_a}{C_{ao}} \right) \right) (a_1 + a_2 T) = 0, \quad (3)
\]
\[
\frac{dc_a}{dt} = K_o C_a + R_o + R_1 \left( T - T_g \right) - \sigma \frac{I_o}{4} \left[ 1 - \alpha_g + b(T_g - T) \right] = 0, \quad (4)
\]

The complete solution derivation for (3) and (4) are given in Appendix D.

The second step is to examine each of the solutions to determine stability by linearizing the model at the equilibrium point and can be carried out by estimating the Jacobian matrix, i.e. determine type of critical points by finding the Jacobian matrix as in equation (5). The Jacobian of a vector function is a matrix of the partial derivatives of that function.

\[
\frac{dT'}{dT'} = \begin{bmatrix} \frac{\partial f_T}{\partial T} & \frac{\partial f_T}{\partial C_a} \\ \frac{\partial f_Ca}{\partial T} & \frac{\partial f_Ca}{\partial C_a} \end{bmatrix} \begin{bmatrix} T' \\ C_a' \end{bmatrix} = 0, \quad (5)
\]

Where the derivatives are

\[
\frac{\partial f_{Ca}}{\partial T} = \frac{\partial \left[ K_o C_{as} + R_o + R_1 T_s - R_1 T_g - \sigma \frac{I_o}{4} \frac{\sigma I_o a_g}{4} + \sigma \frac{I_o b T_g}{4} - \sigma \frac{I_o b T_s}{4} \right]}{\partial T}, \quad (6)
\]

\[
\frac{\partial f_{Ca}}{\partial C_a} = R_1 - \frac{\sigma I_o b}{4}, \quad (7)
\]

\[
\frac{\partial f_{Ca}}{\partial C_a} = \frac{\partial \left[ K_o C_{as} + R_o + R_1 T_s - R_1 T_g - \sigma \frac{I_o}{4} \frac{\sigma I_o a_g}{4} + \sigma \frac{I_o b T_g}{4} - \sigma \frac{I_o b T_s}{4} \right]}{\partial C_a}, \quad (8)
\]

\[
\frac{\partial f_{Ca}}{\partial C_a} = K_o, \quad (9)
\]

Let \( \frac{\partial f_T}{\partial T} = a_{11}, \quad \frac{\partial f_T}{\partial C_a} = a_{12}, \quad \frac{\partial f_{Ca}}{\partial T} = a_{21} \) and \( \frac{\partial f_{Ca}}{\partial C_a} = a_{22} \)

The derivatives obtained can be put into (10) and solved for \( \omega \) to determine the stability of the equilibrium points calculated

\[
\begin{bmatrix} a_{11} - \omega & a_{12} \\ a_{21} & a_{22} - \omega \end{bmatrix} \begin{bmatrix} T' \\ C_a' \end{bmatrix} = 0 \quad (10)
\]
This Jacobian matrix can then be multiplied out to yield

\[(a_{11} - \omega)(a_{22} - \omega) - (a_{21})(a_{12}) = 0,\]  \hspace{1cm} (11)

Let \(S = a_{11} + a_{22}\) and \(\Delta_t = a_{11}a_{22} - a_{12}a_{21}\)

\[\omega^2 - S\omega + \Delta_t = 0,\]  \hspace{1cm} (12)

\[\omega_{1,2} = \frac{S \pm \sqrt{S^2 - 4\Delta_t}}{2}.\]  \hspace{1cm} (13)

The equilibrium points obtained can be classified using the criteria below from (Dijkstra, 2013). The classification points are illustrated in Figure 5.1.

1. Node, real values

\(S^2 - 4\Delta_t > 0\) unstable point; \(S^2 - 4\Delta_t < 0\) stable point

2. Saddle, real values and opposite signs

\(S^2 - 4\Delta_t > 0\) unstable point; \(S^2 - 4\Delta_t < 0\) stable point

3. Focus, complex values

\(S > 0\) unstable; \(S < 0\) stable

4. Center, neutral

\(S > 0.\)
This summarizes the possible types of phase space classifications of the steady state solutions based on their eigenvalues; 

**a.** Here the eigenvalues are complex if with real negative parts then it is a spiral sink and stable but positive real parts, it is a spiral source while a zero is a circle and neutral.

**b.** Here the eigenvalues are positive and negative real numbers so this is an unstable steady state.

**c.** If the eigenvalues are real and positive, then this would be a source node and unstable.

**d.** If the eigenvalues are real and negative, then this would be a sink node and stable.

*Figure 5.1* This summarizes the possible types of phase space classifications of the steady state solutions based on their eigenvalues; **a.** Here the eigenvalues are complex if with real negative parts then it is a spiral sink and stable but positive real parts, it is a spiral source while a zero is a circle and neutral **b.** Here the eigenvalues are positive and negative real numbers so this is an unstable steady state. **c.** If the eigenvalues are real and positive, then this would be a source node and unstable. **d.** If the eigenvalues are real and negative, then this would be a sink node and stable.

Modified from Dijkstra, (2013).
Appendices

Appendix A. Stability Landscape Diagrams

Figure A. 1 Stability landscape diagram to represent changes in climate system stability with changes in external forcings. The bottom shows a possible equilibrium curve for the interglacial to glacial cycles. The upper branch of the curve shows one stable state (interglacial). As conditions change, a bifurcation point is approach depicted as $F_2$, continued change in conditions results in a tip over to an alternative state, a glaciation. If conditions reverse so the system can return to the initial stable state (interglacial), the changes need to go further beyond $F_2$ to the other bifurcation point, $F_1$ indicating the system displays hysteresis.

Modified from Scheffer et al., (2001).
Figure A. 2 The characteristic changes in the equilibrium dynamics as bifurcation is approached.

a. A system far from bifurcation point is depicted by large basin of attraction and a steeper well which represents a fast recovery rate from perturbations. Here the system (ball) kept stable by internal feedback mechanisms. b. A system close to bifurcation point, represented by a small basin of attraction and shallow well, recovery from perturbations are slower, any small change can shift the system to an alternative stable state known as a critical transition.

Modified from Scheffer et al., (2009).
Appendix B. Dominant Feedback Mechanisms of Glacial-Interglacial Transitions

The climate-carbon feedback and ice-albedo are introduced first since these are the principal feedbacks that are reinforced or diminished by the other feedbacks that follow.

The climate-carbon feedback occurs when warming leads to increased greenhouse gases such as carbon dioxide (CO₂) and methane (CH₄) in the atmosphere thereby intensifying the greenhouse gas effect, resulting in further warming, amplifying the initial warming (Friedlingstein, 2016). This feedback is influential in long-term climate transitions where proxy data establish that concentrations were elevated during warm interglacials and decreased during cold glaciations (Pettit et al., 1999, Peacock et al., 2006). The climate-carbon feedback shown in Figure B.1 is also influenced by changes in ocean circulation by significantly influencing atmospheric CO₂ concentration and therefore the strength of the climate-carbon feedback during a transition period (Ewen et al., 2004). Terrestrial feedbacks also influence atmospheric CO₂ concentration and hence the climate-carbon feedback (Zeng, 2003).

![Figure B. 1 Climate-Carbon feedback](image.png)
The second major climate feedback is the ice–albedo feedback shown in Figure B.2 this is a positive feedback mechanism where a small increase in temperature melts some ice reducing the Earth’s albedo leading to further warming and is one of the most significant in promoting abrupt climate changes (Budyko 1969, Sellers 1969, Källén et al. 1979, Dijkstra, 2013). Climate models have consistently demonstrated that a small change in insolation can be effectively amplified by global albedo variations and is responsible for the significant temperature fluctuations observed during climate shifts of ice ages (Budyko, 1969; Sellers, 1969; Paillard, 1998; Crucifix and Loutre, 2002; Calov et al., 2005a).

Figure B. 2 Ice-Albedo feedback

The ocean-climate feedback loop helps to amplify the initial warming brought on by an insolation maximum during terminations (Toggweiler et al., 2006). In this mechanism, increased warming can result in shifting of the Southern Hemisphere Westerlies poleward causing increased upwelling of deep nutrient-rich waters and outgassing of CO₂ to the atmosphere leading to greater warming.
This was confirmed later by data from sediment cores taken from the Southern Hemisphere looking at changes in burial rate of opal that revealed during terminations there is increased upwelling of deep carbon-rich water and outgassing of CO$_2$ to the atmosphere (Anderson et al., 2009). This phenomenon can be reversed during glacial inceptions, where the Southern Westerlies shift north with global cooling, sea ice formation is enhanced around Antarctica, stratification increases between surface and deep ocean, reducing upwelling and outgassing of carbon from the Antarctic deep water dropping its atmospheric concentration and further cooling (Watson and Garabato, 2006).

**Figure B. 3** Ocean-Climate feedback
The \textit{CO$_2$ fertilization – Biosphere feedback} is a negative feedback, when a small temperature increases leads to elevated atmospheric CO$_2$ concentrations, carbon sequestration is enhanced through photosynthesis reducing greenhouse gases and its effect, opposing the original warming. This feedback allows greater terrestrial carbon storage by as much as 288GtC during interglacials (Brovkin et al., 2012). It facilitates improved terrestrial carbon storage especially during interglacials, which can then become frozen in permafrost or buried under glaciers during glaciations (Zeng, 2003). This terrestrial reservoir can release released carbon rapidly during deglaciations as the ice-sheets retreats and respiration increases, further hastening the deglaciation process. During the last termination, atmospheric CO$_2$ increased in pulses of 10-15ppm in less than two centuries, suggesting a source other than the ocean (Marcott et al., 2014). Stable isotope composition $\delta^{13}$C in CO$_2$ from Taylor Glacier confirms that terrestrial carbon sources were key (Bauska et al., 2016).

\textbf{Figure B. 4} CO$_2$ fertilization-Biosphere feedback
The *aged snow-dust deposition feedback* is a positive feedback dominant during glacial rapid terminations and works by reducing the strength of the ice-albedo feedback as aged snow has accumulated dust over the glaciations, decreasing albedo and increasing insolation absorption leading to higher temperatures (Galleé et al., 1992). Model studies show that dust loading enhanced ice-sheet ablation during terminations due to decreased ice-albedo (Peltier and Marshal, 1995). Ice-sheet dynamics and cloud cover are also responsible for the fast transition to a warm period by influencing ice-sheet extent and hence albedo (Källén et al., 1979; Imbrie and Imbrie; 1980; Paillard, 1998).

![Diagram of aged snow-dust feedback](image)

**Figure B. 5** Aged snow-dust feedback
The *Load-Accumulation feedback* between ice sheet mass balance and bedrock isostatic rebound, is crucial in facilitating the asymmetry observed in the glacial-interglacial cycles by enhancing the rate of transitions during terminations (Le Treut and Ghil, 1983). They described the load-accumulation feedback as where accumulation of the ice-sheet over long periods leads to depression of the underlying bedrock, increasing temperature and basal melting decreasing ice-sheet height and consequently precipitation as snow leading to decreased ice accumulation and finally ice-sheet break up. The lithospheric response to ice-sheets showed that isostasy acts as a positive feedback during deglaciations by reducing the height of heavy ice-sheets built up over time and hence the amount of ice deposition (Crucifix et al., 2001, Abe-Ouchi et al., 2013). This feedback is also dominant during glaciations, as the ice-sheet forms its altitude increases further decreasing temperature and ablation rate, therefore allowing net accumulation to increase and further growth of the ice-sheet (Crucifix et al., 2001, Wang and Mysak, 2002, Kageyama et al., 2004).

![Load-Accumulation feedback](image)

**Figure B. 6** Load-accumulation feedback
The **Ice-sheet-Altitude cooling** refers to the feedback where cool summer temperatures allows the buildup of ice volume and as the height increases, the temperature further decreases resulting in greater precipitation as snow further cooling (Kageyama et al., 2004).

![Ice-sheet-Altitude cooling feedback](image)

**Figure B. 7 Ice-sheet-Altitude cooling feedback**

The **Warming-Marine ice-sheet instability feedback** is an ice calving mechanism where a small increase in temperature results in melting ice, higher sea levels, leading to water seeping under the outer layers of ice-sheets, further increasing melting, causing eventual collapse and further warming (Weertman, 1974, Pollard, 1983). This feedback is especially significant in speeding up terminations where the ice calving causes cold oceans, decreased evaporation and precipitation, resulting in reduced ice accumulation, in addition the calving results in even higher sea levels which further undermines the ice-sheet edges and more calving (Watts and Hayder, 1983).
Figure B. 8 Warming-Marine ice-sheet feedback

The *Atmospheric Meridional heat transport feedback* becomes dominant when a larger meridional temperature gradient develops as NH temperature cools resulting in an increase in atmospheric heat transport to the Arctic from 2.88PW to 3.00PW resulting in warming (Jochum et al., 2011). This feedback is significant since the glaciation process is started by an insolation minimum resulting in a temperature drop of 2°C over Canada along with increased precipitation due an enhanced pole-equator temperature gradient, allowing ice-sheet growth and glaciation (Royer et al., 1983). This ice growth is concentrated in the Northern higher latitudes as sediment data from Central Europe showed that decreased insolation caused dry conditions due to shifting of the North Atlantic Drift Current to a southerly position (Sirocki et al., 2005). As persistent annual increases in snow fall continued, this leads to the buildup of ice-sheets over North America (Cortijo et al., 1994, Khodri et al., 2001). This increases the strength of the ice – albedo
feedback, leading to further cooling, amplifying the effects of insolation changes (Calov et al., 2005b), and leading to further cooling towards a glacial state.

Figure B. 9 Atmospheric Meridional heat transport feedback

The Temperature-Precipitation feedback is a negative feedback during termination based on observation data showing increased ice accumulation during warm periods due to increased precipitation (Cuffey and Clow, 1997). Precipitation in general is higher during interglacials than glaciations (Cheddadi et al., 2005, Wu et al., 2007). A small temperature increase results in greater evaporation and precipitation followed by more ice accumulation, ice-sheet growth and enhanced albedo with consequential cooling (Källén et al., 1979, Ghil et al., 1987, Tziperman and Gildor, 2002) opposing the initial warming. However, when temperatures increase past a certain threshold, ablation becomes greater than ice accumulation so that the feedback is no longer dominant (Tziperman and Gildor, 2003). This feedback also results in increased ice-sheet growth as shown by modeling studies of the Antarctic ice-sheet where 1°C uniform warming can
result in an increase of $1.43 \times 10^{11}$ m$^3$ of ice (Huybrechts and Oerleman, 1990). They found increased temperature only results in ice-sheet growth within a range of temperature values. The temperature-precipitation feedback helps ice-sheet growth by increasing high latitude ice accumulation during glaciation inceptions.

**Figure B. 10** Temperature-Precipitation feedback

The *Sea ice-Clouds* feedback is a negative feedback found to be active during glacial inceptions, this is where as the sea ice extends due to cooler temperatures because of reduced summer insolation, the low cloud cover over high latitude regions are reduced as evaporation is hindered, this leads to decreasing albedo and increasing temperature (Jochum et al., 2011).
Figure B. 11 Sea ice - Clouds feedback
Appendix C. Tables of peak variance and temperature for sites analyzed in Chapter 3

Table C. 1 LR04 Data stack

<table>
<thead>
<tr>
<th>Transition event</th>
<th>Peak variance</th>
<th>Peak temperature (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIS 1</td>
<td>36.75</td>
<td>-0.89</td>
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<tr>
<td>MIS 5</td>
<td>35.62</td>
<td>0.03</td>
</tr>
<tr>
<td>MIS 7</td>
<td>14.02</td>
<td>-1.71</td>
</tr>
<tr>
<td>MIS 9</td>
<td>29.02</td>
<td>-0.98</td>
</tr>
<tr>
<td>MIS 11</td>
<td>41.51</td>
<td>1.16</td>
</tr>
</tbody>
</table>

Table C. 2 EPICA Dome C

<table>
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<th>Peak variance</th>
<th>Peak temperature (°C)</th>
</tr>
</thead>
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<td>MIS 1</td>
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<td>-0.41</td>
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<tr>
<td>MIS 5e</td>
<td>22.74</td>
<td>2.84</td>
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<tr>
<td>MIS 7e</td>
<td>8.77</td>
<td>-0.37</td>
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<tr>
<td>MIS 9</td>
<td>18.05</td>
<td>3.62</td>
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<tr>
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Table C. 3 ODP Site 1090

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<th>Peak temperature (°C)</th>
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<td>14.37</td>
<td>17.1</td>
</tr>
<tr>
<td>MIS 7e</td>
<td>2.06</td>
<td>12.46</td>
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<tr>
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<td>14.48</td>
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<tr>
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<td>13.77</td>
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Table C. 4 ODP Site 882

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<th>Peak temperature (°C)</th>
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<td>2.27</td>
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<td>14.57</td>
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<td>13.99</td>
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<td>MIS 9</td>
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<td>12.74</td>
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Table C. 5 ODP Site 846

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Table C. 6 Core MD06-3018

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</tr>
<tr>
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Table C. 7 ODP Site 722

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<th>Peak temperature (°C)</th>
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<td>27.71</td>
</tr>
<tr>
<td>MIS 7e</td>
<td>1.52</td>
<td>27.24</td>
</tr>
<tr>
<td>MIS 9</td>
<td>1.08</td>
<td>27.38</td>
</tr>
<tr>
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<td>27.46</td>
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Table C. 8 Core MD97-2140

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<th>Peak temperature (°C)</th>
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<td>MIS 5e</td>
<td>1.48</td>
<td>29.5</td>
</tr>
<tr>
<td>MIS 7e</td>
<td>0.95</td>
<td>28.53</td>
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<td>MIS 9</td>
<td>0.58</td>
<td>28.9</td>
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### Table C. 9 ODP Site 982

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<th>Peak temperature (°C)</th>
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<td>MIS 1</td>
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<td>14.88</td>
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<td>MIS 9</td>
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</tr>
<tr>
<td>MIS 11</td>
<td>2.83</td>
<td>15.31</td>
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Appendix D. Derivation of Steady States

Detailed derivation of the Climate-Carbon model that can be used to determine critical feedback parameter, stable operating boundaries and critical thresholds given determined conditions.

Energy balance model modified from Yi et al., (2001)
\[
\frac{dT}{dt} = \frac{l_0}{4} [1 - \alpha(T)] - \varepsilon_s(1 - \varepsilon_a)(a_1 + a_2 T), \tag{7.1}
\]
where \(\varepsilon_a = \varepsilon_0 + \varepsilon_T T + \varepsilon_c \ln C_a\),

let \(\ln C_a = \ln \left( \frac{C_a}{C_{a,o}} \right)\) where \(C_{a,o}\) is a reference quantity so that (7.2) becomes
\[
\varepsilon_a = \varepsilon_0 + \varepsilon_T T + \varepsilon_c \ln \left( \frac{C_a}{C_{a,o}} \right) \tag{7.3}
\]
Replacing \(\varepsilon_a\) with (7.3) yields
\[
\frac{dT}{dt} = \frac{l_0}{4} [1 - \alpha(T)] - \varepsilon_s \left( 1 - \varepsilon_0 - \varepsilon_T T - \varepsilon_c \ln \left( \frac{C_a}{C_{a,o}} \right) \right)(a_1 + a_2 T) = f_T(T, C_a) \tag{7.4}
\]
Multiply out the brackets and group similar terms
\[
\frac{dT}{dt} = \frac{l_0}{4} [1 - \alpha(T)] - \varepsilon_s \left( 1 - \varepsilon_0 \right) a_1 - \varepsilon_s \left( 1 - \varepsilon_0 \right) a_2 T + \varepsilon_s \varepsilon_T a_1 T + \varepsilon_s \varepsilon_T a_2 T^2 + \\
\varepsilon_s \varepsilon_c \left( \ln \left( \frac{C_a}{C_{a,o}} \right) \right) a_1 + \varepsilon_s \varepsilon_c a_2 \left[ \ln \left( \frac{C_a}{C_{a,o}} \right) \right] T, \tag{7.5}
\]

Global carbon model
\[
\frac{dc_a}{dt} = K_0 C_a + R_0 + R_1 (T - T_g) - \sigma \frac{l_0}{4} [1 - \alpha(T)] = f_{Ca}(T, C_a), \tag{7.6}
\]
Parameterization of albedo based on temperature from McGuffie and Henderson-Sellers (2005) pg. 67
\[
\alpha(T) = \alpha_g, \quad \text{for } T \leq T_g \text{ (glaciations)}
\]
\[
\alpha(T) = \alpha_i, \quad T \geq T_i \text{ (interglacials)}
\]
\[
\alpha(T) = \alpha_g + b(T_g - T), \quad T_g < T < T_i \text{ (transition)}
\]

Transition stage \(T_g < T < T_i\) (glacial to interglacial)
\[
\alpha(T) = \alpha_g + b(T_g - T) \tag{7.7}
\]
Add (7.7) to (7.4) and (7.6) to obtain (7.8) and (7.9)
\[ C_{\text{dT}} = f(T, C_a) = \frac{1}{4} \left[ 1 - (\alpha_g + bT_g - bT) \right] - \varepsilon_s [1 - \varepsilon_0] a_1 - \varepsilon_s [1 - \varepsilon_0] a_2 T + \varepsilon_s \varepsilon_T a_1 T + \varepsilon_s \varepsilon_T a_2 T^2 + \varepsilon_s \varepsilon_c \left[ \ln \left( \frac{C_a}{C_{a_0}} \right) \right] a_1 + \varepsilon_s \varepsilon_c a_2 \left[ \ln \left( \frac{C_a}{C_{a_0}} \right) \right] T, \]  
(7.8)

\[ \frac{dC_a}{dt} = f_{Ca}(T, C_a) = K_0 C_a + R_0 + R_1 (T - T_g) - \sigma \frac{1}{4} \left[ 1 - (\alpha_g + bT_g - bT) \right], \]  
(7.9)

**Step 1.** Determine equilibrium solutions (steady states) by setting equations = 0, and solving simultaneously for Ts and Cas.

\[ \frac{1}{4} \left[ 1 - (\alpha_g + bT_g - bT_s) \right] - \varepsilon_s [1 - \varepsilon_0] a_1 - \varepsilon_s [1 - \varepsilon_0] a_2 T_s + \varepsilon_s \varepsilon_T a_1 T_s + \varepsilon_s \varepsilon_T a_2 T_s^2 + \varepsilon_s \varepsilon_c \left[ \ln \left( \frac{C_{as}}{C_{a_0}} \right) \right] a_1 + \varepsilon_s \varepsilon_c a_2 \left[ \ln \left( \frac{C_{as}}{C_{a_0}} \right) \right] T_s = 0 \]  
(7.10)

\[ K_0 C_{as} + R_0 + R_1 (T_s - T_g) - \sigma \frac{1}{4} \left[ 1 - (\alpha_g + bT_g - bT_s) \right] = 0 \]  
(7.11)

where \( T_s \) = steady state temperature, 
\( C_{as} \) = steady state atmospheric \( \text{CO}_2 \) concentration

Remove brackets and group similar terms

\[ \frac{1}{4} \left[ 1 - \alpha_g - bT_g \right] + \frac{1}{4} bT_s - \varepsilon_s [1 - \varepsilon_0] a_1 - \varepsilon_s [1 - \varepsilon_0] a_2 T_s + \varepsilon_s \varepsilon_T a_1 T_s + \varepsilon_s \varepsilon_T a_2 T_s^2 + \varepsilon_s \varepsilon_c \left[ \ln \left( \frac{C_{as}}{C_{a_0}} \right) \right] a_1 + \varepsilon_s \varepsilon_c a_2 \left[ \ln \left( \frac{C_{as}}{C_{a_0}} \right) \right] T_s = 0, \]  
(7.12)

\[ K_0 C_{as} + R_0 + R_1 T_s - R_1 T_g - \frac{\alpha_0}{4} + \frac{\alpha_0 \alpha_g}{4} + \frac{\alpha_0 bT_g}{4} - \frac{\alpha_0 bT_s}{4} = 0, \]  
(7.13)

\[ \frac{1}{4} \left[ 1 - \alpha_g - bT_g \right] - \varepsilon_s [1 - \varepsilon_0] a_1 + T_s \frac{1}{4} b - \varepsilon_s [1 - \varepsilon_0] a_2 + \varepsilon_s \varepsilon_T a_1 \right] + T_s^2 \right \varepsilon_s \varepsilon_T a_2 \right] + \ln \left( \frac{C_{as}}{C_{a_0}} \right) \left[ \varepsilon_s \varepsilon_c a_1 + \varepsilon_s \varepsilon_c a_2 T_s \right] = 0, \]  
(7.14)

\[ K_0 C_{as} + T_s \left[ R_1 - \frac{\alpha_0 b}{4} \right] + R_0 - R_1 T_g - \frac{\alpha_0}{4} + \frac{\alpha_0 \alpha_g}{4} + \frac{\alpha_0 bT_g}{4} = 0, \]  
(7.15)

\[ C_{as} = \frac{1}{K_0} \left[ R_1 T_g - R_0 + \frac{\alpha_0}{4} - \frac{\alpha_0 \alpha_g}{4} - \frac{\alpha_0 bT_g}{4} \right] + T_s \left[ \frac{\alpha_0 b}{4} - R_1 \right] \frac{1}{K_0}, \]  
(7.16)

\[ C_{as} = A + BT_s \]  
(7.17)

Where
\[ A = \frac{1}{K_0} \left[ R_1 T_g - R_o + \frac{\sigma l_0}{4} - \frac{\sigma l_o \alpha_g}{4} - \frac{\sigma l_0 b T_g}{4} \right] , \quad B = \frac{1}{K_0} \left[ \frac{\sigma l_0 b}{4} - R_1 \right] \]

Add (7.17) to (7.14) so that \( A + B T_s \) replaces \( C_{as} \)

\[
\frac{l_0}{4} \left[ 1 - \alpha_g - b T_g \right] - \varepsilon_s \left[ 1 - \varepsilon_0 \right] a_1 + T_s \left[ \frac{l_0}{4} b - \varepsilon_s \left[ 1 - \varepsilon_0 \right] a_2 + \varepsilon_s \varepsilon_T a_1 \right] + T_s^2 \left[ \varepsilon_s \varepsilon_T a_2 \right] + 
\ln \left( \frac{A + B T_s}{C_{a_o}} \right) \left[ \varepsilon_s \varepsilon_c a_1 + \varepsilon_s \varepsilon_c a_2 T_s \right] = 0
\]

(7.18)

\[
\ln \left( \frac{A + B T_s}{C_{a_o}} \right) = \ln \left[ \frac{A}{C_{a_o}} \left( 1 + \frac{B T_s}{A} \right) \right] = \ln \frac{A}{C_{a_o}} + \ln \left( 1 + \frac{B T_s}{A} \right)
\]

Taylor series expansion of \( \ln \frac{A}{C_{a_o}} + \ln \left( 1 + \frac{B T_s}{A} \right) \) yields

\[
\ln \frac{A}{C_{a_o}} + \ln \left( 1 + \frac{B T_s}{A} \right) = \ln \frac{A}{C_{a_o}} + \frac{B T_s}{A} - \frac{B^2 T_s^2}{2A^2}.
\]

(7.19)

Using only two terms from the Taylor series expansion, higher order terms are ignored assuming \( \left| \frac{B T_s}{A} \right| < 1 \)

\[
\ln \left( \frac{A}{C_{a_o}} + B T_s \right) = \ln \frac{A}{C_{a_o}} + \frac{B T_s}{A} - \frac{B^2 T_s^2}{2A^2}.
\]

(7.20)

Substituting (7.20) into (7.18) and grouping like terms

\[
\frac{l_0}{4} \left[ 1 - \alpha_g - b T_g \right] - \varepsilon_s \left[ 1 - \varepsilon_0 \right] a_1 + T_s \left[ \frac{l_0}{4} b - \varepsilon_s \left[ 1 - \varepsilon_0 \right] a_2 + \varepsilon_s \varepsilon_T a_1 \right] + T_s^2 \left[ \varepsilon_s \varepsilon_T a_2 \right] + 
\left( \ln \frac{A}{C_{a_o}} + \frac{B T_s}{A} - \frac{B^2 T_s^2}{2A^2} \right) \left[ \varepsilon_s \varepsilon_c a_1 + \varepsilon_s \varepsilon_c a_2 T_s \right] = 0
\]

(7.21)

Continue grouping and multiply out brackets

\[
\frac{l_0}{4} \left[ 1 - \alpha_g - b T_g \right] - \varepsilon_s \left[ 1 - \varepsilon_0 \right] a_1 + T_s \left[ \frac{l_0}{4} b - \varepsilon_s \left[ 1 - \varepsilon_0 \right] a_2 + \varepsilon_s \varepsilon_T a_1 \right] + T_s^2 \left[ \varepsilon_s \varepsilon_T a_2 \right] + 
\left[ \ln \left( \frac{A}{C_{a_o}} \right) \right] \varepsilon_s \varepsilon_c a_1 + \left[ \ln \left( \frac{A}{C_{a_o}} \right) \right] \varepsilon_s \varepsilon_c a_2 T_s + \frac{B T_s}{A} \varepsilon_s \varepsilon_c a_1 + \frac{B T_s}{A} \varepsilon_s \varepsilon_c a_2 T_s - \frac{B^2 T_s^2}{2A^2} \varepsilon_s \varepsilon_c a_1 - 
\frac{B^2 T_s^2}{2A^2} \varepsilon_s \varepsilon_c a_2 T_s = 0
\]

(7.22)

Move all terms to the other side of equation to avoid a negative \( a T_s^3 \), and group similar terms
\[ \varepsilon_s [1 - \varepsilon_0] a_1 - \frac{I_0}{4} [1 - \alpha_g - b T_g] - \left[ \ln \left( \frac{A}{C_{a_o}} \right) \right] \varepsilon_s \varepsilon_c a_1 + T_s \left[ \varepsilon_s [1 - \varepsilon_0] a_2 - \frac{I_0 b}{4} - \varepsilon_s \varepsilon_T a_1 - \left[ \ln \left( \frac{A}{C_{a_o}} \right) \right] \varepsilon_s \varepsilon_c a_2 - \frac{B \varepsilon_s \varepsilon_c a_1}{A} \right] + T_s^2 \left[ \frac{B^2 \varepsilon_s \varepsilon_c a_1}{2A^2} - \varepsilon_s \varepsilon_T a_2 - \frac{B \varepsilon_s \varepsilon_c a_2}{A} \right] + T_s^3 \left[ \frac{B^2 \varepsilon_s \varepsilon_c a_2}{2A^2} \right] = 0 \] (7.23)

\[ a T_s^3 + b T_s^2 + c T_s + d = 0 \] where

\[ a = \frac{B^2 \varepsilon_s \varepsilon_c a_2}{2A^2}, \]

\[ b = \frac{B^2 \varepsilon_s \varepsilon_c a_1}{2A^2} - \varepsilon_s \varepsilon_T a_2 - \frac{B \varepsilon_s \varepsilon_c a_2}{A}, \]

\[ c = \varepsilon_s [1 - \varepsilon_0] a_2 - \frac{I_0 b}{4} - \varepsilon_s \varepsilon_T a_1 - \left[ \ln \left( \frac{A}{C_{a_o}} \right) \right] \varepsilon_s \varepsilon_c a_2 - \frac{B \varepsilon_s \varepsilon_c a_1}{A}, \]

\[ d = \varepsilon_s [1 - \varepsilon_0] a_1 - \frac{I_0}{4} [1 - \alpha_g - b T_g] - \left[ \ln \left( \frac{A}{C_{a_o}} \right) \right] \varepsilon_s \varepsilon_c a_1. \]

Finding the roots of this cubic equation to determine the value of steady state temperature, \( T_s \) at this stage.

Divide (7.24) by \( a \)

\[ T_s^3 + \frac{b}{a} T_s^2 + \frac{c}{a} T_s + \frac{d}{a} = 0 \] (7.25)

Let \( p = \frac{b}{a}, q = \frac{c}{a}, r = \frac{d}{a} \)

\[ T_s^3 + p T_s^2 + q T_s + r = 0 \] (7.26)

Use the substitution \( T_s = y - \frac{p}{3} \) to reduce to the normal form

\[ y^3 + a_{-1} y + b_{-1} = 0 \] (7.27)

where

\[ a_{-1} = \frac{1}{3} (3q - p^2), \]

\[ b_{-1} = \frac{1}{27} (2p^3 - 9pq + 27r) \]

\[ y^3 + a_{-1} y + b_{-1} = 0 \] has three roots:
\[ y_1 = A_1 + B_1 \]  \hspace{1cm} (7.28)
\[ y_2 = -\frac{1}{2}(A_1 + B_1) + \frac{i\sqrt{3}}{2}(A_1 - B_1) \]  \hspace{1cm} (7.29)
\[ y_2 = -\frac{1}{2}(A_1 + B_1) - \frac{i\sqrt{3}}{2}(A_1 - B_1) \]  \hspace{1cm} (7.30)

where \( i = \sqrt{-1} \) and
\[
A_1 = \sqrt[3]{-\frac{b_1}{2} + \sqrt{\frac{b_1^2}{4} + \frac{a_1^3}{27}}} \\
B_1 = \sqrt[3]{-\frac{b_1}{2} - \sqrt{\frac{b_1^2}{4} + \frac{a_1^3}{27}}}
\]

If \( p, q \) and \( r \) are real, then three cases exist.

1. \( \Delta = \frac{b_1^2}{4} + \frac{a_1^3}{27} > 0 \) (positive): one real root \( y = y_1 \) and two conjugate imaginary roots.
   - \( y_1 = A_1 + B_1 \)
   - \( y_2 = A_1 + B_1i \)
   - \( y_3 = A_1 - B_1i \)

2. \( \Delta = \frac{b_1^2}{4} + \frac{a_1^3}{27} = 0 \) : Three real \( y \) roots of which at least two are equal
   Let \( n2 = \sqrt[3]{-\frac{a_1}{3}} \)
   - \( y_1 = -2 * n2 \) \hspace{0.5cm} \text{If } b_1 > 0 \\
   - \( y_2 = n2 \) \\
   - \( y_3 = n2 \)

   - \( y_1 = 2 * n2 \) \hspace{0.5cm} \text{If } b_1 < 0 \\
   - \( y_2 = -n2 \) \\
   - \( y_3 = -n2 \)

   - \( y_1 = 0 \) \hspace{0.5cm} \text{if } b_1 = 0 \\
   - \( y_2 = 0 \) \\
   - \( y_3 = 0 \)

3. \( \Delta = \frac{b_1^2}{4} + \frac{a_1^3}{27} < 0 \) (negative): three real and unequal roots
\[ n = \sqrt{\frac{b_1^2}{a_1^3}} \]

\[ \cos \phi = -n, \text{ if } b_1 > 0, \text{ therefore } \phi = \cos(-n) \]

- \[ y_1 = 2 \sqrt{-\frac{a_1}{3}} \cos \left( \frac{\cos(-n)}{3} \right) \quad k = 0 \]
- \[ y_2 = 2 \sqrt{-\frac{a_1}{3}} \cos \left( \frac{\cos(-n)}{3} + \frac{2\pi}{3} \right) \quad k = 1 \]
- \[ y_3 = 2 \sqrt{-\frac{a_1}{3}} \cos \left( \frac{\cos(-n)}{3} + \frac{4\pi}{3} \right) \quad k = 2 \]

\[ \cos \phi = n, \text{ if } b_1 < 0, \text{ therefore } \phi = \cos(n) \]

- \[ y_1 = 2 \sqrt{-\frac{a_1}{3}} \cos \left( \frac{\cos(n)}{3} \right) \quad k = 0 \]
- \[ y_2 = 2 \sqrt{-\frac{a_1}{3}} \cos \left( \frac{\cos(n)}{3} + \frac{2\pi}{3} \right) \quad k = 1 \]
- \[ y_3 = 2 \sqrt{-\frac{a_1}{3}} \cos \left( \frac{\cos(n)}{3} + \frac{4\pi}{3} \right) \quad k = 2 \]

Derived steady state temperatures during transition stage

- \[ T_{s1} = y_1 - \frac{p}{3} \quad (7.31) \]
- \[ T_{s2} = y_2 - \frac{p}{3} \quad (7.32) \]
- \[ T_{s3} = y_3 - \frac{p}{3} \quad (7.33) \]

Derived steady state atmospheric CO2 during transition stage

- \[ C_{as1} = A + BT_{s1} \quad (7.34) \]
- \[ C_{as2} = A + BT_{s2} \quad (7.35) \]
- \[ C_{as3} = A + BT_{s3} \quad (7.36) \]

**Step 2.** The second step is to linearize the model at the equilibrium point by estimating the Jacobian matrix, i.e. determine type of critical points by finding the Jacobian matrix. Examine each of the solutions to determine stability according to the eigen value criteria shown in Figure 5.1.

\[
\frac{dT'}{d\tau} \left[ \begin{array}{c} T' \\ C_a' \end{array} \right] = \left[ \begin{array}{cc} \frac{\partial f_T}{\partial T} & \frac{\partial f_T}{\partial C_a} \\ \frac{\partial f_{Ca}}{\partial T} & \frac{\partial f_{Ca}}{\partial C_a} \end{array} \right] \left[ \begin{array}{c} T' \\ C_a' \end{array} \right] = 0, \quad (7.37) \]

From (7.12) \[ \frac{1}{4} \left[ 1 - \alpha_g - bT_g \right] + \frac{1}{4} bT_s - \varepsilon_s [1 - \varepsilon_0] a_1 - \varepsilon_s [1 - \varepsilon_0] a_2 T_s + \varepsilon_s \varepsilon \varepsilon_T a_1 T_s + \varepsilon_s \varepsilon_T a_2 T_s^2 + \varepsilon_s \varepsilon_c \left[ \ln \left( \frac{C_{as}}{C_{ao}} \right) a_1 + \varepsilon_s \varepsilon_c a_2 \left[ \ln \left( \frac{C_{as}}{C_{ao}} \right) \right] T_s \right] = 0 \]
\[
\frac{\partial \tau}{\partial T} = \frac{1}{4} \left[ 1 - \alpha_s - bT_s - \varepsilon_s (1 - \varepsilon_0) \alpha_1 + T_s \left( \frac{1}{4} - \varepsilon_s (1 - \varepsilon_0) \alpha_2 + \varepsilon_s \varepsilon_1 \alpha_1 \right) + T_s^2 \left( \varepsilon_s \varepsilon_1 \alpha_2 + \varepsilon_s \varepsilon_2 \alpha_1 \ln(C_{as} - \ln(C_{ao})) + \varepsilon_s \varepsilon_3 \alpha_2 \ln((C_{as} - \ln(C_{ao}))T_s \right) ]
\]

\[
\frac{\partial \tau}{\partial \varepsilon_s} = \frac{1}{4} \left[ b - \varepsilon_s (1 - \varepsilon_0) \alpha_2 + 2 \varepsilon_s \varepsilon_1 \alpha_1 + 2 \varepsilon_s \varepsilon_1 \alpha_2 T_s + \varepsilon_s \varepsilon_2 \alpha_2 \ln(C_{as} - \ln(C_{ao})) \right],
\]

\[
\frac{\partial \tau}{\partial \varepsilon_2} = \frac{1}{4} \left[ 1 - \alpha_s - bT_s - \varepsilon_s (1 - \varepsilon_0) \alpha_1 + T_s \left( \frac{1}{4} - \varepsilon_s (1 - \varepsilon_0) \alpha_2 + \varepsilon_s \varepsilon_1 \alpha_1 \right) + T_s^2 \left( \varepsilon_s \varepsilon_1 \alpha_2 + \varepsilon_s \varepsilon_2 \alpha_1 \ln(C_{as} - \ln(C_{ao})) + \varepsilon_s \varepsilon_3 \alpha_2 \ln((C_{as} - \ln(C_{ao}))T_s \right) ]
\]

\[
\frac{\partial \tau}{\partial \varepsilon_1} = \varepsilon_s \varepsilon_1 \alpha_1 \left( \frac{1}{C_{as}} \right) + \varepsilon_s \varepsilon_2 \alpha_2 \left( \frac{1}{C_{as}} \right) T_s,
\]

\[
\frac{d}{dx} \ln(x) = \frac{1}{x}
\]

From (7.13)

\[
\frac{\partial C_a}{\partial T} = \frac{\partial}{\partial T} \left[ K_0 C_{as} + R_0 + R_1 T_s - R_1 T_s - R_1 T_s \frac{\sigma_{l_0} a g}{4} + \frac{\sigma_{l_0} b T_s}{4} - \frac{\sigma_{l_0} b T_s}{4} \right],
\]

\[
\frac{\partial C_a}{\partial \alpha_s} = R_1 - \frac{\sigma_{l_0} b}{4},
\]

\[
\frac{\partial C_a}{\partial \alpha_2} = \frac{\partial}{\partial \alpha_2} \left[ K_0 C_{as} + R_0 + R_1 T_s - R_1 T_s - R_1 T_s \frac{\alpha_{l_0} a g}{4} + \frac{\sigma_{l_0} b T_s}{4} - \frac{\sigma_{l_0} b T_s}{4} \right],
\]

\[
\frac{\partial C_a}{\partial \alpha_1} = K_0,
\]

Let \( \frac{\partial \tau}{\partial T} = a_{11}, \frac{\partial \tau}{\partial \varepsilon_s} = a_{12}, \frac{\partial \tau}{\partial \varepsilon_2} = a_{21} \) and \( \frac{\partial \tau}{\partial \varepsilon_1} = a_{22} \)

Plug in the derivatives obtained into (7.46) and solve for \( \omega \) to determine the stability of the equilibrium points calculated.
\[
\begin{bmatrix}
  a_{11} - \omega & a_{12} \\
  a_{21} & a_{22} - \omega
\end{bmatrix}
\begin{bmatrix}
  T' \\
  C_{a'}
\end{bmatrix} = 0
\]

(7.46)

Multiply out the Jacobian matrix
\[
(a_{11} - \omega)(a_{22} - \omega) - (a_{21})(a_{12}) = 0,
\]

(7.47)

\[
\omega^2 - \omega(a_{11} + a_{22}) - a_{21}a_{12} + a_{11}a_{22} = 0.
\]

(7.48)

Let \( S = a_{11} + a_{22} \) and \( \Delta_t = a_{11}a_{22} - a_{12}a_{21} \)
\[
\omega^2 - S\omega + \Delta_t = 0,
\]

(7.49)

\[
\omega_{1,2} = \frac{S \pm \sqrt{S^2 - 4\Delta_t}}{2}.
\]

(7.50)
References


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http://dx.doi.org/10.1098/rsta.2013.0096