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NON-METRIC SCALING OF LOUDNESS

by

ALAN M. RICHARDS

A dissertation submitted to the
Graduate Faculty in Speech in partial
fulfillment of the requirements for the
degree of Doctor of Philosophy,
The City University of New York.

1971

This manuscript has been read and accepted for the University Committee in Speech in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.

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Abstract

NON-METRIC SCALING OF LOUDNESS

by

Alan M. Richards

Advisor: Professor Harry Levitt

Determination of loudness scales for 1000 Hz stimuli by conventional ratio scaling methods have yielded loudness functions which grow as approximately the 0.54 power of sound pressure. Thus, two-fold loudness differences are equivalent to approximately 10 dB across the auditory continuum. The unidimensional representation of loudness as a power function of sound intensity implies that if A is twice as loud as B, which, in turn, is twice as loud as C, then A will be four times as loud as C. In order to test this implication across the auditory continuum loudness ratio estimates were obtained from four 7x7 matrices of 1000 Hz stimuli with differing inter-stimulus spacings and ranges.

Two types of data analysis were used in comparing the obtained ratio estimate results with those implied by the 10 dB rule. The first was a multidimensional representation of the data based upon Shepard's Analysis of Proximities [R. N. Shepard, Psychometrika, 27, 125-140, 210-246 (1962)]. From these analyses simple two-dimensional configurations were found which adequately represented the data. In general, these configurations indicated that the obtained estimates did not conform to the configurations implied by the 10 dB rule, i.e.,

a straight line in space, but curved upwards indicating increasing non-additivity with increased inter-stimulus differences. It was further found that as the stimulus range of a matrix decreased, the ratio estimates associated with common stimulus pairs increased.

The second type of analysis was designed to plot the obtained ratio estimate data as a unidimensional function of intensity, which, in turn, would yield linear spatial configurations. The results of this analysis yielded loudness scales which could be directly compared to conventional power functions. It was found that for two matrices (B and C) that power functions were obtained, although the slope for Matrix C (15 dB range) was high relative to the conventional scale, while the slope of Matrix B (30 dB range) was quite similar to the conventional function. With a 60 dB stimulus range (Matrix A) a power function was not obtained.

Loudness growth was further investigated with other than moderate to intense 1000 Hz tones. Ratio estimates were obtained from four new matrices which contained either 250 Hz tones, white-noise, or low sensation level 1000 Hz and 4000 Hz tones. Each of these matrices was analyzed by the Analysis of Proximities, and the obtained configurations compared to the results implied from earlier findings concerning the stimuli of interest.

A monaural test of loudness recruitment was suggested utilizing ratio estimates combined with the Analysis of Proximities.

CHAPTER I
INTRODUCTION AND HISTORY

The relative advantages and limitations of the various types of scales are well known (Guilford, 1954; Stevens, 1951, 1958). Four types exist: nominal, ordinal, interval, and ratio. The nominal scale represents the simplest form because only the classification of attributes is considered, with no metricizing or ordering. Among some of the psychophysical problems lending themselves to determination by nominal scales are absolute and differential threshold, and the equation of magnitude, such as found in equal loudness contours.

Ordinal scales are one step removed from nominal scales in that stimuli are ranked relative to some attribute. These scales, however, are not designed to indicate the distance present between two attributes, nor do they contain true zero points. Psychophysical determination of ordinal scales conventionally is accomplished by using the methods of Rank Order, Rating, or Paired Comparisons.

Interval scales are another step removed from nominal scales in that the distances between two attributes can be determined quantitatively (Guilford, 1954; Stevens, 1951). They also lend themselves to conventional statistical analyses. However, no true zero point is present. The psychophysical procedure used most frequently in erecting interval scales is that of bisection (Stevens & Volkman, 1940).

Ratio scales represent the most advanced scales in that they contain an interval scale within themselves, as well as have true zero points (Guilford, 1954; Stevens, 1951, 1958,

1960). By absolute zero point here is meant a point which is representative of "neither more nor less than none of the property represented by the scale" (Guilford, 1954, p. 16).

The most important aspect of ratio scales, as concerns subjective magnitudes, is that preserve information about the ratio s between sensations. Thus, it is possible to indicate that one subjective sensation is twice as great, or one-half as great, as another.

The perception of loudness, as implied from previously obtained loudness functions (Stevens, 1955, 1956, 1957a, 1958, 1959, 1960), shows properties which are found only in ratio scales. Thus, it is assumed that ratio scaling procedures are applicable to loudness-intensity relations (Stevens, 1960). These methods fall into five classes: Magnitude Estimation, Magnitude Production, Ratio Production, Ratio Estimation, and Numerical Magnitude Balance.

Magnitude Estimation of loudness can be accomplished by using either of two techniques. The first method requires that a number (modulus) be assigned to a stimulus by the experimenter (E). E then presents other stimuli to the subject (S), and S estimates the loudness of these subsequent stimuli relative to the modulus. In the other variation, no modulus is included, and S simply reports the percept by any number which he feels is proportional to the loudness. Numerous investigations have used either method in the development of loudness scales (Hellman & Zwislocki, 1961, 1963; Jones & Woskow, 1966; J. C. Stevens, 1958; Stevens & Tulving, 1957).

Although the method of Magnitude Estimation leads to a ratio scale of loudness (Stevens, 1958), it is instructive to note that an equal-interval scale can be erected from the same obtained data when different properties of the data are analyzed. For example, Garner (1952) constructed an "equal discriminability" scale based upon the dispersion of loudness judgments of various stimuli to different response categories. The scale was constructed so that equal scale distances represented equal tendencies to judge two stimuli in the same category. The subjects were instructed to judge individually presented 1000 Hz tones on a scale which ranged from "0" to "20", where "20" was the loudest tone heard, and "1" was the lowest tone heard ("0" was a stimulus which was not heard). A visual stimulus preceded the presentation of each of the stimuli. Four different experimental conditions were tested, each with different stimulus spacings and ranges (5-100 dB in 5 dB intervals; 55-100 dB SPL in 5 dB intervals; 5-50 dB SPL in 5 dB steps, and 5-100 dB SPL in equal loudness increments). Garner found that the obtained equal discriminability (E.D) scale grew as a linear function of log-intensity over most of its' extent. Thus, the form of the E.D. scale agreed well with scales based upon cumulating difference limens (DL's), Fechnerian scales, but did not agree with the form of ratio scales derived from the various direct ratio scaling methods (Stevens, 1958) (With the modality of loudness perception the ratio scales are generally linear on log-log coordinates).

Magnitude Production is the inverse of Magnitude Estimation in that S adjusts a variable intensity control to produce a subjective loudness impression which is proportional to a number suggested by E (Stevens, 1958). This technique has not been utilized as frequently as Magnitude Estimation. Hellman and Zwischlocki (1963) have shown, however, that Magnitude Production give comparable results to Magnitude Estimation at sensation levels (SL's) above 40 dB. At lower SL's, the two yield more divergent data, but these differences become negligible as the SL's approach threshold.

In Ratio Production (the method of Fractionation and Multiple Stimuli, among others) S is required to produce a prescribed ratio between two stimuli. Generally, two psychophysical methods are used to gather Ratio Production data: (1) the Method of Adjustment and, (2) the Method of Constant Stimuli. Typically, with the Method of Adjustment, a standard stimulus is presented, subsequently followed by a variable stimulus which can be controlled by S. The S's task is to adjust the variable stimulus to the prescribed ratio set by E. With the Method of Constant Stimuli, a standard is presented, which is subsequently followed by one of several comparison stimuli. The S's task is to report whether the comparison stimulus is equal to, or less than, the prescribed loudness ratio set by E. For example, if S were instructed to make one-half loudness judgments E, for each judgment, would present the standard

stimulus followed by one of several comparison stimuli (usually 4 to 7) presented in a pre-arranged random order. The S then indicates whether the comparison stimulus is greater than, or less than, the prescribed standard/comparison ratio. The half-loudness point is taken as the level of the comparison stimulus which yield 50% "greater" judgments and 50% "less" judgments.

The Fractionation and Multiplication procedures have often been used in the determination of 2:1 and 1:2 loudness-intensity relations (Churcher, King, and Davies, 1934; Garner, 1952, 1954, 1959; Geiger and Firestone, 1933; Ham, 1956; Ham, Biggs, and Cathey, 1962; Pollack, 1951; Richards, 1968; Robinson, 1953, 1957; Rschevkin & Rabinovitch, 1936; Stevens, Rogers, & Herrnstein, 1955).

The technique of Ratio Estimation, although advocated for constructing ratio scales (Stevens, 1958), has generally not been used. The procedure involves the presentation of two or more stimuli, with S estimating the loudness ratio between the two sensations. McRobert, Bryan, and Tempest (1965) obtained ratio estimates of fifteen pairs of tones at 1000 Hz. The greatest intensity difference between the two tones in a pair was 50 dB (30-80 dB SPL), whereas the smallest intensity difference was 10 dB. Twenty-five estimates per pair were obtained. To insure a minimum of bias, each S made only one loudness estimate. McRobert et. al. (1965) found that the obtained

ratio estimates were not consistent with those estimates predicted by previously obtained loudness functions when the inter-stimulus differences were large, but were found to be somewhat less. It was further found that the differences between their data and previous data increased as the inter-stimulus differences increased. When the stimulus differences within a pair was approximately 15 dB or less, previous findings could be approximated by their obtained data.

Within the last decade the Method of Numerical Magnitude Balance has been developed for the scaling of loudness (Hellman & Zwislocki, 1963, 1964, 1968; Rowley & Studebaker, 1969). The method consists of first obtaining magnitude estimates of several stimulus intensities. No modulus is assigned, and S is free to choose whatever number he feels is proportional to the loudness. In a subsequent test session, a Magnitude Production procedure is initiated. The numbers in this latter session, in turn, are chosen from the group medians obtained from the previous magnitude estimates. The data from the two procedures are combined by taking their geometric means.

Generally, the results from the above studies dealing with 1000 Hz tones have indicated that loudness at this frequency grows as a power function (linear on log-log coordinates) of the stimulus intensity (Stevens, 1957c), and that 2:1 loudness changes are equivalent to approximately 10 dB from 30-100 dB SL (Robinson, 1957; Stevens, 1955, 1956, 1957a, 1957b; Stevens & Foulton, 1956; J. C. Stevens, 1958; J. C. Stevens &

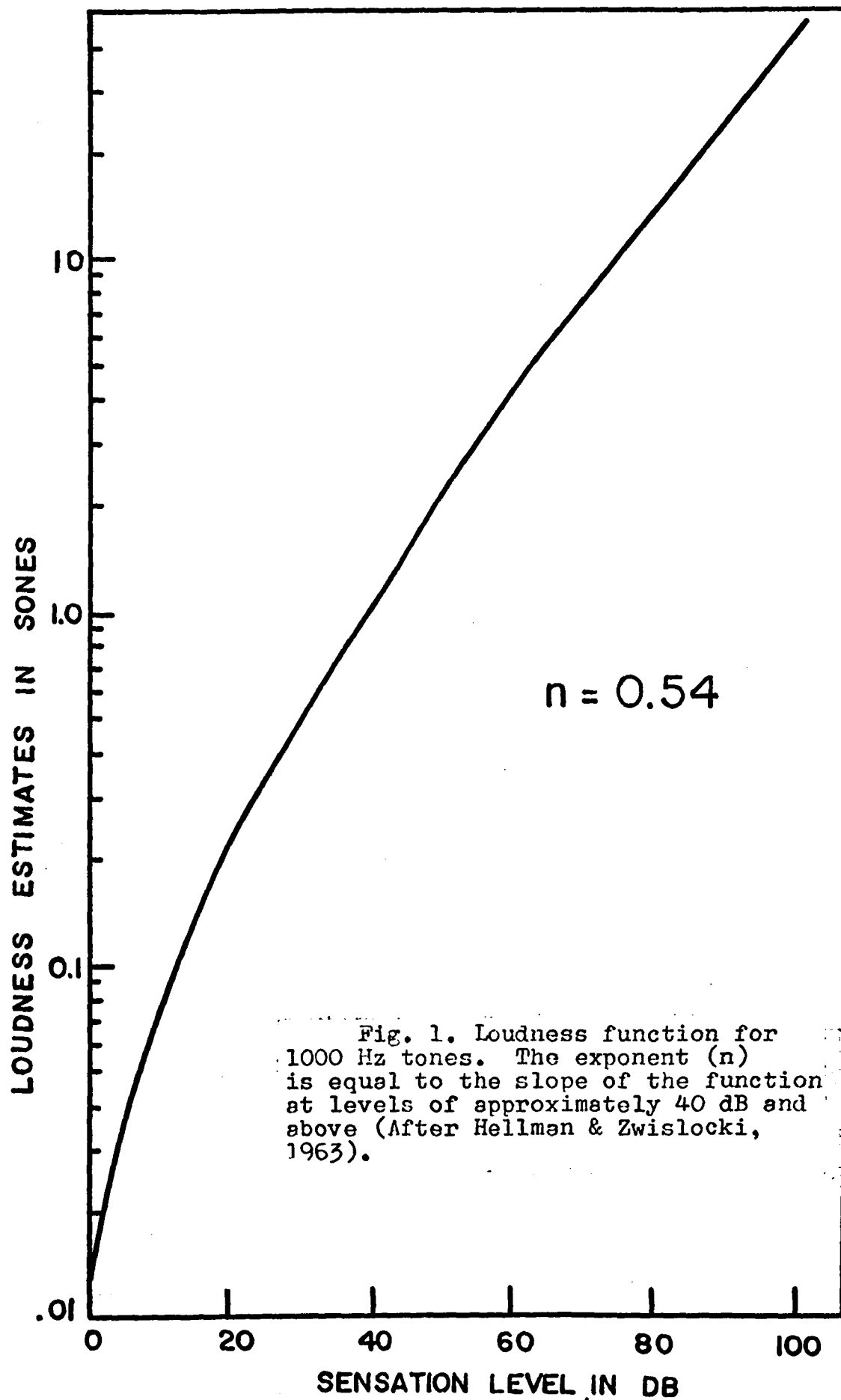
Tulving, 1957). In this regard, the exponent of the loudness power function has been accepted as approximately 0.54 for sound pressure, and 0.27 for power (Hellman & Zwisllocki, 1963; Lochner & Burger, 1962; Rowley & Studebaker, 1969). Figure 1 shows the loudness function for 1000 Hz tones.

The form of the loudness function implies that it is possible to predict loudness relations between any two points on the intensity continuum. Accordingly, as the 10 dB rule predicts a 10 dB inter-stimulus difference would be perceived as a 2:1 loudness ratio, a 20 dB difference as a 4:1 loudness ratio, and a 30 dB difference as an 8:1 ratio, and so on.

Certain nuisance parameters arise in the scaling of loudness by the various techniques. The most influential of these are: order effect, centering effect, stimulus range, context, and magnitude of the standard stimulus.

The order effect occurs when a pair of sounds are heard consecutively. When this occurs the first stimulus will appear less loud than the second, even if both are of the same intensity. Robinson (1957) found that the order effect increased as the loudness level increased, although the perceived differences at the higher loudness levels (100 phons) did not amount to more than 2 dB. He further indicated that the order effect is evident in experiments dealing with equal-loudness judgments. These effects, in turn, can be balanced by presenting the stimuli in random order.

The centering effect is considered a subjective resistance



to remote levels of a comparison stimulus often found using the Fractionation and Multiplication procedures. For instance, if during a Multiplication procedure the S was required to double the loudness of an intense standard, the dB increase for 2:1 loudness would be low relative to the difference obtained when the standard was of moderate intensity. Using a very low standard, the reverse would occur. Thus, the centering effect may be interpreted as a preference for moderate intensities (Robinson, 1957; Stevens, 1955).

It is often seen (Ham, Biggs, & Cathey, 1962; Stevens, 1955) that there are systematic departures between the dB changes relating 2:1 and 1:2 loudness along the intensity continuum. The differences (the fractional/multiple anomaly) can be explained in terms of the centering effect (Robinson, 1957). The differences between Fractionation and Multiplication are seen particularly when the standard stimulus to be fractionated or multiplied is low (in dB SL), or high, with the differences decreasing as the standards approach about 85 dB SL at 1000 Hz (Robinson, 1957). Stevens (1957a), in an experiment which combined halving and doubling, showed the same effect. At low levels the dB changes were greater for doubling loudness than for halving, and at the high levels the reverse was true. Near the middle of the range halving and doubling of loudness tended to agree. Stevens (1955) indicated that the centering error could be ascertained and averaged out by a

balanced procedure where each stimulus served once as the standard and once as the variable.

Robinson (1957) indicated that the centering effect was not the same as the order effect. The former was directly dependent upon the intensity level, whereas the latter was not.

Several investigations dealing with the judgment of loudness (Engen & Levy, 1958; Stevens, 1956; Tabory & Thurlow, 1959) have found differences in their subject's subjective estimates associated with the stimulus range of their experiments.

Stevens (1956) found that when the range of the variable stimuli on which magnitude estimates could be made was increased from 70 to 90 dB, differential results were obtained. The variable stimuli ranged from 30-100 dB SPL in the first experiment, and 30-120 dB SPL in the second. All stimuli were 1000 Hz tones. A standard stimulus of 80 dB SPL was assigned the modulus "10" in the first experiment, and a 90 dB SPL tone the same modulus number in the second experiment. Stevens found that when the range was increased the subject's estimates were over-estimated (re the 10 dB rule) at the higher intensities and under-estimated at the lower intensities. Engen and Levy (1958) found that the stimulus range did not affect the form of the obtained loudness functions when the "Constant-Sum" technique was used. In another experiment, however, Engen and Levy (1958) found that when the stimulus range was curtailed from 25-75 dB SL to 55-75 dB SL, and all

judgments were made relative to a fixed standard (75 dB SL), the exponent of their obtained loudness functions increased.

In a somewhat different vein, Tabor and Thurlow (1959) found that a subject's magnitude estimates could be influenced by his range expectancies. The experiment involved two stimulus ranges (30-90 dB SL and 70-90 dB SL) of stimuli at 1000 Hz. Two groups of subjects were employed. At the beginning of a test session both groups were presented the end-points of each range, and told that the lower stimulus was called "50", the louder "60." However, a different set of instructions were given to each group. One group was told that some subsequent stimuli might fall outside the 50-60 range (open set), the other group that the subsequent stimuli would be contained within the prescribed numerical range (closed set). It was shown that for both stimulus ranges that lower magnitude estimates were obtained from the group with the open set of instructions.

Stimulus spacing appears to have little effect upon the judgment of loudness when using the methods of Magnitude Estimation (Beck & Shaw, 1965; J. C. Stevens, 1958; Stevens, 1956) and Ratio Estimation (Engen & Levy, 1958). On the other hand, the effect of stimulus spacing is quite large when the method of Constant Stimuli is used to determine half-loudness judgments (Garner, 1954).

J. C. Stevens (1958), using magnitude estimates of white-noise, found that the obtained loudness functions were relatively

insensitive to stimulus spacing. In the experiment four stimulus spacing arrangements were included, each covering a range of from 40-100 dB SPL. The modulus for all the ranges was 80 dB SPL, and was assigned the number "10." Engen and Levy (1958), using ratio estimates of 1000 Hz tones also found that the inter-stimulus difference (either 5 or 10 dB) had little effect upon the obtained loudness functions.

In a study designed to show how the stimulus ensemble could bias the judgment of one-half loudness judgments using the Method of Constant Stimuli, Garner (1954) used the same standard (90 dB SPL at 1000 Hz) in each of three non-overlapping stimuli ranges (55-65, 65-75, and 75-85 dB SPL). He found that for the three ranges the obtained half-loudness points approximated the middle of the comparison stimulus range.

The magnitude and placement of the standard in the Method of Magnitude Estimation is another bias encountered in the scaling of loudness. Jones and Woskow (1966) found that a remote standard relative to the variable stimuli invariably reduced the power function exponent. Hellman and Zwislocki (1961) found that low sensation level tones which were assigned high modulus numbers produced loudness functions which were steeper below the reference than above it. On the other hand, when high sensation level tones were assigned low modulus numbers, the reverse occurred.

Placement of the standard stimulus also appears to have some differential effects using the Fractionation and Multiplication procedures. Robinson (1957) found that when the dB change to halve or double the loudness of a 1000 Hz tone was measured, differential results were found at various points along the auditory continuum. The dB changes were maximal at about 55 dB SL, and minimal at about 90 dB SL. Stevens (1957a) found the change to be highest at 60 dB SL, and lowest at 90 dB SL. Stevens (1957a), however, only obtained these results from doubling judgments, and not from halving.

Stevens (1955) indicated that the practice of averaging decibels was apt to produce a bias because the loudness function was not linearly related to dB. To obtain an unbiased mean, he indicated that it was better to transform all dB values to loudness units (sones), and then average. Robinson (1957) pointed out, however, that this procedure might be erroneous since it assumed a common loudness function for all individuals. Robinson indicated that a subject's median score should be utilized in such instances.

The scaling of loudness by the conventional approaches has not been restricted only to 1000 Hz tones. Other stimuli which have been scaled include 250 Hz tones, 1000 Hz tones at low sensation levels, and white-noise, among others.

Hellman and Zwischki (1968) used the method of Numerical Magnitude Balance in the scaling of 250 Hz tones. Initially,

each S made magnitude estimates of nine stimuli from 4-70 dB SL. In the Magnitude Production phase, nine number derived from the obtained magnitude estimates were given to the Ss. When the obtained loudness scale for the 250 Hz tones were plotted on the same coordinates as the 1000 Hz function, the former function was displaced by approximately 13 dB upward in intensity. This, of course, was due to the higher absolute threshold for the 250 Hz stimuli. However, both functions grow at approximately the same rate, i.e., have the same slopes. Melnick (1969) obtained magnitude estimates of 250 Hz tones for both normal and pathological hearing subjects. The obtained functions were found to be curvilinear on log-log axes to 40 dB SL for both groups of subjects. Above this level both groups exhibited power functions. The slope for the normal hearing subjects was 0.39, and for the abnormal subjects (stapedectomized), 0.34. Melnick (1969) concluded that magnitude estimates of loudness could not differentiate between the normal and pathological groups. Hellman and Zwislöcki (1968) obtained a slope of 0.51 for their 250 Hz loudness function.

Several investigations (Pollack, 1951; Poulton & Stevens, 1955; J. C. Stevens & Tulving, 1957; Stevens, 1955, 1961) have shown that loudness relations for white-noise stimuli are quite similar to 1000 Hz tones. Pollack (1951) instructed his subjects to adjust the intensity of a variable stimulus to sound one-half or twice as loud as several standards.

The results indicated that 2:1 loudness changes were nearly equivalent to 10 dB for the entire stimulus range. Stevens (1955) and Stevens and Poulton (1956) both indicated that the dB changes necessary for a 2:1 loudness ratio of white-noise stimuli were relatively independent of the standard intensity level, and that the mean value was approximately 8.4 dB.

Using the method of Magnitude Estimation with and without a modulus, Stevens and Tulving (1957) found that the loudness function for white-noise was the same as that of 1000 Hz stimuli, and that a twofold increase in loudness was equivalent to 10 dB from 50-110 dB SPL. Stevens (1961) indicated that the loudness function for white-noise from 30-100 dB SPL is approximately 12-15 phons greater than the 1000 Hz function. However, at these levels the slopes are nearly equivalent.

Although the general shape of the loudness function is well established for 1000 Hz tones at 40 dB SL and above, the shape of the function below 40 dB is somewhat in doubt. Hellman and Zwisllocki (1961) found that below 30 dB SL the loudness function became progressively steeper. Hellman and Zwisllocki (1963) further found that in the vicinity of threshold the loudness was directly proportional to the stimulus level. Lochner and Burger (1961) indicated that the curved section of the loudness function below 30 dB SL was due to masking by physiological noise. They hypothesized that under quiet conditions physiological noise determines the threshold

of audibility for a particular stimulus. This physiological noise, in turn, reduces the loudness for a particular stimulus by a constant amount across the intensity continuum. In the area of the audible threshold, a constant loudness reduction is much more influential than at the higher levels, and this relation is indicated by the steeper functions below 40 dB SL.

Within the last decade a new method of multidimensional scaling has been developed (Shepard, 1962a, 1962b). The procedure, the Analysis of Proximities requires only that some number be assigned to represent the psychological distance between each of the pairs of stimuli along the continuum of interest. These numbers, called "proximity measures", are said to be greatest in proximity when both stimuli in a pair are judged to be relatively the same magnitude, and have less proximity as the distances between the subjective sensations increase. Shepard (1962b) indicated that the objective of the analysis was to find "an appropriate spatial configuration of the N stimuli, represented as points in Euclidian space of minimum dimensionality. By an "appropriate" configuration here is meant one in which the distances between points are monotonically related to the original proximity measures". (Shepard, 1962b, p.210). Accordingly, those stimuli along the continuum which are judged most similar should be separated in space by the smallest distance, and vice versa.

As concerns the role of the Analysis of Proximities in

the scaling of loudness, it offers features not encountered using the conventional scaling techniques. Of prime importance in this regard is that each stimulus point can be related to every other stimulus point directly in the form of the spatial configuration (or proximity plot). This, of course, can only be implied using conventional scales.

The present study utilized a modification of the Analysis of Proximities which was designed to find a spatial configuration whose points were proportionally related to the original proximity measures. The proximity measures were obtained from ratio estimates made between all stimulus combinations in 7x7 matrices. The modification differed from the original analysis in two aspects. The first was that a fixed transformation was imposed relating the plotted distance to the proximity measures. The transformation imposed defined the proximity measure as \log_2 of the obtained ratio estimate. Utilization of this transform was based upon previously obtained loudness functions, whose linear unidimensional form on log-log coordinates implied that a straight line would be obtained as a solution to the Analysis of Proximities if the log-transform were used, i.e., each time the loudness were doubled would be equivalent to another distance unit in space, and the loudness relation would be completely additive. (By additive here is meant that ratio estimates made over longer inter-stimulus spacings would be the sum of its component estimates). Thus, the log-transform insured a direct comparative analysis between the

obtained proximity plots and the configurations implied by the 0.54 power function. In contrast, Shepard's original analysis first found the transformation, and the points were plotted with relation to it.

The second feature of the modified Analysis of Proximities was a standard statistical test which considered the subject's response variability, thereby providing a measure of the quality of the proximity analysis as a description of the data. The test used was the Chi Square, and it evaluated the differences between the predicted distances (as measured on the spatial configurations) and the obtained ratio estimates. The measure of variability included within the test was the variance of the mean ratio estimates between subjects for each matrix cell. Shepard's original analysis did not include data on variability, but simply utilized mean values.

The objective of the present study was to utilize the modified Analysis of Proximities in the scaling of loudness in order to see if the 0.54 power function provided an adequate description of loudness relations between stimuli using the method of Ratio Estimation. Each stimulus condition tested (1000 Hz tones with various inter-stimulus spacings, white-noise, 250 Hz tones, and 4000- and 1000 Hz tones at low sensation levels) was represented by seven stimulus intensities, and ratio estimates were made between each of these values in 7x7 matrices.

In those conditions where moderate to intense 1000 Hz tones were utilized, several matrices which differed in their inter-stimulus intervals and overall ranges were included. These matrices were designed to test the effects of stimulus spacing and range on the judgment of loudness. Also, from these matrices analyses of both inter- and intra-subject variability was obtained. Included in these latter analyses were measures of the response variability within one session, between sessions, and in the imposed judgmental response mode placed upon a subject, i.e., the first tone judged relative to the second tone, or vice versa.

Matrices containing other than moderate to intense 1000 Hz tones were included for various reasons. Those containing 250 Hz and white-noise stimuli were included to see if loudness power functions of these stimuli provided an adequate description of ratio estimates made between various points on the intensity continuum.

Those matrices containing low sensation level 1000- and 4000 Hz tones were designed to test several loudness relations. The first objective was to investigate the rapid growth of loudness at close to threshold levels for 1000 Hz tones using the Ratio Estimation method combined with the Analysis of Proximities. Whether the obtained proximity plots for 1000 Hz tones differed for high and low sensation levels was of prime importance because previous investigations (Hellman & Zwislacki, 1961; Lochner & Burger, 1962) have shown loudness at this frequency to grow more rapidly at low sensation

levels than at moderate to intense SL's.

The second objective of using low SL stimuli was to compare the 1000- and 4000 Hz proximity plots in order to ascertain whether the two configurations differed among normal hearing individuals. These data could be used as baselines in the evaluation of aural pathology, because recruitment of loudness usually occurs at levels just above threshold for individuals with cochlear disorders. Since recruitment is most prevalent at the higher frequencies, it might be expected that a subject with cochlear pathology, who performs ratio estimates at low sensation levels, would give essentially the same results as normals at 1000 Hz, but differential results at 4000 Hz.

CHAPTER II
METHOD

II. METHOD

SUBJECTS

Twenty normal hearing subjects (Ss), as determined by Bekesy sweep-frequency audiograms, were used. Ten of the subjects were used in Experiment I, Matrices A-D, and the remaining ten in Experiment II. All subjects at the outset were naive as regards the judgment of loudness. The mean age for both groups was approximately 23 years. All were either graduate or undergraduate college students.

PROCEDURE

Prior to the initiation of a test session, it was necessary to first determine each subject's threshold for the stimulus of interest, and then adjust the test stimuli to the correct sensation levels. Two methods were used. In those phases of the study where moderate to intense 1000 Hz or 250 Hz were used, threshold was determined by Bekesy fixed-frequency audiograms (Grason-Stadler E-800 Bekesy Audiometer), where the median of the excursions was used as the threshold. Sensation levels were then set relative to the obtained sound pressure levels at the S's threshold. A somewhat different technique was used for the white-noise stimuli and the low SL 1000 Hz and 4000 Hz tones. During these phases threshold was found, and SL set, by a tape recorded series of 0.5-sec presentations of the appropriate

stimuli (25 msec rise-fall time). These stimuli were, in turn, recorded at the highest intensity that a particular stimulus tape contained. By attenuating the level of these tone presentations to threshold, SL could easily be set by decreasing the attenuation "X" dB. For example, if the highest stimulus level within a phase were to be 70 dB SL, this level could be obtained by decreasing the attenuation 70 dB after the initial threshold determination. This latter technique of threshold determination was especially useful using low SL stimuli because the desired level could be set immediately before the actual test session, and on the same recording playback system. This, of course, is of utmost importance when considering the effects of threshold shift at low sensation levels.

Generalized Administration of Experiments I and II. Both experiments consisted of monaurally presented stimuli. In order to eliminate any possible biases introduced by transients, all stimuli had 25 msec rise-fall times (Grason-Stadler 829E Electronic Switch). Experiment I included 1000 Hz tones of moderate to intense levels, and Experiment II included white-noise, 250 Hz tones, and 1000 Hz and 4000 Hz tones at low sensation levels.

Stimuli were presented as pairs of tones (or noise) of differing intensity. The subject's task was to perform a ratio estimate, i.e., to estimate the loudness of the first tone relative to the second tone, or vice versa. The

stimuli pairs were presented randomly, and were chosen from 7x7 matrices of stimuli.

A stimulus sequence was initiated with a 1.5-sec tone (or noise), followed by 0.5-sec of silence, and then by another 1.5-sec tone. The inter-sequence time was 5-sec. During this time the subject recorded his ratio estimate on a provided answer sheet. Following every 10th pair a 10-sec pause was introduced before the next pair to aid the S in coordinating his answers with the actual test pair.

Presentation of the matrices differed somewhat from Experiments I to II. During Experiment I four test sessions were included, whereas in Experiment II only two test sessions were included. Each test session consisted of the entire stimulus matrix being presented to S twice, with S judging Tone A re Tone B (or B re A) on both occasions. During two of the test sessions in Experiment I, S judged Tone A re Tone B, and in the remaining sessions Tone B re Tone A. In Experiment II, one A re B and one B re A session were tested. The first complete presentation of a matrix within a session is referred hereafter as Replication 1, and the second as Replication 2.

Ten practice trials preceded a new day's sessions, and two sessions per day were tested.

Experiment I. Included within this experiment were four 7x7 matrices of stimuli at 1000 Hz. Each matrix differed as regarded its stimulus range, and inter-stimulus spacing.

However, each of the matrices contained stimuli which were common to the other three. Matrix A covered a range of from 30-90 dB SL in 10 dB intervals. Matrix B ranged from 40-70 dB SL in 5 dB steps; Matrix C was from 40-55 dB SL in 2.5 dB steps, and Matrix D covered a 30-90 dB SL range in irregular steps (30, 40, 45, 47.5, 55, 70, and 90 dB SL). Figure 2 shows all matrix ranges used in Experiment I.

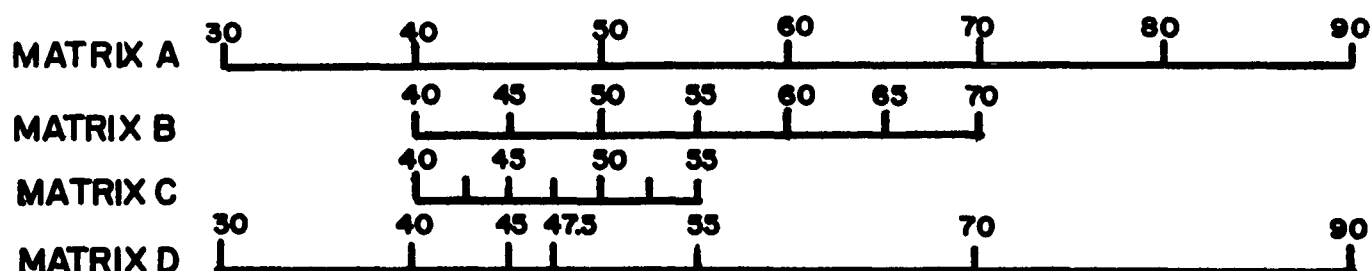


Fig. 2. Matrix ranges for Experiment I.

During the administration of Experiment I, the matrices were presented in a preselected random order. Thus, Matrix A was presented first, followed by Matrices C, D, and B, respectively.

Experiment II. Four 7x7 matrices were included in this experiment. As before, the Ss made their ratio estimates on provided answer sheets. Matrix E consisted of 250 Hz tones which covered a stimulus range of from 10-70 dB SL in 10 dB steps. Matrix F was composed of white-noise stimuli from 40-70 dB SL in 5 dB steps; Matrix G of 4000 Hz tones

from 10-40 dB SL in 5 dB intervals, and Matrix H of 1000 Hz tones from 10-40 dB SL, also in 5 steps.

APPARATUS

All stimuli were recorded on an Ampex PR-10 tape recorder. A block representation of the recording equipment may be seen in Figure 3.

The stimulus recording apparatus was set up so that Interval Timer 1 was triggered by the controlled output of Interval Timer 2, and vice versa. When triggered, these timers activated their respective electronic switches, which, in turn, passed the stimulus through the attenuator and filter to the input of the tape recorder. Thus, the left hand side of the circuit controlled the length and intensity of the first tone, and the right side, the relevant parameters of the second tone.

During the recording of a stimulus matrix the highest stimulus value that could be introduced onto the tape without producing distortion was found with no attenuation, and the other six values were set at "X" dB down from that point. For example, if the highest stimulus value was to be 90 dB SL, this value was arbitrarily assigned the highest non-distorting input voltage. A stimulus at 70 dB SL would then be recorded by the introduction of 20 dB attenuation. The foregoing process insured the utilization of the complete dynamic range of the tape.

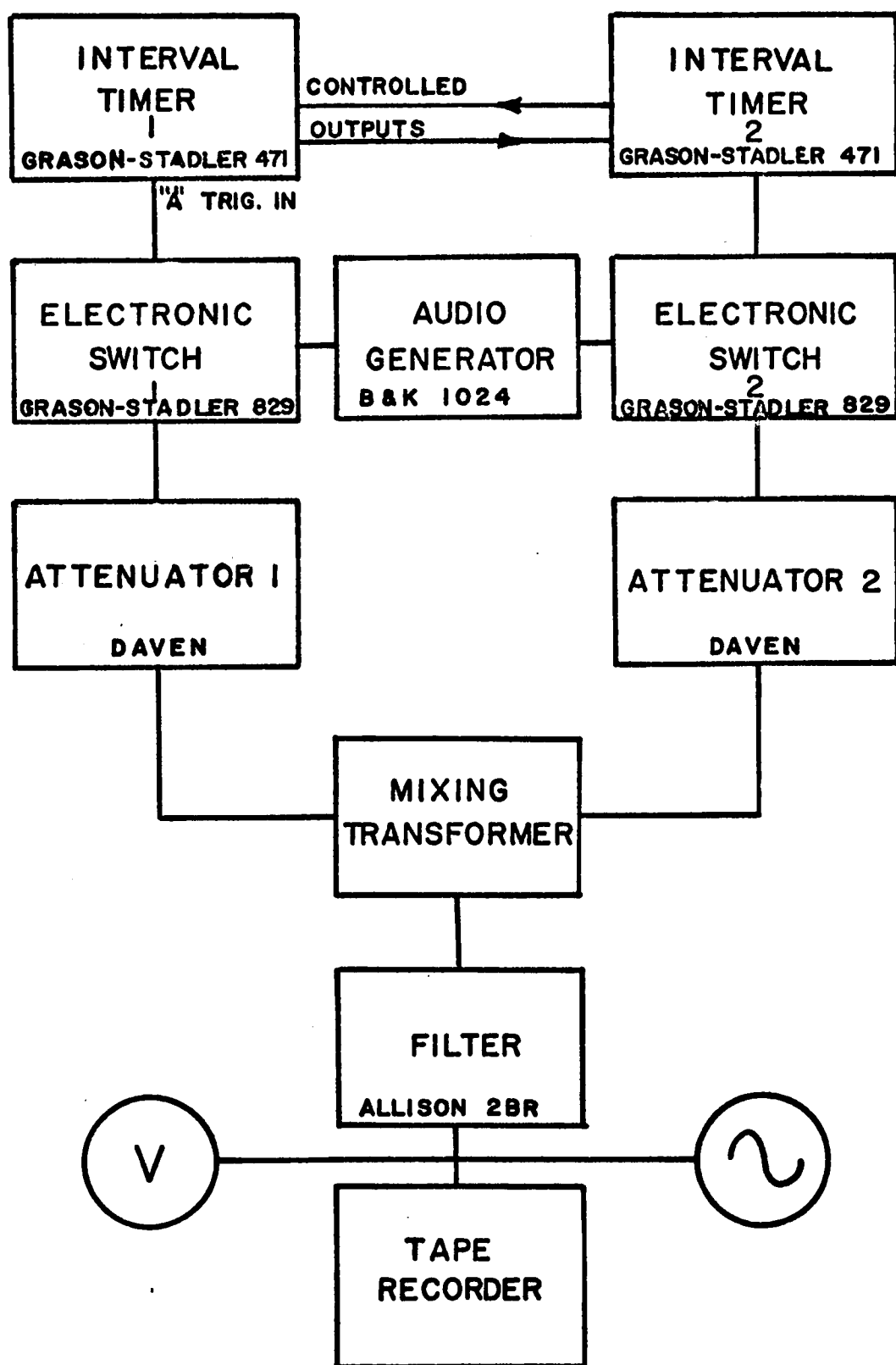


Fig. 3. Block representation of the recording equipment.

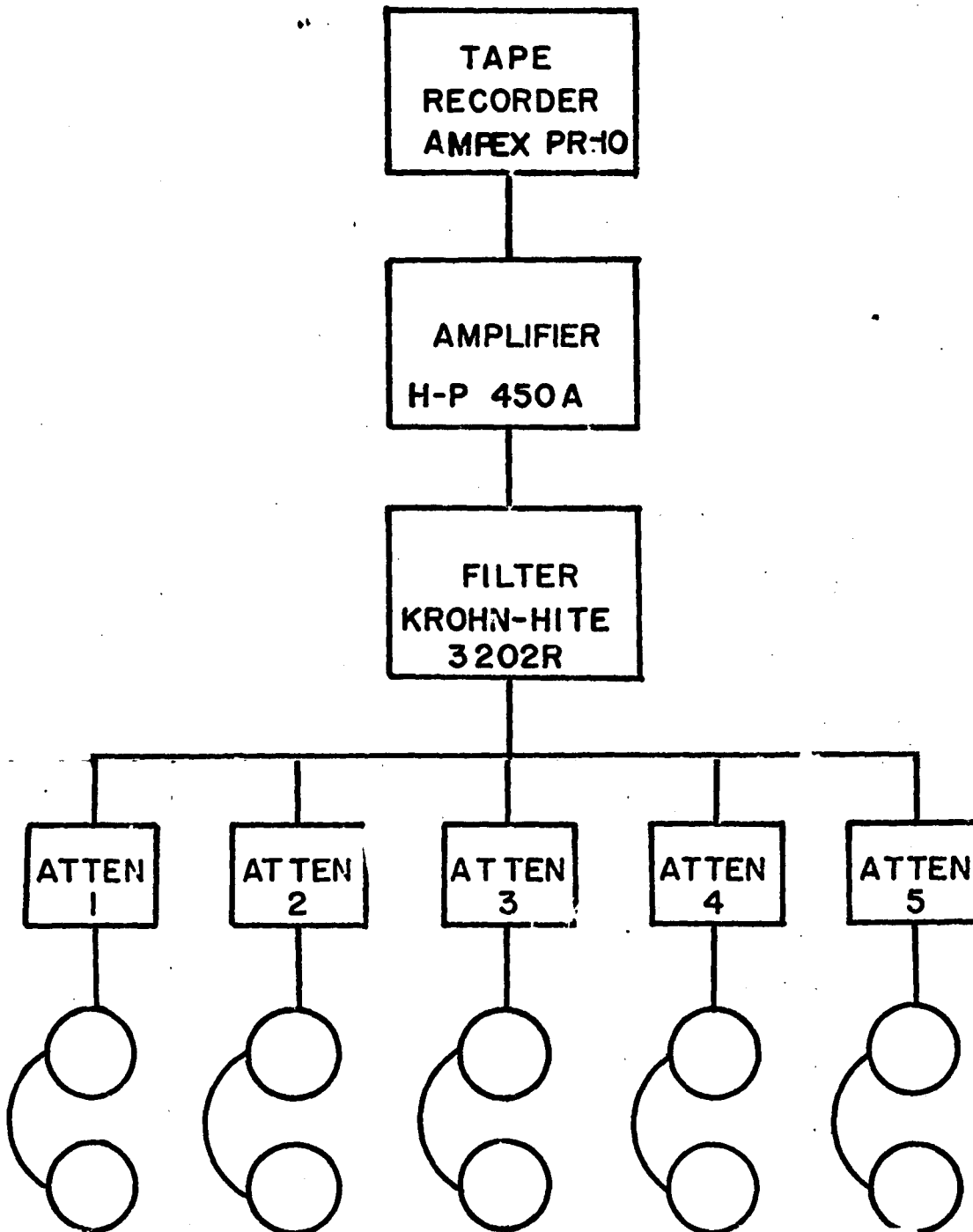
Calibration of the attenuators at the input to the tape recorder showed an error of approximately ± 1.0 dB to -50 dB re an input voltage of 1.47 volts. Mean values may be seen in Table 1.

Table 1
Calibration of Attenuators at Input of
Tape Recorder

dB re 1.47 volts	Voltage	Error
0 dB	1.47 v.	
-20 dB	0.15 v.	0.4 dB
-40 dB	0.017 v.	1.3 dB
-50 dB	0.005 v.	1.0 dB

A block diagram of the playback apparatus is seen in Figure 4. The equipment was set up so that five individuals could be run simultaneously.

In order to check the attenuation characteristics of each stimulus tape through the entire playback system, calibration tones were recorded at each of the seven intensities associated with a given matrix. These calibration tones preceded the test stimuli, and were 30-sec each. Table 2 shows the sound pressure levels (B&K 2203 SPL meter and Octave Band Filter) recorded at each of the five earphones



ALL EARPHONES: TDH-39's IN MX 41/AR CUSHIONS
ATTENUATORS: H-P DC-IMC AND DAVEN

Fig. 4. Block diagram of the playback apparatus.

with increasing attenuation for Matrix A (1000 Hz).

Table 2 shows that all earphones displayed good linearity in their SPL responses to increasing attenuation. Also, the largest difference displayed between any of the earphones at 0 dB attenuation was only 3 dB.

Each attenuator was checked by playing a 1000 Hz tone at 0 dB attenuation (0.44 v. at the earphone) through the system, and then increasing the attenuation in 10 dB steps. This procedure showed each attenuator to be within ± 1.0 dB to -60 dB.

For those phases of the study where 250 Hz, 4000 Hz, and white-noise stimuli were used similar calibrating procedures were adopted. These results also showed little deviation from linearity as the stimuli were attenuated.

All earphones were Telephonics TDH-39's mounted in MX-41/AR cushions.

Data collection was made in large audiometric suite with an ambient noise level of 33 dBC.

Table 2
 Attenuation Characteristics of Stimulus Tapes and
 Responses of Earphones (in dB SPL) at 1000 Hz

	Earphone				
	1	2	3	4	5
Atten. (0dB=0.44v)	102.5	102.0	100.5	99.5	102.5
-10 dB	92.5	92.0	90.5	89.5	92.5
-20 dB	82.0	82.0	80.5	79.5	82.5
-30 dB	72.0	72.0	70.5	69.5	72.5
-40 dB	62.0	62.0	61.0	59.5	62.5
-50 dB	52.0	52.0	51.0	49.5	52.5
-60 dB	42.5	42.0	41.0	39.0	42.5

CHAPTER III

RESULTS

III. RESULTS

Prior to the presentation of the data, it is instructive to present the format for the specification of a matrix cell. Each cell will be designated by a number which ranges from 1-28 in the top half of the matrix, and 1L to 28L in the lower half. Figure 5 shows how each cell in a 7x7 matrix is classified. It can be seen that the opposite cells in both the upper and lower halves are designated by the same number, with the lower half cell having the number followed by the letter "L". Throughout the remainder of the study, only the upper half cells of a matrix will be presented for clarity, with the understanding that the lower half cells have been included in the result. In each instance a specification of the statistical procedures used to equate the value of the lower cell to that of the upper cell, or vice versa, will be given. This procedure is used to keep the values of the ratio estimates at 1.00 and above, and so avoid confusion by the use of fractions.

Experiment I. Tables 3-6 show the overall mean ratio estimates for Matrices A-D, respectively. These values were obtained by taking the reciprocal of the Tone A re Tone B sessions for each subject, and then averaging these results with the values of the Tone B re Tone A sessions. To convert all values to 1.00 or greater the reciprocals of the remaining fractional estimates were taken, and used in the

DB SL LOUDER TONE

DB SL LOWER TONE

1	2	3	4	5	6	7
2L	8	9	10	11	12	13
3L	9L	14	15	16	17	18
4L	10L	15L	19	20	21	22
5L	11L	16L	20L	23	24	25
6L	12L	17L	21L	24L	26	27
7L	13L	18L	22L	25L	27L	28

Fig. 5. Format for the specification of each cell in a 7x7 matrix.

Table 3
Grand Means for Matrix A

		DB SL Louder Tone						
		30	40	50	60	70	80	90
DB SL Lower Tone	30	1.00	1.34	2.02	2.61	3.25	4.61	7.90
	40		1.00	1.43	1.97	2.37	3.77	7.23
	50			1.00	1.47	2.24	3.25	5.88
	60				1.00	1.58	2.71	5.58
	70					1.00	2.00	4.45
	80						1.00	2.95
	90							1.00

Table 4
Grand Means for Matrix B

		DB SL Louder Tone						
		40	45	50	55	60	65	70
DB SL Lower Tone	40	1.01	1.15	1.60	2.19	2.49	3.29	3.90
	45		1.00	1.19	1.74	2.24	2.78	3.27
	50			1.00	1.23	2.03	2.36	2.99
	55				1.01	1.30	2.03	2.67
	60					1.00	1.41	2.21
	65						1.00	1.45
	70							1.00

Table 5

Grand Means for Matrix C

		DB SL Louder Tone						
		40	42.5	45	47.5	50	52.5	55
DB SL Lower Tone	40	1.01	1.33	1.47	1.83	2.27	2.75	3.03
	42.5		1.01	1.13	1.52	1.91	2.36	2.81
	45			1.02	1.15	1.71	2.00	2.45
	47.5				1.01	1.32	1.88	2.09
	50					1.04	1.34	1.75
	52.5						1.02	1.31
	55							1.03

Table 6

Grand Means for Matrix D

	DB SL Louder Tone							
	30	40	45	47.5	55	70	90	
DB SL Lower Tone	30	1.00	1.62	2.06	2.71	3.69	9.06	10.30
	40		1.01	1.19	1.40	2.08	3.49	8.60
	45			1.01	1.09	1.63	2.69	8.03
	47.5				1.00	1.44	2.76	8.74
	55					1.01	2.12	7.09
	70						1.00	5.14
	90							1.00

grand averages. Sixteen responses per subject per cell were obtained, and each average represents the mean of 160 responses.

To illustrate the above process, a brief example will be given. When the subjects were judging the stimuli A re B, the top half of a matrix contained the fractional estimates, and the lower half, the estimates of 1.00 or more. To combine the data, first the reciprocals of the entire matrix judged under the A re B condition were taken. Thus, two matrices were obtained with the greater than 1.00 estimates estimates in the upper half and fractional estimates in the lower half. To rid the data of these remaining fractional estimates, their reciprocals were taken. This final process, in essence, folded both matrices. The data from the two response conditions could then be added for each stimulus combination. For example, if for the 30-90 dB SL stimulus combination the subject reported ratio estimates of 0.20 and 5.00 for the upper and lower cells (respectively) under the A re B condition, and 5.00 and 0.20 for the upper and lower cells under the B re A condition, the data for the A re B condition would be first inverted to insure that the greater than 1.00 judgments were in the upper matrix half. To rid the data of the two remaining 0.20 estimates, their reciprocals would be taken, and added to the two 5.00 estimates. The above example is illustrative of the situation where only one ratio estimate per cell is obtained before the inverting

and folding process. In the study four ratio estimates per cell were obtained before inversion and folding, and therefore 16 measurements for each stimulus combination were obtained.

Table 3 (Matrix A) shows the greatest mean ratio estimate given was for the 30-90 dB SL stimulus combination (7.90), and that no reversals in the mean ratio estimates between adjacent stimuli were obtained, i.e., as the louder tones increased in intensity for a particular lower tone, the ratio estimates became larger. In like manner, as the lower tones increased, the ratio estimates for the louder tones decreased. Tables 4, 5, and 6 show the same relationships as Table 3, although a reversal does occur at cell 22 in Matrix D. The highest ratio estimate in Matrix B is 3.90, which occurs with the stimulus combination 40-70 dB SL. In Matrix C a ratio estimate of 3.03 is obtained from the most divergent stimuli, and Matrix D shows a mean judgment of 10.30 for the 60 dB inter-stimulus difference.

Table 7 compares the mean ratio estimates of stimulus pairs common to Matrices A-D. The table is arranged so that the top value within a cell indicates the judgment for Matrix A, the second line, Matrix B, and so on. A straight line within a cell denotes that particular matrix did not contain that stimulus combination. It is seen that for the common stimulus pairs the results for Matrix C are always greater than Matrix B, which is greater than Matrix D, and, in turn,

39

		DB SL Louder Tone												
		30	40	42.5	45	47.5	50	52.5	55	60	65	70	80	90
30	1.00	1.34		-	-	2.02		-	2.61		3.25	4.61	7.90	
	-	-		-	-	-		-	-		-	-	-	
	1.00	1.62		2.06	2.71	-		3.69	-		4.06	-	10.30	
40		1.00	-	-	-	1.43	-	-	1.97	-	2.37	3.77	7.23	
		1.01	-	1.15	-	1.60	-	2.19	2.49	3.29	3.90	-	-	
		1.01	1.33	1.47	1.83	2.27	2.75	3.03	-	-	-	-	-	
		1.01	-	1.19	1.40	-	-	2.08	-	-	3.49	-	8.60	
42.5			-	-	-	-	-	-						
			1.01	1.13	1.52	1.91	2.36	2.81						
45				1.00	-	1.19	-	1.74	2.24	2.78	3.27		-	
				1.02	1.15	1.71	2.00	2.45	-	-	-		-	
				1.01	1.09	-	-	1.63	-	-	2.69		8.03	
47.5					-	-	-	-			-		-	
					1.01	1.32	1.88	2.09			-		-	
50					1.00	-	-	-			-		-	
					1.00	-	-	1.46	-	2.24	3.25	5.88		
					1.04	1.34	1.75	2.03	2.36	2.99	-	-		
					-	-	-	-	-	-	-	-		
52.5						-	-	-						
						1.02	1.31	-						
55							-	-						
							1.01	1.30	2.03	2.67		-		
							1.03	-	-	-		-		
60							1.01	-	-	2.12		7.09		
								1.00	-	1.58	2.71	5.58		
								1.00	1.41	2.21	-	-		
								-	-	-	-	-		
65										1.00	1.45			
										-	-			
										-	-			
70											1.00	2.00	4.45	
											1.00	-	-	
											-	-	-	
80											1.00	-	5.14	
											-	-	2.95	
											-	-	-	
90													1.00	
													-	

greater than Matrix A. Thus, disregarding Matrix D (for it has irregular inter-stimulus spacing), it may be seen that as the inter-stimulus spacing is decreased, the higher the ratio estimate for a common pair of stimuli. For example, the 40-50 dB SL stimulus combination yields judgments of 1.43, 1.60, and 2.27 for Matrices A, B, and C, respectively.

Figures 6-8 show the results of Tables 3-6 graphically. In these figures the data are plotted so that each row of the matrix is shown with the lower stimulus being the parameter of the curve, and the higher tone the abscissa. The obtained ratio estimates are plotted logarithmically. Figure 6 (Matrix A) indicates that four of the curves (Lower Tones = 30, 40, 50, and 60 dB SL) can be represented by a linear function to approximately 75 dB SL. At that point these four curves appear to exhibit a knee, and then accelerate at an even greater slope. In those curves where the lower stimulus is equal to either 80 or 90 dB SL, no knee is exhibited. The slopes for Matrix A increase from 0.27 to 1.00 as the value of the lower stimulus increases.

Figure 7 indicates that the six functions for Matrix B can all be represented by power functions (linear functions on log-log coordinates). No knee is seen for any of the curves. Again, the slopes of adjacent slopes increase as the intensity of the lower stimulus increases. The range of the slopes is from 0.40 to 0.64.

Figure 8 (Matrix C) shows, once again, that the functions

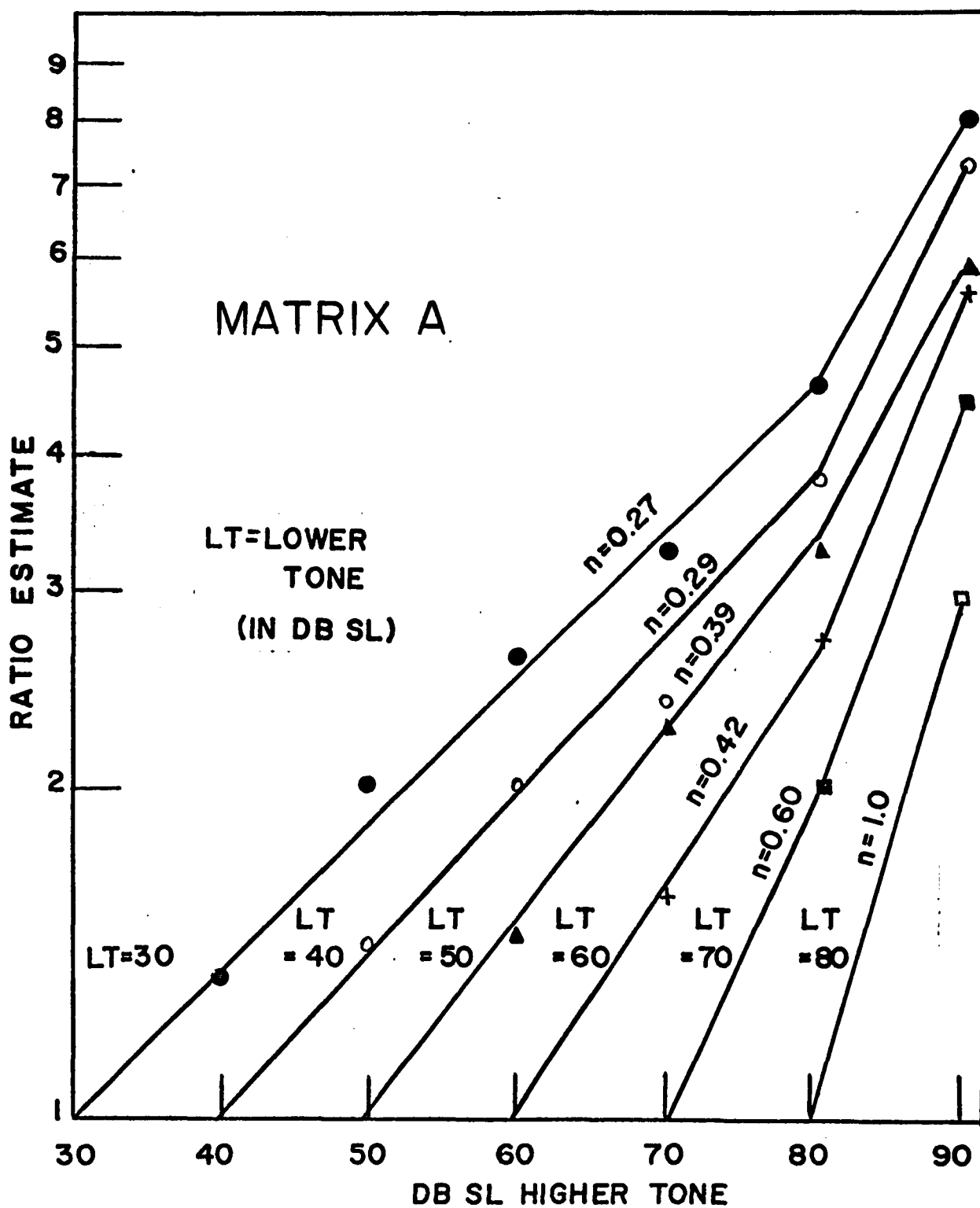


Fig. 6. Ratio estimates for Matrix A as a function of the dB SL of the higher tones. The lower tone is the parameter of each curve. The exponent (n) is equal to the slope of the curve.

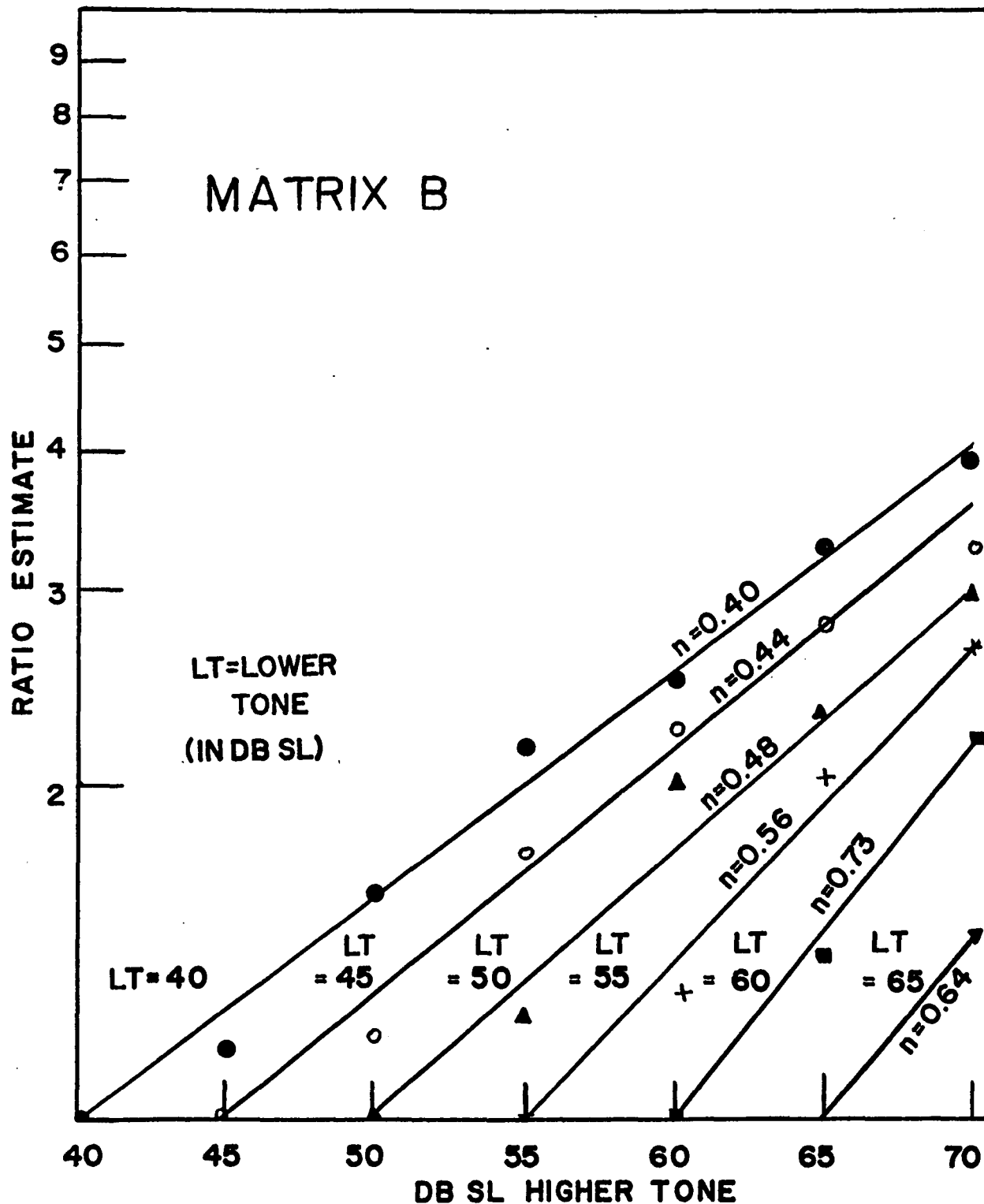


Fig. 7. Ratio Estimates for Matrix B as a function of the dB SL of the higher tones. The lower tone is the parameter of each curve. The exponent (n) is equal to the slope of the curve.

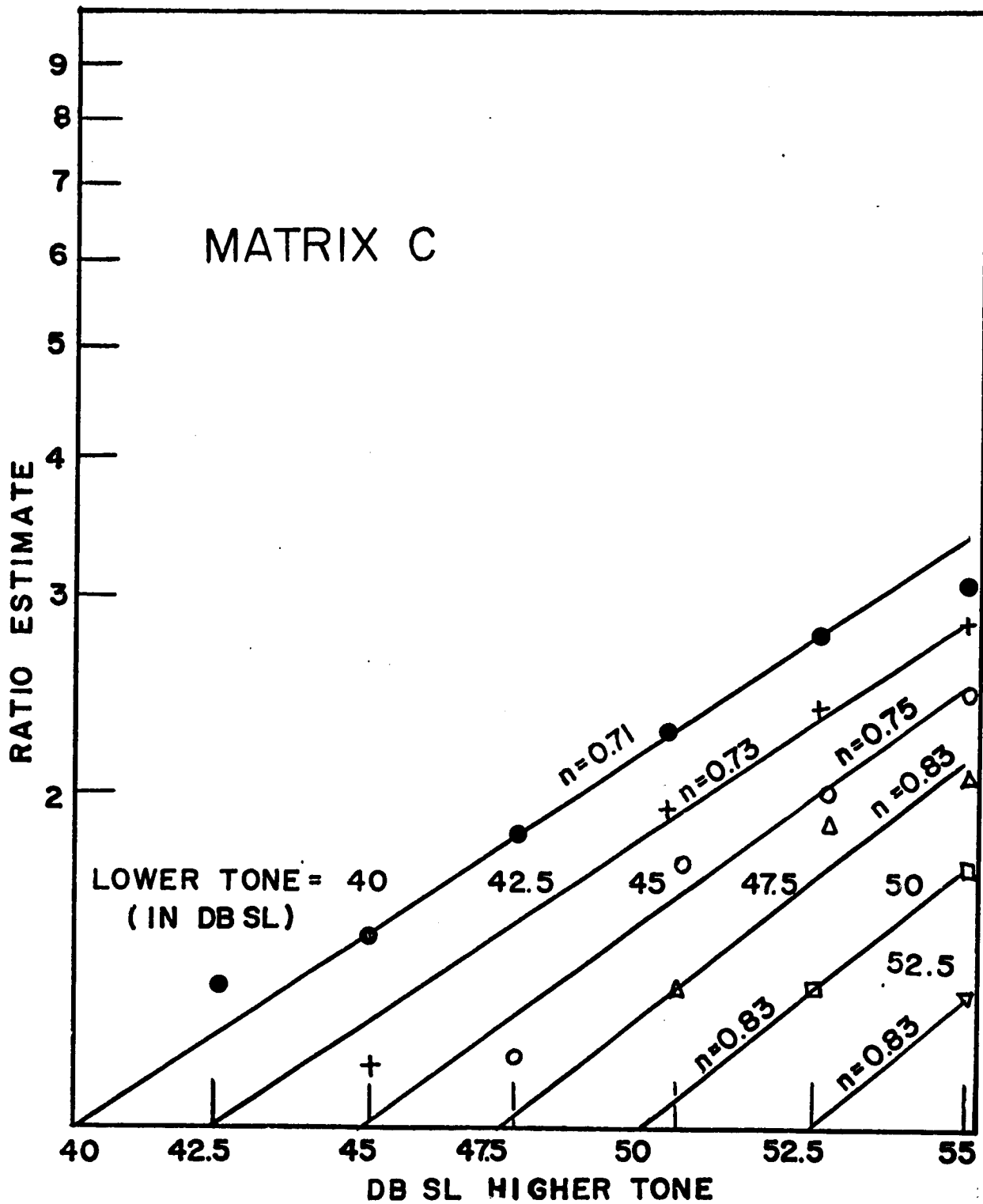


Fig. 8. Ratio estimates for Matrix B as a function of the dB SL of the higher tones. The lower tone is the parameter of each curve. The exponent (n) is equal to the slope of the curve.

are linear, and that the slope increases. However, the range of the slope variation is not as great as in Matrices A and B.

It is seen that the median slope for Matrix C is 0.77; Matrix B, 0.54, and Matrix A, 0.40.

Tables 8-10 show the variance associated with the mean ratio estimates obtained from each of the ten subjects for Matrices A-C. Generally, all three tables show that the variance of the mean ratio estimates increase as the differences between the stimuli in a pair increase. The greatest is in cell 7 for all matrices, with values of 2.46, 1.62, and 1.06, in Matrices A-C, respectively. A feature of the variability is that it shows a relation similar to those found for ratio estimates of stimulus pairs common to two, or all of the matrices. For common pairs of stimuli, the variability of Matrix C is greater than Matrix B, which, in turn, is greater than A. For example, with the inter-stimulus pair 40-50 dB SL the variance for Matrix C is 0.73, for B, 0.42, and A, 0.25.

Tables 11-13 show the obtained F-ratios of a four-way Analysis of Variance for each subject in Matrices A, B, and C. Each factor consisted of two levels. The factors were: (1) Upper or Lower half of the matrix (U), (2) Replications (R), (3) Sessions (S), and (4) Judgments A re B or B re A (J).

The Upper/Lower factor (U) referred to whether the subject

Table 8

Variance of Ten Mean Ratio Estimates (Matrix A)

		DB SL Louder Tone						
		30	40	50	60	70	80	90
DB SL Lower Tone	30	0.003	0.16	0.34	0.44	0.65	1.08	2.46
	40		0.00	0.25	0.35	0.40	1.10	2.58
	50			0.00	0.20	0.39	0.93	2.09
	60				0.00	0.21	0.60	1.76
	70					0.00	0.43	1.77
	80						0.00	1.10
	90							0.00

Table 9

Variance of Ten Mean Ratio Estimates (Matrix B)

		DB SL Louder Tone						
		40	45	50	55	60	65	70
DB SL Lower Tone	40	0.04	0.14	0.42	0.57	0.60	1.28	1.62
	45		0.01	0.08	0.47	0.57	0.75	1.22
	50			0.00	0.17	0.49	0.63	0.95
	55				0.01	0.26	0.40	0.93
	60					0.00	0.22	0.53
	65						0.00	0.24
	70							0.00

Table 10
 Variance of Ten Mean Ratio Estimates (Matrix C)

		DB SL Louder Tone						
		40	42.5	45	47.5	50	52.5	55
DB SL Lower Tone	40	0.02	0.24	0.33	0.49	0.73	0.98	1.06
	42.5		0.03	0.14	0.38	0.43	0.64	1.19
	45			0.02	0.14	0.41	0.52	0.75
	47.5				0.02	0.21	0.46	0.48
	50					0.04	0.24	0.41
	52.5						0.02	0.23
	55							0.04

Table 11
Analysis of Variance Results for Matrix A

Subj.	Source of Variation F-Ratio													
	U	R	UR	S	US	RS	URS	J	UJ	RJ	URJ	SJ	USJ	RSJ
1	1.62	1.99	0.89	1.80	1.63	1.67	0.91	1.37	1.51	1.50	1.11	1.17	1.10	2.05
2	5.13*	3.05*	2.81*	3.40*	1.02	0.97	2.67*	9.25*	2.37	3.57*	2.06	3.34*	3.22*	3.40*
3	4.80*	2.73*	1.43	2.06	2.36	1.82	2.26	2.54*	2.78*	2.62*	2.39	1.21	3.51*	2.26
4	4.26*	1.19	2.08	1.27	2.13	1.13	1.07	4.10*	1.41	3.30*	1.64	2.34	1.97	2.15
5	1.89	0.40	0.44	0.65	0.70	0.46	0.95	1.63	1.27	0.69	0.67	0.61	0.77	1.67
6	1.32	1.23	0.40	0.61	2.04	0.69	1.07	1.16	1.95	1.40	1.09	1.97	1.17	0.71
7	7.71*	3.39*	3.89*	10.50*	1.36	4.58*	4.47*	5.48*	1.93	2.97*	2.05	8.57*	3.32*	3.86*
8	3.31*	3.20*	0.89	1.91	1.37	2.10	1.57	1.90	3.32*	1.96	1.27	3.31*	0.91	2.08
9	2.84*	1.22	0.51	2.17	1.13	0.89	0.75	3.78*	1.80	1.09	0.55	3.19*	1.54	1.14
10	2.18	0.41	0.72	1.93	0.79	2.90*	0.53	1.45	0.91	1.56	0.89	2.67*	0.50	2.17
% Sig. (10 Ss)	60	40	20	20	0	20	20	50	20	40	0	50	30	20
% Sig. (8 Ss)	50	25	0	0	0	10	0	40	25	25	0	40	10	0

* Significant at α 0.01 df=28/28 (2.46)

Table 12
Analysis of Variance Results for Matrix B

Source of Variation F-Ratio														
Subj.	U	R	UR	S	US	RS	URS	J	UJ	RJ	URJ	SJ	USJ	RSJ
1	2.35	1.54	1.06	3.01*	1.18	1.49	0.80	1.37	0.79	1.31	1.09	0.75	0.57	1.17
2	4.37*	1.22	0.93	1.34	3.15*	0.68	1.43	1.07	2.67*	1.97	1.46	2.03	0.72	0.94
3	3.03*	1.47	1.08	0.98	1.33	2.51*	1.28	1.64	2.61*	1.02	1.45	0.86	1.01	1.35
4	1.36	0.45	0.59	0.70	1.18	0.87	0.66	3.01*	1.01	0.82	0.79	1.07	0.56	0.45
5	1.80	1.58	1.36	0.74	0.74	0.82	1.19	1.79	0.64	1.03	0.48	0.71	0.89	0.66
6	5.62*	1.57	1.99	0.62	0.71	1.71	0.75	0.66	1.35	0.70	0.46	1.08	0.60	0.91
7	3.06*	3.84*	2.81*	6.31*	1.26	4.45*	4.25*	4.01*	2.13	2.20	2.39	5.03*	1.40	2.52*
8	1.76	1.28	0.85	2.30	1.15	1.21	0.84	1.59	1.55	0.80	1.23	0.41	1.03	1.32
9	1.91	0.57	0.44	0.84	0.77	1.88	0.88	1.23	2.06	0.81	0.69	0.71	0.67	1.25
10	7.86*	1.42	0.42	1.21	0.97	1.40	0.97	0.70	1.13	0.72	1.22	0.81	0.89	1.15
% Sig. (10 Ss)	50	10	10	20	10	20	10	20	20	0	0	10	0	10
% Sig. (9Ss)	45	0	0	11	11	11	0	11	22	0	0	0	0	0

* Significant at $\alpha=0.01$ df=28/28 (2.46)

Table 13
Analysis of Variance Results for Matrix C

Source of Variation F-Ratio														
Subj.	U	R	UR	S	US	RS	URS	J	UJ	RJ	URJ	SJ	USJ	RSJ
1	2.49*	2.28	1.40	3.63*	1.25	2.11	1.47	2.04	1.42	2.65*	1.29	2.31	0.74	1.78
2	7.26*	1.39	1.87	1.60	1.00	2.16	0.98	2.94*	2.38	1.69	1.43	1.29	1.77	1.12
3	1.24	1.38	0.87	1.85	0.61	1.73	1.21	1.28	1.35	1.27	1.44	2.10	0.99	1.21
4	0.95	1.11	0.89	1.00	0.60	0.77	0.46	0.69	1.13	2.42	0.94	0.81	1.34	0.67
5	3.66*	1.66	1.21	1.48	2.04	1.76	1.88	1.77	1.28	0.84	1.10	1.36	1.35	1.42
6	5.56*	0.61	1.35	1.40	0.62	1.61	0.86	0.39	1.57	2.47*	1.39	1.56	1.13	1.68
7	4.27*	1.39	1.57	1.66	0.78	1.05	1.46	1.88	2.15	0.89	1.34	3.24*	1.52	1.33
8	1.18	0.67	0.18	0.69	0.78	0.79	0.38	0.49	0.37	0.67	0.79	0.89	0.60	0.55
9	1.40	0.47	1.03	0.58	0.74	0.51	0.83	0.60	0.47	0.46	1.37	0.46	0.55	1.06
10	32.02*	1.99	3.07*	2.29	2.05	2.63*	2.76*	2.32	2.46*	1.52	1.31	1.36	1.33	1.28
% Sig. (10 Ss)	60	0	10	10	0	10	10	10	10	20	0	10	0	0

* Significant at $\alpha = 0.01$ df=28/28 (2.46)

was able to make equivalent responses to stimuli in analogous cells, one in the upper half of the matrix, the other in the lower half. Depending upon the judgment condition (A re B or B re A) the half of the matrix containing fractional estimates was inverted, and then the comparison made. The Replications Factor (R) referred to either the first or second presentation of a matrix within a test session. The Sessions (S) factor was divided into two levels; the results of the first day being compared to the results of the second day. The factor A re B/ B re A referred to the judgment mode that the subjects responded under, and to any differences between the two.

Each entry in Table 11 is the F-ratio obtained when considering all cells in Matrix A. An asterisk above an entry indicates that the F-ratio is significant at the 0.01 level. The table shows that 60% of the subjects had significant results associated with judgments made between the upper and lower halves of the matrix. Also, 50% showed significance on the A re B/ B re A factor, and 40% on the Replications factor.

Upon closer examination of Table 11, however, it can be noticed that some of the percentages may be superfluously high. This is because two subjects (Number 2 and 7) exhibited significance in nearly all the sources of variation. These two subjects had extremely small residual error terms, which caused their obtained F-ratios to be inflated.

Since the percentages might have been spuriously high due to the inclusion of the results of these subjects, their data were eliminated, and the results re-analyzed. These results are shown at the bottom of Table 11. The new values indicate that 50% of the eight remaining subjects still showed significance on the Upper/Lower factor, and 40% for the A re B/ B re A factor. Also, many of the interactions which showed 20% significance with ten subjects dropped to 0% with the exclusion of subjects 2 and 7.

Table 12 shows the results for Matrix B. When the significance is observed over all ten subjects, 50 % show significance for the Upper/Lower factor, with the remainder of the sources showing 20% or less. As in Matrix A, subject 7 shows large F-ratios on many of the sources of variation. Subject 2, however, does not exhibit large F-ratios, and his data are in line with the other eight subjects. If subject 7's data are removed, it is seen that 45% of the subjects still show significance on the Upper/Lower factor.

Table 13 shows the percentages of subjects showing statistical significance for Matrix C. Both subjects 2 and 7 are consistent with the patterns of the other subjects. As in the two other matrices, the greatest percentage of significance occurs on the Upper/ Lower factor (60%), with the remainder of the percentages being negligible.

In light of the findings that a high percentage of the subjects showed statistical significance on the Upper/Lower

factor, further analysis to show where the significance lay was performed. For all subjects the mean of each of the 49 cells in a matrix was computed separately for both the A re B and the B re A conditions. A comparison was then made between the ratio estimates given for analogous pairs of stimuli, one stimulus pair in the upper half of a matrix, the other stimulus pair in the lower half of the matrix. Depending upon the response condition, the reciprocals of the half of the matrix with fractional estimates was taken, and compared to the unaltered analogous cell. Figures 9-14 show the results of the foregoing process. Each point is the mean ratio estimate for the ten subjects. The numbers associated with each data point is the cell number of the upper cell and its analogous lower cell. The line represents a perfect reciprocal relationship between the upper and lower cells.

Figure 9 shows the results for Matrix A under the Tone A re Tone B response condition. It is seen that almost all of the points lie to the right of the reciprocal relation line, indicating that the reciprocals of the ratio estimates for the upper cells are less than would be predicted from the lower cells. The cells showing the most divergence from the perfect reciprocal relationship are numbers 5, 7, 12, and 13. Although a few points lie to the left of the line, most appear to be quite close to it. One exception appears to be cell 25.

Figure 10 shows the results of Matrix A under the B re

A response condition. The figure indicates that in all those cells containing either an 80- or 90 dB SL stimulus that the points fell to the left of the line. Thus, in those cells, the mean ratio estimates of the upper cells are greater than the analogous reciprocals. On the other hand, when a cell does not contain an 80- or 90 dB SL stimulus, the reciprocal of the lower cell is greater than the ratio estimate of the upper cell.

The results for Figure 11 (Matrix B under the A re B response condition) are similar to Matrix A. Thus, all stimulus points either lie on the line, or to the right of it. This, of course, indicates that the mean ratio estimate of the lower cell is greater than the reciprocal of the upper cell. The divergence from perfect linearity increases as the inter-stimulus differences become greater.

In Figure 12 (Matrix B under the B re A judgment condition) the points lying to the left of the line are representative of matrix cells which have 65- or 70 dB SL as one of their stimuli. The points to the right of the line contain stimuli other than 65- or 70 dB. Exceptions to this general relation occur at cells 12 and 24.

The A re B judgments for Matrix C are shown in Figure 13. A strong similarity between this figure and those obtained for Matrices A and B under the same conditions is seen. The effect is most prevalent with those stimulus pairs having at least one of the higher intensities to be found in the matrix.

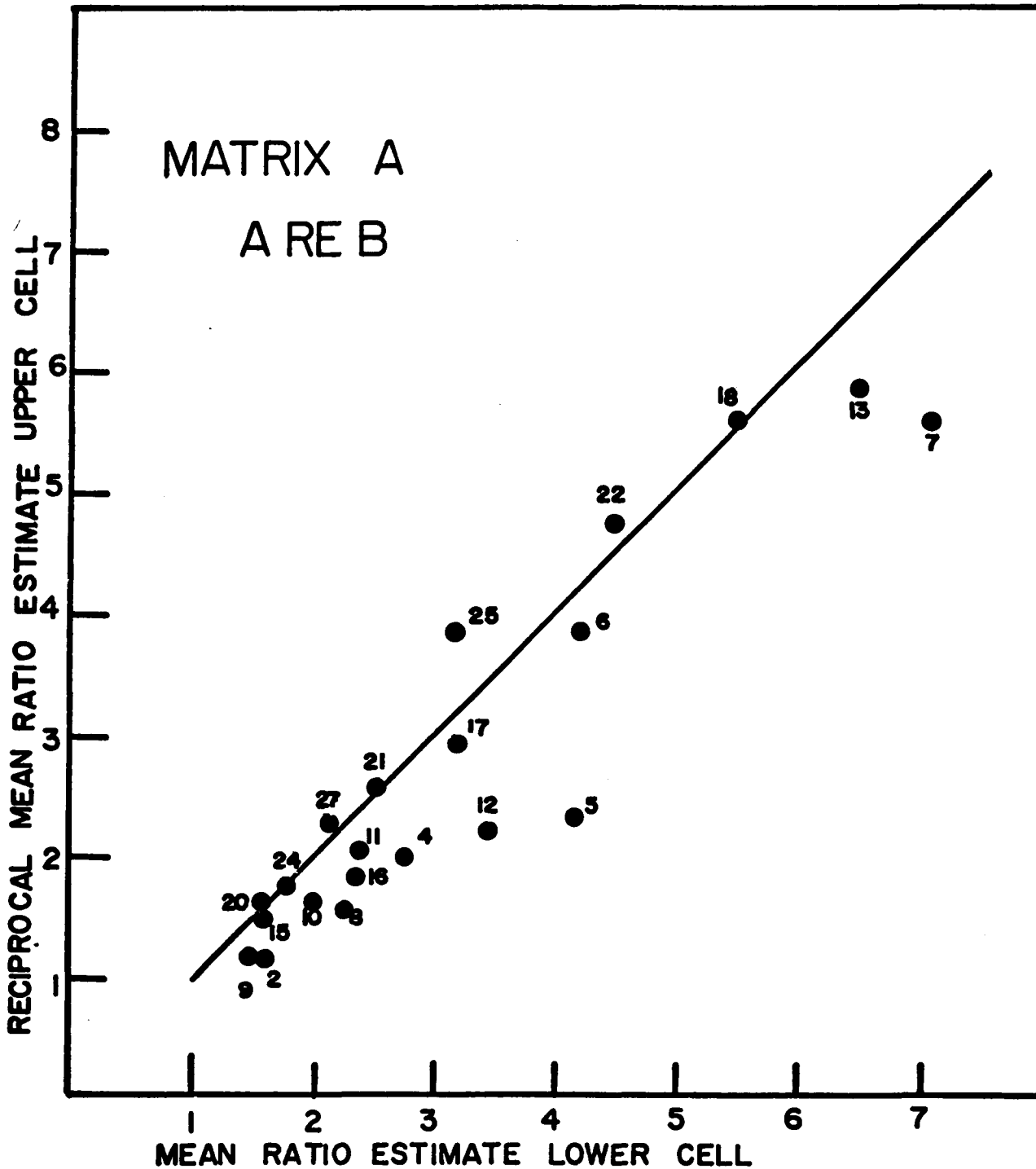


Fig. 9. Mean ratio estimate of the lower cell as a function as a function of the reciprocal of the mean ratio estimate of the upper cell for Matrix A under the A re B response mode.

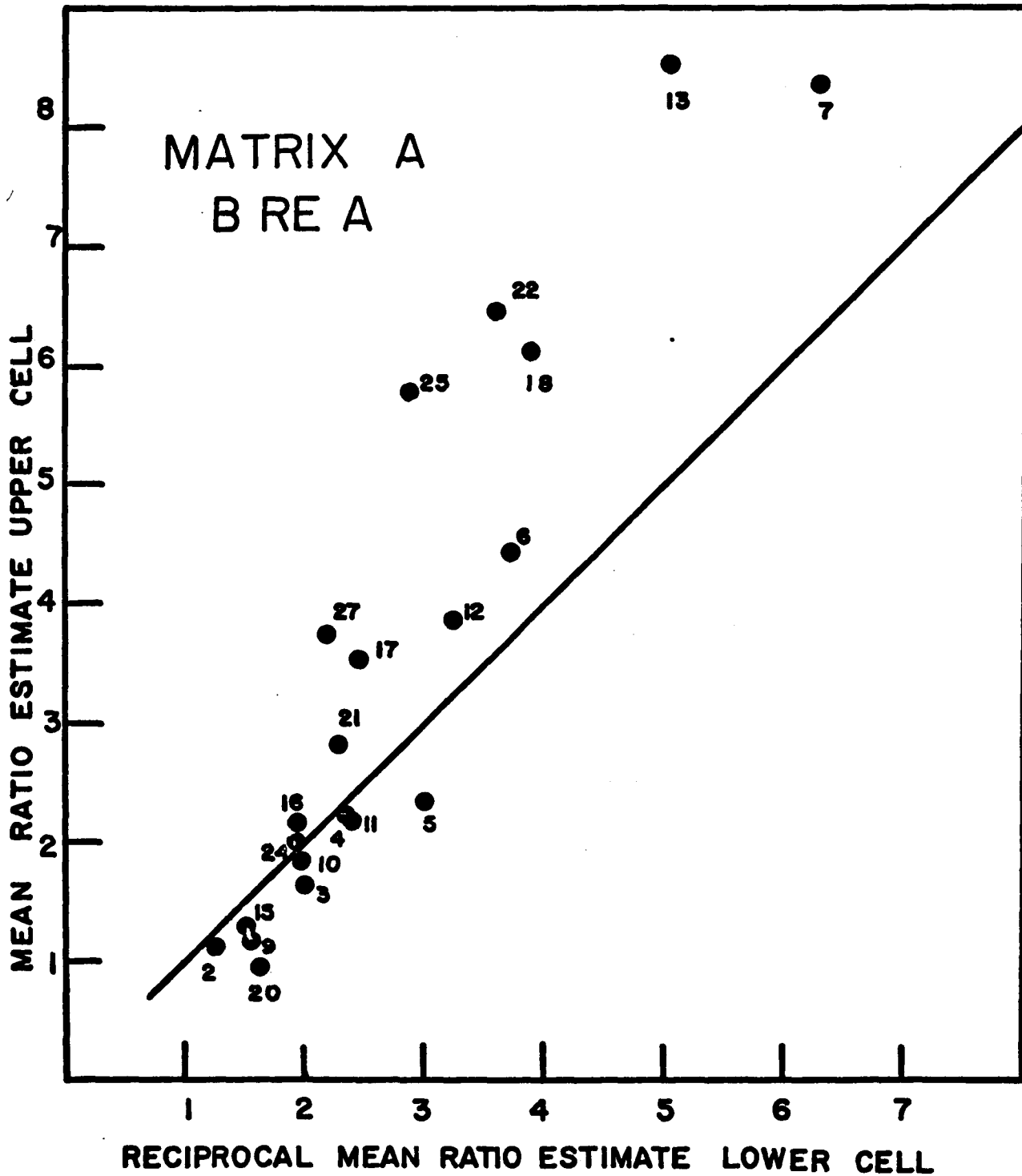


Fig. 10. Reciprocal of the mean ratio estimate of the lower cell as a function of the mean ratio estimate of the upper cell for Matrix A under the B re A response mode.

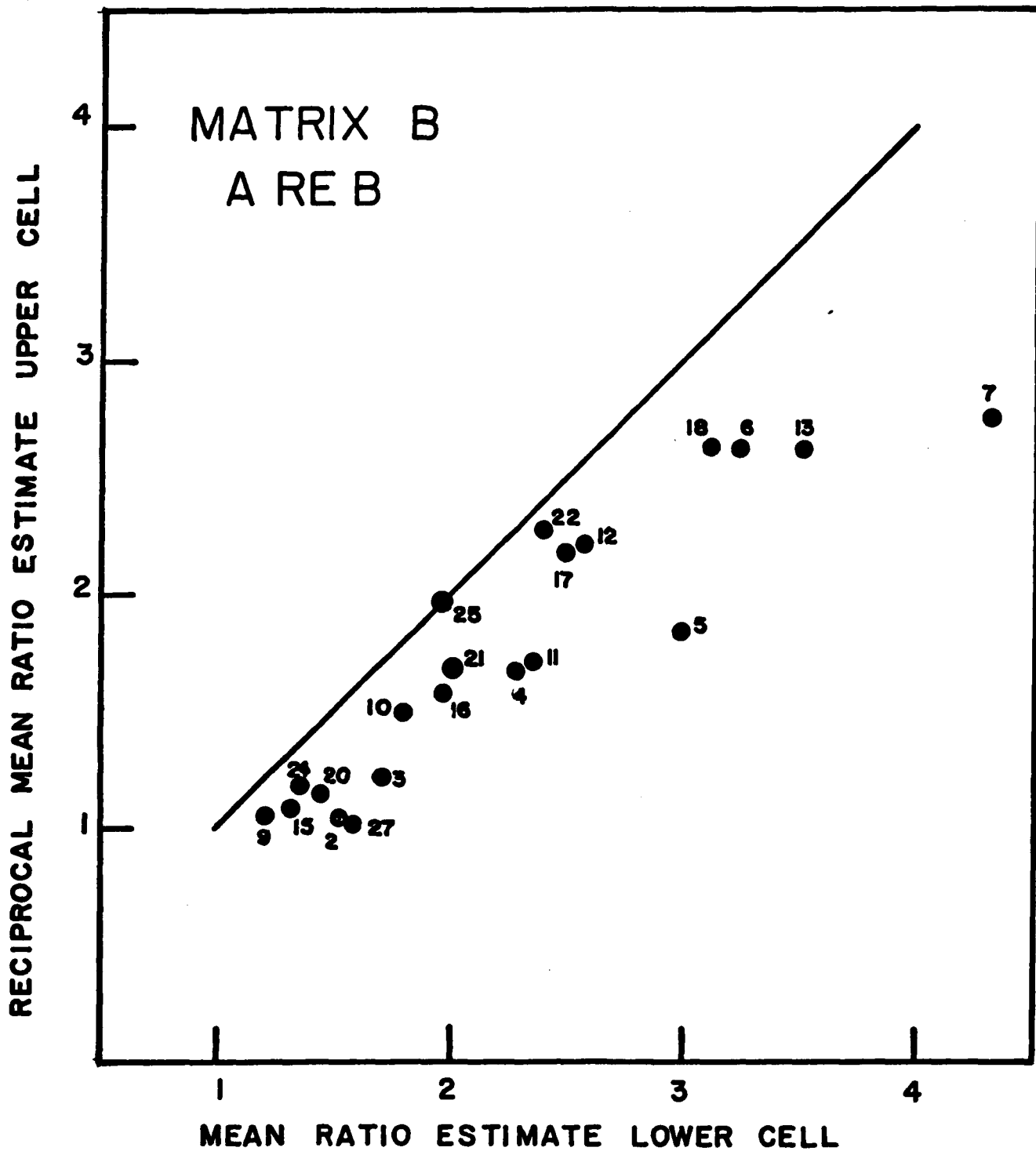


Fig. 11. Mean ratio estimate of the lower cell as a function of the reciprocal of the mean ratio estimate of the upper cell for Matrix B under the A re B response mode.

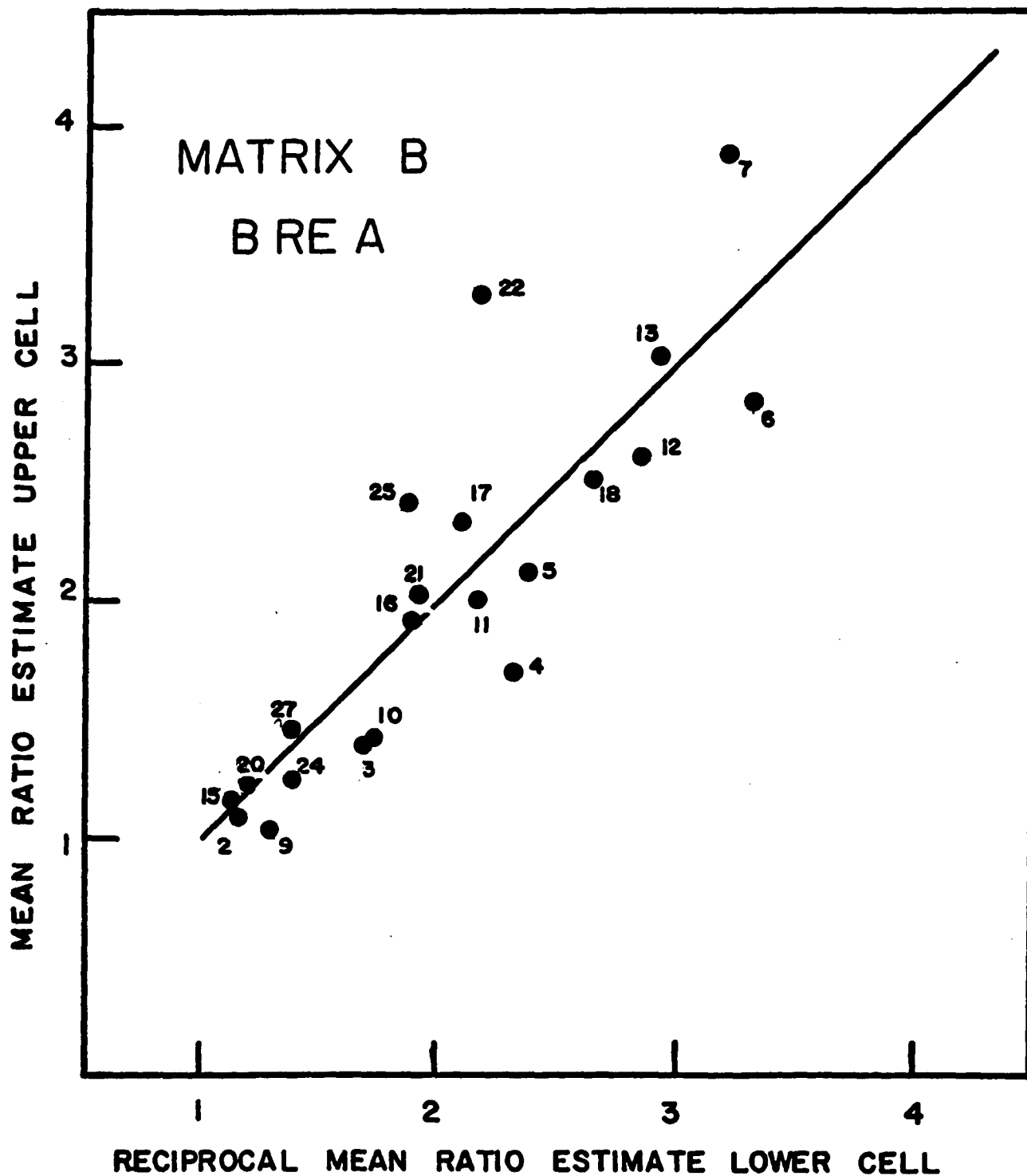


Fig. 12. Reciprocal of the mean ratio estimate of the lower cell as a function of the mean ratio estimate of the upper cell for Matrix B under the B re A response mode.

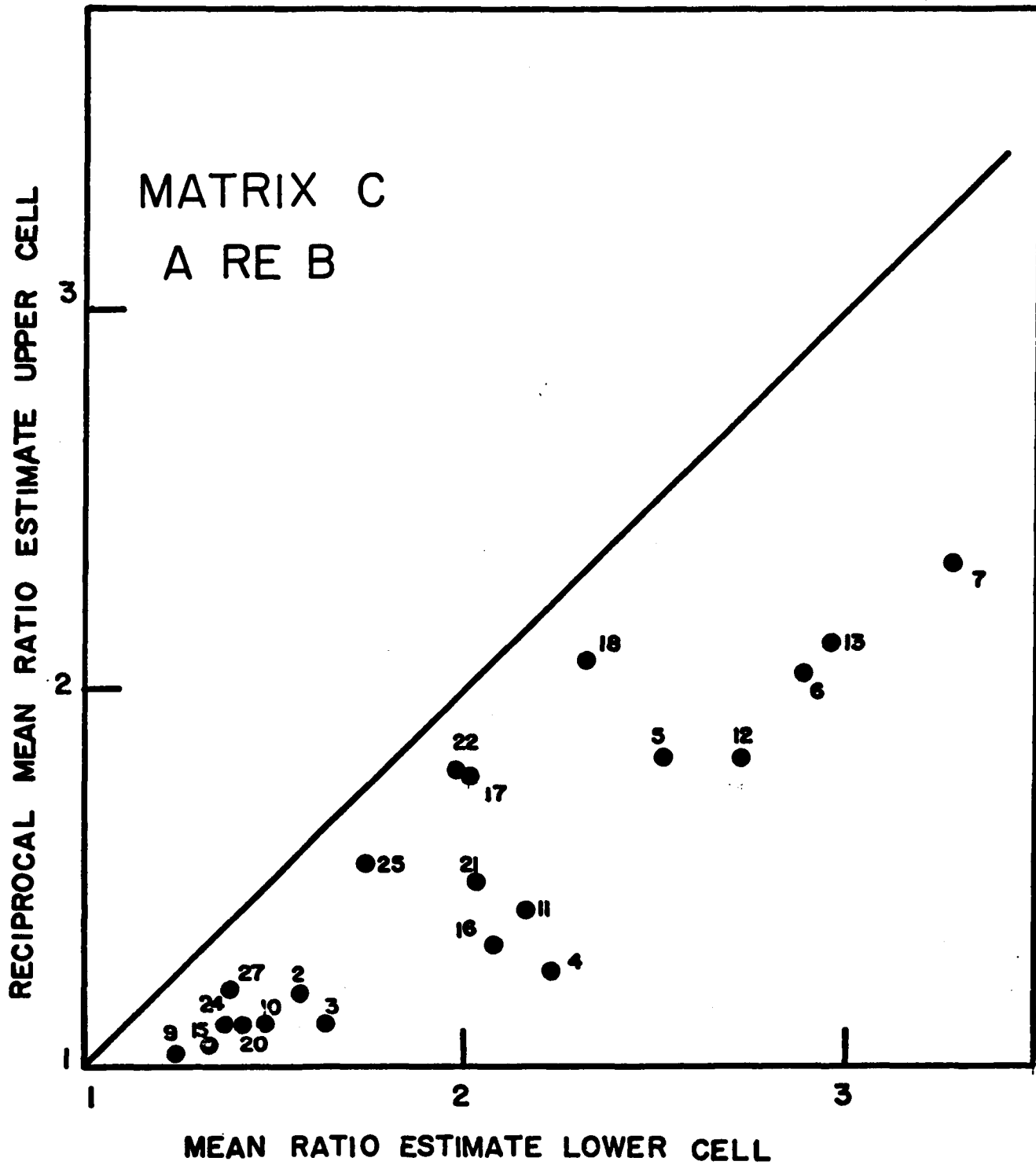


Fig. 13. Mean ratio estimate of the lower cell as a function of the reciprocal mean ratio estimate of the upper cell for Matrix C under the A re B response condition.

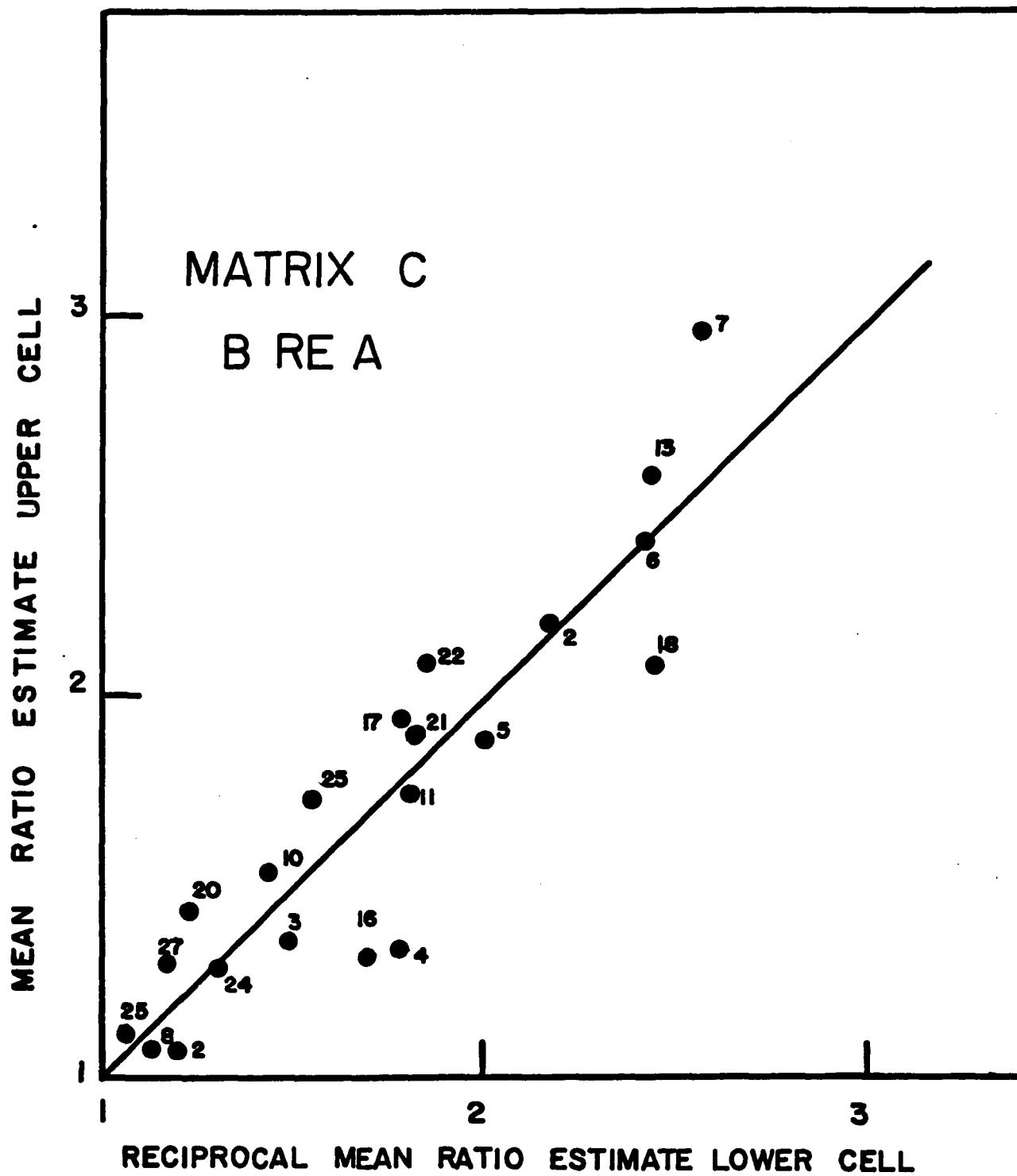


Fig. 14. Reciprocal of the mean ratio estimate for the lower cell as a function of the mean ratio estimate of the upper cell for Matrix C under the B re A response mode.

Thus, the cells containing stimuli from 47.5-55 dB SL are most affected.

Figure 14 is the plot of the results for Matrix C under the B re A response condition. Again, as in the two other matrices, those points lying to the left of the line are representative of those stimulus combinations containing at least one of the higher intensities to be found in a matrix.

In order to ascertain which of several stimulus parameters caused the above reciprocal relations between the halves of the matrices under both response conditions, a partitioning of several effects was performed. The partition was based upon the schema for a two-factor Analysis of Variance, with two levels per factor. Under these conditions, when one measurement per cell is obtained, that score can be viewed as a deviation from the grand mean of the array. Further, this deviation is composed of three components: (1) a deviation of the row mean from the grand mean, (2) a deviation of the column mean from the grand mean, and (3) a residual interaction.

Figure 15 illustrates the above schema, and shows how the effects were partitioned. The data were obtained by taking the mean ratio estimate for a particular cell, and the reciprocal of the opposite cell, under both the A re B and B re A conditions. These four values were placed in a 2x2 table (as in Figure 15) so that two levels of one factor

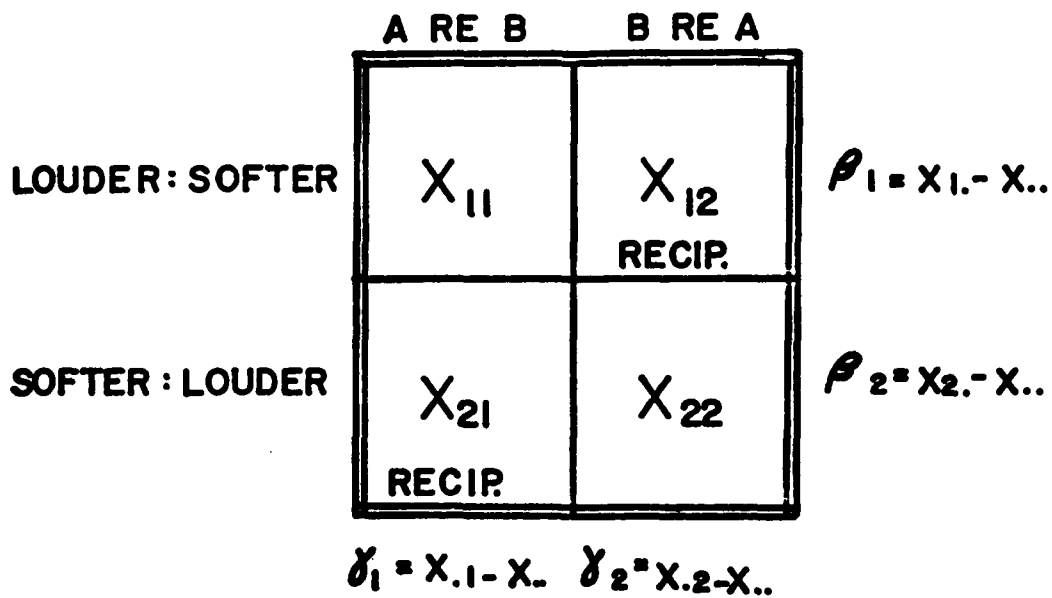


Fig. 15. Schema for partitioning the effects due to response condition and the intensive order of the tones.

were A re B and B re A, and the two levels of the other factor were the louder tone leading the lower tone, or vice versa. For convenience, each row and column were assigned Greek letters. β_1 corresponds to the louder tone heard first, β_2 the opposite. For example, under the A re B condition, if the louder tone was heard first a ratio estimate of greater than 1.00 was obtained. Under the B re A response condition, when the louder tone was heard first, a fractional (less than 1.00) was obtained. Therefore, under the B re A condition, it was necessary to invert the obtained value in order to make a comparison. Thus, β_1 represents the mean of the unaltered value and the reciprocal value less the grand mean of the array. δ_1 represents the situation where the stimuli are judged A re B, δ_2 , B re A. β_2 and δ_2 are equal and opposite in sign from β_1 and δ_1 , respectively. Thus, β_1 is the deviation of the first row's mean ($X_{1.}$) from the grand mean ($X_{..}$), and δ_1 the deviation of the first column's mean ($X_{.1}$) from the grand mean.

Tables 14-16 indicate the increment (deviation) due to the louder tone leading the lower tone (β_1) for Matrices A, B, and C, respectively. The most evident feature of Table 14 (Matrix A) is that when the level of the louder stimulus reaches either 80- or 90 dB SL, a negative deviation from the mean is obtained. This deviation is most pronounced when the level of the louder tone is 90 dB. For a given row of the lower intensity stimuli, the increments

Table 14

Increment (ρ_1) Due to Louder Tone Leading (Matrix A)

	DB SL Louder Tone						
	30	40	50	60	70	80	90
30	0.00	0.13	0.28	0.22	0.61	-0.09	-0.14
40		0.00	0.16	0.11	0.12	-0.10	-0.73
50			0.00	0.14	0.06	-0.21	-0.60
60				0.00	0.07	-0.16	-0.80
70					0.00	-0.01	-0.90
80						0.00	-0.42
90							0.00

Table 15

Increment (ρ_1) Due to Louder Tone Leading (Matrix B)

	DB SL Louder Tone						
	40	45	50	55	60	65	70
40	0.00	0.13	0.40	0.31	0.34	0.28	0.22
45		0.00	0.12	0.14	0.20	0.14	0.20
50			0.00	0.06	0.09	0.02	0.15
55				0.00	0.07	0.07	-0.24
60					0.00	0.09	-0.14
65						0.00	0.12
70							0.00

Table 16

Increment (β_1) Due to Louder Tone Leading (Matrix C)

	DB SL Louder Tone						
	40	42.5	45	47.5	50	52.5	55
40	0.00	0.13	0.17	0.38	0.21	0.22	0.15
42.5		0.00	0.06	0.07	0.20	0.22	0.17
45			0.00	0.06	0.28	0.01	0.15
47.5				0.00	0.05	0.09	-0.01
50					0.00	0.08	0.00
52.5						0.00	0.01
55							0.00

were positive to 70 dB SL, and then were negative.

Table 15 shows the β_1 results for Matrix B. All values in the table are positive, except in two cases where the louder stimulus reaches 70 dB. Generally, as the louder tones increased for a given lower SL stimulus, the increments increased at first, and then fell off.

Table 16 shows the results for Matrix C. For a given row of lower intensity stimuli, an increment in β_1 is generally seen at first, which is followed by a decline. A minor exception occurs at one point when the lower stimulus is 45 dB. In this case, as the louder tone increases there is a decrease at 52.5 dB, with a subsequent increase at 55 dB.

Figures 16-18 are graphic representations of the increments due to β_1 in Matrices A-C, respectively. Each figure is plotted so that the lower (softer) tone is the parameter of the curve, and the abscissa is the level of the louder tone in dB re the dB SL of the lower tone.

With the exception of the 80 dB SL curve (which is composed of only two data points), each of the functions in Figure 16 (Matrix A) increases initially, and then drop to negative values as the intensity of the louder tone increases. The initial increase in β_1 is shown to decrease as the level of the softer tones becomes more intense. Also, as the softer tones become more intense, the fall-off in the function occurs earlier. As the softer tone increases from 30-70 dB, the fall-off occurs at approximately +40, +30, +15, +10, and

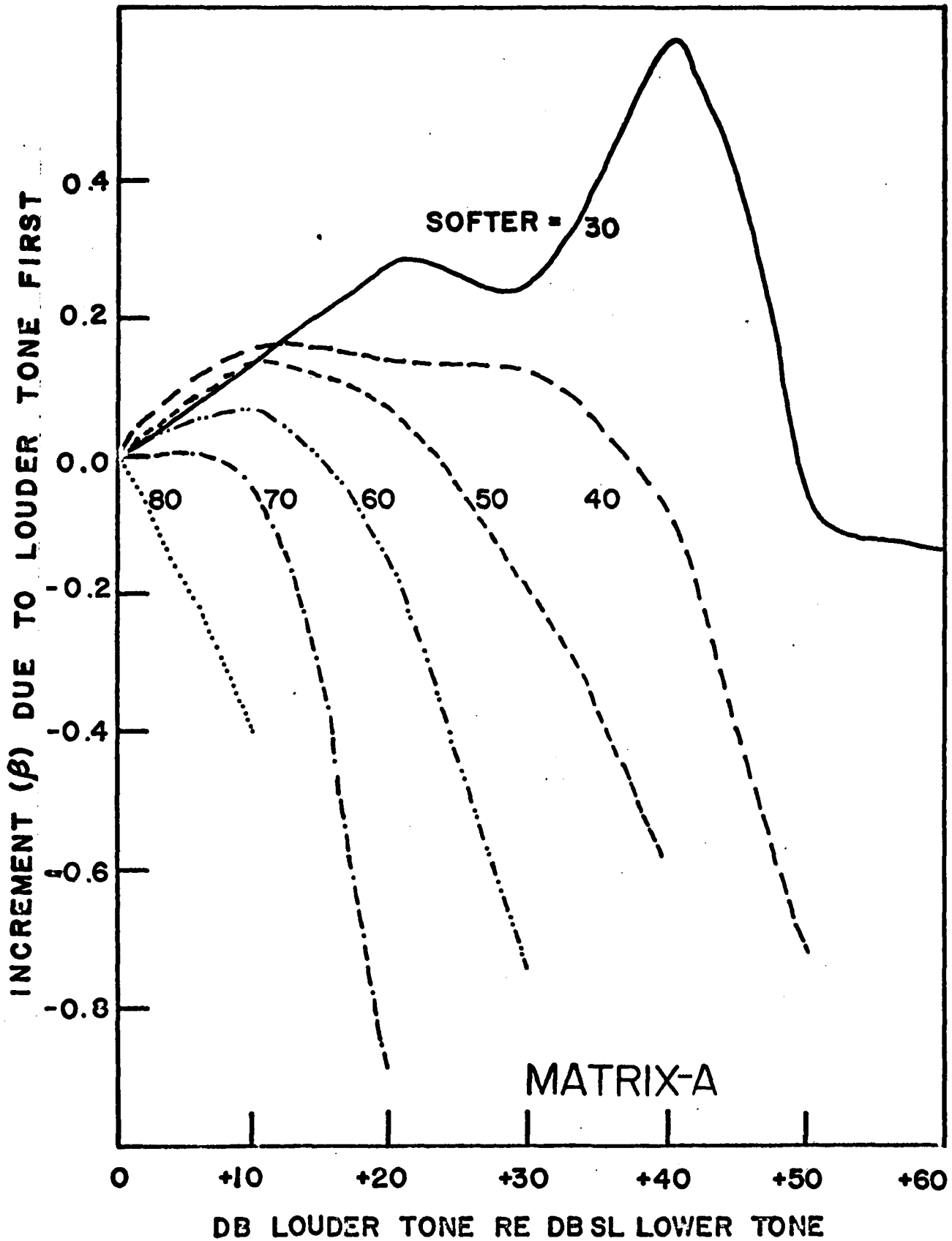
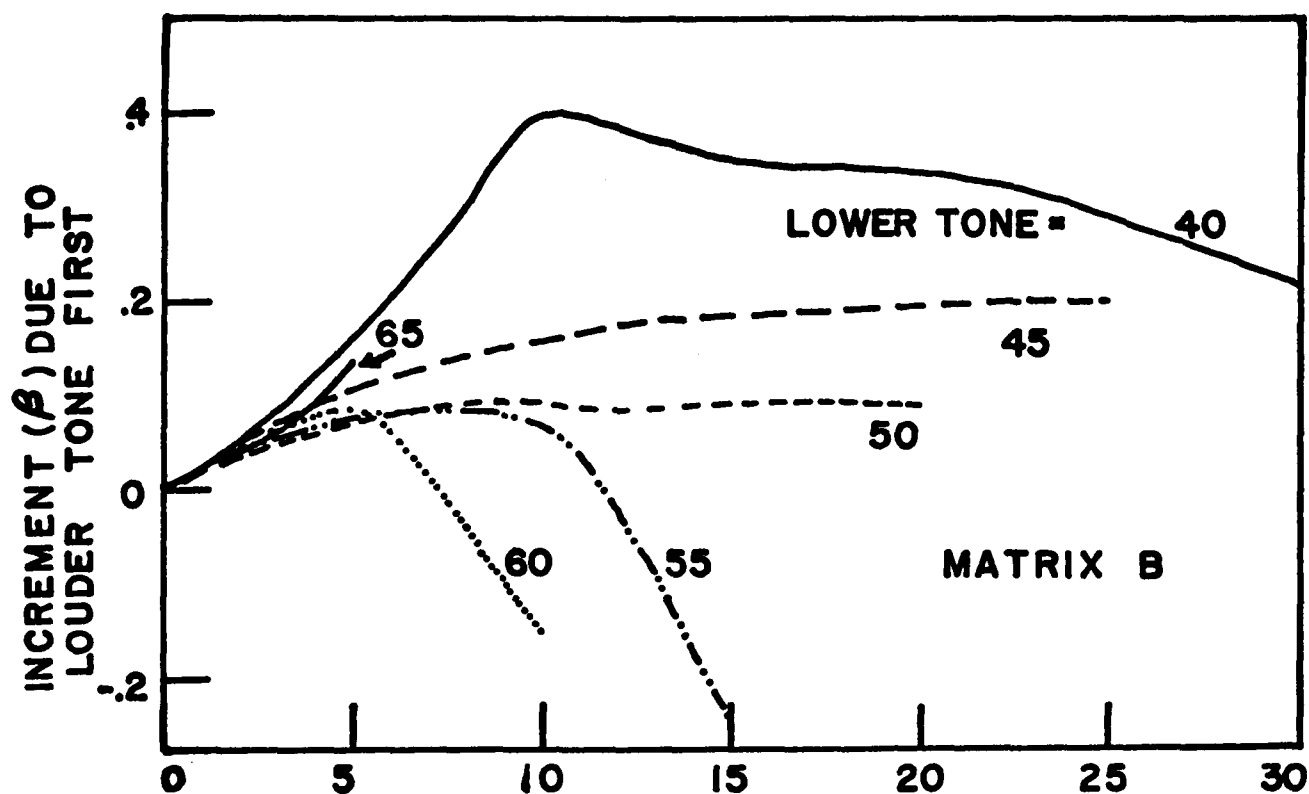
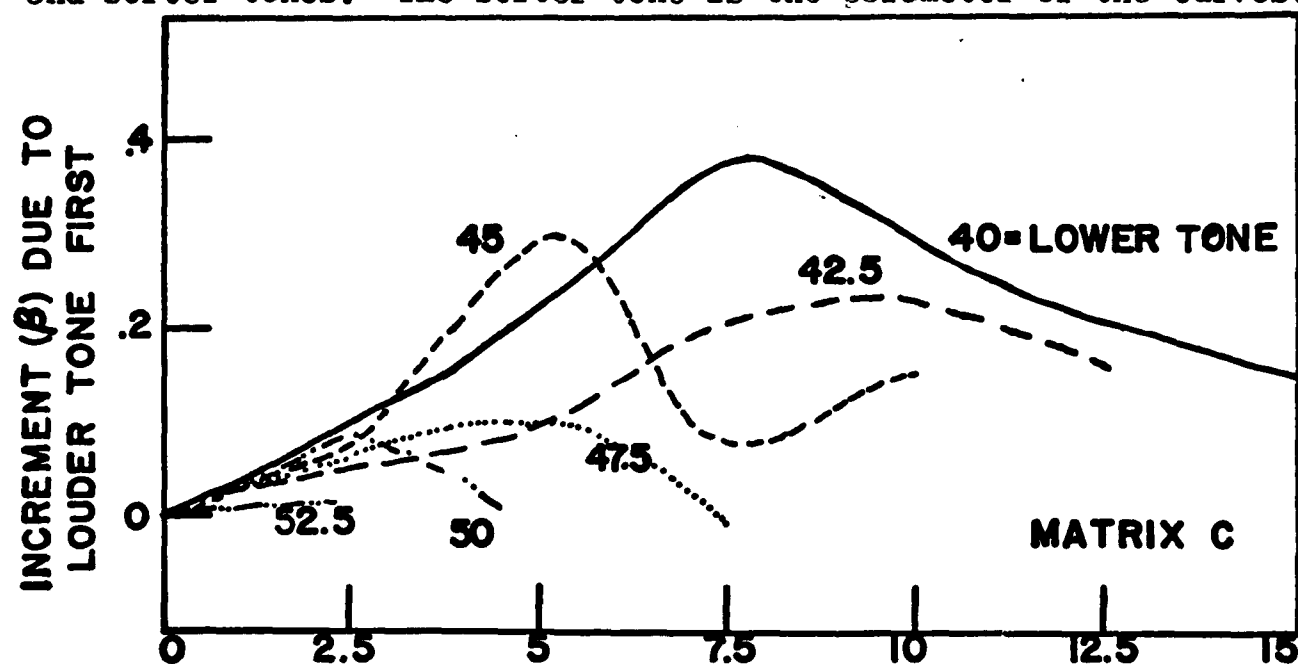


Fig. 16. Increments (β_1) for Matrix A due to the louder tone heard first as a function of the difference in dB between the louder and softer tones. The softer tone is the parameter of the curves.



DB SL LOUDER TONE RE DB SL LOWER TONE

Fig. 17. Increments (β) for Matrix B due to the louder tone heard first as a function of the difference in dB between the louder and softer tones. The softer tone is the parameter of the curves.



DB SL LOUDER TONE RE DB SL LOWER TONE

Fig. 18. Increment (β) for Matrix C due to the louder tone heard first as a function of the difference in dB between the louder and softer tones. The softer tone is the parameter of the curves.

+8 dB re the dB SL of the lower tone, respectively.

Figure 17 (Matrix B) shows in three curves (Softer Tones = 40, 55, and 60 dB SL) an initial increment, subsequently followed by a fall-off as the level of the louder tone increases. The fall-offs, however, are not as great as was seen for Matrix A. In two instances (Softer Tones = 45 and 50 dB SL) the functions increase initially, and then appear to level off.

The functions for Matrix C (Figure 18) are consistent in form to those obtained for Matrices A and B. Initially, an increase in β_1 is observed, which is subsequently followed by a decrement. Also, it appears that as the lower tone increases, the fall-off in β_1 occurs earlier. Some exceptions to the above relationships do occur, but these are not serious deviations. β_1 , for all purposes, does not extend below zero as was seen in Matrices A and B.

Tables 17-19 show the increments due to the response condition Tone A re Tone B ($\delta \beta_1$) for Matrices A, B, and C. The most evident feature of Table 17 (Matrix A) is that for those stimulus pairs where the louder tone is 70 dB SL or less, near zero increments are found. However, when the level of the louder tone reaches 80- or 90 dB SL, negative increments prevail. Apparently, the influence of the judgmental response mode does not have considerable effects until the level of the louder stimulus in a pair reaches approximately 80 dB SL. Regarding the results for Matrix B (Table 18),

Table 17

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Increment (δ_1) Due to A re B Presentations (Matrix A)

	DB SL Louder Tone						
	30	40	50	60	70	80	90
	30	40	50	60	70	80	90
DB SL Lower Tone	0.00	0.10	0.06	0.06	0.30	-0.02	-0.49
40		0.00	-0.01	-0.05	-0.03	-0.10	-0.29
50			0.00	0.01	0.04	0.05	0.26
60				0.00	0.00	-0.02	-0.21
70					0.00	-0.11	-0.40
80						0.00	-0.40
90							0.00

Table 18

Increment (δ_1) Due to A re B Presentations (Matrix B)

	DB SL Louder Tone						
	40	45	50	55	60	65	70
	40	45	50	55	60	65	70
DB SL Lower Tone	0.00	0.07	-0.04	-0.02	0.07	-0.07	0.00
45		0.00	-0.02	0.04	-0.02	-0.18	0.05
50			0.00	0.02	-0.06	0.06	0.10
55				0.00	0.04	-0.05	-0.19
60					0.00	-0.02	-0.08
65						0.00	-0.06
70							0.00

Table 19

Increment (γ_1) Due to A re B Presentations (Matrix C)

	DB SL Louder Tone						
	40	42.5	45	47.5	50	52.5	55
40	0.00	0.12	-0.02	0.07	0.12	0.02	0.03
42.5		0.00	0.01	0.07	0.00	0.04	0.02
45			0.00	0.06	0.10	0.01	-0.03
47.5				0.00	-0.01	-0.05	-0.01
50					0.00	0.08	0.00
52.5						0.00	0.01
55							0.00

it is seen that the majority of the γ_1 values are extremely close to zero. Placement of a cell within a matrix appears to have negligible effects upon the obtained γ_1 value. The same situation prevails in Matrix C (Table 19), and no differential increment or decrement in γ_1 is observed as a function of the matrix cell.

Figures 19-22 are the spatial configurations (proximity plots) obtained from the Analyses of Proximities for Matrices A to D. The transformation imposed upon the analyses specified that one distance unit (DU) was equivalent to the value of the \log_2 of the ratio estimate 2.00. Thus, one DU measured on the configurations is equal to the ratio estimate 2.00, and the ratio estimate associated with any other distance is the \log_2 of that judgment. Table 20 provides the ratio estimates associated with the measured distances (in DU's). One DU on each of the Figures 19-22 is indicated.

Figure 19 is the configuration for Matrix B. Presentation of the results for Matrix A will be deferred for the present. It is seen that the data points progress in an orderly fashion, with the points curving upwards when the level of the stimuli reach approximately 55 dB SL. From 40-55 dB, each of the points can be joined by a straight line. Thus, the total loudness between the points 40 and 55 dB SL is the sum of the log-ratio estimates 40-45, 45-50, and 50-55 dB SL. As noted, after 55 dB the configuration curves upwards, indicating that the points 60, 65, and 70 dB SL are not the

TABLE 20

Conversion of Distance Units to Ratio Estimates

RATIO EST.	DIST. UNITS	RATIO EST.	DIST. UNITS	RATIO EST.	DIST. UNITS	RATIO EST.	DIST. UNITS	RATIO EST.	DIST. UNITS
0.050000	-4.321928	0.100000	-3.321928	0.150000	-2.736966	0.200000	-2.321928	0.250000	-2.000000
0.300000	-1.736965	0.350000	-1.514573	0.400000	-1.321928	0.449999	-1.152003	0.500000	-1.000000
0.550000	-0.862496	0.600000	-0.736965	0.649999	-0.621488	0.700000	-0.514573	0.750000	-0.415037
0.800000	-0.321928	0.850000	-0.234465	0.899999	-0.152003	0.950000	-0.074000	1.000000	-0.000000
1.049999	0.070388	1.100000	0.137503	1.150000	0.201633	1.200000	0.263034	1.250000	0.321927
1.299999	0.378511	1.350000	0.432959	1.400000	0.485426	1.450000	0.536052	1.500000	0.584962
1.549999	0.632267	1.600000	0.678071	1.650000	0.722465	1.700000	0.765534	1.750000	0.807354
1.799999	0.847996	1.849999	0.887524	1.900000	0.925999	1.950000	0.963474	2.000000	0.999999
2.050000	1.035623	2.099999	1.070389	2.150000	1.104336	2.200000	1.137503	2.250000	1.169924
2.300000	1.201623	2.349999	1.232660	2.400000	1.263034	2.450000	1.292781	2.500000	1.321928
2.550000	1.350497	2.599999	1.378511	2.650000	1.405797	2.700000	1.432959	2.750000	1.459431
2.800000	1.485426	2.849999	1.510961	2.900000	1.536052	2.950000	1.560714	3.000000	1.584962
3.050000	1.608808	3.099999	1.632267	3.150000	1.655351	3.200000	1.678071	3.250000	1.700439
3.300000	1.722465	3.349999	1.744160	3.400000	1.765534	3.449999	1.786596	3.500000	1.807354
3.550000	1.827818	3.599999	1.847996	3.650000	1.867876	3.699999	1.887524	3.750000	1.906890
3.800000	1.925999	3.849999	1.944858	3.900000	1.963474	3.949999	1.981852	4.000000	1.999999
4.050000	2.017921	4.100000	2.035623	4.150000	2.053111	4.199999	2.070388	4.250000	2.087462
4.300000	2.104336	4.350000	2.121015	4.400000	2.137503	4.449999	2.153805	4.500000	2.169924
4.550000	2.185866	4.600000	2.201633	4.650000	2.217230	4.699999	2.232660	4.750000	2.247927
4.800000	2.263034	4.850000	2.277984	4.900000	2.292781	4.949999	2.307428	5.000000	2.321928
5.050000	2.336283	5.100000	2.350497	5.150000	2.364572	5.199999	2.378511	5.250000	2.392317
5.300000	2.405992	5.350000	2.419538	5.400000	2.432959	5.449999	2.446256	5.500000	2.459431
5.550000	2.472487	5.600000	2.485426	5.650000	2.498250	5.699999	2.510962	5.750000	2.523561
5.800000	2.536052	5.850000	2.548436	5.900000	2.560715	5.949999	2.572889	6.000000	2.584962
6.050000	2.596934	6.100000	2.608809	6.150000	2.620586	6.199999	2.632267	6.250000	2.643856
6.300000	2.655351	6.350000	2.666756	6.400000	2.678071	6.449999	2.689279	6.500000	2.700439
6.550000	2.711494	6.600000	2.722465	6.649999	2.733354	6.699999	2.744161	6.750000	2.754887
6.800000	2.765534	6.850000	2.776103	6.899999	2.786595	6.949999	2.797012	7.000000	2.807354
7.050000	2.817623	7.100000	2.827818	7.149999	2.837943	7.199999	2.847996	7.250000	2.857980
7.300000	2.867896	7.350000	2.877744	7.399999	2.887524	7.449999	2.897240	7.500000	2.906889
7.550000	2.916476	7.600000	2.925999	7.649999	2.935459	7.699999	2.944858	7.750000	2.954196
7.800000	2.963473	7.850000	2.972692	7.899999	2.981852	7.949999	2.990954	8.000000	3.000000
8.050000	3.008988	8.100000	3.017921	8.149999	3.026799	8.200000	3.035624	8.250000	3.044393
8.300000	3.053111	8.350000	3.061775	8.399999	3.070389	8.450000	3.078951	8.500000	3.087462
8.550000	3.095924	8.600000	3.104336	8.649999	3.112699	8.700000	3.121015	8.750000	3.129282
8.800000	3.137503	8.850000	3.145677	8.899999	3.153804	8.950000	3.161827	9.000000	3.169924
9.050000	3.177917	9.100000	3.185866	9.149999	3.193771	9.200000	3.201633	9.250000	3.209453
9.300000	3.217230	9.350000	3.224965	9.399999	3.232660	9.450000	3.240314	9.500000	3.247926
9.550000	3.255500	9.600000	3.263033	9.649999	3.270528	9.700000	3.277984	9.750000	3.285401

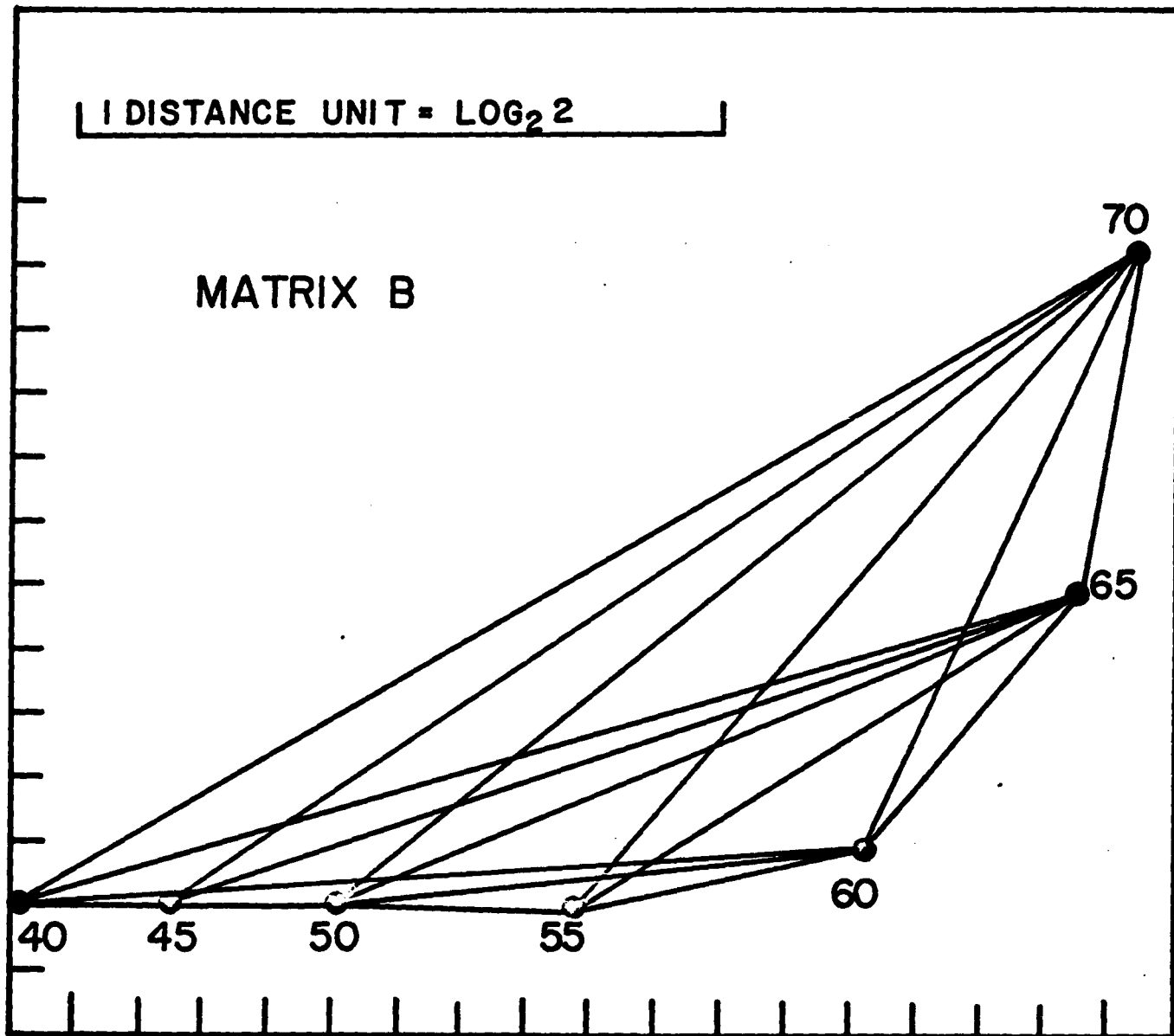


Fig. 19. Analysis of Proximities spatial configuration for Matrix B.

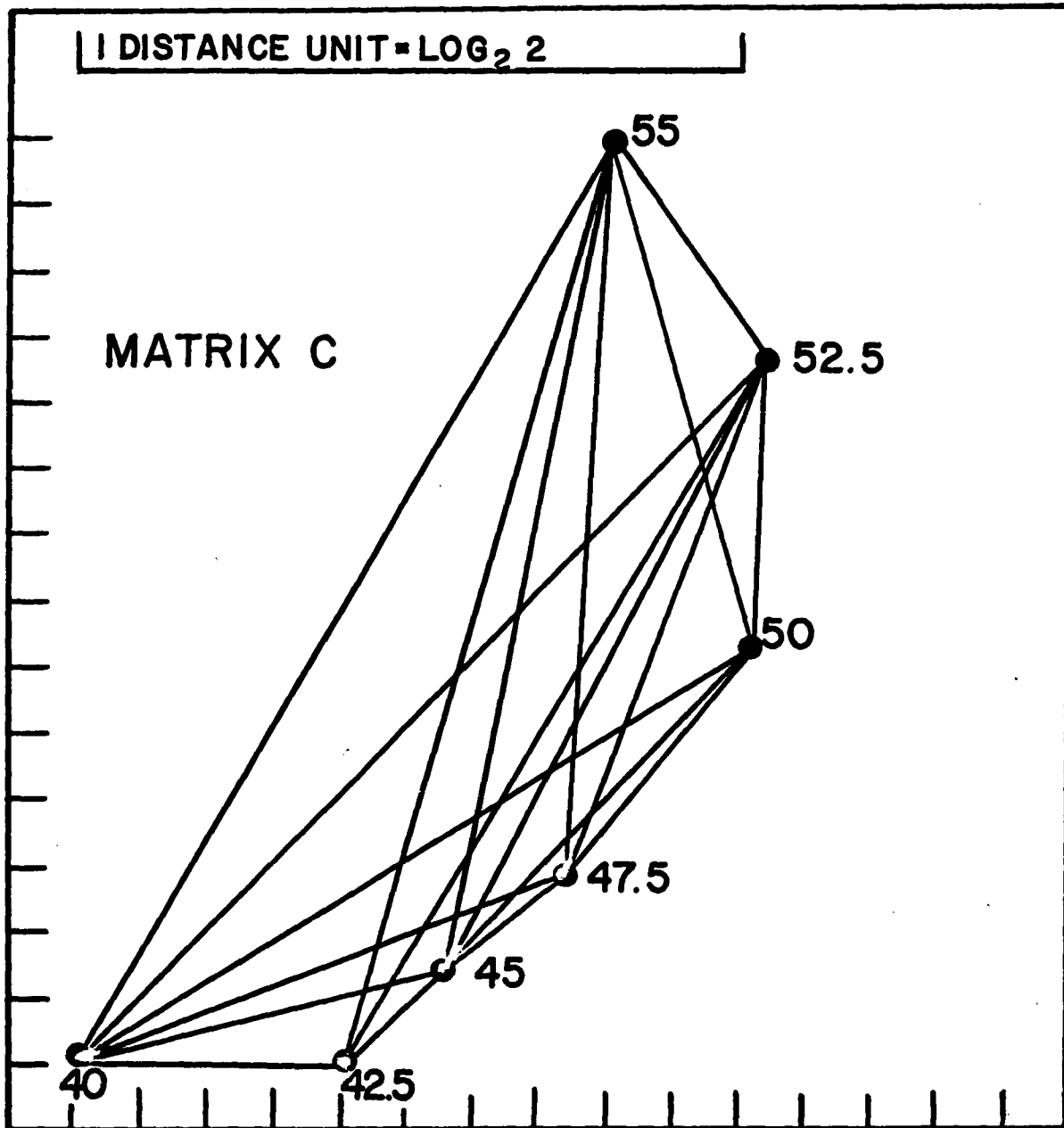


Fig. 20. Analysis of Proximities spatial configuration for Matrix C.

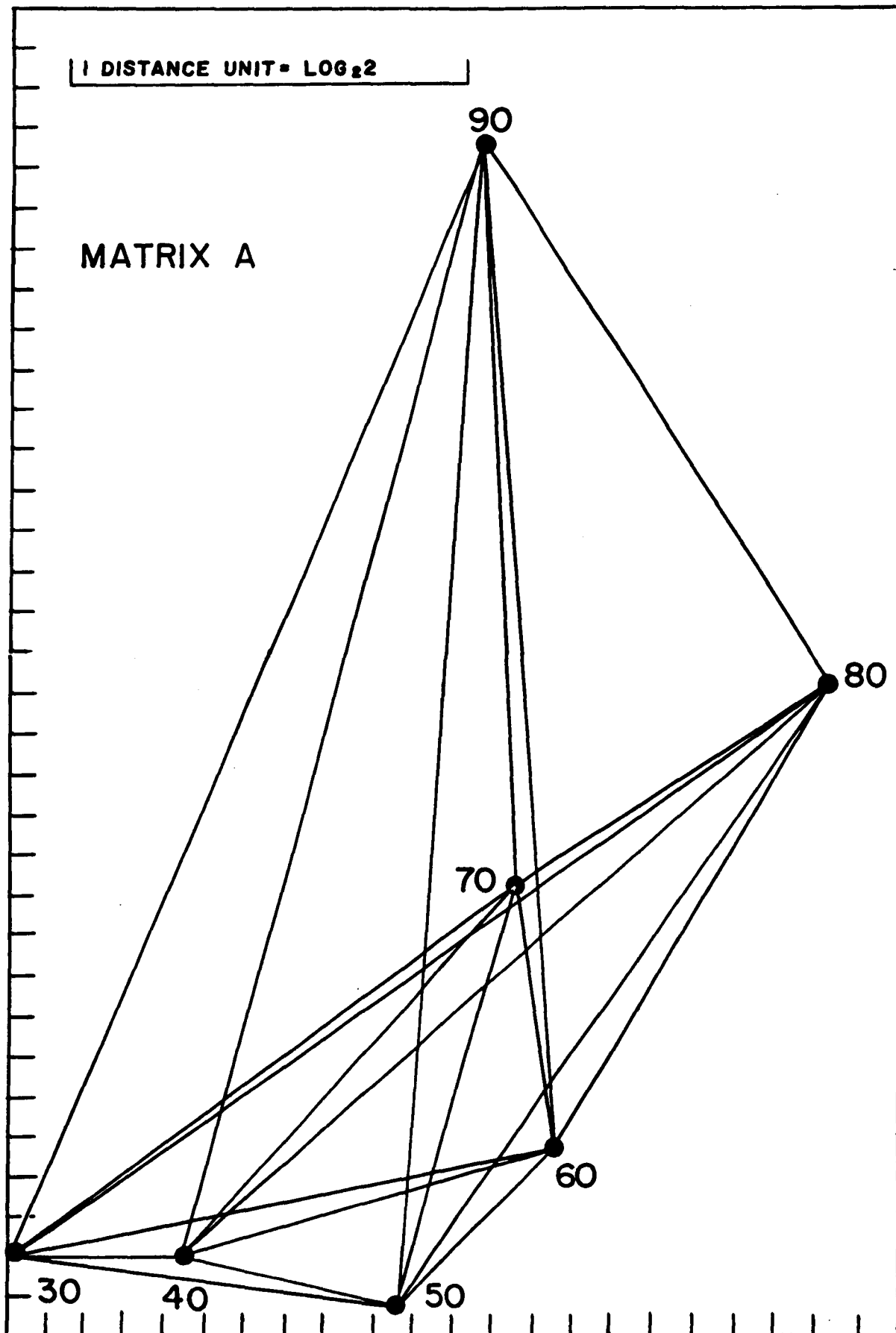


Fig. 21. Analysis of Proximities spatial configuration for Matrix A.

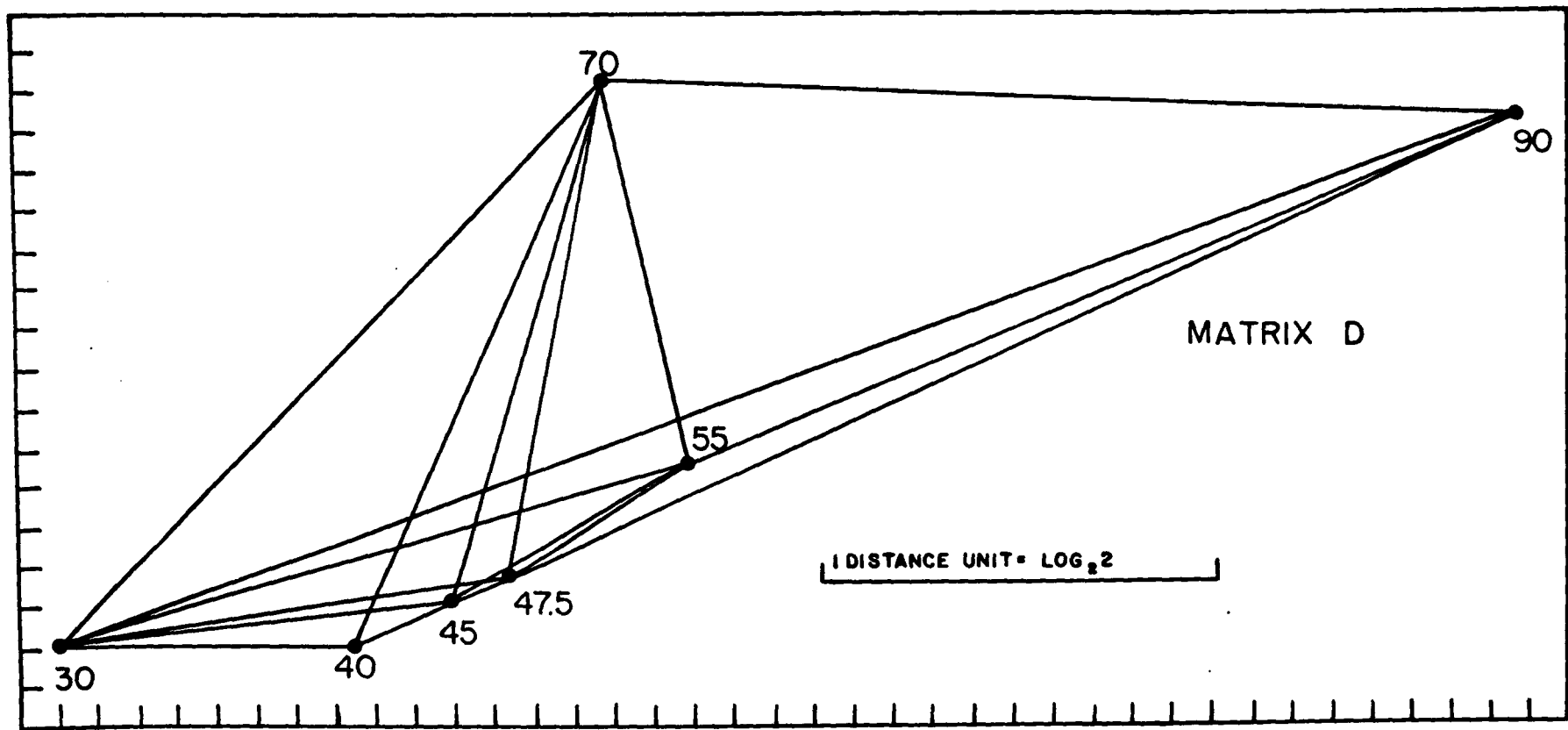


Fig. 22. Analysis of Proximities spatial configuration
for Matrix D.

sums of all previous log-ratio estimates, but somewhat less. This, non-additivity increases as the stimulus intensity increases. It is further seen that the loudness estimate between 60 and 70 dB is not the sum of the log-ratio estimates 60-65 and 65-70 dB, but less. The distance between each of the adjacent stimulus points from 40-70 dB increases, indicating that the loudness at 5 dB intervals increases with increasing intensity.

Figure 20 is the proximity plot for Matrix C. The form of the configuration is similar to that found for Matrix B. Additivity exists from 42.5-50 dB SL, but outside that region the figure bends, indicating non-additivity of the loudness judgments at the higher matrix intensities. The ratio estimate made between 50-55 dB is not the sum of the log-estimates 50-52.5 and 52.5-55 dB. Also, there is a general increase in the distance between adjacent 2.5 dB intervals.

Figure 21 shows the results for Matrix A. The configuration appears to be different for those found for Matrices B and C in that the regular upward pattern appears to be broken at 70 dB. An additive (linear) relation is evident, however, between 30 and 50 dB SL. Regarding those stimulus points between 30 and 70 dB, it is seen that a similarity exists between these five points and the figures obtained in Matrices B and C.

Figure 22 is the proximity plot generated by the data

in Matrix D. Between 30-70 dB a pattern similar to Matrices B and C is seen, with the additive section occurring between 40-55 dB SL. When comparison is made between the distances found for common stimulus pairs in Matrices A and D, the distances are always greater for Matrix D. Further, Table 21 shows that the distance difference increases as the inter-stimulus difference increases.

In order to plot the obtained loudness estimate results as a unidimensional function of dB SL, which, in turn, would yield complete additivity if plotted as a proximity plot, several process were initiated.

To understand the processes involved, it is instructive to view each 7x7 matrix (A-C) as a double classification Analysis of Variance (as in the β_1 results) with one sampling unit per cell. In the present schema, one variable consists of the seven louder tones (designated $\lambda_1 - \lambda_7$), and the other variable consists of the seven lower tones (designated as $\rho_1 - \rho_7$). Using this paradigm each cell entry can be viewed as consisting of three components: (1) a deviation of the row mean from the grand matrix mean, (2) a deviation of the column mean from the grand matrix mean, and (3) a residual interaction term. Thus, $\lambda_1 - \lambda_7$ and $\rho_1 - \rho_7$ represent the column means and row means, respectively.

To insure complete additivity in space two requirements had to be met. The first was that the interaction sum of squares of each matrix had to be reduced to zero, and the

Table 21
Comparison of Distances in Matrices A and D
for Common Stimulus Pairs

Common Pair	Distance Matrix A (DU)	Distance Matrix D (DU)	Distance Difference (D-A)
30-40	0.45	0.75	0.30
30-70	1.55	2.00	0.45
30-90	3.00	3.90	0.90
40-70	1.23	1.56	0.33
40-90	2.84	3.25	0.51
70-90	1.85	2.30	0.45

second that corresponding estimates of λ and ρ had to be equal and opposite in sign, thus setting the matrix diagonal to zero.

To accomplish these requirements, a two-stage process was initiated. The first stage consisted of using Kruskal's (1968) Monanova technique on the \log_2 values of each matrix. Since this analysis is designed to reduce the interaction sum of squares to zero (or nearly so), the first requirement for additivity was met. The second stage of the process involved a modification of the Monanova analysis (through an iterative technique) which set corresponding estimates of λ and ρ equal and opposite in sign. This second operation fulfilled the second requirement for additivity.

Figure 23 illustrates the above processes by the use of a fictitious 4x4 stimulus matrix, which shows how additivity was attained. In the figure each of the requirements for a linear proximity plot have been met, i.e., diagonals of the matrix set to zero, and all corresponding λ and ρ values equal and opposite. It is also assumed that the interaction sum of squares for the matrix has been reduced to zero, so that each cell value is simply the sum of its respective main effects. For example, the 30-60 dB SL stimulus combination (ratio estimate =4.0) is the sum of the main effects λ_3 and ρ_1 , or 1.0+3.0. As stated, if this matrix were plotted as an Analysis of Proximities, a linear proximity plot would result because the ratio estimate found within each of the

		dB SL Louder Tone				
		30	45	60	70	
dB SL Lower Tone	30	0	2	4	6	$\rho_1 = +3$
	45	-2	0	2	4	$\rho_2 = +1$
	60	-4	-2	0	2	$\rho_3 = -1$
	70	-6	-4	-2	0	$\rho_4 = -3$
		$\lambda_1 = -3$	$\lambda_2 = -1$	$\lambda_3 = +1$	$\lambda_4 = +3$	

Fig. 23. Illustration of matrix requirements for linear proximity plots.

cells would be equal not only to its main effects (as above), but also to its component ratio estimates. Thus, the loudness relation is completely additive in nature.

Figures 24-26 show the lambda (λ) value results for Matrices A-C as a function of dB SL. Also shown is the subjective loudness in $\log_2 A$ (additive) units. This latter scale represents a simple transformation of the lambda ordinate which was included to make the obtained data more comparable with previously reported unidimensional loudness scales, which commonly assign a 40 dB SL stimulus the scale value "1", i.e., the sone scale. Thus, in each of the figures the λ value which corresponded to 40 dB SL was arbitrarily assigned the value "1" (or "0" in \log_2 terms), and the remainder of the scale with reference to this point. Thus, a direct comparison between the power function based upon the 10 dB rule (with 40 dB SL = 1.0 as a reference point) and the present additive scale could be accomplished.

Figure 24 indicates that the $\log_2 A$ (additive) scale does not approximate the conventional 10 dB rule results for Matrix A over the entire intensity continuum. However, the loudness from 80-90 and 30-40 dB SL does appear to grow at approximately the same rate.

Figure 25 (Matrix B) shows that both the conventional power function based on the 10 dB rule and the $\log_2 A$ scale

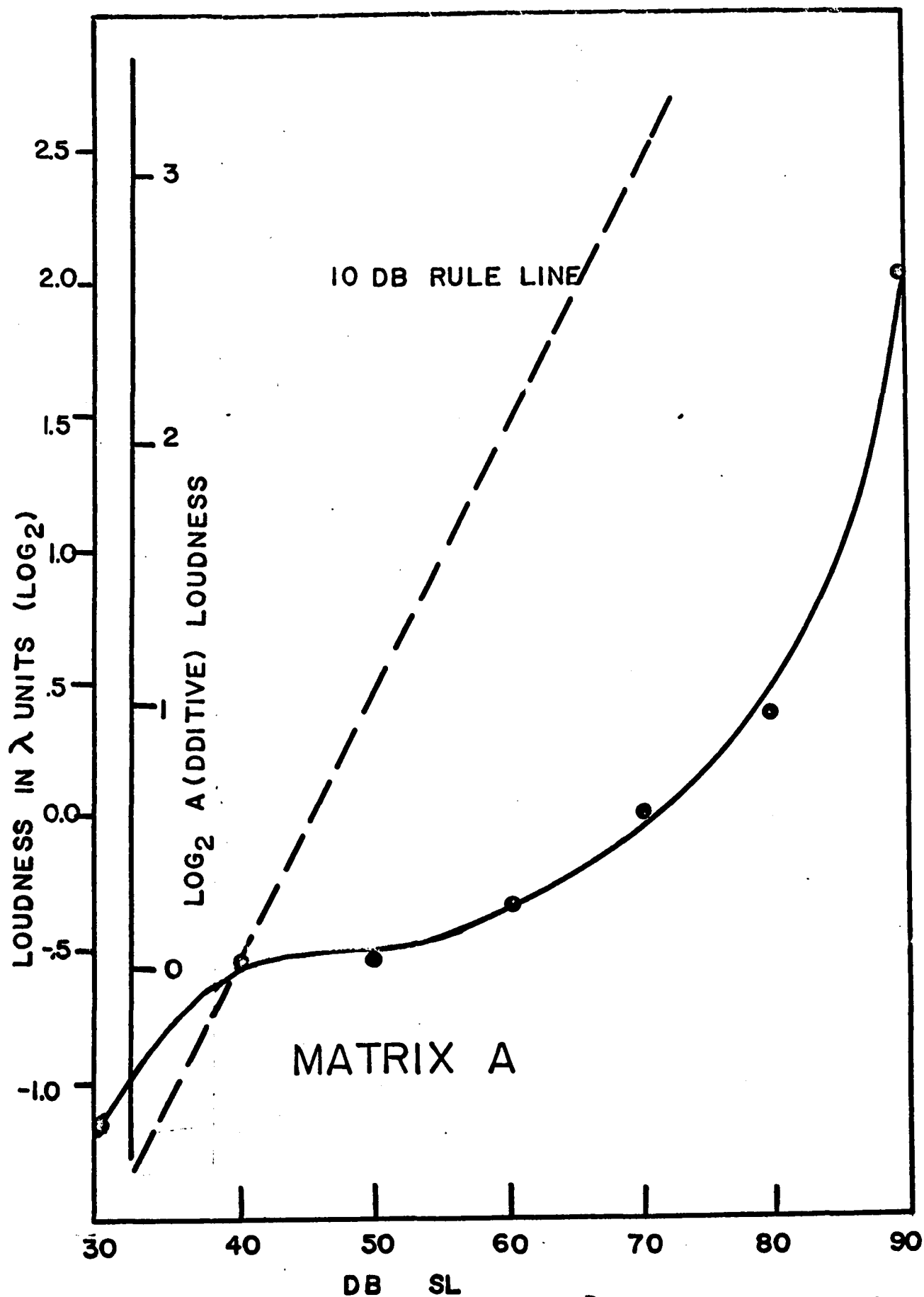


Fig. 24. Subjective loudness in lambda (λ) units and log₂A units as a function of dB SL for Matrix A.

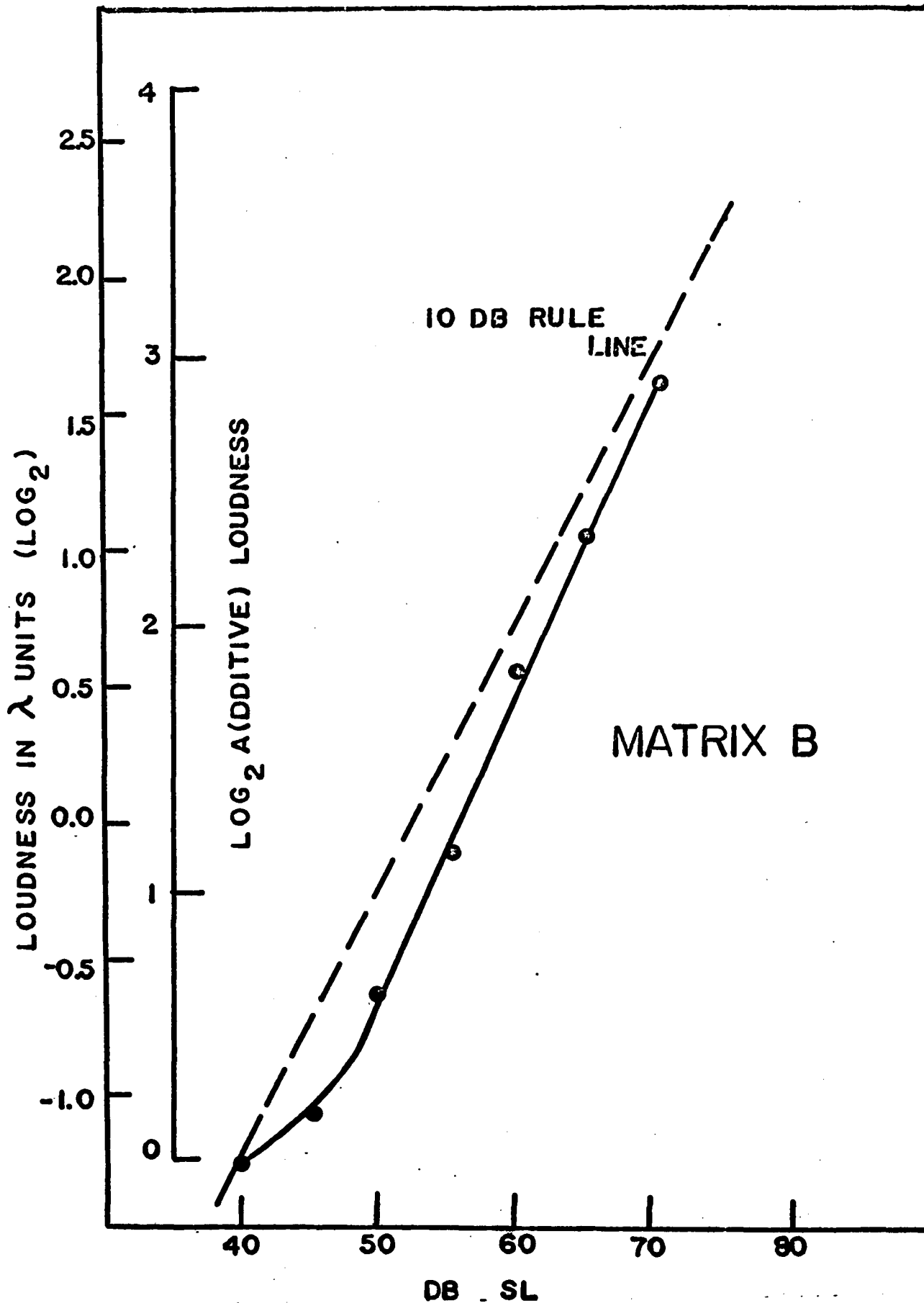


Fig. 25. Subjective loudness in lambda (λ) units and $\log_2 A(DDITIVE)$ units as a function of dB SL for Matrix B.

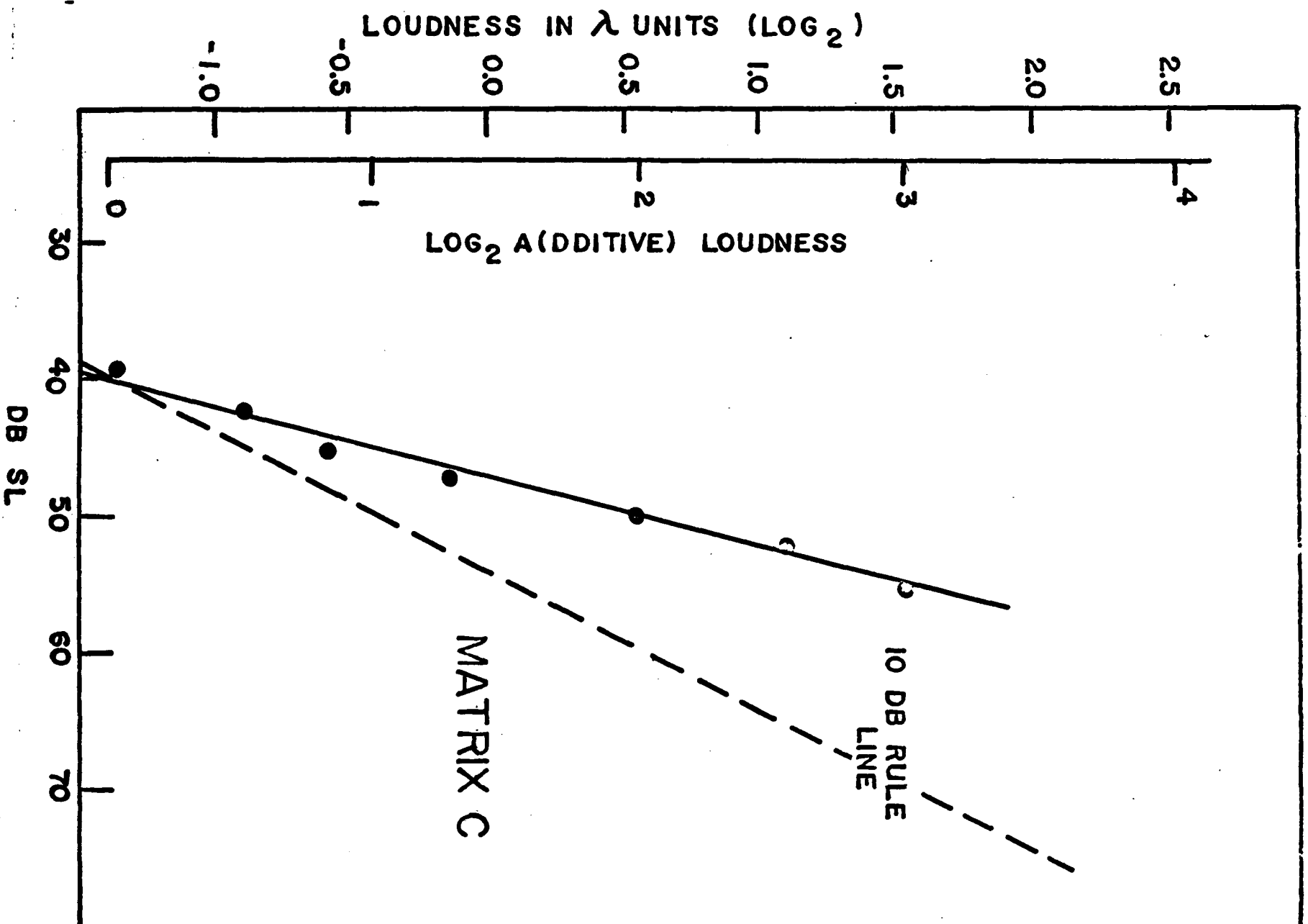


Fig. 26. Subjective loudness in λ units and $\log_2 A(DDITIVE)$ units as a function of dB SL for Matrix C.

grow at the same rate, although the additive units are displaced toward the higher intensities by approximately 4 dB, and show a flatter function between 40 and 45 dB.

Figure 26 shows that the additive function for Matrix C approximates a power function (as does Matrix B), whose slope is greater than that of the conventional function.

Figures 31-35 (Appendix) show the individual lambdas (λ) and $\log_2 A$ values for each subject in Matrices A-C. In both Matrices B and C only the data points are plotted because all subjects show nearly equivalent λ values. Thus, the slopes of the individual subject functions do not differ appreciably. The results for Matrix A are a bit more variable. Therefore, the individual subject functions are plotted (Figures 31-33). It is seen that subjects 1-8 in Matrix A exhibit functions which are quite similar in that they generally accelerate at the same rate. Subjects 9 and 10 differ from the pattern somewhat, although their results are not greatly different from the other eight subjects.

Experiment II. Tables 22-25 show the overall mean ratio estimates for Matrices E-H, respectively. These values were obtained by taking the reciprocals of the Tone A re Tone B session results, and averaging these values with the results of the Tone B re Tone A session. To convert all values to 1.00 or greater, the reciprocals of the fractional estimates were taken, and included within the grand average.

Table 22

Grand Means for Matrix E (250 Hz)

	DB SL Louder Tone						
	10	20	30	40	50	60	70
	10	20	30	40	50	60	70
DB SL Lower Tone	1.00	1.36	1.81	2.20	2.80	3.42	4.94
20		1.00	1.17	1.75	2.30	2.87	4.30
30			1.00	1.22	1.93	2.52	4.09
40				1.00	1.42	2.19	3.70
50					1.00	1.45	2.94
60						1.00	2.19
70							1.00

Table 23

Grand Means for Matrix F (White-Noise)

	DB SL Louder Tone						
	40	45	50	55	60	65	70
	40	45	50	55	60	65	70
DB SL Lower Tone	1.00	1.30	1.68	2.35	2.63	2.89	3.02
45		1.00	1.27	1.74	2.20	2.47	2.61
50			1.00	1.25	1.94	2.25	2.23
55				1.01	1.22	1.53	1.71
60					1.02	1.09	1.25
65						1.00	1.04
70							1.00

Table 24

Grand Means for Matrix G (4000 Hz)

	DB SL Louder Tone						
	10	15	20	25	30	35	40
DB SL Lower Tone							
10	1.03	1.26	1.64	1.97	2.14	2.55	2.93
15		1.00	1.24	1.64	1.88	2.11	2.44
20			1.01	1.23	1.49	1.71	2.19
25				1.01	1.13	1.53	2.09
30					1.00	1.19	1.69
35						1.00	1.18
40							1.00

Table 25

Grand Means for Matrix H (1000 Hz)

	DB SL Louder Tone						
	10	15	20	25	30	35	40
DB SL Lower Tone							
10	1.00	1.23	1.40	1.82	2.11	2.40	2.48
15		1.00	1.13	1.56	1.83	2.08	2.60
20			1.00	1.18	1.70	1.97	2.33
25				1.00	1.22	1.68	2.12
30					1.01	1.28	1.94
35						1.00	1.33
40							1.00

Matrix E (Table 22) shows the results obtained when the 250 Hz tones were judged from 10-70 dB SL. The largest estimate was between the 10 and 70 dB SL stimuli (4.94). As the level of the louder tone increased, the ratio estimates for each of the lower intensity rows increased. No reversals between adjacent cells were evidenced.

Table 23 gives the analogous data for the white-noise stimuli (Matrix F) which were judged in 5 dB steps from 40-70 dB SL. It is seen that the highest obtained ratio estimate was 3.02 for the 40-70 stimulus combination. A regular progression in the values is seen as the level of the louder tone increases.

Tables 24 and 25 indicate the results of both the 1000 and 4000 Hz tones judged from 10-40 dB SL in 5 dB steps. A comparison between the two shows that the loudness of the 4000 Hz tone is greater when the stimulus pair 10-40 dB SL is judged (2.93 v. 2.48). It is also seen that the 4000 Hz stimuli are (except in cell 16) are always judged greater than the 1000 Hz tones in those cells containing a lower stimulus intensity of from 10-20 dB SL. However, when the level of the lower stimulus reaches 25 dB and higher, the relationship changes, and the judgments for the 1000 Hz pairs are greater.

Figures 27-30 show the obtained Analysis of Proximities spatial configurations for Matrices E-H. Figure 27 shows the relations for the 250 Hz tones. As in Matrices B and C,

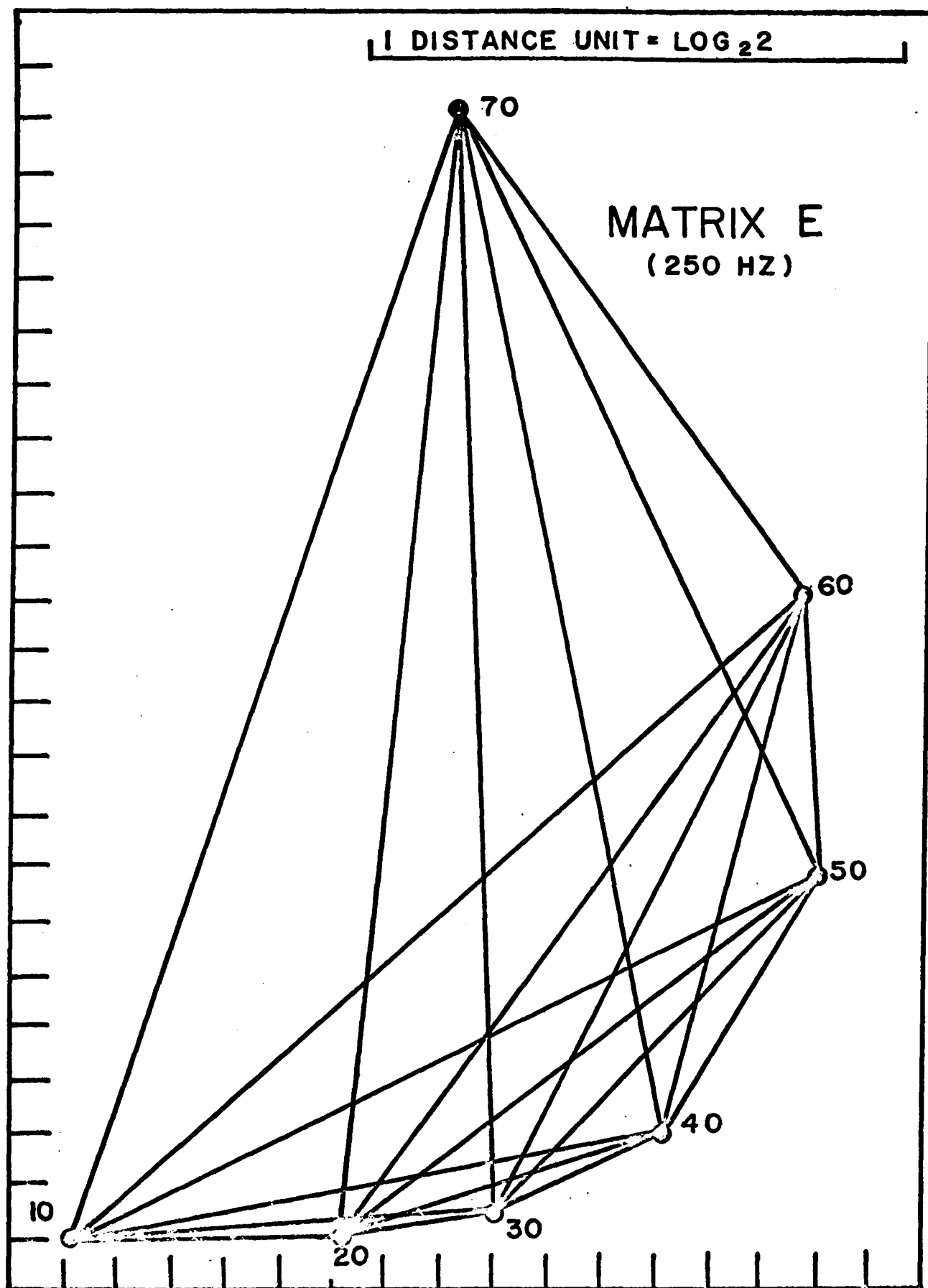


Fig. 27. Analysis of Proximities spatial configuration for Matrix E (250 Hz tones).

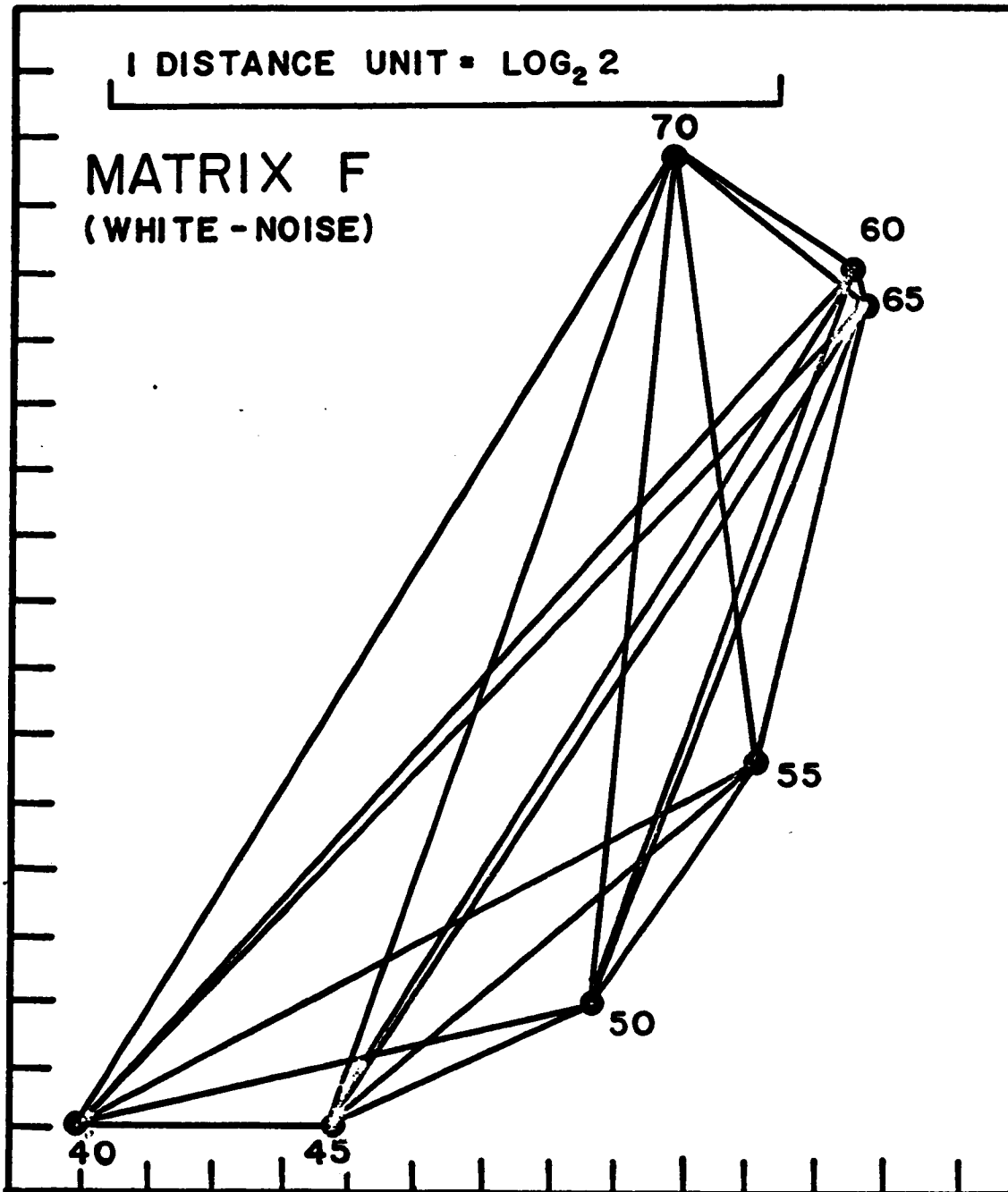


Fig. 28. Analysis of Proximities spatial configuration for Matrix F (white-noise).

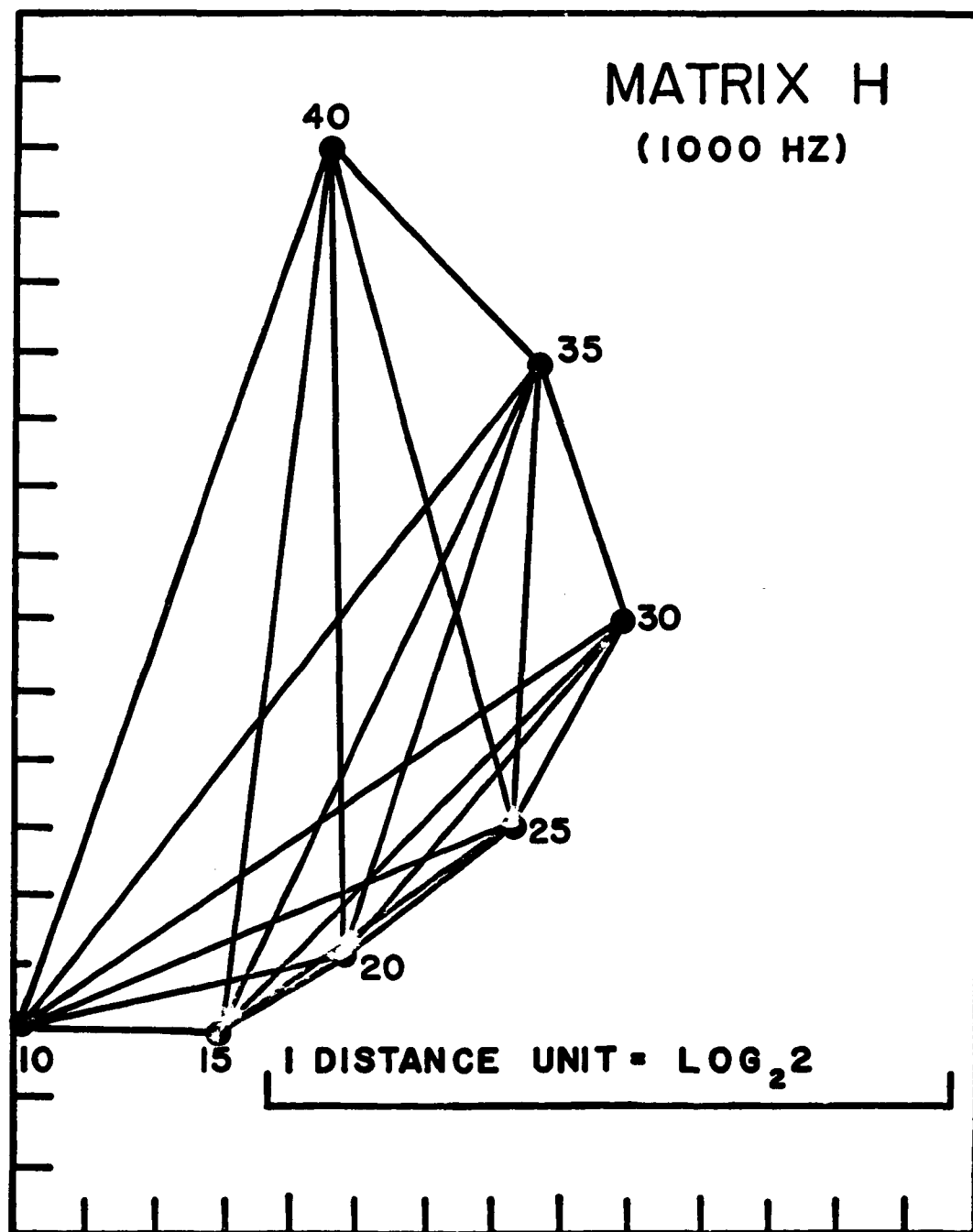


Fig. 29. Analysis of Proximities spatial configuration for Matrix H (1000 Hz tones).

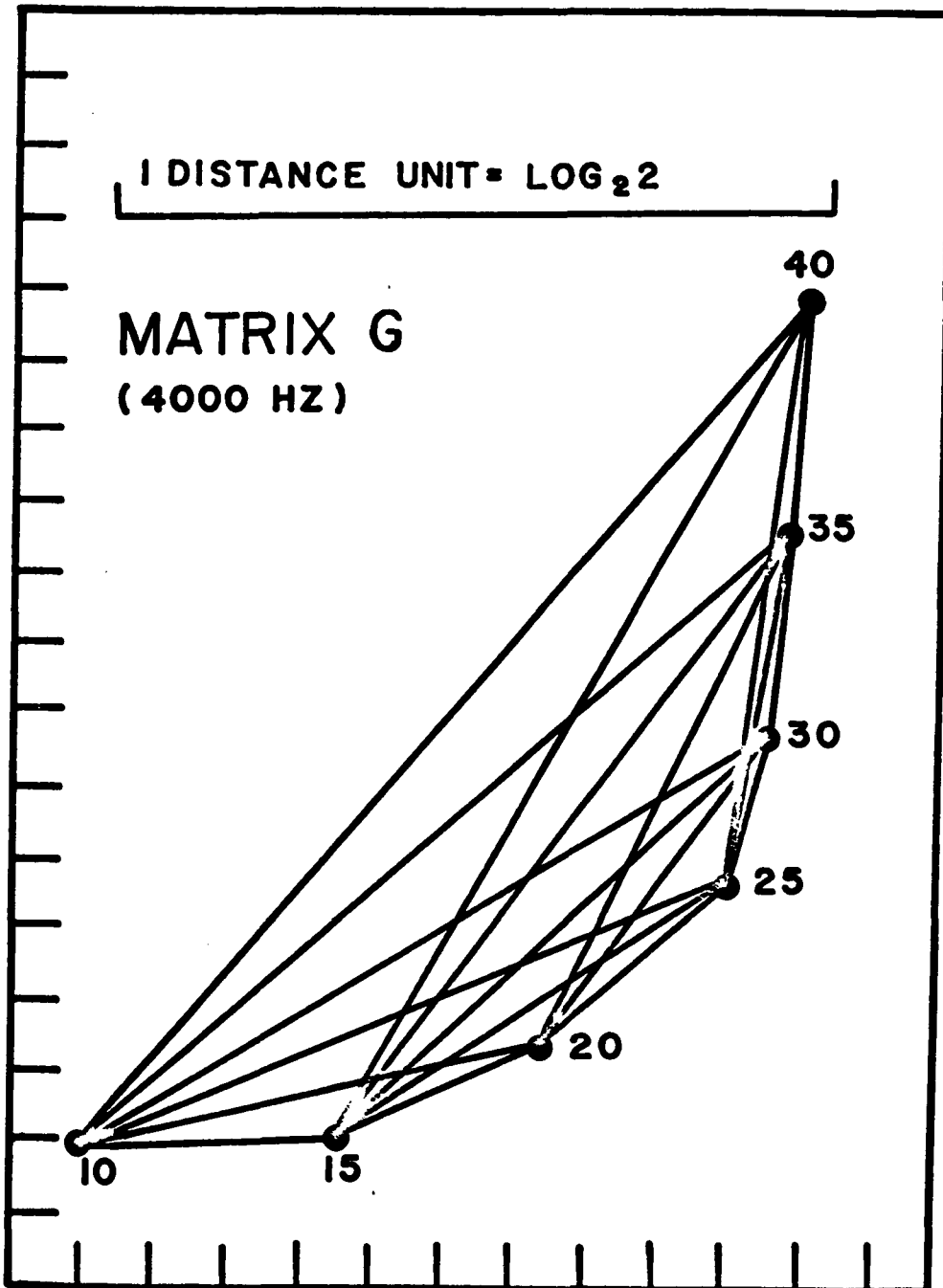


Fig. 30. Analysis of Proximities spatial configuration for Matrix G (4000 Hz stimuli).

the configuration curves upwards after an area where additivity prevails. In this instance, additivity is seen from approximately 10-35 dB SL. The log-ratio estimates are not additive in nature above 40 dB.

Figure 28 (white-noise) shows some peculiarities not evident in the regular configurations generated by most of the foregoing matrices. The most obvious difference is that the 65 and 70 dB SL points reverse themselves. However, this results could be due to experimental error. Additivity is predominant from approximately 52-70 dB SL.

Figures 29 and 30 show the configurations obtained for the 1000 Hz and 4000 Hz tones under the same low sensation level conditions. Both figures show regular patterns, although a comparison between the two shows some differences. The main difference lies in the points above 25 dB SL. At these intensities, the configuration for the 1000 Hz tones bends back, indicating less additivity in the log-ratio estimates between distant inter-stimulus pairs. On the other hand, the points above 25 dB for the 4000 Hz judgments are additive from 25-40 dB SL. Between 15 and 25 dB SL both figures show additivity of the log-ratio estimates.

CHAPTER IV

DISCUSSION

IV. DISCUSSION

Although the present paper has been quite comprehensive in scope, several main points may be made. Therefore, the initial section of the present chapter is devoted to a general summary of the major study objectives and findings. Detailed analyses of these findings, and their implications may be found later in the discussion under specific sub-headings.

Summary of Major Objectives and Findings

As stated, the primary purpose of the present investigation was to explore whether the 10 dB rule (or the 0.54 power function, which approximates the 10 dB rule at 40 dB SL and above) provided an adequate description of loudness relations for stimulus pairs judged in 7x7 stimulus matrices. The vehicle for judging the adequacy of this unidimensional loudness relation was the modified Analysis of Proximities, which, through the log-transformation, defined one distance unit on each of the obtained proximity plots as the \log_2 of the ratio estimate. This transformation insured that if the 10 dB rule held for the obtained ratio estimates, a straight line proximity plot would be obtained. In this regard, every doubling of loudness at 10 dB intervals would be equivalent to the addition of one distance unit in space.

Obviously, the obtained spatial configurations for Matrices A-C (as well as the other stimulus conditions) are not straight lines. Therefore, the predictions made from the 10 dB rule regarding ratio estimates between various points on the intensity continuum do not provide an adequate description of the obtained judgments. However, it should be noted that each of the obtained plots contain a region of relative additivity before the configuration curves upwards. Also, as will be further explored in a subsequent section, the inter-stimulus differences in this additive section are more similar to the estimates implied by the 10 dB rule as the stimulus range of a matrix is decreased to moderate ranges.

Another major consideration of the study was to transform the obtained data so as to yield a unidimensional function relating judged loudness to intensity, which, in turn, would provide a linear proximity plot. This goal was met by using the *Monsenne* technique combined with the transformation of each of the matrix diagonals to zero.

The results of the above process yielded the $\log_2 A$ (additive) loudness scale (described earlier), which could be directly compared to the unidimensional power function based on the 10 dB rule. Regarding these data, the major finding was that for Matrices B and C, the unidimensional $\log_2 A$ plots were power functions of the stimulus intensity.

The slope of the Matrix B scale was quite similar to that of the conventional power function, and the slope for Matrix C was greater than the conventional rule function. The Matrix A data did not yield a power function, and the shape of its unidimensional plot was complex. Generally, however, the growth of the plot for Matrix A was much slower than for the conventional 10 dB rule function.

A consolidation of both the Analysis of Proximities and the $\log_2 A$ data indicate that the overall range of a stimulus matrix is quite influential in determining how loudness-ratios grow, and that the more constricted the range, the more rapid the loudness-ratio growth. In general, it appears that when the stimulus range reaches approximately 30 dB, the results obtained using the present technique compare quite closely to those implied by the conventional unidimensional scale implied by the 10 dB rule.

Relation Between Matrix A Ratio Estimate Values
and Those Predicted from the 10 dB Rule

Matrix A was designed to investigate how ratio estimates of loudness compared to predicted estimates (as implied by the 10 dB rule) over a wide stimulus range.

The most significant outcome regarding the obtained data for Matrix A is the large deviation between the predicted and observed values. The obtained data are always

below the predicted values, with the differences between the two increasing as the inter-stimulus differences increase. The largest ratio estimate (7.90) falls far short of the predicted estimate of 64.00 for the 30-90 dB SL stimulus combination.

An explanation of the incongruity between the present data and those found by other investigators may be accounted for by the fact that the overwhelming majority of studies investigating loudness have concentrated on only moderate intensity intervals, with little attention being paid to loudness estimates over longer or shorter distances. In this regard, many of the more influential studies have been based almost entirely on halving or doubling determinations (Robinson, 1957; Stevens, 1955, 1957). McRobert, Bryan, and Tempest (1965) have even noted that in those studies which have tried to reduce the constraints to a minimum (magnitude estimation without a modulus), in effect, do not measure loudness over wide inter-stimulus intervals. This situation arises because all the estimates are made within a short time period, and as each estimate is made, another reference point is created. Therefore, a subject's later responses are made with reference to the nearest estimates, rather than to the original stimulus, i.e., the first tone in the experiment. "This is important in the case of levels which are far from the reference level [original tone], since the observer may make his judgment

with reference to the nearest tone he has so far heard, rather than to the reference tone. By this process all his judgments are reduced to judgments of small intervals and the results of the experiment do not really answer the question, which is basic to any loudness scale, of whether the observer can make self-consistent estimates of large and small intervals" (p. 393).

In one of the only studies dealing with loudness estimates over larger inter-stimulus distances, McRobert et. al. (1965) obtained data similar to those found in the present study. Fifteen pairs of stimuli at 1000 Hz were presented, and the task of the subject was to estimate the loudness of the second tone relative to that of the first tone. Unlike the present study, the intensity of the second tone was always greater than the first. The largest inter-stimulus difference was 50 dB (30-80 dB SPL). Their data indicated that when the inter-stimulus difference was approximately 15 dB or less, the ratio estimates were similar to those predicted by the conventional power function. However, when the inter-stimulus difference increased beyond 15-20 dB, the ratio estimates became increasingly more divergent from the conventional scale predictions. They concluded that "although the "sone" type of scale relating loudness to intensity (at 1000 c/s) represents an average of the data available, almost all of the published data has in fact been effectively obtained over small intervals (up to

about 20 dB) on intensity. The question of validity of the scale over larger intervals has been left quite open. The present data, together with that of Stevens and Poulton (1956) [They used Stevens & Poulton (1956) to compare their data with 7, suggests that the "sone" type of scale is a poor approximation to the true relation between loudness and intensity over wide ranges..." (p.399).

Another important relation to be found in the present data for Matrix A (Table 3), and in those obtained by McRobert et. al. (1965) concerns the relation between ratio estimates obtained over large inter-stimulus intervals, and the sum of the component ratio estimates contained within those intervals. McRobert compared the sum of several ratio estimates comprising a given inter-stimulus interval with a direct estimate of the end points of that interval. Their results indicated that the sum of the component estimates were always greater than the direct estimate. The same relationship may be seen in Table 3. For example, the 30-90 dB SL stimulus pair yields a mean ratio estimate of 7.90 when judged directly, although the sum of its component estimates (30-40, 40-50, 50-60, 60-70, 70-80, and 80-90 dB SL) is only 10.77.

Relation Between Ratio Estimates Obtained for
Matrices A, B, and C

An examination of the results for Matrices A, B, and C (Tables 3-5) indicates that as the stimulus range approaches 30 dB the obtained ratio estimates become more like the 10 dB rule predictions. One index of this process is the ratio estimate associated with common stimulus pairs within each of the matrices. As described previously (Table 7), the ratio estimate increases for each of the common pairs as the overall stimulus range of a matrix is reduced, and the Matrix B values for each of these common pairs are more similar to the 10 dB rule predictions than either Matrices A or C.

Another index of the convergence of the data to 10 dB rule predictions with increasing constriction to 30 dB is an examination of those cells (Tables 3-5) where the inter-stimulus difference is 10 dB. When this is accomplished it is seen that the median value for the 10 dB difference in Matrix A is 1.53, 2.03 for Matrix B, and 2.36 for Matrix C. Since the 10 dB rule predicts a ratio estimate of 2.00 for a 10 dB inter-stimulus difference, the median for Matrix B was, virtually, equal to the predicted value.

A final index of the approach to 10 dB rule predictions using a 30 dB stimulus range can be seen in Figures 6-8. If these figures were plotted as the conventional 0.54 power function would imply, each of the figures would have six parallel lines with a slope of 0.54. The median slope of the obtained functions for Matrix B is 0.54.

Variability of Ratio Estimates. The results of the Analyses of Variance (Tables 11-13) lend support to several investigations which have studied individual variability of loudness estimates of time. Generally, the results of these analyses for Matrices A-C show that ratio estimates do not differ when the judgments are made over several different days. A mean of only 7% of the subjects for all matrices showed significance on the Sessions factor. These results support McGill (1960), who found that an individual's loudness function will not differ greatly over a period of at least one week. These results also support the 0.53 correlation coefficient obtained by Stevens and Guirao (1964), who correlated the power function exponents obtained from an individual on two separate occasions.

The common practice of using naive subjects in experiments concerning loudness perception is supported by the low percentage of subjects showing statistical significance for both the Sessions and Replications factors (Tables 11-13). However, twenty-five percent of the subjects did show significance on the Replications factor for Matrix A. Since this was the first matrix presented to the subjects, a small learning factor effect may have been present. However, the influence of such an effect appears to be rather small. The above findings support studies by J. C. Stevens and Tulving (1957), Stevens and Poulton (1956), and McRobert et. al. (1965), who all showed that naive subjects could give loudness

estimates which differed in no significant manner from those reported by more sophisticated observers.

A primary finding concerning the variability of the ratio estimates between subjects is that it becomes larger as the inter-stimulus differences increase (Tables 8-10). This relation was expected in light of the findings of McGill (1960) and McRobert et. al. (1965), who both found increasingly variable estimates for individuals with increased intensity.

Analyses of Proximities for Matrices A-D

The underlying cause of the characteristic upswing in each of the proximity plots, and the divergence from a purely additive relationship is based upon the fact that loudness over long inter-stimulus intervals is not the sum of its component loudnesses, as the unidimensional scale based upon the 10 dB rule would imply. As alluded to previously, McRobert et. al. (1965) recognized this relationship, although they did not have the vehicle to present the data in concise form. In contrast, the Analysis of Proximities adequately describes the obtained ratio estimates made over wide inter-stimulus differences, as well as the sum of the individual component estimates.

Although showing the characteristics inherent in the other spatial configurations, i.e., additivity at the

lower intensities and an upward curvature, Matrices A and D differed significantly in shape from the other configurations.

As noted, Matrix A was different from Matrices B and C in that the regular upward pattern appeared not to be maintained at 70 dB SL (Fig. 21). This appearance, however, may be misleading, for the familiar pattern is maintained if one regards the points from 30-70 dB. These five points show the usual additive relation between 30-50 dB, and then upward swing.

What appears to be different at 70 dB may, in fact, be caused by the relationship between all the 80- and 90 dB SL stimulus combinations. Evidence attesting to this relationship may be seen in the data for Matrix A (Table 14) as compared to the analogous β increment data for Matrices B and C (Tables 15-16). Generally, in Matrices B and C, when the louder tone leads the lower tone in time, an increment is added to the grand mean of the cell to account for the obtained ratio estimate. Unlike the other two matrices, the obtained β values for Matrix A are consistently negative when the intensity of the louder tone reaches 80 dB SL. Therefore, it appears likely that the placement of the 70 dB point in space was influenced by what might be termed a "differential high intensity response mode," which was in juxtaposition to the mode of response at the

lower intensities.

The reason governing the rather irregular appearance of Matrix D was, of course, the disproportionately loud 90 dB SL stimulus, which acted as an "anchor" (Woodworth & Schlosberg, 1954). The quantitative effects of this anchor can be seen by regarding Table 21, which shows the obtained differences for common stimulus pairs in Matrices A and D. In all instances, the distances are greater for the Matrix D configuration.

Unidimensional Scaling of Matrices A-C

Figures 24-26 indicate that unidimensional functions may be found for the data contained in Matrices A-C, and it is thus possible to plot each of these matrices as a linear proximity plot.

When each of the $\log_2 A$ scales are compared directly to the scale implied by the 10 dB rule, a rather clear relation is seen, which, once again, supports the hypothesis that when the stimulus range is in the vicinity of 30 dB, the ratio estimates approach conventional predictions. This is evidenced by the minimal transformation necessary to obtain the conventional power law predictions from the $\log_2 A$ scale for Matrix B.

Ratio Estimates and Analysis of Proximity Plots
for 250 Hz and White-Noise Stimuli

The obtained ratio estimates for 250 Hz and white-noise stimuli (Tables 22-23) are low relative to what would be predicted from their respective power function exponents (as found by previous investigators). Also, both spatial configurations show the regular upward pattern evidenced in the previous matrices, as well as show areas of relative loudness additivity at the lower matrix intensities.

The configuration for Matrix F (white-noise) is somewhat irregular in that a reversal of points occurs when the noise reaches approximately 60 dB. However, this does not affect the overall form of the configuration, and may simply be due to experimental error.

An important aspect of the proximity plots for the 250 Hz and white-noise stimuli, as well as the other matrices, is that local areas of the plots can generally be described in terms of the sum of the local log-ratio estimates. For example, a general approximation to the distance between 50-70 dB SL for the 250 Hz tones can be obtained by taking the sum of the log-ratio estimates of the 50-60 dB and the 60-70 dB intervals. Although the sum in most instances will be greater than the direct estimate, a fair approximation can be made over a limited range.

Constraints Imposed by the Method

The present technique of using ratio estimates as the primary measure of the perceived loudness difference between two points on the intensity continuum sought to minimize the constraints placed upon a subject. Since Zwislocki (1967), Hellman and Zwislocki (1961), and Stevens (1956) have noted that minimizing the constraints causes the obtained loudness functions to be less variable, the present subjects were free to choose whatever ratios they thought appropriate. This, of course, is in juxtaposition to the Ratio Production methods (Fractionation and Multiplication), where S is required to satisfy some specified ratio, and to the Magnitude Estimation procedure which includes a prescribed modulus. As described in the Introduction, all of the above techniques are subject to biases which are introduced via the imposed constraints. The present procedure eliminated the biases produced by a fixed relation between any stimulus and a given modulus, and did not specify any particular ratio to be fulfilled.

One constraint which was placed upon a subject, however, concerned the judgmental response mode (Tone A re Tone B, or vice versa). When the overall magnitudes of the obtained ratio estimates are considered under both response modes there appears to be little difference (except perhaps in Matrix A where 40% of the subjects showed significance on

the judgmental factor (J)). On the other hand, an effect which was produced by the judgmental mode was evidenced in the reciprocal relations in each of the matrices. A first indicator of this effect was the percentage of subjects who were found to have significant F-ratios for the Upper/Lower factor for Matrices A, B, and C. These results were further investigated by plotting Figures 9-14, which indicated where the differences lay.

When considering the Tone A re Tone B reciprocal relations (Figures 9, 11, 13) for Matrices A, B, and C, a consistent pattern is exhibited. In each of these figures the mean ratio estimates of the lower unaltered cells are greater than the reciprocals of the opposite cells (Although in Matrix A a small divergence is seen). Further, if lines are connected between each of the points representing the cells of a given lower SL stimulus (cells 2-7, 9-13, 15-18, 20-22, and 24-25) it is seen that a closer reciprocal relation is achieved as the level of the lower stimulus increases.

When the subjects are responding under the A re B response mode, those cells which are inverted for subsequent analyses always have the lower SL stimulus presented before the louder stimulus. Thus, the level of this first lower tone has a direct influence upon the reciprocal relation. If the first tone is a low stimulus intensity for the matrix, the judged loudness differences between it

and the other stimuli will be low (as evidenced by its reciprocal) relative to the judged difference if the stimuli had been presented in the other order. However, as the level of the lower stimulus increases, the judged differences between the stimuli presented in either order will become similar, i.e., the reciprocal and the unaltered values will approach each other.

The situation is more complex when the subjects are judging the stimuli under the B re A response mode, although general patterns are evident (Figures 10, 12, and 14). For each of the matrices it is found that the reciprocals are generally larger than the unaltered values. However, when the levels of the louder stimuli in a pair reach the highest matrix intensities, the trend reverses itself.

If lines are drawn, as above, between each of the points representing those cells for a given lower SL stimulus, it is once again seen that the reciprocals approach the unaltered values as the level of the lower stimulus increases (except when the level of the louder tone is at the higher matrix intensities). Under the B re A mode, however, those cells which are inverted always have the louder tone leading the softer tone. Therefore, the effect of the louder tone leading is generally reduced as the level of the lower stimulus increases.

In all matrices judged under the B re A mode when the level of the first tone is at the highest matrix intensities

the reciprocals become less than the unaltered opposite cell. The reason for this divergence is unclear.

In general, then, under both response modes, as the level of the lower stimulus is increased, the reciprocals will tend to become similar to the value obtained for the unaltered opposite cell. Whether or not the reciprocal will be greater or less than the opposite cell, and how much this divergence will be, is directly contingent upon the judgmental response mode, and the level of the louder stimulus.

Although the present study was designed to minimize the response constraints placed upon a subject, a possible source of bias could have been the effect of a previous stimulus pair upon the ratio estimate of a subsequent pair, i.e., sequential dependencies. Since the possibility of substantial sequential dependencies being present within the data was an afterthought, only a brief analysis was performed.

The analysis consisted first of taking four stimulus pairs which contained intensities at either the high or low ends for each of Matrices A-C (designated hereafter as the initial pairs). These initial stimulus pairs were designated as High-High, Low-Low, High-Low, and Low-High. For example, in Matrix B, the initial pairs were: High-High (65-65 dB SL); High-Low (65-40 dB SL); Low-High (40-65 dB SL), and Low-Low (40-40 dB SL). The analysis further consisted of taking

the median ratio estimate values in those cells following each of these initial pairs which were common to two or more of the initial pairs. For example, in Matrix B, the 50-65 dB SL stimulus combination median ratio estimate was calculated three times because it followed the Low-High, High-Low, and High-High initial pairs in the random stimuli presentations. Therefore, the median ratio estimate for a common subsequent stimulus pair could be compared after differential initial pair combinations.

Since the cell order in which the stimulus pairs were randomly presented in Matrices A-C were identical, it was possible to compare any differential sequential dependencies between matrices. Tables 26-28 present the results of the foregoing analyses.

The results show that there was a tendency for the High-High initial pair in each of the matrices to reduce the value of the subsequent stimulus pair. This effect was almost as great as the effect of the louder tone leading the lower tone in time (β_1 results in Figures 16-18).

Ratio Estimate Data and Spatial Configurations for
1000 Hz and 4000 Hz Low Sensation Level Tones

Relation Between the Obtained 1000 Hz Low SL Ratio
Estimates and Previous Investigations. The problem of how loudness grows at low sensation levels has not been investigated

Table 26

Test for Sequential Dependencies in Matrix A

	Stimulus Order of Preceding Pair			
	Low-High (30-80 dB)	High-Low (80-30 dB)	Low-Low (30-30 dB)	High-High (80-80 dB)
Median Rat. Est. Following Pair				
50-80 dB	3.50	2.75	---	3.50
60-80 dB	2.65	---	2.50	2.00
30-60 dB	2.00	---	---	2.00
80-70 dB	1.75	1.50	---	---

Table 27

Test for Sequential Dependencies in Matrix B

	Stimulus Order of Preceding Pair			
	Low-High (40-65 dB)	High-Low (65-40 dB)	Low-Low (40-40 dB)	High-High (65-65 dB)
Median Rat. Est. Following Pair				
50-65 dB	2.51	2.00	---	2.00
55-65 dB	2.00	---	2.00	1.51
40-65 dB	2.00	---	---	1.75
65-60 dB	1.46	1.62	---	---

Table 28

Test for Sequential Dependencies in Matrix C

	Stimulus Order of Preceding Pair			
	Low-High (40-52.5 dB)	High-Low (52.5-40dB)	Low-Low (40-40 dB)	High-High (52.5-52.5 dB)
Median Rat. Est. Following Pair				
45-52.5 dB	2.00	2.00	---	2.00
47.5-52.5 dB	2.00	---	1.75	1.23
40-47.5 dB	1.00	---	---	1.12
52.5-50 dB	1.00	1.25	---	---

as frequently as the growth at moderate to intense levels. Hellman and Zwisllocki (1961), however, were able to plot the 1000 Hz loudness function to 4 dB SL. These authors found that when the stimulus level reached approximately 30 dB or less, the slope of the function increased. Using the functions plotted by Hellman and Zwisllocki (1961), Lochner and Burger (1962) found that the loudness function could adequately be described by the equation $\psi = K(I^n - I_0^n)$ (where I_0 is the threshold intensity and $n=0.27$), rather than the power function ($\psi = KI^n$) from the lowest to the highest intensities. This new equation accounted for the increased steepness at levels below approximately 30 dB SL, whereas the power function did not.

Table 29 shows the loudness ratio estimates which are predicted from the $\psi = K(I^n - I_0^n)$ equation if subjects were asked to judge all pairs in a 1000 Hz matrix ranging from 10-40 dB in 5 dB increments. These values were obtained by taking the ratio between the sone values associated with different intensities, as reported by Lochner and Burger (1962).

A comparison of the predicted data by Lochner and Burger (1962) (Table 29) and the obtained data (Table 25) show that the predicted values are high for all matrix cells. However, the two sets of data do show close similarities when the level of the lower tone reaches approximately 30 dB SL.

Clinical Implications of Low Sensation Level 1000 Hz

and 4000 Hz Spatial Configurations. The clinical utilization of the present method of using ratio estimates combined with the Analysis of Proximities is as a test of loudness recruitment. The advantage of such a technique is that the proximity plots would relate all intensities to one another with, theoretically, differing configurations for pathological and normal hearers.

Another prime advantage of such a test is that it would be completely monaural. This, of course, eliminates the necessity of requiring that a patient have a unilateral loss using the Alternate Binaural Loudness Balance (ABLB) (Dix, Hallpike, & Hood, 1948; Fowler, 1928). It also eliminates the prerequisite that a patient have normal, or near normal, threshold at one frequency, as required by the Alternate Monaural Loudness Balance test (AMLB) (Rager, 1936).

A test based upon ratio estimates would yield direct loudness assessments, and would not be based upon indirect measures such as the difference limen (Bekesy, 1947; Denes & Neunton, 1950; Jerger, 1953, 1959; Luscher & Zwislocki, 1949).

Specifically, the test would include low sensation level tones from just above threshold to moderate stimulus levels. This, of course, is because recruitment manifests itself at low SL's, with loudness perception becoming more normal as the intensity is increased (Fowler, 1928; Steinberg &

Gardner, 1937). Since cochlear pathology is associated with recruitment (Davis & Silverman, 1970), with the audiometric effects being more prevalent at the higher frequencies (Jerger, 1959), the proposed test should include one frequency which is likely to be affected by pathology (4000 Hz), and one that is less likely to be affected (1000 Hz).

To this effect, Matrices G and H (4000 Hz and 1000 Hz, respectively) tested a normal group of subjects at the proposed low sensation levels (10-40 dB SL) in order to determine if there were any inherent differences between judgments made at these frequencies for normal listeners.

Obviously, the configurations differ from each other. It is seen that for those stimulus pairs where one of the tones is 20 dB SL or less, greater ratio estimates are obtained from the 4000 Hz tones. On the other hand, when the level of the lower stimulus in a pair reaches 25 dB SL or greater, the 1000 Hz estimates are greater. Also, additivity occurs at different points on each configuration. The fact that the two configurations differ from one another does not negate their usefulness, but simply points to an inherent difference in the perception of loudness for normal listeners at two sound frequencies.

If such a test were administered to patients with purely conductive losses, the obtained proximity plots should not differ in any essential manner from those found

in normal subjects. A subject with a cochlear lesion, on the other hand, might give differential results due to recruitment.

The development of such a test rests upon the variability of a subject's ratio estimates as well as upon the variability

Table 29

Predicted Ratio Estimates Obtained from the Equation

$$\psi = K(I^n - I_0^n) \text{ (After Lochner \& Burger (1962))}$$

		DB SL Louder Tone						
		10	15	20	25	30	35	40
DB SL Lower Tone	10	1.00	1.80	2.86	4.33	6.34	8.95	12.80
	15		1.00	1.60	2.42	3.54	5.00	7.15
	20			1.00	1.51	2.21	3.12	4.45
	25				1.00	1.46	2.06	2.95
	30					1.00	1.41	2.01
	35						1.00	1.43
	40							1.00

of the spatial configurations. Evidence attesting to the feasibility of the proposed test may be seen by the invariance of the $\log_2 A$ functions for each S under Matrices A-C. Since each subject is able to approximate the scale generated by a group of subjects, it appears that baseline functions can be specified for normals. This, of course, is of utmost importance in the diagnosis of cochlear pathology.

Summary and Conclusions

Loudness scales for 1000 Hz tones, as determined by conventional scaling procedures have yielded loudness functions which grow as approximately the 0.54 power of the sound pressure. That is, a 10 dB increase in intensity corresponds roughly to a doubling of loudness across the auditory continuum. By plotting the loudness as a unidimensional function of intensity, it is implied that if A is twice as loud as B, which, in turn, is twice as loud as C, then A is four times as loud as C. In order to test this prediction loudness ratio estimates were obtained from 10 subjects on four 7x7 matrices of stimuli at 1000 Hz with differing inter-stimulus spacings (30-90 dB SL in 10 dB steps; 40-70 dB SL in 5 dB steps; 40-55 dB SL in 2.5 dB steps, and 30-90 dB SL in irregular intervals). Several types of data analyses were employed in making the comparisons between the present results and those predicted by the 0.54 power function. The first was a multidimensional representation of the data based upon Shepard's (1962a, 1962b) Analysis of Proximities. From these analyses simple two-dimensional spatial configurations were found for each matrix which adequately represented the data.

In general, the results of these four matrices showed two important features. The first was that the spatial

configurations did not conform to the plots implied by the power function, i.e., a straight line in space. Each configuration showed a region of loudness additivity at the lower intensities, and then the figures curved upwards, indicating increasing non-additivity as the inter-stimulus distances increased. The second feature was that in the matrix where the stimulus range was great (Matrix A), the obtained ratio estimates were much below those predicted by the 0.54 power function. On the other hand, when the stimulus range was short (Matrix C), the obtained estimates were high relative to the power function. Only when the stimulus range of the matrix approached 30 dB (Matrix B) were the obtained values comparable to the predicted values.

The divergence of the results obtained with extremely short and long stimulus ranges from the 0.54 power function predictions was hypothesized to be directly attributable to the lack of studies in the past which have investigated loudness over wide and narrow inter-stimulus distances. The overwhelming majority of past investigations have utilized halving or doubling data, or methods which are strongly influenced by context, thereby reducing loudness judgments to moderate ranges.

In order to present the obtained ratio estimates as a unidimensional function of intensity, which, in turn, would yield linear proximity plots, several processes were initiated. These included Kruskal's (1968) MONONOVA

technique combined with the setting of the matrix diagonals to zero. The results of this process yielded the $\log_2 A$ (additive) scale, which could be compared directly to the results predicted by the 0.54 power function. It was found that for Matrices B and C power functions could be obtained. The slope for Matrix B was nearly equivalent to that predicted by the conventional 0.54 power function, and the slope for Matrix C was higher than the conventional function. It was further found that the data for Matrix A did not yield a power function, but that the loudness growth was quite slow. These data further supported the hypothesis that the conventional loudness function (0.54 slope) does not adequately describe loudness over long or short distances.

To further investigate how loudness grows with stimuli other than moderate to intense 1000 Hz tones, ratio estimates were obtained for four new matrices (250 Hz tones from 10-70 dB SL in 10 dB steps; white-noise from 40-70 dB SL in 5 dB steps; and 1000 Hz and 4000 Hz tones from 10-40 dB SL in 5 dB steps). Each of these matrices was evaluated relative to earlier findings concerning the stimuli of interest. The 250 Hz and the white-noise stimulus matrices were evaluated relative to the power functions previously found for these stimuli, whereas the low sensation level 1000 Hz tones were evaluated relative to the form of the loudness function near threshold.

Those stimuli which were compared with the power law

predictions (250 Hz and white-noise) yielded lower ratio estimates, and had spatial configurations which curved upwards after a region of relative additivity. The obtained data for the low SL 1000 Hz tones were also low relative to the predicted estimates.

A monaural test of loudness recruitment was suggested which would utilize ratio estimates of loudness combined with the Analysis of Proximities. The test would consist of low SL 1000 Hz and 4000 Hz tones. Some advantages of the test would be the elimination of the prerequisite conditions necessary for the ABLB and AMLB procedures. Further, the test would be a direct loudness assessment, and would not be based upon indirect measures such as the difference limen.

APPENDIX

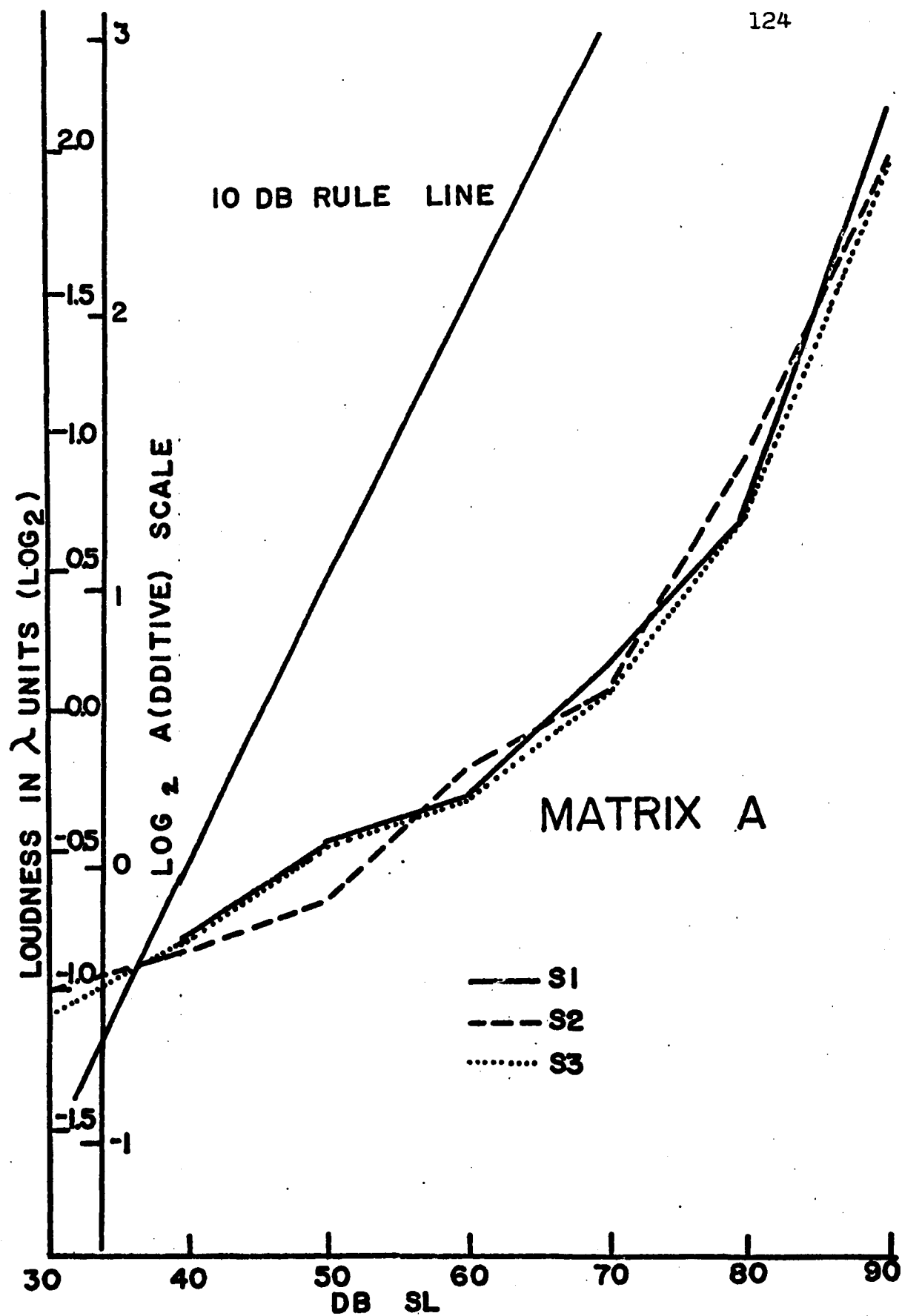


Fig. 31. Individual $\log_2 A$ functions for Matrix A (Subjects 1-3).

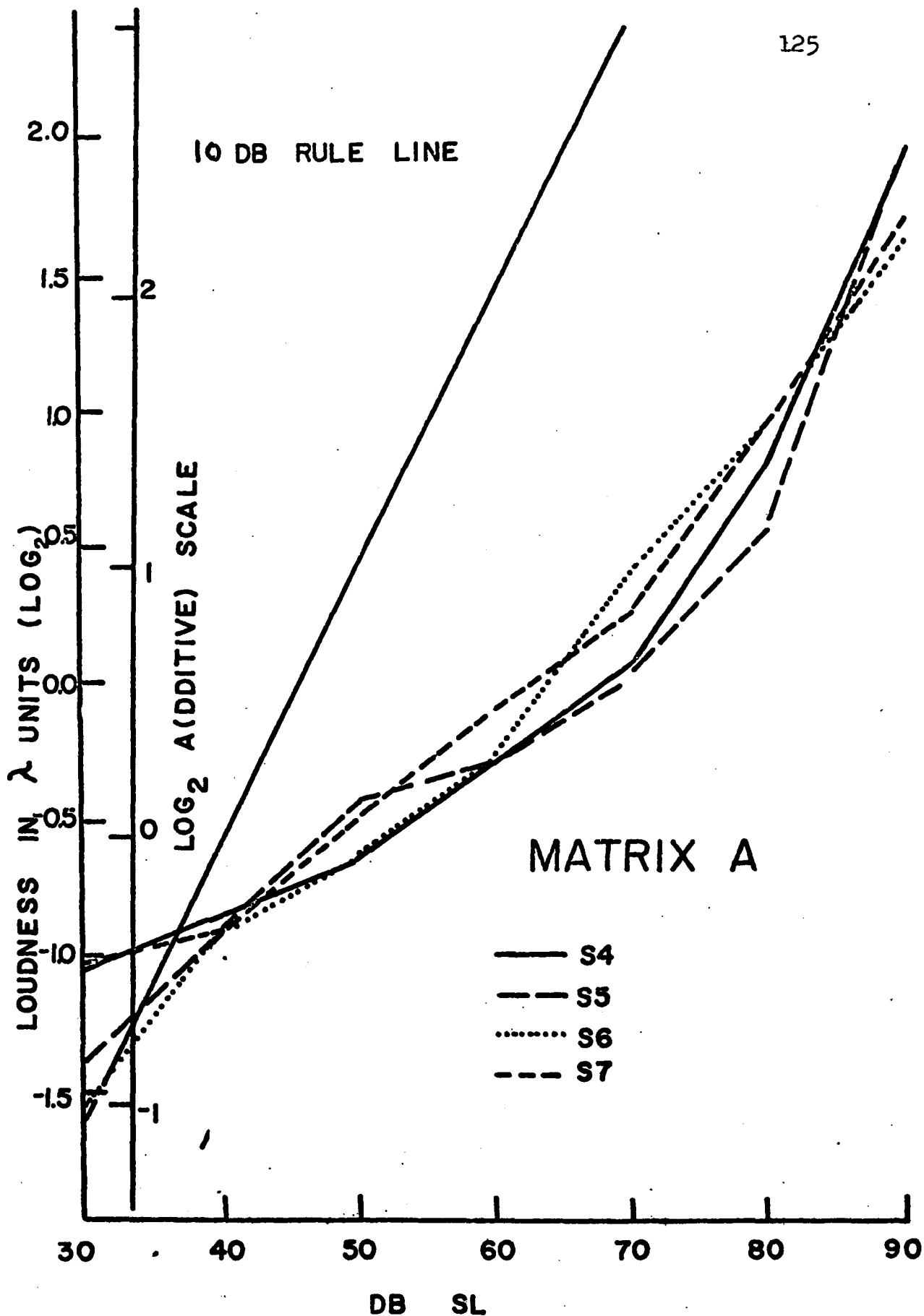


Fig. 32. Individual $\log_2 A$ functions for Matrix A (Subjects 4-7).

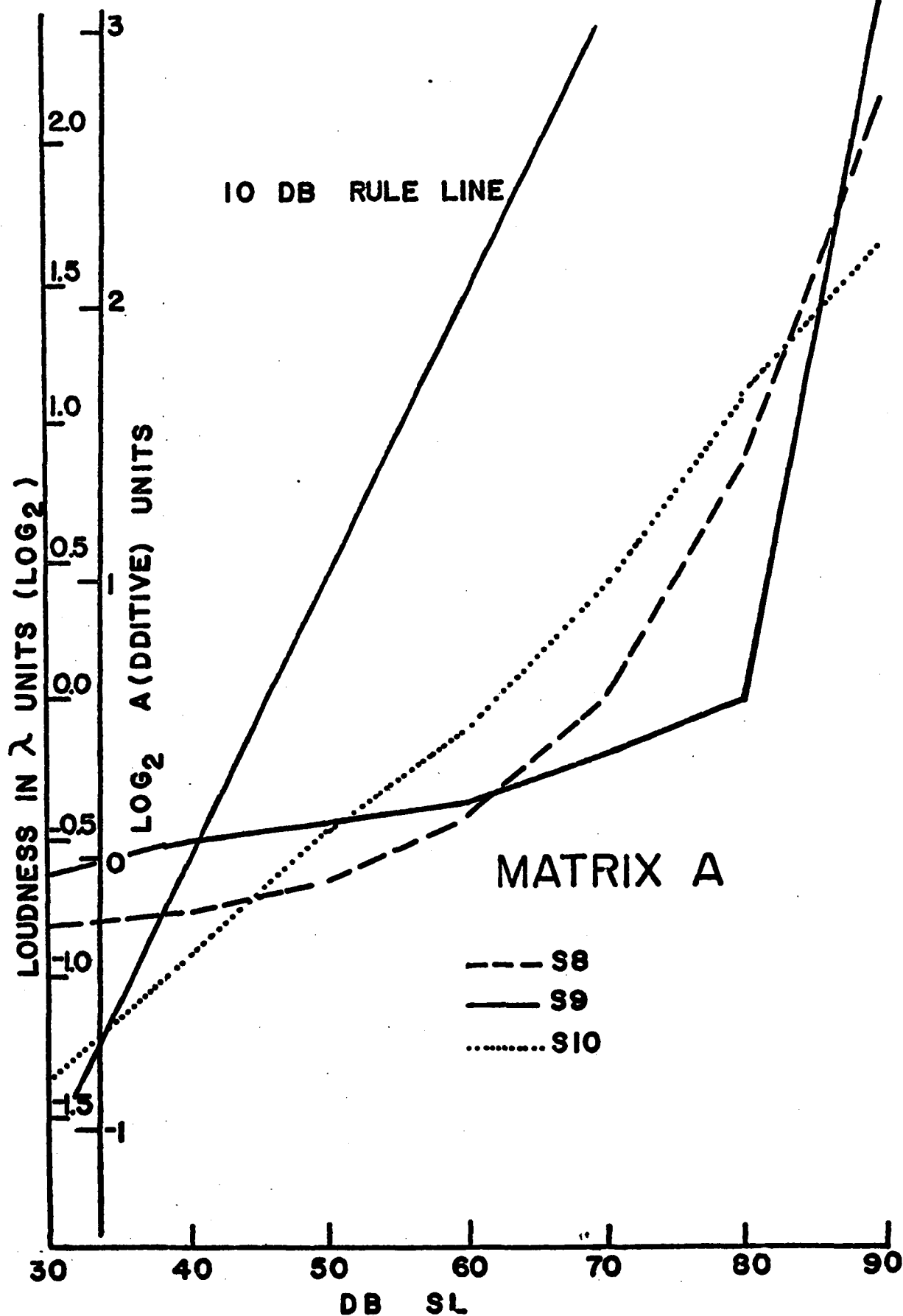


Fig. 33. Individual $\log_2 A$ functions for Matrix A (Subjects 8-10).

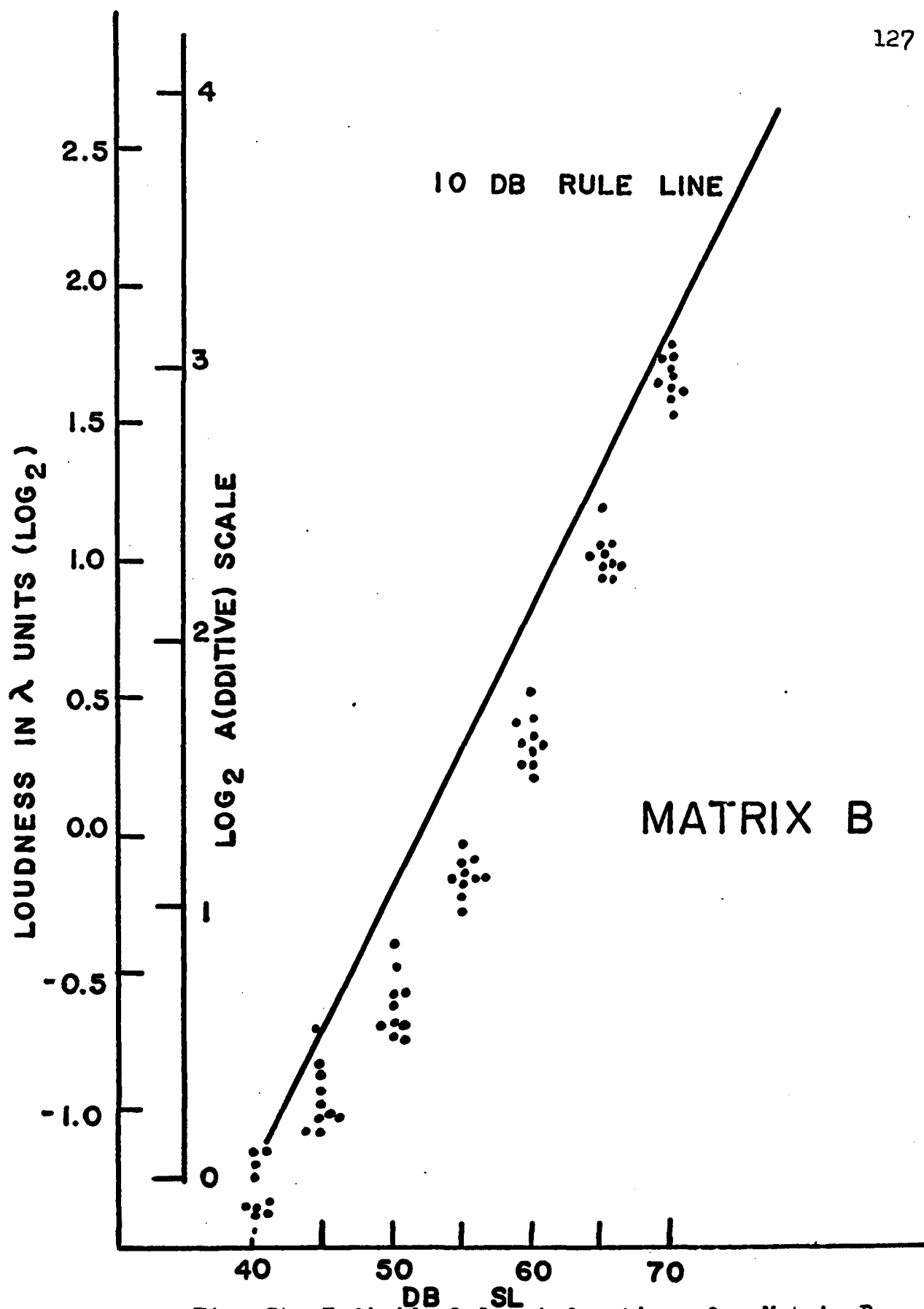


Fig. 34. Individual $\log_2 A$ functions for Matrix B.

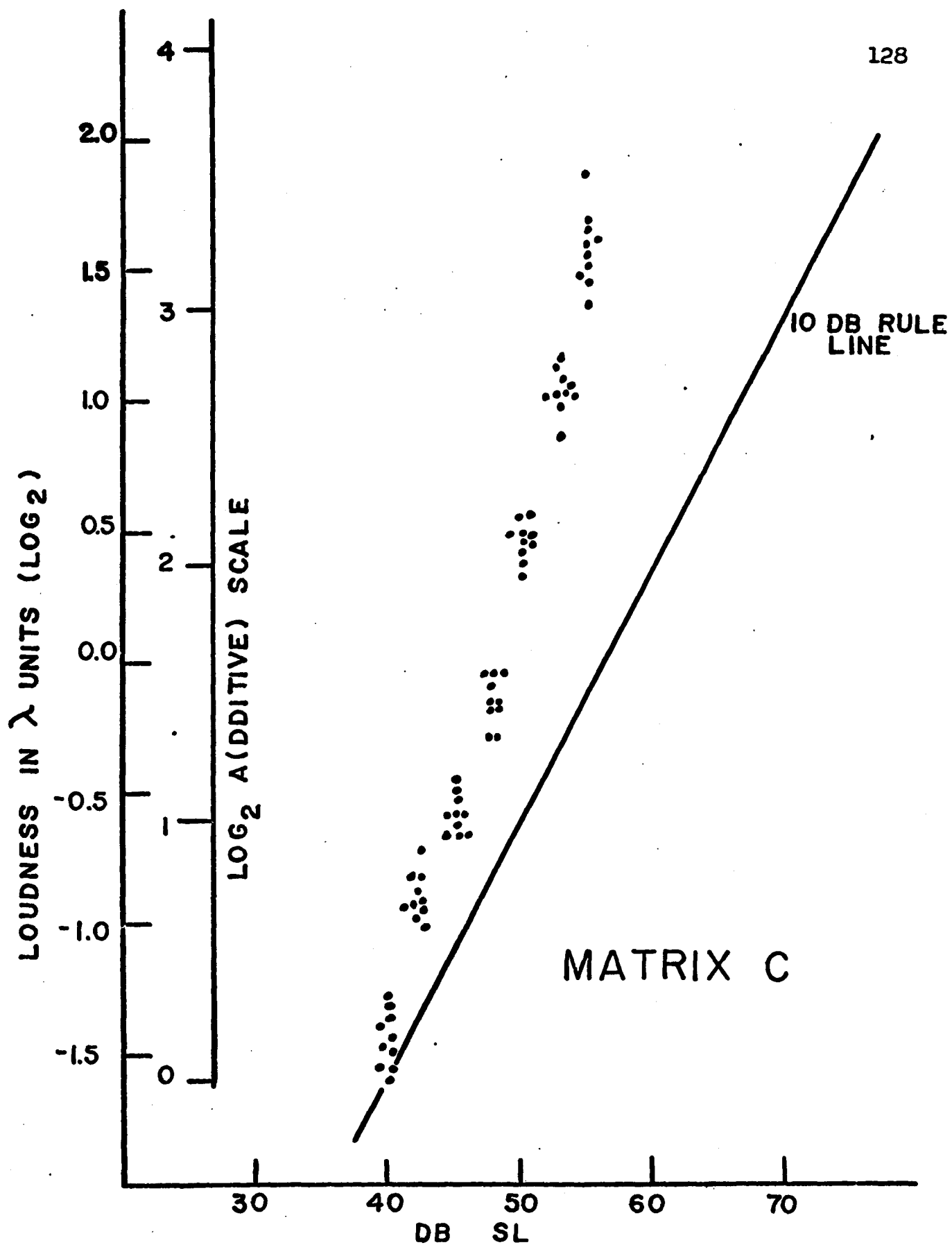


Fig. 35. Individual $\log_2 A$ functions for Matrix C.

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