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# Structural and Topological Contributions to the Electronic Properties of Inhomogeneous Media (Fractal, Electrode, Impedance) 

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# STRUCTURAL AND TOPOLOGICAL CONTRIBUTIONS 

## TO THE ELECTRONIC PROPERTIES

OF INHOMOGENEOUS MEDIA
by

## MICHAEL KRAMER

A dissertation submitted to the Graduate Faculty in Physics in partial fulfillment of the requirements for the degree of Doctor of Philosophy, The City University of New York.

This manuscript has been read and accepted for the Graduate Faculty in Physics in satisfaction of the dissertation requirement for the degree of Doctor of Philosophy.


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## ABSTRACT

# STRUCTURAL AND TOPOLOGICAL CONTRIBUTIONS <br> TO THE ELECTRONIC PROPERTIES OF INHOMOGENEOUS MEDIA <br> by 

MICHAEL KRAMER

Adviser: Professor Micha Tomkiewicz

Recent theoretical developments in the theory of inhomogeneous media have enabled Physicists to explore various properties of random composites. We demonstrate how these techniques can be applied to the treatment of electrochemical systems, specifically porous structures. A Random Network Model is introduced to represent the porous electrode immersed in electrolyte, and using numerical simulation the a.c. impedance behavior of the system is explored. We explore how various compositional and structural parameters contribute to the electrical properties of the system. The topology of our model is analyzed, and the effect of fractal structures on impedance behavior is considered. To examine the electrode topology during electrode operation, another model is introduced, the dissolution-precipitation model. We show how the model correctly predicts the concentration profile of dissolved particles in solution and how the reaction produces fractal structures on the surface of the model electrode.

# Dedicated to my dear wife, Debby <br> and to <br> Yehoshua, Yair and Shoshana 

## ACKNOWLEDGEMENTS

I wish to express my greatest appreciation to my adviser, Professor Micha Tomkiewicz. Without his guidance, support, prodding and encouragement, this work would never have seen the light of day. His valuable friendship and his constant interest in my progress has given me more than one could hope for in a mentor. My thanks to Professor Fred Pollak for sharing his vast knowledge and insight from the days of my undergraduate studies and throughout my graduate career, including his participation in my doctoral committee. To the other members of the committee, Dr. Morrel Cohen of Exxon Research and Engineering, Dr. Myriam Sarachek of City College, and to Dr. Yedidyah Langsam of Brooklyn College my sincerest thanks for agreeing to serve on my committee and for your helpful advice and encouragement. I particularly want to thank Dr. Cohen for numerous discussions during which he patiently took the time to illuminate some obscure points. Special thanks also to Dr. Langsam for always being available to assist me in getting my computer to jump through new loops. I am deeply indebted to Dr. Phil Seiden of IBM Thomas J. Watson Research Center and to Dr. Itzhak Webman of Exxon Research and Engineering for their valuable guidance in the initial phases of this work, and to the Office of Naval Research for providing the funding. I have been most fortunate to have had the opportunity to work closely with Dr. Joseph Lyden and Dr. Benedict Aurien-Blajeni. Dr. Lyden helped inaugurate me into the world of experimental

Physics, and it was his experimental work that helped serve as the inspiration for the topic of this thesis. Dr. Aurien-Blajeni has helped me in numerous discussions in which he has enhanced my grasp of electrochemistry, and the work of chapter IV on the dissolution-precipitation model is a result of our collaboration. I also appreciate the help of colleagues in graduate school past and present; Dr. Pinchas Laufer, Dr. Orest Glembocki, and Dr. Padman Paryanthal stand out in this regard. Many members of the Physics Department faculty provided me with much knowledge and support. I would particularly like to thank Professor Joe Krieger for his constant encouragement and his wonderful lectures.

This research involved very CPU-intensive computer processing, and my special thanks go to Bill Gruber, Director of Consulting and Education in the University Computer Center (UCC) of the City University of New York who, from the outset, took a special interest in insuring that $I$ was able to obtain the computer resources that $I$ needed to do this work, and who always seemed to have the answers to questions of "what went wrong?" or "how do I do this?". Without his help in moving mountains of bureaucracy these computations could not have been done. Thanks are also due to Ben Klein, Director of UCC for his full cooperation as well. Thank you to Julio Berger, and Larry Schweitzer from the Brooklyn College Computer Center (BCCC) who were instrumental in insuring that $I$ obtained full support in the local computing facilities. Joey Sussman of BCCC provided me with much technical support and assistance for which I am grateful.

In general, all staff members of both UCC and BCCC were wonderful, and too numerous to mention.

To paraphrase a hebrew idiom "last mentions are dearest to the heart" ("acharon acharon chaviv"). Words cannot express my appreciation to and admiration for my wife, Debby for her support and sacrifice during the sometimes grueling years of my graduate career. This work is as much of a reflection of that as of my efforts. Thanks also to my parents and in-laws as well as all our brothers and sisters for their help and support. And lastly to my children Yehoshua, Yair and Shoshana my thanks for livening up our lives, always being ready to play (whether I needed a break or not) and putting up with the demands of my work.

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## Introduction

The focus of Physics in the 19 th century was on the world of the visible and experiential - gravity, mechanics, electricity and magnetism. The Newtonian and Maxwellian theories are extraordinarily successful in describing the world of our senses. With the advent of the twentieth century and the birth of quantum mechanics much of the physicist's attention shifted to the opposite extreme the microscopic world of atoms and subatomic particles. The quantum theory of solids has brought with it a deep understanding of the behavior of materials from first principles. The behavior of electrons in the solid's crystal potential, the interaction of electromagnetic radiation with the solid, the identification of phonons and their effect on elastic, thermal and electrical properties are all relatively well understood and have sparked our technological explosion.

The existence of multi-phase granular systems presents new challenges, due to the fact that neither of the above approaches are suitable for an accurate treatment of this problem. To approach the electronic properties of the system- on the one hand, each phase of the material can be well characterized by local electronic properties (the inhomogeneity is manifest on a length scale larger than the electronic mean free path). On the other hand, the system is macroscopically inhomogeneous, and horrendous boundary conditions confront anyone who would attempt to solve Maxwell's equations
across the different phases. Recent studies into phenomena based on random aggregation and the clustering of small particles have demonstrated that by utilizing statistical techniques, the aggregate properties of composite materials may be derived from the local behavior of the system's constituents.

This work is an attempt to apply the study of inhomogeneous media to electrochemical systems, particularly porous structures, and to explore the contributions of the electrode morphology to the electronic properties of the system. The basis of the research is a computer model of the electrode system that captures the composite media character of the electrode. Chapter I presents an overview of the theory of conduction in heterogeneous media, and Chapter II describes the porous electrode-electrolyte system and explains the applicability of the composite media description. Chapter III introduces the Random Network Model- the structure of the system, the impedance characteristics and the relationship between them. Chapter IV details an alternative simulation for the construction of structures on the surface of porous electrodes and sketches how this may be used to further explore the system.

## Electronic Conduction in Composite Media:

## Theoretical Tools

## A. Introduction

Electronic conduction in randomly disordered systems has been explored using the tools of percolation theory. If the system under study is composed of an insulating phase and a conducting phase, then by varying the composition of the system, a sharp change in the conductivity of the system is observed. The critical point (of volume fraction) at which this occurs is termed the percolation threshold and is determined by the dimensionality of the system. Percolation is actually a general process that applies to many systems in which a random media can exhibit long range connectivity. If the composition of the system is varied, the point at which this long range connectivity disappears is the percolation threshold. In this manner, percolation has been used as a model to describe the flow of liquid in a porous media, stochastic star formations in spiral galaxies, and dilute magnetic systems, to name just a few areas of application ${ }^{1}$.

Percolation can be illustrated by considering a lattice of N sites (which we will consider in the limit as $N \rightarrow \infty$ ), in which some of the sites are randomly selected as being "allowed", and the remaining sites are vacant. In addition to this site percolation, one can formulate the problem in terms of allowed and missing bonds to arrive at bond percolation. If $c$ is defined as the concentration of
allowed sites, then when $c$ is very small, allowed sites occur as scattered small islands in the lattice. As c increases, the clusters grow larger and larger, until we reach $c^{*}$, where the mean cluster size approaches the size of an infinite cluster (i.e. it provides a path of connectivity through the whole space). If $P(c)$ is the ratio of the number of sites in the infinite cluster to the number of sites on the lattice, $P(c)$ becomes the probability that a given site will be part of the infinite cluster. A plot of $P(c)$ as a function of $c$ for a 3D lattice is shown in Figure 1.1, and the existence of a critical point is seen from the shape of the curve as $c \rightarrow c^{*}$. Near threshold (from above), the behavior of $P(c)$ may be characterized by a power law,

$$
\begin{equation*}
P(c)=\left(c-c^{*}\right)^{s} \tag{1.1}
\end{equation*}
$$

where $s$, for a cubic lattice, has been found to be approximately

$$
0.3 \leq s \leq 0.4,^{2}
$$

and $c^{*} \approx .3$ (site percolation) ${ }^{66}$ and .25 (bond percolation) ${ }^{67}$.

An alternate quantity of interest is the correlation length, $\xi$, which measures the average cluster size. As the concentration approaches the critical concentration, the formation of the infinite cluster described above is represented by the divergence of $\xi$, such that

## 3D Cubic Latt. Bond Percolation $G(c), P(c)$ and EMT



Figure 1.1 - 3D Bond Percolation, after Kirkpatrik (Ref. 3). c is the \% bond concentration of conducting bonds. The data for $P(c)$ (dotted line) and $G(c)$ (triangles) are obtained by a computer simulation on a cubic lattice of size $15 \times 15 \times 15$. $P(c)$ is the probability of a given site being part of the infinite cluster, $G(c)$ is the conductance of the system, and the solid line is the Effective Medium Prediction for G(c). The arrow under the x axis corresponds to the location of $c^{*}$, the percolation threshold.

$$
\begin{equation*}
\xi-E_{0}\left(c-c^{*}\right)^{-v} \tag{1.2}
\end{equation*}
$$

where $\xi_{0}$ is the size of the granules of which the composite material is made. As one approaches the percolation threshold, the correlation length becomes the dominant length scale of the system. The exponent $v$ will prove to be of some theoretical importance, as we shall see later.

It has been shown, however, that $P(c)$ is not suitable for representing the dependance of the conductivity on the composition of the system ${ }^{3}$. Although an allowed site may be part of a cluster, it is not necessarily a participant in the transport properties of the system. Many cluster "dead ends" will increase the probability of an infinite cluster, but will not themselves be participating in conduction. In order to formulate the percolation problem in terms of conductivity, $\sigma(c)$, Kirkpatrick ${ }^{5}$ introduced the Random Resistor Network Model.

## B. The Random Resistor Network Model

We assume that the system is microscopically inhomogeneous but that the dimensions of the conducting regions are large with respect to the electronic mean free paths. We may therefore define a local conductivity, $\sigma\left({ }^{\prime}\right)$, at a position $H$ in material. The system is considered to have two phases of microscopic conductances $\sigma_{1}(r)=0$, and $\sigma_{2}(r)=1$, and we suppose that we have a detailed knowledge of how
the two phases are arranged. We can calculate the macroscopic conductivity of the system using the 0 hm 's law equation

$$
\begin{equation*}
j(r)=-\sigma(r) \nabla V(r) \tag{1,3}
\end{equation*}
$$

and the continuity equation

$$
\begin{equation*}
\nabla \cdot j(r)=0 \tag{1.4}
\end{equation*}
$$

By introducing a cubic mesh of points $\left\{r_{i}\right\}$ with spacing $\Delta r$, these continuum equations may be solved using a finite difference approximation:

$$
\begin{equation*}
\sum_{j} g_{i j}\left(v_{i}-v_{j}\right)=0 \tag{1.5}
\end{equation*}
$$

with $g_{i j}=\Delta r \sigma\left[\left(r_{i}+r_{j}\right) / 2\right]$, and $i, j$ being neighboring sites on the mesh. Kirkpatrick noted ${ }^{5}$ that Equation 1.5 is identical to the Kirchoff's law equations for a 3D network of resistors, and Equations 1.3-1.5 may be solved by numerical simulation. We define a cubic mesh of resistors, and using a random number generator a specified fraction, $c$, of these resistors are assigned conductivity 1 while the remaining resistors are assigned conductivity 0 . $\sigma(c)$ as a function of the relative concentration $c$, is also shown in Figure 1.1. Again it is determined that near the threshold (as approached from above),

$$
\begin{equation*}
\sigma(c) \propto\left(c-c^{*}\right)^{t} \tag{1.6}
\end{equation*}
$$

where $t \approx 1.94 \pm .1$ for both bond and site percolation ${ }^{68}$. (The value of $c^{*}$ remains $\approx .25$ (bond percolation) and .3 (site percolation)). Thus far the numerical simulations considered only discrete lattice percolation; by introducing correlations between neighboring bonds, Webman, Jortner and Cohen ${ }^{6}$ transformed the problem into a continuous percolation model, and showed that $c^{*} \approx .15$ (bond percolation).

A generalization of the above result ${ }^{7,8}$ is for $\sigma_{2}=b$ with a probability $c$ and $\sigma_{1}=a$ with $a$ probability (1-c), where $a$ and $b$ are both greater than zero. If we assume that $a \ll b$, then we have:

$$
\begin{array}{ll}
\sigma \propto b\left(c-c^{k}\right)^{t} & \text { for } c>c^{*} \\
\sigma \propto a\left(c^{*}-c\right)^{-s} & \text { for } c<c^{*} \tag{1.7b}
\end{array}
$$

Equation $1.7 b$ may be explained by observing that although a is the low conductance phase of material, at very low concentrations of "b" the current is forced to traverse a path through "a". As the concentration of " $b$ " increases, the current shorts through clusters of " $b$ ", resulting in an apparent divergence in the conductivity as one approaches threshold $\left(c^{*}\right)$. At $c=c^{*}$, however, there is no divergence of the conductivity due to the fact that the current does not follow all the twists and turns of the conducting phase and travels through the more numerous paths of the "a" phase, even though it is more poorly conducting. (There is also no divergence
in the correlation length). If we assume that'at $c^{*}$ the power laws in Equations 1.7 are no longer applicable, and $\sigma$ is almost constant in a small region $\delta=\left|c-c^{*}\right|$, we may equate Equations 1.7 at $c=c^{*} \pm \delta$ :

$$
\sigma=a \delta^{-s}=b \delta^{t}
$$

we find that the "crossover region" is defined by

$$
\delta=\left(\frac{a}{b}\right)^{1 /(t+s)}
$$

and that near $c^{*}$, the conductivity is defined as:

$$
\begin{equation*}
\sigma \propto a^{u} b^{1-u}, \quad u=t /(s+t) \tag{1.7c}
\end{equation*}
$$

Thus far we have considered only systems containing a mixtures of pure conductors and non-conductors. Bergman and Imry ${ }^{9}$ considered the case of a heterogeneous mixture of a conducting phase and an insulating dielectric phase, The bulk effective dielectric constant, $k_{e}$ is

$$
\begin{equation*}
k_{e}=\varepsilon_{e}+\frac{4 \pi \sigma_{e}}{i \omega} \tag{1.8}
\end{equation*}
$$

where $\varepsilon_{e}$ and $\sigma_{e}$ are the bulk static dielectric constant and conductivity respectively, and $\omega$ is the frequency of the excitation signal. For $\kappa_{2}=\varepsilon_{1}$ and $\kappa_{2}=4 \pi \sigma_{2} / i \omega$, with $\left|\kappa_{2}\right| \ll\left|k_{2}\right|$ and near
the percolation threshold $c_{2}+c^{*}, \sigma_{e}$ scales as $\left(c-c^{*}\right)^{\alpha}$ while $\varepsilon_{e}$ increases as $\left(c-c^{*}\right)^{-\gamma(1-\beta)}$. They postulated that the large enhancement of the dielectric constant as one approaches $c^{*}$ from below is due to the presence of long paths of conducting material separated by thin barriers, whose effective capacitance is enormous. In addition, at $c=c^{*}$,

$$
\begin{equation*}
\sigma_{e}-\omega^{\beta} \text { and } \varepsilon_{e}-\omega^{\beta-1} \text {, where } \beta=0.73 \pm .05 \tag{1.9}
\end{equation*}
$$

Equations 1.7 present a scaling behavior for the conductivity, and exponents $s, t$ and $u$ are the critical exponents that characterize this scaling relation. Values for these exponents have been obtained from numerical simulation, and for 3D bond percolation are:

$$
s=.75 \quad t=1.94 \quad u=.67 \pm .08 .69
$$

A correlation length for the resistor network has been defined by Bass and Stephen ${ }^{11}$ as follows: apply a potential $V_{0}$ at vertex 0 , and measure the average potential $\left\langle V_{n}\right\rangle$ at vertices $n$ links away. Then

$$
\begin{equation*}
\xi^{-1}=-\lim _{n \rightarrow \infty} n^{-2} \ln \left\langle V_{n} / V_{0}\right\rangle \tag{1.10}
\end{equation*}
$$

## C.Finite Size Scaling and Real Space Renormalization

The existence of the scaling relationships of Equations 1.7 has prompted the application of scaling theories ${ }^{12}$ which have been successfully applied to critical phenomena, such as magnetic spin systems (the simplest of them being the Ising Model). Real space renormalization is applied to map the system into regions of physical parameters where the correlations are easy to compute. Following this, one matches these transformed values to the original correlations, and calculates the correlation function. This allows us to identify the correlation length $\xi$ (which is the scale on which, for a particular concentration, the system first appears homogeneous ${ }^{13}$ ), and to calculate the critical exponent $v$ from Equation 1.2. (A detailed example calculation is presented in Stauffer, p. 15). This renormalization is allowed due to the divergence of $E$ near $c^{x}$, so that the system's properties are invariant no matter which finite length scales we use to investigate the system. "Thus it should not matter at $c^{*}$ on what length scale we are investigating the system; apart from simple scale factors the system looks similar whether we look at it with the eye, with a magnifying glass, or with an optical microscope"14.

Systems that exhibit such properties of self-similarity under transformations of scale are prime candidates for classification as fractal systems. In fact, a major theoretical contribution to the study of critical behavior in heterogeneous systems due to Mandelbrot ${ }^{15}$ is the concept that the scaling property of the
clusters result from the analytic properties of the cluster's geometry. Although the clusters constitute a random disordered system, when the clusters are treated as fractals their geometric properties become simply describable through their fractal dimension, which is related to the exponent $v$ of Equation 1.2.

## D. Fractals

The traditional Euclidean geometry defines geometric constructs as occurting in a Euclidean space $E$ (the dimensions of the space in which it is embedded), and having a topological dimension, $D_{T}$ (e.g. a line or a circle have $\mathrm{D}_{\mathrm{T}}=1$ ). In connection with some systems, however, mathematicians have found it useful to define other dimensions, one such being the Hausdorff dimension, D. The Hausdorff dimension describes how the measure of an object changes under the application of different scales of measurement. In general,

$$
\begin{equation*}
D_{T} \leq D \leq E \tag{1.11}
\end{equation*}
$$

and specifically, Mandelbrot advances the following definition:

A fractal object is one whose Hausdorff dimension is strictly greater than it's topological dimension. $D$ is termed the object's fractal dimension, which is in general not an integer.

The classic example of a fractal dimension is the Richardson Effect (Figure 1.2). If we wish to measure the length of an island coastline, we can take a ruler and start to measure the length of the land-water border. It quickly becomes apparent though, that the resulting length measure is totally dependent on the size of the ruler with which we measured the coast (the resolution of the measurement). As we decrease the size of our measuring stick we are able to measure the coast with finer and finer detail. The result is the Richardson curves of Figure 1.2 , where the relationship between the length scale used and the total length is linear when viewed on a log-log plot. If $\varepsilon$ is the resolution of the measurement, and $L(\varepsilon)$ is the total length measured, then

$$
\begin{equation*}
L(\varepsilon) \propto \varepsilon^{1-D} \tag{1.12}
\end{equation*}
$$

where $D$ is the fractal dimension. Thus, coastlines are representative of a fractal curve. Mandelbrot argues that $D$ is the only measure of convincing physical significance, since it is the only scale-invariant quantity in the problem. In the case of the coast of Britain, the Richardson slope is approximately -.2, which means that $\mathrm{D}=1.2$, which is indeed greater than the topological dimension of the coastline curve $\left(D_{T}=1\right)$, thus satisfying the condition for a fractal.

An alternate definition of fractals is the relationship between measure of contained mass and radius, (which implies a relationship


Figure 1.2- The Richardson curve illustrating that the coast of Britain (outlined points) is a fractal curve. The Richardson Curve for a circle (solid points) has a slope of 0.0 , indicating that its effective dimension is equal to its topological dimension. (After Mandelbrot, Ref. 15)
between length and volume, or length and area). The amount of mass, $M$, contained in an object of radius $R$ can be expressed as:

$$
\begin{equation*}
M(R) \propto R^{D} . \tag{1.13}
\end{equation*}
$$

(We are well acquainted with the classic cases for which $D=3$ (e.g. a homogeneous sphere) or $D=2$ (e.g. a homogeneous 2-D square plate). It is this relationship which may be utilized to calculate the fractal dimension of a percolating cluster. The straightforward procedure ${ }^{14}$ consists of taking circles or spheres of increasing size and measuring their contents or mass (i.e, how much of the percolating cluster do they contain at different sizes), and using Equa(i.) .: . tion 1.11 to directly calculate D. Kirkpatrick ${ }^{16}$ employs a variation of this method for calculating a fractal dimension of the "backbone" of the infinite cluster.

An alternate expression of Equation 1.11 is the scaling property of the density-density correlation function ${ }^{19,20}$;

$$
\begin{equation*}
C(r) \equiv N^{-2} \sum_{\mathbf{r}^{\prime}} \rho\left(r^{\prime}\right) \rho\left(r^{\prime}+r\right) \quad \propto \quad r^{D-d} \equiv r^{-A} \tag{1.14}
\end{equation*}
$$

where $d$ is the spatial (Euclidean) dimension ( $E$ ), and $D$ is the fractal dimension. The density $\rho(r)$ is defined to be 1 for occupied sites and 0 for empty sites. The calculation is performed by computing $C(r)$ for a given $r$ in a given direction over all the particles in the lattice (or in the cluster), and then $C(r)$ is
averaged over different directions. This is repeated for a number of discrete lengths $r$, and the exponent $A$ is evaluated from the slope of $\log (C(r))$ plotted as a function of $\log (r)$. The utility of this calculation is that it is easy to perform in computer aggregation simulations. This relationship is also directly verifiable using scattering experiments such as neutron or X-ray scattering. The structure factor, $S(k)$ which is measured in a scattering experiment, is none other than the Fourier transform of $C(r) .20,21$ Thus, we may relate the scattering intensity, $I$, and the wavenumber, $k$, by

$$
\begin{equation*}
I(k) \propto k^{-x} \tag{1,15}
\end{equation*}
$$

The interpretation of this exponent, $x$, is dependant on the scattering system. For "volume fractals" (1.e. polymer-like structures), x is none other than $D$ from Equation 1.13; for scattering from surfaces, $x=6-D$, where $D$ is the fractal dimension of the surface ( $D=2$ corresponds to a flat surface). For fractally porous materials $22,23,24, x=7-Y$, where $Y$ is the exponent describing the distribution of pores of radius $r$,

$$
\begin{equation*}
P(r)=r^{-Y} \tag{1.16}
\end{equation*}
$$

Real life objects do not exhibit fractal behavior over all length scales. In practice, there is an inner and outer cutoff for $\varepsilon$ and $R$, within which Equations 1.12 and 1.13 hold - corresponding to the
range where these systems exhibit self-similarity. In the percolation problem the bounds are from the lattice size, $a$, to the correlation length, $\xi^{16}$

## E.Scaling of the Conductance on Percolating Fractal

By combining Equation 1.2 and the conductance scaling relationship of Equation 1.7 b , we find that the conductivity may be expressed as

$$
\begin{equation*}
a \propto \xi^{-\mu / \nu} \tag{1.17}
\end{equation*}
$$

Using finite-size scaling arguments due to the self-similarity of the system, Palevski and Deutscher ${ }^{25}$ show that for a system of size $L \ll \xi:$

$$
\begin{equation*}
\sigma \propto L^{-\mu / \nu} \tag{1.18}
\end{equation*}
$$

Thus, effective dimensions for the impedance properties of the system may be calculated using methods similar to those for the classic fractal relationships reviewed above. In fact, Palevski and Deutscher measure the dependance of $R(L)$ as a function of $L$ (where $R(L)$ is the resistance in a segment of the percolating gold clusters and $L$ is length of the side of a square within which they measure the resistance). As they increase the area of the square within which they measure $R$, they observe a linear relationship between log
$R(L)$ and $\log L$, Computation of the slope of that line yields the ratio $\mu / \nu$.

## F.Effective Medium Theory (EMT)

While the Random Resistor Network approach treats the electrical properties of the system as explicitly arising from the system's microscopic constituents, EMT takes almost the opposite approach. We consider a composite system made up of two materials having conductivity $\sigma_{1}$ and $\sigma_{2}$. Our objective is to calculate $\sigma_{e}$, an effective conductivity of the entire composite. We begin ${ }^{28}$ by considering a small (spherical) grain inside the material, and treat that grain as if it is embedded in a homogeneous effective medium, of conductivity $\sigma_{e}$, to be computed self-consistently (see Figure 1.3). Let the field and current density far from the central grain be $E_{0}$ and $J_{0}=\sigma_{e} E_{0}$. We can therefore calculate the fields and current density within the grain to be

$$
\begin{align*}
& E_{i n}=\left[3 \sigma_{e} /\left(\sigma_{i}+2 \sigma_{e}\right)\right] E_{0}  \tag{1,19}\\
& J_{i n}=\sigma_{i} E_{i n} \tag{1.20}
\end{align*}
$$

where $\sigma_{1}$ is either $\sigma_{1}$ or $\sigma_{2}$. (the microscopic assumptions and geometrical considerations necessary to produce Equation 1.19 are somewhat controversial, although all derivations produce this result). ${ }^{26,27,28}$ The self consistency for computing $\sigma_{e}$ comes from the assumption that


(c)

Figure 1.3- Illustration of Effective Medium Theory. (After Stroud, Ref. 28).
(a) Schematic of a two-component composite in an applied electric field, EO.
(b) An illustration of the self-consistent embedding procedure which defines the effective conductivity, $\sigma$.
(c) Schematic behavior of de as a function of concentration, $f$ (from the application of Equation 1.23 .

$$
\begin{equation*}
\left\langle J_{i n}\right\rangle=\sigma_{e}\left\langle E_{i n}\right\rangle . \tag{1.21}
\end{equation*}
$$

Assuming that our composite is made up of the fraction $f$ of component 1 and (1-f) of component 2, we can substitute Equations 1.19 and 1.20 into 1.21 to yield,

$$
\begin{equation*}
\frac{3 f \sigma_{1}}{\sigma_{2}+2 \sigma_{e}}+\frac{3(1-f) \sigma_{2}}{\sigma_{2}+2 \sigma_{e}}=\sigma_{e}\left(\frac{3 f}{\sigma_{2}+2 \sigma_{e}}+\frac{3(1-f)}{\sigma_{2}+2 \sigma_{e}}\right) \tag{1.22}
\end{equation*}
$$

which simplifies to:

$$
\begin{equation*}
f \frac{\sigma_{1}-\sigma_{e}}{\sigma_{1}+2 \sigma_{e}}+(1-f) \frac{\sigma_{2}-\sigma_{e}}{\sigma_{2}+2 \sigma_{e}}=0 . \tag{1.23}
\end{equation*}
$$

Although EMT has proved to be a useful approximation for dealing with systems about which we have very little structural information, the theory fails when a percolating system approaches the percolation threshold. EMT predicts a much higher critical concentration, $c^{*}$, then is observed experimentally or by numerical simulation ${ }^{3}$. This may be explained ${ }^{29}$ by noting our bias for spherical conducting geometries. If in fact our system consists of flattened spheroids or elongated conducting components, the probability of connectivity through the system is enhanced at lower conductor concentrations. In a random system, there is no apriori reason to assume the favoring of spherical structures over elongated ones. This illustrates that some knowledge of the internal geometry of a particular system is necessary for the application of EMT such a system.

## G. Summary

We have presented a multiplicity of techniques for the theoretical treatment of conduction in heterogeneous media. The current presentation has centered on a simple application, the impedance characterization of a two-phase random mixture of conductor and non-conductor. It should be pointed out however, that these theories may be adapted to treat:more complex systems, not only systems with multiple component phases, but also systems with unique non-random correlations, and systems with specialized geometries. Moreover, although this work focuses on conduction in composite media, these are general tools equally applicable for the treatment of a wide range of transport and static problems in stochastic systems, all revolving around the properties of clustering and aggregation in these systems. A full discussion of all the ramifications of these theories are beyond the scope of this work (and me). In the coming chapters we will employ these concepts and techniques to our problem at hand - the stochastic morphology of the porous electrode and it's contribution to the electronic conduction.

## II. The Chemistry and Physics of Porous Electrodes

## A.Solid-Liquid Interfaces

When two phases of material are brought into contact with each other, the difference in their free energy develops a potential difference across the interface. When these two phases are metal (electrode) and electrolyte, the potential difference induces a surface charge on the metal and region of net charge of opposite sign in the electrolyte. In the Gouy-Chapman (GC) model of the interface, there is a diffuse region of local net charge that extends a distance into the electrolyte. If we imagine a microscopically thin lamina of electrolyte at a distance $x$ from the interface, utilizing Boltzmann statistics the charge density in that region can be written as

$$
\begin{equation*}
\rho(x)=\text { nze } \exp \left[-z \operatorname{e\varphi }(x) / k_{B} T\right] \tag{2.1}
\end{equation*}
$$

where $n$ is the ion concentration, $z$ is the net charge on each ion, $\varphi(\mathrm{x})$ is the electrostatic potential at position x in the solution with respect to the interface, $k_{B}$ is the Boltzmann constant, and $T$ is the absolute temperature. Combining this with the Poisson equation:

$$
\begin{equation*}
\nabla^{2} \varphi=\frac{4 \pi}{\varepsilon} \rho \tag{2.2}
\end{equation*}
$$

the approximate solution is 30

$$
\begin{equation*}
\varphi=\varphi_{0} \mathrm{e}^{-\kappa x} \tag{2.3}
\end{equation*}
$$

$k$ has units of inverse length and $\kappa^{-1}$ is the characteristic expanse of the charge distribution. Thus, we can consider all the charge in solution to be located on a plane parallel to the interface and located at $\mathrm{d}_{\mathrm{GC}}=K^{\cdot 1}$. The behavior of this double layer is therefore identical to that of a parallel plate capacitor of capacitance

$$
\begin{equation*}
C_{G C}=\frac{\varepsilon_{0} A}{G C} \tag{2.4}
\end{equation*}
$$

d
where $\varepsilon_{\text {, }}$ is the permittivity of free space and $A$ is the cross sectional area of the interface. It is the electric field at the interface, represented by the capacitor, that constitutes the driving force for charge transfer across the interface and which represents the electrode potential (in comparison with other electrodes in solution). Another field at the interface is generated by adsorbed water molecules and unsolvated ions on the interface and it is also associated with a capacitance $C_{H}$, the Helmholtz Capacitance. The total capacitance of the interface is due to two capacitors in series, and the total capacitance is:

$$
\begin{equation*}
\frac{1}{\mathrm{C}}=\frac{1}{\mathrm{C}_{\mathrm{H}}}+\frac{1}{\mathrm{C}_{\mathrm{GC}}} \tag{2.5}
\end{equation*}
$$

Figure 2.1 is a diagram of the charge distribution at the interface and the location of the two planes of effective charge. At high


Schematic of the electrified interface. The IHP (inner Helmholtz plane)is due to the adsorbed surface species, and the OHP (outer Helmholtz plane) is due to the diffuse charge distribution in solution.
solution concentrations the GC layer is very narrow, and as a result of the capacitance behavior of Equation $2.4, C_{G C}$ becomes large and may be ignored in comparison with $\mathrm{C}_{\mathrm{H}}$. At low electrolyte concentrations $C_{G C}$ dominates (the Helmholtz capacitance is independent of concentration).

Thus far we have considered no faradaic current flow across the interface, an assumption of an Ideally Polarizable Electrode (IPE). More generally (depending on the catalytic properties of the metal and on the solution), the metal electrolyte interface is represented by the equivalent circuit model of Figure 2.2 , where $R_{p}$ represents a leakage current (a deviation from IPE behavior). In Figure $2.2 \mathbf{R}_{s}$ is the resistance of the metal - which is usually negligible.

If there is an insulator or semiconductor build up on the electrode due to some chemical reaction at the surface, then the metal-solution capacitance remains, but the value will be altered by the dielectric properties of the new layer. In this case, the $R_{s}$ of Figure 2.2 is the bulk resistance of the insulating or semiconducting layer on the interface.

The bulk electrolyte solution has an ohmic conductivity due to the movement of ions, and for a given volume of solution we can represent the current carrying capacity of the solution using an equivalent resistance,


## Figure 2.2

Equivalent circuit model of the solid-liquid interface.

$$
\begin{equation*}
R_{e}=\frac{D_{e^{l}}}{A} \tag{2.6}
\end{equation*}
$$

where the electrolyte volume is defined by the length $l$ and cross-sectional area $A$, and the electrolyte resistivity is determined from the relationship ${ }^{31}$

$$
\begin{equation*}
o_{e}=\frac{1}{n z \mu} \tag{2.7}
\end{equation*}
$$

$n$ is the number of ions in the given volume, $z$ is the charge per ion, and $\mu$ is the mobility determined by the characteristic species in solution.

A useful experimental technique for the study of electrochemical systems is the use of a.c. impedance measurements ${ }^{32}$. These measurements allow for nondestructive in-situ characterization of the system. Impedance measurements are a natural choice given the theoretical behavior of these systems as simple passive circuit components. Using very low signal a.c. excitations, we can measure both static and kinetic electrochemical processes.

## B.Typical Application: A Secondary Battery Cell

Perhaps the most important technological application of these electrochemical principles is in battery electrodes. A typical example is the Silver-Zinc cell of Figure $2.3^{33}$. A silver oxide
electrode and a Zinc electrode are placed in a solution of KOH . The overall reaction is a two step one ${ }^{34}$ :

$$
\begin{align*}
& \mathrm{Ag}_{2} \mathrm{O}_{2}+\mathrm{H}_{2} \mathrm{O}+\mathrm{Zn} \underset{\substack{\text { discharge }}}{\stackrel{\text { charge }}{ }} \mathrm{Ag}_{2} \mathrm{O}+\mathrm{Zn}(\mathrm{OH})_{2}  \tag{2.8}\\
& \mathrm{Ag}_{2} \mathrm{O}+\mathrm{H}_{2} \mathrm{O}+\mathrm{Zn} \underset{\text { discharge }}{\stackrel{\text { charge }}{ }} 2 \mathrm{Ag}+\mathrm{Zn}(\mathrm{OH})_{2}
\end{align*}
$$

In the discharge process the Zn electrode is oxidized, liberating electrons to flow through the external circuit, and forming a layer of ZnO on the electrode. The half-cell reaction is:

$$
\begin{array}{r}
\mathrm{Zn} \longrightarrow \mathrm{Zn}^{2+}+2 e^{-}  \tag{2.9}\\
\mathrm{Zn}^{2+}+2 \mathrm{H}^{-} \longrightarrow \mathrm{Zn}(\mathrm{OH})_{2}
\end{array}
$$

On the silver electrode the AgO is reduced to produce $\mathrm{Ag}^{35}$ :

$$
\begin{align*}
& \mathrm{Ag}_{2} \mathrm{O}_{2}+\mathrm{H}_{2} \mathrm{O}+2 \mathrm{e}^{-} \longrightarrow \mathrm{Ag}_{2} \mathrm{O}+2 \mathrm{OH}^{-}  \tag{2.10}\\
& \mathrm{Ag}_{2} \mathrm{O}+\mathrm{H}_{2} \mathrm{O}+2 \mathrm{e}^{-} \longrightarrow 2 \mathrm{Ag}+2 \mathrm{H}^{-}
\end{align*}
$$

or schematically:

$$
2 \mathrm{Ag}^{+}+2 \mathrm{e}^{-} \longrightarrow 2 \mathrm{Ag}
$$

Focusing on the Zn electrode, the net result of the discharge process is the buildup of ZnO on the surface of the electrode. The state of charge of the electrode can be determined from the amount of ZnO on the surface, and as previously mentioned this changes the capacitance of the interface.


## Figure 2.3

A typical secondary Battery cell during discharge. During discharge, a layer of ZnO is forming on the anode.

The most important characteristics of any battery are its storage capacity, its energy density and its power density. The energy density can be increased by the choice of two electrodes with large differences in their electrode potential, and by producing a very high current density on discharge. To achieve this, a large real surface area is needed between the metal and the electrolyte which can be obtained by using a porous electrode structure. Indeed, all commercial batteries are based on porous morphology. Although the electrochemical theory discussed thus far assumes flat surfaces, it is a logical starting point for the treatment of rough surfaces associated with porous electrodes. It is clear though, that some corrections and modifications are needed to account for the new morphological effects that accompany the increased current density.

## C. Porous Electrodes - Classical Approach

DeLevie ${ }^{36}$ introduced the first coherent treatment of the effects of surface roughness. He considered a single pore with a uniform cross section, homogeneously filled with electrolyte, and assumed that the resistance of the electrode material is negligible. The actual potentials in the pore are replaced by their average values in planes perpendicular to the pore axis. This may be represented by the transmission line equivalent circuit model of Figure 2.4, which transforms the problem into a one-dimensional one. $R$ is the solution resistance per unit pore length ( $R$ has dimensions of ohm/cm), and $Z$ is the impedance of the electrode-electrolyte interface per unit pore length ( $Z$ has dimensions of ohm-cm and $Z / d x$ is the


Figure 2.4- DeLevie's aingle pore transmission line model, for a pore of length 1 . $R$ is the electrolyte resistance per unit length, and $Z / d x$ is the impedance of the interface (a microscopic section of length dx ).
interfacial impedance of a tiny expanse of the pore of length $d x$ this makes sense if you consider an example of a pure capacitance on the interface of value ( $C$ per unit pore length) ; the impedance of a length dx of that will be $1 /(\mathrm{i} \omega \mathrm{Cdx})$ ]. Solving the current-voltage equations for the transmission line yields the impedance of the entire pore of length $\ell$ :

$$
\begin{equation*}
Z_{0}=(R Z)^{1 / 2} \operatorname{coth}(\rho l) \tag{2.11}
\end{equation*}
$$

where $\rho=(R / Z)^{1 / 2}$.

This leads to the definition of a characteristic penetration depth for the current in the pore, $\lambda=1 / \rho$. When $\ell \gg \lambda \operatorname{coth}(\rho \ell) \approx 1$ which DeLevie refers to as the "semi-infinite pore" where there are effectively no "edge effects" due to the pore end. Another important result of this model is the fact that although $Z$ represents the interfacial impedance of a unit flat surface, the presence of the pore produces a $Z^{i / 2}$ dependance for the entire rough surface. For an arbitrary a.c. signal, this translates into a frequency dependance of

$$
\begin{equation*}
Z \propto \omega^{-1 / 2} \tag{2.12}
\end{equation*}
$$

and in the literature this has been taken to be the "signature" of a rough surface ${ }^{37}$, as opposed to an $\omega^{-1}$ dependance for a flat surface. (This signature, however is not unique since one can show, for example, that the Warburg impedance which is a pure diffusion phenomenon unrelated to morphology, is also characterized by $w^{-1 / 2}$ ).

The definition of penetration depth in Equation 2.11 also illustrates the fact that at high frequencies the a.c. current penetrates very little into the pores. The ratio of $Z / R$ controls the penetration due to the fact that at $Z \gg R$, the current tends to flow through the "top" of the transmission line in Figure 2.4 (through $R$ exclusively). On the other hand, when $Z \ll R$, then the current tends to flow in Figure 2.4, right through the first "down" path that it finds (through $Z$ ). At high frequencies, $Z$ is low and the current short circuits "down" early in its flow through the pore.

## D. Critique of Classical Model-A Search for Alternatives....

There are numerous problems with this approach to rough surface impedance behavior. The most striking difficulty is that there is a large body of experimental evidence to the fact that Equation 2.12 is simply not a unique characterization of surface roughness. Some manifestly rough surfaces have impedance behavior of a smooth surface ${ }^{38}$. In addition, experimental studies have found many different frequency dependances not at all limited to exponents of -. $5 .^{39,40}$ A general dependance of

$$
\begin{equation*}
z \propto \omega^{-\beta} \tag{2.13}
\end{equation*}
$$

where $0 \leq \beta \leq 1.0$, emerges from the Literature, with the deviation from $\beta=1$ attributed to surface roughness ${ }^{41,42 \text {. In various studies }}$ this fractional frequency scaling has been called a Constant Phase Angle (CPA) ${ }^{41}$, Constant Phase Element (CPE) ${ }^{43}$, or Fractional Power


Figure 2.5 - A.C. impedance measurement of polycrystalline CdSe showing that both the real ( $R$ ) and the imaginary (X) components of the impedance present a power-law frequency dependence. The CPA behavior is evident at low frequencies where $R$ and $X$ have identical frequency scaling exponents.

Frequency Dependence (FPFD) ${ }^{44}$, with little new theoretical light shed on the subject. (We will hereafter refer to this behavior as CPA behavior; the constant phase angle refers to the fact that both the real and the imaginary parts of the impedance scale with the same exponent.)

Lyden, Cohen and Tomkiewicz, in an experimental study on the impedance characterization of polycrystalline CdSe, demonstrated a characteristic scaling (CPA) behavior of the impedance as a function of frequency ${ }^{45}$ (Figure 2.5). The authors identified this as a percolation-type behavior which they traced directly to the polycrystalline semiconductor electrolyte interface, with the critical exponents within the range of the Imry-Bergman treatment (see Equation 1.9). The percolation behavior may be associated with the morphology of the system, an analysis which arises from the identification of the interface region as a composite medium. However, the model that was employed ${ }^{46}$ to explain the percolation mechanism is very specific to that system and has little general applicability. Despite this fact, the work is most noteworthy because it is a first attempt at applying the ideas of the study of composite media to the porous electrode and rough surface problem.

## E. ....Which Brings Us Back to Fractals

Very recent studies have shown that CPA behavior arises as a natural consequence of a fractal interface structure.

Liu ${ }^{47}$ has constructed a model electrode-electroiyte interface out of a "textbook" fractal structure - a Cantor bar ${ }^{48}$ (Figure 2.6a). He assumes that the interface is ideally polarizable and that each "prong" in the bar can be represented by a series $R-C$ element that scales with the "prong" width (Figure 2.6b). By solving for the total impedance of this, equivalent circuit he obtains a CPA behavior. The exponent $\beta$ that he measures (Equation 2.13) is related to the fractal dimension of the bar by $\beta=1-D$. It should be noted that the shape of the impedance curve that he obtains is as follows (In chapter 3 we will show very similar results from the Random Network model): On a plot of $\log (Z)$ vs. $\operatorname{LOG}(\omega)$, at low frequency the real impedance is a constant value, and the imaginary impedance is of slope -1. In mid-frequency both the real and imaginary slopes are $-\beta$. Finally at high frequency the real impedance is again constant, and the imaginary impedance has a slope of -1 . Liu gives a very intuitive explanation of the scaling in the mid frequency range. If we view the a.c. signal in light of it's penetrability (see the description of penetration depth above) then the lower the frequency the finer is the length scale that is spanned by the current. In the limit of high irequency, the current is unable to penetrate the pores at all due to the large length scale that the current averages over. At low frequencies the current is able to penetrate everywhere due to the fact that it scales very microscopic lengths. It is in the mid-frequency regime in which the length scale of the "current yard stick" is varying, and that is when it is measuring the interface length with varying length yardsticks and the

(a) Liu's cantor bar model of the electrode-electrolyte interface with a fractal surface. The electrolyte is the dark area.
(b) Equivalent circuit representation of the cantor bar geometry.
(From Ref. 47, reprinted by permission of the author.)
self-similarity ratio emerges directly, as in the Richardson curve of Figure 1.3.

More generally, Nyikos and Pajkossy ${ }^{49}$ have shown how resistance and capacitance scale with length (for a self-similar distribution of R-C networks), so that when the total admittance is calculated from the combination of $R$ and $C$ it scales as $\omega^{\beta}$, where $\beta$ is related to the fractal dimension by

$$
\begin{equation*}
\beta=1 /(D-1) \tag{2.14}
\end{equation*}
$$

This treatment builds on many of the assumptions in our Random Network Model except that no specific model structure is chosen; the authors simply argue that one can choose a fractal surface that may be represented by the self-similar R-C network, and calculate from there. We will consider one specific possible implementation of this idea in chapter IV.

The work of LeMehaute ${ }^{50}$ is also noteworthy; he assumes that most real life electrochemical systems have a fractal morphology on the interface, and derives the contributions of a fractal interface using a highly mathematical TEISI model. In this treatment he computes fractal time derivatives of kinetic phenomenon and illustrates scaling relationships.

The DeLevie model is a convenient one dimensional description of trends in the a.c. impedance behavior of porous electrodes. What the model lacks is the morphological sophistication of real-life systems. Similarly, the treatment of Liu has built in to it a realistic model of a rough interface, but it is still two dimensional and not connected with real structures. In the next section, we present our Random Network Model, a theoretical treatment of porous electrodes that mimics the three dimensional heterogeneous distribution of metal and electrolyte, while still providing a means of exhaustive calculations of its a.c. impedance behavior.
III. Porous Electrode Simulation

## A. The Random Network Model

The Random Network model is designed to merge the techniques of handing conduction in inhomogeneous media (chapter I) with the theory of the porous electrode-electrolyte system (chapter II). The result is effectively a four phase inhomogeneous system composed of air, electrolyte, metal, and semiconductor. (In this chapter we refer to the phase that is present on the interface as a semiconductor, although following the theory presented in Chapter II the same electrical properties of the interface would arise from the existence of the metal-electrolyte interface.) The electrode problem may actually be divided up into two subcategories: a) The rough electrode is a description of the entire system consisting of the bulk electrolyte and the bulk metal of the electrode, and everything in between (the whole of Figure 3.1). b) The porous electrode is a description of the system from the point of view of a simple heterogeneous distribution of metal and electrolyte, (area (B) in Figure 3.1), and is the focus of this study.

Following the methodology of Equations 1.3-1.5, we assume that our electrode system is composed of a heterogeneous mixture of metal, electrolyte and semiconductor microscopic components. These components are assumed to be large enough to be represented by an average resistivity or complex dielectric constant, the same values as for bulk quantities of the materials. We use the equivalent circuit representations introduced in Chapter II to represent the electronic


Schematic of a rough electrode surface. Regions $A$ and $C$ are comprised of bulk metal and electrolyte respectively, while region B may be viewed as a composite inhomogeneous system.
properties of these phases. Our objective is to obtain the total macroscopic impedance of the system. With a detailed knowledge of the system's composition (since after all, we built it ourselves), we set out to correlate the impedance information with the system's microscopic characteristics. Due to the charge storage behavior of the insulating phase (semiconductor) on the metal-electrolyte interface, the system's impedance is frequency dependent, and much useful information is hypothesized to lie in the impedance vs. frequency relation. This theoretical study mimics the a.c. impedance characterization of electrochemical systems that is widely utilized experimentally ${ }^{32}$. It is important to note that this simulation is not meant to model the d.c. operation of the electrode and to follow in detail the mechanisms of Equations 2.8-2.10, but merely to simulate the electronic behavior of the porous electrode system under small signal a.c. excitations at any given in moment in time.

## B.The Algorithm

We define a three dimensional cubic lattice, $A(i, j, k)$, where $i, j$ and $k$ range from 1 to $N, N$ being the total number of nodes. $N$ is chosen so that it is sufficiently large to approximate an infinite system. Construction of the electrode is simulated using a random number generator to place "metal" at various points in the lattice, until the desired porosity is reached (plane $k=N$ is all metal as in the back electrode contact). A check is made of the resulting electrode to insure that it is continuous and that no, pieces of
metal are "hanging" in mid-air. Any disjointed pieces of metal are removed, and are randomly replaced on the remaining available lattice points. This process is repeated until a continuous electrode of the desired porosity is obtained. Lattice plane $k=1$ is defined as electrolyte, to represent the solution; the electrode itself begins at lattice plane $\mathrm{k}=2$. The electrolyte is allowed to "seep into" the pores in the electrode until all accessible pores are filled, by searching for continuous electrolyte paths from the $\mathrm{k}=1$ plane throughout the electrode. Any point on the lattice which has been left empty (i.e. no metal was placed there, and electrolyte was precluded from penetrating to that site) is considered to be "air".

The geometric assumptions in this model are that the various components consist of volume filling cubes centered on the lattice points $A(i, j, k)$. Two adjoining cubes of metal (or even two on the diagonal with each other) prevent the electrolyte from seeping past them. The result of this constraint is that the fluid flow through the system shows classic percolation behavior. If the electrolyte is introduced at $k=1$, then at low porosities it flows only a small part of the way through to $k=N$. At porosities above the percolation threshold the electrolyte does indeed flow throughout the system. There is, in fact, a whole body of literature that includes this complication of pore connectivity in an analysis of electrode structure. 54 We have minimized these effects by limiting our consideration of the system to be above the fluid flow percolation threshold. An alternative to this approach would be to consider the
system components as closely packed spheres that permit fluid flow in the interstices, which effectively eliminates all "air" in the system (electrolyte flows everywhere that there is an absence of metal). This introduces discontinuities in the electrolyte connectivity as far as the cubic lattice is concerned, and great pains must be taken to insure that the computer will recognize electrical connectivity betweens islands of electrolyte. This can be done using a more laborious algorithm for calculating the electrical conductances, but we have not done so in this phase of the work.

Each lattice point represents a node in a three dimensional circuit network (See Figure 3.2), and each pair of nodes defines a circuit branch in the network. The impedance of each branch is determined by the characteristics of the two surrounding nodes. Thus, for example, if $A(1,1,1)$ is electrolyte and $A(1,2,1)$ is also electrolyte, the circuit branch defined by those nodes will consist of two series resistors of resistance $R_{e}$ (the resistance of a microscopic section of the electrolyte). If $A(1,1,2)$ is metal and $A(1,2,2)$ is metal, then the circuit branch defined by those two nodes will be two series resistors of value $R_{m}$ (the resistance of a microscopic section of the metal). Finally, the branch defined by $A(1,1,1)$ - electrolyte, and $A(1,1,2)$ - metal, is represented by an $R_{m}$ and an $R_{e}$ resistor in series, with the addition of a parallel $R-C$ element in series with them to represent the semiconductorelectrolyte interface that results (see Figure 3.2). We refer to "semiconductor" on the interface, although the interface could more generally have an insulating layer or a simple Gouy-Chapman layer on


Figure 3.2 - The Random Network Model of a porous electrode. Figure 3.2b shows our idealized picture of the electrode's structure (the metal was distributed using a random number generator), and 3.2a shows a portion of this structure represented by a network of circuit elements.


## Electrolyte <br> -m~

 $\mathrm{R}_{\mathrm{e}}=5 \Omega$
## Semiconductor



Figure 3.3 - Equivalent circuit representation of the three different phases in our electrode system, and typical values chosen to represent physical processes in realistic systems. Note that the interface is describes as semiconducting, and appropriate circuit element values were chosen to reflect this. To represent an insulating phase on the interface these values may be suitably altered.
it. Any circuit branches leading into nodes that are defined as "air" are taken to be of infinite resistance.

A typical set of values that were chosen as unit impedances for the individual components of metal, electrolyte, and semiconductor are presented in Figure 3.3. These values were obtained by taking the magnitudes of bulk impedance for $1 \cdot \mathrm{~cm}^{3}$ sized samples, (we chose numbers characteristic of Zn and ZnO ). In the interest of generating physically meaningful numbers, we defined a unit pore of arbitrary diameter $10 \mu \mathrm{~m}$ and scaled the impedance values to represent the dimensions of the unit pore. In effect we consider each of the microscopic components (metal or electrolyte) as being comprised of a cube whose side is of length $10 \mu \mathrm{~m}=10^{-3} \mathrm{~cm}$. When we normalize the 1 $\mathrm{cm}^{3}$ values given in Figure 3.3 to this shrunken size, we have to multiply the impedance of the $1 \mathrm{~cm}^{3}$ "resistor chunks" by a factor of $10^{3}$. This is due to the fact that resistors have a dimensional dependance of $\ell / A$ where $\ell$ is the longitudinal length and $A$ is the cross-sectional area. The determination of the scaling factors for the input parameters that represents the semiconductor is not so straightforward. As far as the geometry of the model goes, the semiconductor-electrolyte interface is "pure". surface area with no thickness (i.e. we do not devote any lattice coordinates to the interface- the computer just determines where it is and calculates an enhanced impedance corresponding to the semiconductor on the interface at that point). The thickness of the semiconductor deposit is implicit in the values that were chosen for $R_{p}, R_{s}$, and C. (The numbers chosen in Figure 3.3 approximate a 100 Angstrom
layer of moderately doped semiconductor of area $1 \mathrm{~cm}^{2}$ ). Thus, when we normalize the components from 1 cm to $10^{-3} \mathrm{~cm}$, we multiply $R_{s}$ and $R_{p}$ by $10^{6}$ and divide the capacitance $C$ by $10^{6}$ because of the reduced surface area. These components are then assembled into a system, as described above (see Figure 3.2) and the total impedance of the system is calculated. The total impedance is then rescaled by an appropriate factor that normalizes the whole system to a $1 \mathrm{~cm}^{3}$ size. Specifically, for an, $11 \times 11 \times 11$ system there is $10 \times 10^{-3} \mathrm{~cm}$ per side. Thus, we scale the total calculated impedance by $10^{4}$. This final normalization assures that no matter what size we choose for our lattice size, the numbers will be comparable.

Once the electrical network has been defined, the computer sets up the Kirchoff's law equations,

$$
\begin{equation*}
G \cdot V=I \tag{3.1}
\end{equation*}
$$

where $G$ is the conductance matrix (see Equation (1.3); here: $G_{i j}=\varepsilon_{j} g_{i j}$ and $G_{i j}=-g_{i j}$ ), $V$ is a vector containing the voltages on each node, and $I$ is a similar vector containing the net current inputs to each node. Due to the fact that our model includes reactive circuit elements, the quantities $G, V$, and $I$ in Equation 3.1 are complex, and may be represented as $2 N-1$ equations of the form:

$$
\left(\begin{array}{cc}
G_{r}^{r} & -G_{i}  \tag{3.2}\\
G_{i} & G_{r}
\end{array}\right)\binom{V_{r}^{r}}{V_{i}^{r}}=\binom{I}{I_{i}^{r}}
$$

where the subscripts $r$ and $i$ stand for the real and imaginary
components respectively.

A known current is sent uniformly into all the bottom nodes, which is designed to insure an equipotential surface on the bottom plane. We have verified that the resulting voltage fluctuations on the bottom plane are in fact very small. The impedance of the system is calculated between one node on the electrolyte plane ( $1,1,1$ ) and one node of the top electrode plane ( $N, N, N$ ) by solving for the $V$ 's on all the nodes. For a system of $\mathrm{N}=15$ (a lattice of size $15 \times 15 \times 15$ with 3375 nodes) there are up to 6748 equations to be solved. This is a formidable computer problem. Indeed, the task would be prohibitive without taking advantage of the fact that the conductance matrix, $G$, is a sparse-symmetric matrix.

A number of techniques exist for the solution of this class of problems. A widely used method for solving the simultaneous equations in the Random Resistor lattice is the Gauss Seidel iteration procedure with over-relaxation ${ }^{6}$. This is a very economical method, in both execution time and storage space. A serious drawback of this technique is the relatively strict requirements that it places on the coefficient matrix in order for convergence to be achieved, namely that the matrix be either diagonally dominant, or at least positive definite ${ }^{51}$. For Kirchoff's law problems with purely resistive components, diagonal dominance is assured, since the diagonal elements of the conductance matrix are simply the sum of the off diagonal elements. In our problem, with complex impedances, we find that at frequencies for which the real conductance
approaches the same order of magnitude as the imaginary conductance, the Gauss-Seidel procedure does not converge. This is due to the form of the Equations 3.2 where there are many additional off-diagonal elements due to the $G_{i}$ values. When these imaginary components are large, the matrix is no longer diagonally dominant, and becomes indefinite. For most of our computations, we have resorted to the use of a Gaussian elimination routine which takes advantage of both the sparseness and the symmetry of the $G$ coefficient matrix ${ }^{13}$. Only the nonzero elements in the upper half triangle of the matrix are stored, and an efficient pivoting strategy is chosen to minimize non-zero matrix fill during the pivoting and to minimize the number of multiplications required in the solution. In practice, the technique is approximately an order of magnitude more expensive to use than Gauss-Seidel in both speed and storage requirements. The advantage of using the Gaussian elimination technique is that a solution is guaranteed for all but the most ill-conditioned problem. The solutions obtained from the Gauss-Seidel procedure (in the range in which convergence is reached) are in complete agreement with the results obtained using the modified Gaussian elimination technique.

The final step in "fine tuning" the model is to choose a minimum lattice size that provides some degree of statistical significance but is still within reasonable limits of available computer time. If the random network model is a true statistical model then the calculation should not be sensitive to the lattice size of the system. Thus the minimum lattice size was chosen by solving a


Figure 3.4 - The impedance of the system in the RNM as a function of lattice size. Although the solution is converging, numerical limitations prevented us from solving for larger systems, and some statistical deviation is assumed to result.
single problem on increasingly large lattice sizes until the answer converges to a fixed value. The results of this study is presented in Figure 3.4. There are two significant difficulties that we encountered in trying to solve for the impedance of our system. Firstly, we were not able to eliminate all "dead end" conductive paths as Kirkpatrik did in his composite media simulations. He was able to do this because all he was concerned about was the system's degree of connectivity. Thus, he was able to choose only the cluster's "backbone" and measure the resistance of that to decide on the conductivity of his system. In our system, the capacitance of all surfaces contributes to the impedance, and it is not clear which current paths may be discarded, if any. Secondly, as already mentioned, the complex impedance doubled the size of the network, and turned the problem from a well-behaved calculation into an ill-conditioned one. For most of the results in this study a system of size $11 \times 11 \times 11$ was used. Computational limitations forced us to choose this minimum size, although continuous boundary conditions were employed to minimize finite-size effects.

## C. Homogeneities \& Systematic Correlations

From the foregoing description, it is obvious that our model differs in a number of important ways from the purely random conductor-insulator distributions of Chapter I. For one thing, although the distribution of metal is governed by a random number generator, we have modified the random statistics to require a connectivity in the metal for a realistic representation of an electrode (although some metal powder - electrolyte electrode
systems have been experimentally explored). One effect that this has on the electronic properties of the system is that metal connectivity is assured across the system - it always lies above the percolation threshold. Another constraint introduced is the fact that the three phases electrolyte, semiconductor and metai are not mixed homogeneously, but are distinctly separated by an orderly juxtaposition to each other (see Figure (3.3) for a schematic representation of this). Even in the pores where the paths are contorted and the surface rough and complicated, there is always a continuous distribution of electrolye and metal respectively on either side of an interface. It is thus impossible to formulate an effective medium theory for this arrangement. Any treatment of local inhomogeneity in an average medium such as in chapter I cannot be made to preserve this structured distribution around an interface.

The RNM simulation is a valid approximation of conduction in a porous electrode system, and superior to previous treatments of electrodes precisely because it preserves the stochastic nature of the system to the extent that it is present. Thus far, that aspect has not been properly treated in the literature.

It may be more precise to say that, unlike the conductor-insulator problem which presents a bulk inhomogeneity in two or three dimensions, the porous electrode system is a problem of a quasi two dimensional inhomogeneity of the interface. Although the system's bulk component composition will also exhibit bulk properties and its
behavior will be governed by these bulk-related phenomena, it is the properties due to the random morphology of the rough surface that is of greatest interest to us, and we will attempt to show when these interfacial contributions dominates the system's properties. We define the surface (or interface) in the model as being those metal sites that are adjoined by one or more electrolyte sites.

## D. The Model Parameters

The input parameters of the model consist of the porosity, $p$, the electrolyte resistance, $R_{e}$, and the metal electrode resistance, $R_{m}$ (all bulk impedance values are for a typical $1 \mathrm{~cm}^{3}$ ). By varying the system input parameters, we can simulate various physical conditions in our electrode system. Variations in $\mathrm{R}_{\mathrm{e}}$ correspond to different electrolyte concentrations. $Z_{s c}$, the impedance of the semiconductor is represented by a resistor, $R_{p}$, in parallel with a capacitor, $C_{p}$, which are both in series with a resistor, $R_{s}$ (Figure 3.3). Varying these parameters correspond to variations in the state of charge of the electrode (i.e. the thickness of the semiconductor buildup), and the doping level of the semiconductor buildup (determined by the reaction mechanisms at the electrode surface during electrode discharge).The resulting (frequency dependant) impedance of the interface is:

$$
\begin{equation*}
Z_{s c}=R_{s}+\frac{R_{p}}{1+\left(\omega \tau_{p}\right)^{2}}-j \frac{\omega R_{p}^{2} C_{p}}{1+\left(\omega \tau_{p}\right)^{2}} \tag{3.3}
\end{equation*}
$$

where $\tau_{p}=R_{p} C_{p}$. $R_{s}$ corresponds to the resistivity of the bulk semiconductor, $\cdot R_{p}$ to the faradaic current leakage through the
interface (a space charge region if it is a semiconductor), and $C_{p}$ to the capacitance of the interfacial charge storage region. We obtained the $C_{p}$ value by assuming a semiconductor buildup of approximate thickness $\ell^{\prime}=100 \mathrm{~A}$, an area of $a=1 \mathrm{~cm}^{2}$ and the relationship

$$
\begin{equation*}
C=\frac{\varepsilon_{0} \varepsilon a}{\ell^{\prime}} \tag{3.4}
\end{equation*}
$$

Where $E_{0}$ is the permittivity of free space and $\varepsilon$ is the dielectric constant of the material on the interface (we chose $\varepsilon=10$ representative of a ZnO deposit). As previously mentioned, the primitive size of our lattice was taken to be $10 \mu$, such that for a lattice of size $11 \times 11 \times 11$ we have a cube of side $100 \mu$.

## E.Model Output- Bulk Contributions

Figure 3.5 is a typical result from the electrode RNM at a given $\mathrm{p}=.4$, using the input values of Figure 3.3. We may loosely categorize the imaginary impedance vs. frequency behavior as being composed of three regimes: At low frequency (f<100 Hz) we have a slope 1 behavior on the $\log -\log$ plot, at high frequency ( $f>10^{5} \mathrm{~Hz}$ ) we see a slope ( -1 ) dependance, and in the middle frequency regime is some sort of transition region or regions. The low and high frequency behavior is the same as would be observed from measuring the impedance of an $\mathrm{R}-\mathrm{C}$ circuit, however, when an attempt is made to calculate the capacitance, it is observed that the capacitance has different values at low and high frequency. The measure of capacitance is usually taken as an indication of a measure of some active interfacial area, and thus it would appear that there are two

## IMPEDANCE VS. FREQUENCY-P192



Figure 3.5 - A typical data set from the RNM computation. The data points are from the simulation, and the solid line drawn through the points is a fit to the data (see section III.G).
characteristic areas being measured in this system; one at high frequencies and another at low frequencies.

DeLevie ${ }^{36}$ had suggested that for cylindrical pore electrodes the penetration depth of the ac signal decreases with increasing frequency (see Chapter II). If we accept this as applicable to our electrode model, this would account for the lower capacitance being measured at higher frequencies- the current is not penetrating far into the pores and hence the current is scanning less area. It is easy to verify that in fact that is what is occurring in our RNM electrode. Figure 3.6 depicts the current distribution as it flows through the semiconductor (labeled $S$ ), electrolyte (E) and metal (M). These values were obtained by actually calculating the current flow throughout the nodes in our model. At low frequency (Figure 3.6a) all the semiconductor interfaces are passing an appreciable amount of current; all of the electrolyte paths are contributing to the current flow as well (in order for the current to reach all the interfaces it must go through all the electrolyte). In contrast to this, at high frequency (Figure 3.6b) we note that only a small minority of all the semiconductor interfaces are passing any current at all.

Figure 3.1 schematically depicts the process. The current flows through the bulk electrolyte (labeled C) and tries to get through to (A), the metal of the electrode. Before it can do that, it must pass through the interface (labeled B). How (and where) it passes through the interface is the determining factor in the ac

capacitance. If the interface is an easier path for the current to $g 0$ through than continuing through the electrolyte in the pores (since the pores may be narrow the resistance could be high), then the current will "short" right through (B) into (A) at it's earliest opportunity (near $k=1$ in our model). If on the other hand, the journey through $R_{e}$ is the least-resistive path, the current will travel through the electrolyte deep into the pores before being forced to enter the interface into (A).

The low frequency capacitance value would then be a measure of the total surface area of the electrode (at least as far as the current got), and the low frequency impedance data may be taken to be a measurement of the electrode surface area. Further verification of this may be obtained by increasing the electrolyte resistivity and watching if the value of $C_{1 f}$ changes as the electrolyte becomes more and more unsuited for performing this area measurement. Sure enough, Figure 3.7 shows how we can make the value of $C_{1 f}$ approach $C_{\text {hf }}$ simply by raising the $R_{e}$. Conversely, we can raise the value of $C_{h f}$ simply by lowering $R_{e}$ which improves the penetrability of the current even at high frequency. Figure 3.7 shows this as well, but we were prevented from making $C_{h f}=C_{\text {lf }}$ due to numerical difficulties.

To further test the current's area scanning capability, we varied the porosity and measured the corresponding $C_{1 f}$. Figure 3.7 shows how closely the low frequency capacitance is able to follow trends in the roughness factor. (The roughness factor, $p$, is a measure of

Capacitance vs. Electrolyte Resistance


Figure 3.7 - The change in the measured low frequency and high frequency capacitance as Re is varied. At high Re , even the low frequency capacitance fails to measure the apparent surface area of the pores.
total interfacial area of a rough electrode to the geometric surface area- the area of a flat $1 \mathrm{~cm}^{3}$ electrode. The capacitance in our system is also normalized to a $1 \mathrm{~cm}^{3}$ value by our simulation so we may compare $p$ and $C_{1 f}$ directly.) The value of $\rho$ in Figure 3.8 was computed by simply counting up all the metal-electrolyte unit interfaces in the computer model for each porosity. Figure 3.8 is a conclusive demonstration of the utility of the value of the low frequency capacitance from the impedance measurement. At the same time it also demonstrates that $C_{h f}$ is a measurement corresponding to the geometric surface area. Another interesting feature of the $\rho$ vs. porosity plot is that $p$ is not a unique measure of porosity, A roughness factor of .9 may be obtained by a porosity of 0.36 or 0.62 . The bell shaped curve is due to a competition of two opposing factors for the increased surface area. On the one hand, as the volume fraction of metal increases from zero, the more available metal surfaces there are for metal-electrolyte interfaces. On the other hand, as the volume fraction of metal increases beyond .5 or so, the probability of having a metal site bordered by another metal (and hence no metal-electrolyte interface) also increases!

To quantify the penetrability of the current with decreasing frequency, DeLevie had introduced the penetration depth, $\lambda=(|Z| / R)^{\frac{1}{2}}$ (see chapter II; this $R$ is related to our parameter $R_{e}$ ). The concept should be applicable here as well, except that a single-valued penetration depth is of little meaning in this 3-D system, as opposed to DeLevie's one-dimensional pore. The current may penetrate deeply into wide necks of the pores, but get choked off in a


Figure 3.8 - A comparison of the actual surface roughness and the measured low frequency capacitance illustrates that the low frequency capacitance may be used to measure the apparent surface area of the pores. For comparison, the high frequency capacitance is shown, and it is not sensitive to changes in electrode surface.
narrow horizontal cross link. Nevertheless, the value of lambda is useful if thought of as a measure of the current "penetrability". In Figure 3.9, $\sqrt{Z}_{s c}$ is plotted as a function of frequency. We may draw a horizontal line across the figure corresponding to any value of $\sqrt{ } \mathrm{R}_{\mathrm{e}}$ that we choose to introduce into our system. At the frequencies that the $\mathrm{R}_{\mathrm{e}}$ line falls below $\mathrm{Z}_{\mathrm{sc}}$, then the current will flow through the electrolyte. At frequencies for which $Z_{s c}$ falls below $R_{e}$, then the current will short through the interface.

## F.Duplicating the DeLevie results as a Special Case of RNM:

To demonstrate that our RNM is a more general case of the DeLevie model, we can numerically simulate DeLevie's cylindrical pore electrode by constructing uniform pores with our random resistors. The DeLevie parameters $R$ and $Z$ differ from our $Z_{s c}$ and $R_{e}$ due to the fact that the former quantities are normalized to the unit pore length. We may consider a resistance and impedance per unit length as (see Figure 3.10):

$$
\begin{equation*}
R=\frac{\rho}{A} \quad \text { and } \quad Z=\frac{\rho^{\prime} \ell^{\prime}}{4 \sqrt{A}} \tag{3.5}
\end{equation*}
$$

where $A$ is the geometric area of the pore, $\ell^{\prime}$ is the thickness of the semiconductor layer, and $\rho^{\prime}$ is an effective (complex) resistivity of the semiconductor. Consider an ideally polarized interface where

$$
\left|Z_{s c}\right|=1 /(\omega C)
$$



Figure 3.9- The impedance of the interface alone as a function of frequency.


Rectangular pores set up to explore aimple pore behavior. (a) is a view of the 9 pores in a "top view" cross section, and (b) is a "side view" cross section. (c) illustrates the dimensions of each pore.
and a dielectric solid, for which we may define

$$
c=\frac{\varepsilon_{0} \varepsilon a}{d}
$$

where $\varepsilon$ is the static dielectric constant, $\varepsilon_{0}$ is the permitivity of free space. From Figure $3.9 \mathrm{a}=4 \sqrt{ } \mathrm{Al}, \mathrm{d}=\ell \mathrm{l}$ so that

$$
\begin{equation*}
Z_{s c}=\frac{\ell^{\prime}}{\omega \varepsilon_{0} \varepsilon 4 \sqrt{A} \omega}=\frac{\rho^{\prime} \ell^{\prime}}{4 \sqrt{A}} \tag{3.6}
\end{equation*}
$$

Therefore the impedance per unit length is $Z_{s c} / \ell$ and

$$
\begin{equation*}
\lambda=\left(\frac{Z}{R}\right)^{\frac{1}{2}}=\left(\frac{\ell^{\prime} \sqrt{A}}{4 \varepsilon_{0} \varepsilon \rho \omega}\right)^{\frac{1}{2}} \tag{3.7}
\end{equation*}
$$

taking the values (as before), $\ell^{\prime}=100 \mathrm{~A}=10^{-8} \mathrm{~m}, A=10^{-10} \mathrm{~m}$, and $\mathrm{c}=10$, we get a penetration depth of

$$
\lambda=\begin{array}{ll}
2.8 \times 10^{-3} & \text { at } 1 \mathrm{~Hz} \\
9.0 \times 10^{-5} \mathrm{~m} & \text { at } 1 \mathrm{KHz} \\
2.8 \times 10^{-6} & \text { at } 1 \mathrm{MHz}
\end{array}
$$

For our system of (as previously mentioned) size $100 \mu$ on a side, this corresponds to a current flow throughout the electrode at under 1 KHz , which is in agreement with the location of the low frequency regime in Figure 3.5.

To construct a simple cylindrical pore system with the RNM, we arrange our resistors so that they are distributed as in Figure 3.10, with the shaded area denoting the metal and the white area being the pores where the electrolyte flows. We solve for the current distribution in the system, and plot $I$, the current flow in


CURRENT FLOW THROUGH PORE - HI FREQ


Lecion onmurn - theorr
Figure 3.11- Magnitude of current flow into the pores as a function of distance, using the pore structure of Figure 3.10. The data points are the results of the calculation, and the line is a fit from the DeLevie model.
the $Z$ direction, as a function of $z$, the distance into the electrolyte (Figure 3.11). Figure 3.11a is the result for high frequencies (1 MHz with an $R_{e}$ of $1 \mathrm{~K} \Omega$ ). The shape of the curve is exponential, and following DeLevie ${ }^{52}$ we look for a result of the form

$$
\begin{equation*}
I=I_{0} \exp ^{-z / \lambda} \tag{3.8}
\end{equation*}
$$

Upon fitting this function to the data of 3.11a, we arrive at $\lambda=.6$ which indicates that at high frequency the penetration depth is not even through to the first lattice point. Figure 3.11b shows the results at low frequency, and we may also try to fit that curve to Equation 3.8. However, if $\lambda \gg z$ then the exponential may be expanded as

$$
\begin{equation*}
I_{0}=I(1-z / \lambda) \tag{3.9}
\end{equation*}
$$

We perform a linear fit of this function to the data in Figure 3.11b and we obtain $\lambda=10.4$. This verifies the frequency behavior is as DeLevie postulated, and the exponential behavior of the current in the pores.

## G.Other Features of the Impedance Curve

Returning to the impedance curve of Figure 3.4, we identify more features that may be correlated with the input parameters. As an alternative to the high frequency and low frequency capacitance that we have been using thus far to model the frequency response of the system, we can construct an equivalent circuit model for the system


## Figure 3.12

Equivalent circuit model of the total electrode impedance.
that is valid at all frequencies. Figure 3.12 introduces some more circuit elements; but this time they correspond to the output parameters of the system (the total impedance calculated) and hence are denoted by the ' (prime) notation. The curve drawn through the data in Figure 3.5 is a theoretical fit to the impedance dispersion curve, using the circuit of Figure 3.12. By fitting the theoretical impedance curve, we evaluate the output parameters (the circuit element magnitudes).

We have varied the input parameters, and correlated that with changes in the equivalent circuit output parameters. $R_{s}^{\prime}$ is determined from the values of the real impedance at high frequency. Its value is usually governed (dominated) by the value of the input electrolyte resistivity, $R_{e} \cdot R_{p}^{\prime}$ is determined by examining the system's real impedance at low frequency. The space charge resistance, input parameter $R_{p}$, of the semiconductor on the interface usually decides the value of $R_{p}^{\prime}$. The peak in the imaginary component of the impedance occurs at $\omega \tau_{p}=1$, where $\tau_{p}=R_{p} C_{p}$. These dependencies can be altered. For example, if the value of $R_{e}$ is made very small, then even at high frequencies it is the $R_{s}$ of the interface, the bulk impedance of the insulator, that will dominate the high frequency $R_{s}^{\prime}$. If $R_{e}$ is made very large, then at low frequencies it is $R_{e}$ that will dominate the real impedance, and not the interface's $R_{p}$.

## H.Summary - Bulk Properties of the System

We have shown how the system's bulk properties may be related to
it's microscopic composition. The prominance of the electronic properties of the interface is largely due to the semiconductor on the interface being the sole contributor to the charge storage in the system. There is an interplay between the deliverance of the current to the interface by the electrolyte and the interface's ability to pass current (and we have shown that this interplay is highlighted by the frequency dependance of the impedance). The penetrability of the current is solely determined by the complex impedance of the interface (Figure 3.8) and the electrolyte's concentration. . The increased electrode surface area can be determined from the low frequency impedance data, whereas the high frequency impedance typifies flat electrode behavior of the area outside the pores. Thus far, however, we have not focused on the ramifications of the porous electrode's surface inhomogeneities and irregularities other than as a means of increasing the total surface area available for the electrochemical reaction. This information is contained in the middle frequency impedance data (see section II.E), and it remains for us to attempt to extract some unique interpretation from it.

## I.Model Output-Surface Contributions

In an attempt to assess the surface roughness contributions to the impedance, we address two questions:

1) Can a characteristic topology and morphology of the RNM porous electrode surface be identified? Can the faatures of the surface be quantified?
2) Can a characteristic morphology so identified be correlated with the electrode's impedance characterization?

To answer the second question first, recall that in section I.E we cited the work of Deutscher et. al. in showing a fractal impedance behavior of metal film fractal structures, and in section II.E we cited the work of Liu who used a similar approach in calculating the impedance behavior of a fractal electrode interface. These studies both utilized systems whose impedance was linearly dependent on area, so that a self-similar area-length relationship translated into a self-similar impedance-length relationship. Thus, Liu chose an ideally polarized electrode interface so that the interfacial impedance was that of a pure capacitor, to obtain the simple area dependent impedance. Following suit, we change the RNM interfacial impedance from the general non-IPE treatment of Figure 3.3 to that of an IPE by eliminating $R_{p}$. It follows that if we identify the electrode surface as fractal, we can directly correlate it with a CPA appearing in the impedance spectrum.

Solving the RNM for the ideally polarized interface presents some additional numerical and computational difficulties. This is due to the fact that at low frequencies the impedance of the interface is so high. This prevents us from examining the results of all input parameters, including the configuration of figures 3.3 and 3.5. By reducing the capacitance from $10^{-6}$ to $10^{-4}$ Farads, we succesfully obtain the results of Figure 3.13, which indeed shows a CPA behavior in the intermediate frequency range. From Figure 3.13b we evaluate


MID FREQUENCY CPA - P220


Figure 3.13- CPA behavior of the impedance in the RNM when we assume an Ideally polarized electrode.
the slope of the $\log -\log$ plot as -.67 . We find, however, that the value of the CPA slope may be altered simply by changing the impedance values of the constituents of the system. Changing the values of $C_{p}$ or of $R_{m}$ (the metal resistance) varies the CPA slope from . 6 to . 8 . This strongly supports the notion that the appearance of a CPA dependance is not uniquely determined by the purely structural morphology of the electrode.

To further explore the possible fractal nature of our surface, we utilize Equation 1.14 , to calculate the density-density correlation function for the surface metal sites. In our calculation we averaged the two particle correlations over the six nearest neighbor directions, and we did not employ continuous boundary conditions. The correlation function as a function of radius does not show a linear relationship on a log-log plot (Figure 3.14). In fact, the behavior is not unlike that of a Richardson curve for a circle (Figure 1.3). We verified that this was not due to finite-size effects by showing that the same behavior was obtained irrespective of system size. The same non-fractal correlation is obtained irrespective of porosity (Figure 3.15) which suggests that our RNM porous electrode surface is not fractal. The fact that our electrode does not exhibit fractal properties further undermines the Liu hypothesis which attempts to uniquely tie CPA behavior to fractal morphologies.


Figure 3.14- Correlation function of our RNM lattice.

# 2 Particle Correlation Func. for various porosities C(r) 



Figure 3.15- All porosities are measured at lattice size 11.

## IV. Dissolution-Precipitation Reactions

Many electrode processes involve the formation of deposits on an electrode from solution and the dissolution of material back into solution. Corrosion, deposition, dendritic growth are all complex processes which involve a mass transfer between the solution and the electrode surface and back ${ }^{58}$. The reactions involved are diverse and complex, and we present an idealized computer simulation of the process in order to explore the physical structures formed on the electrode and to present methods for characterizing this surface. The simulation does not concern itself with the specifics of the chemical mechanisms involved, but rather treats the dissolution and precipitation as random processes on a large number of particles which are governed by average reaction and diffusion rates.

The use of computer simulation to study this class of phenomena is not new. Witten and Sander ${ }^{19}$ were the first to apply this approach to the study of diffusion-limited aggregation, in which particles were allowed to form in spherical clusters growing from one initial seed site. They measured a correlation exponent (equation 1.14) of .34 on a square lattice. Meakin ${ }^{55}$ has extended the original study by considering varying sticking probabilities, and clusters in three and four dimensional space. In two dimensions, Meakin lists the correlation exponent as .29 for a sticking probability of 0.25 , and . 26 for a sticking probability of 0.1 . There has also been numerous experimental verification of these ideas, that indeed aggregation phenomena build fractal structures ${ }^{58-62}$. More recently, Voss and

Tomkiewicz ${ }^{56,57}$ have applied the same methodology to the study of dendritic growth in electrodeposition, and were able to correlate their theoretical calculations of dendritic growth rates with experiment. Thus far, however, the simulations have considered only the aggregation of particles onto a substrate. The more general situation in which particles are allowed to dissociate back into solution has not yet been treated. It is precisely dissolution precipitation that has been identified ${ }^{63}$ as a reaction mode in porous electrodes (as in the anodic formation of ZnO on Zn electrodes in KOH ), and in order to achieve a better understanding of the type of surface structures that result from this system, we have extended the previous models. This simulation is an alternative approach to the simple random metal distribution that we utilized in the previous chapter for constructing our electrode, and it also serves as a more realistic picture of the surface as the electrode's deposit of insulating material builds during discharge.

## A. The Simulation

A two dimensional NxN lattice is defined on which two types of particles reside- solid particles are fixed (=solid phase), and dissolved particles are free to move through the lattice (xliquid phase) under Brownian motion. We define a dissolution probability, $P_{d}$, and a precipitation probability, $P_{p}$, (corresponding to the rate constants of those respective reactions) and a moving probability, $\mathrm{P}_{\mathrm{m}}$ (corresponding to the diffusion of dissolved particles). Dissolved particles precipitate only if they are part of a cluster of
minimum size $N_{m}$. Each time step in the simulation every particle in the system is examined in a random order, and the following occurs:

1) If the particle is in the bulk of the solid then it remains stationary.
2) If the particle is part of the "solid", and it resides on the surface, it is given a chance to dissolve into solution with probability $\mathrm{P}_{\mathrm{d}}$.
3) If the particle is dissolved, it moves one step in a random direction with probability $p_{m}$. If the lattice site that it tries to move to is occupied, the particle remains in place.
4) If the particles is dissolved in solution but adjoins the solid surface, and is part of a cluster bigger than $N_{m}$, it is given a chance to precipitate with probability $p_{p}$. When the particle precipitates, the adjoining cluster also precipitates.

These checks are made sequentially, so that a particle may be dissolved in step (2) and reattached in step (4). The simulation is begun with rows 1 to $N / 2$ populated with a flat-surfaced solid and with the top half of the lattice empty.

The simulation proceeds for $t$ time steps, and a schematic representation of what the system might look like after time $t$ is shown in Figure 4.1. The particles in solution are represented by the stars, the particles of the bulk are represented by squares, and the particles on the interface are highlighted by the thick continuous line. As illustrated in Figure 4.1, quantitative measures of possible interest for the precipitated particles are the maximum,


Figure 4.1- Schematic of the DP model at a given time $t$. Some quantities of possible theoretical interest are labeled: The surface of the solid is indicated by the heavy line.


Figure 4.2- A typical system configuration of the DP model.
This result is for a dissolution probability of .5 , a precipitation probability of .5 after 1000 time - teps.
minimum and average growth heights, as well as the surface roughness and the possible fractal dimension of the interface. The solution may be characterized by the diffusion behavior of the dissolved particles. Figure 4.2 shows an actual simulation result at tal 000 for $p_{d}=p_{p}=5$, and for $p_{m}=1.0$. It remains for us to attempt to quantify the system and ultimately to correlate the solute behavior and the grown structures with the input parameters of the simulation.
B. Solution characterization-Diffusion Behavior

It is to be expected that if the simulation is realistic, the particles in solution should follow the diffusion equation:

$$
\begin{equation*}
\frac{\partial C(x, t)}{\partial t}=D \frac{\partial^{2} C(x, t)}{\partial x^{2}} \tag{4,1}
\end{equation*}
$$

with the following boundary conditions:

$$
\begin{align*}
c(x, 0) & =c_{*}^{*} \quad, x>0 \\
c(\infty, t) & \left.=c^{( }\right)  \tag{4.2}\\
D\left(\frac{\partial C}{\partial x}\right)_{x=0} & =k_{1} C(0, t)-k
\end{align*}
$$

where $D$ is the diffusion coefficient, $C(x, t)$ is the time and position dependent concentration, $c^{*}$ is the bulk solution concentration, $k$ is the dissolution constant and $k_{1}$ is the precipitation constant. The third boundary condition establishes a fictitious flat plane at $\mathrm{x}=0$ at which all the dissolution-precipitation process is said to occur at all times, $t$. As the simulation progresses and the surface gets to be rough, this is no longer true. With this approximation, we assume that one can always find a plane at $x=x_{0}$ where the dissolution precipitation process effectively occurs. The
value of $X_{0}$ is a parameter to be determined in fitting our data to these calculations.

The differential equation 4.1 may then be solved to yield:

$$
\begin{align*}
C(x, t)= & c^{*}+\left(\frac{k-k_{1} c^{*}}{k_{1}}\right)\left\{\operatorname { e r f c } \left(\frac{x}{\left.2(D t)^{\frac{1}{2}}\right)}\right.\right.  \tag{4.3}\\
& \left.-\exp \left(\frac{k_{1}\left(x+k_{1} t\right)}{D}\right) \quad \operatorname{erfc}\left(\frac{x}{2(D t)^{\frac{1}{2}}}+k_{1} \sqrt{t_{D}}\right)\right\}
\end{align*}
$$

Solving for the total number of particles in solution, from Fick's first law,

$$
\begin{align*}
& \frac{\partial N}{\partial t}=-D \frac{\partial C}{\partial x}  \tag{4.4}\\
& \left.\frac{\partial N}{\partial t}\right|_{x=0}=k-k_{1} C(0, t) \tag{4.5}
\end{align*}
$$

These diffusion equation have been successfully utilized to predict the behavior of experimental dissolution-precipitation systems ${ }^{64}$. In our simulation $c^{*}=0$, and equation 4.4 may be integrated to give the total number of particles in solution at time $t$ as:

$$
\begin{equation*}
N(t)=\frac{k D}{k_{1}^{2}}\left\{\exp \left(\frac{k_{1}^{2} t}{D}\right) \operatorname{erfc}\left(k_{1} \sqrt{\frac{t}{D}}\right)+2 k_{1}\left(\frac{t}{\pi D}\right)^{\frac{1}{2}}-1\right\} \tag{4.6}
\end{equation*}
$$

Figure 4.3 is a plot of concentration as a function of position from the surface into the the solution at $\mathrm{t}=1000$ time steps. The dotted line separates the x axis into two regions. In region I is the

555171000


Figure 4.3- The concentration in solution as a function of the distance away from the surface. Region I is still within the heterogeneous mixture of solid and solute. Region II is above the height of the tallest solid peak.


Figure 4.4- The total number of dissolved particles in solution as a function of time. The straight line is a fit of the data to equation 4.6 .
rough interface where there is an inhomogeneous mixture of solid and dissolved particles, which leads to large fluctuations in the dissolved particle concentration. In region II is the bulk of the solution, and we see the particle concentration begin to behave similar to what we would expect from equation 4.3. The total number of particles in solution as a function of time is shown in figure 4.4, and the solid line is a fit from equation 4.6. This is a confirmation of the validity of the simulation, and from the fit parameters we obtain the theoretical $k, k_{1}$, and $D$. Using this methodology, it is possible to correlate the simulation input parameters $P_{p}, P_{d}, P_{m}$ with the more realistic diffusion equation parameters.

## C. Solid Characterization - Rough Surface

We can also explore how surface topology is altered by variations in the previously mentioned input parameters. The surface properties can be quantified by calculating the density-density correlation function as in chapter III, or the more straightforward Richardson curve for the surface length under different measuring scales. (The Richardson curve measurement on a discrete lattice is easily implemented on a two dimensional surface as opposed to the 3D Random Network Model of chapter III.) Figure 4.5 is the result of such a calculation for the system of figure 4.2. The calculation is done by considering pair correlations only for the surface particles. First, the computer identifies which particles are on the interface (the solid line in Figure 4.1) and then discards all the other points. The pair correlation for each radius $r$ is computed, and
this is averaged over the four nearest-neighbor directions. Figure 4.5 shows a power law behavior in the relationship between $C(r)$ and $r$, with a correlation exponent of 0.22 . $C(r)$ is shown at $t=600,800,1000,2000$ time steps, and the power law behavior is stable for r<9.0. Finite size effects seem to dominate for r>9.0. From equation 1.14 the fractal dimension of this two-dimensional system is $\mathrm{D}=1.78$. This value is considerably higher than that calculated by Witten, Sander and Meakin, but it is worthwhile to note that the geometry of our system is very different from the spherical cluster aggregation that they considered. It is hoped that a further understanding of the significance of the values in this system may be realized by examining the fractal dimension under different dissolution and precipitation probabilities, as well as at 1onger times.

The surface constructed using the dissolution-precipitation model is an alternative starting point for our Random Network Model. Instead of taking the electrode configuration as constructed using a simple random number generator, we can utilize the dissolution-precipitation lattice as a template for assembling our system of resistors and capacitors. This is a more realistic system than the Cantor Bar model, and is an example of exactly the type of self-similar R-C network that Nyikos and Pajkossky proposed to construct for the examination of fractal interfaces (see section II.E).


Figure 4.5- Correlation function calculation for the surface of the DP model for the system shown in Figure 4.2 .

## V. Summary and Conclusions

Recent developments in the field of composite media and disordered systems have availed us with powerful techniques for characterizing stochastic multi-phase systems. Analytic methods such as Effective Medium Theory offer a simplicity of concept and execution, but are more limited in their ability to model some of the salient features of real life systems. Computer simulations offer more flexibility in this regard, and provide a powerful means of exploring the behavior of systems with detailed constraints and/or complicated geometries. The drawback to numerical simulations is that one is limited by a finite accuracy that may not be sufficient in some cases (the "round off error"), and by the fact that as fast as today's computer's are, sophisticated models and large systems require huge amounts of computer time.

The simulations presented in this work are physical simulations involving the construction of a model structure to mimic the topological features of a system. The process is really a two step one:

Firstly, the system must be constructed in the computer using a realistic set of rules and input parameters. We have presented two examples of this - the random network model (RNM) and the dissoloution-precipitation (DP) model. These representations are very different from each other and complementary; the RNM model is an attempt to depict a typical porous metal structure in an electrolyte with the major topological contribution arising from the
metal's porosity, while the $D P$ model is an attempt to depict the topology of an electrode surface as it is roughened by chemical deposition and dendritic growth. An "ideal" examination of porous electrodes might utilize both of these configurations simultaneously. To be sure, both of these models introduce simplifying assumptions, and additional variations can be considered to explore how different pictures of the geometry and different constraints would affect the results.

Secondly, once the system has been constructed, the computer may be utilized to analyze and perhaps quantify the structure of the model. In addition, other properties of the system may be calculated and an attempt can be made to correlate these with the system's topology. In this work we examined the impedance behavior of the system and it's dependance on morphology. The electrical impedance is very well suited for this type of exploration, due to the fact that impedance in addition to being dependent on the internal structure of the material is highly dependent on it's macroscopic shapes and lengths. The frequency dependance of the impedance is especially useful in this respect as well, for it allows us a consistent way of weighting various surface and bulk effects with respect to each other and seeing the result. It is also a measurement that is well open to experimental exploration and verification.

In addition to providing sample implementations of the above methodology, we have provided some new insight into the porous electrode system. We have demonstrated the utility of the low frequency
capacitance values in measuring the apparent area of the rough electrode, and provided a caveat concerning "what is called low frequency?"- showing that it is dependent on the relative magnitudes of the interfacial impedance and the electrolyte resistivity in each electrode system. In this regard, for all it's simplicity, the DeLevie model is an excellent representation of the current flow in a porous structure. We have found that in an ideally polarized rough electrode there is a fractional power law relationship between $Z$ and $\omega$ with an identical scaling exponent for the real and the imaginary components (the CPA behavior), but we have shown that it is not necessarily related to fractal structures, as suggested by Liu and others. All in all, the RNM seems to be a useful addition to an electrochemist's toolbox. The DP model of Chapter IV is an attempt to explore the morphology of the electrode which results from the electrochemical reactions on the interface. We have shown that the model is valid by verifying the predicted diffusion behavior of the dissolved particles. The $D P$ model also produces a fractal surface which is fertile ground for additional research.

## Future Work

We have treated a single geometry in the RNM, and an extension of that to alternate geometries is warranted. One example is allowing the fluid to flow through the cracks between adjoining metal sites. The DP model is as yet largely unexplored - we have not yet examined how the fractal geometry of the system changes under varying sticking and dissolution probabilities. An important further consideration is to attempt to construct the system in different geometries
(for example on a triangular lattice) to verify that the results are independent of the numerical implementation of the model. The merging of the RNM and the DP model is an important further step to be explored. We can thus construct a realistic picture of the electrode under operating conditions, and solve for the impedance of such a structure. New techniques are being utilized for the calculation of network impedances utilizing a transfer matrix algorithm ${ }^{68,69 \text {. The advantage of this method is greater accuracy as }}$ well as greater efficiency in utilizing computer storage. This method has already been applied to 2-D complex impedances ${ }^{70}$, and extension to three dimensions should be trivial. Any further work in network simulations would benefit by utilizing these transfer matrix techniques instead of the simultaneous equation solutions.

## APPENDIX

PROGRAM LISTINGS
POROOON10
POROON20
啇宽
으웅

응
0600080

$\stackrel{N}{C}$







```
        1D1M4=NT1
        101M5=1BOT
        101M6 = NRD
        1D1M7 = IB01
C
TOTAL STORAGE REQUIREMENT IS:
    12*N + 2*NT + NTI + IBOT + 2*NRD + 18O
        NSTOR = 12*1DIM2 + 2*IDIM3 + IDIM4 + IDIM5 + 2*IDIM6 + IDIM7
        NBYTES = 4 * NSTOR
        HRITE (6,1) MAXSIZ, ISMIN, ISMAX, NT, NTI, IBOT, NRD, IBOI, NSTOR, NBYTES
    1 FORMAT (' STORAGE ALLOCAT IONS- IDIMI-->IDIM7:, MAXSIZE= , 12,//,
        C! N=, 110, ISMAX= 110,'NT= , 110,' NTI=, , 110,%,
        C' 1BOT = ', 110,' NRD= ,110,' IBO1= ,110,
        C//:MIN TOTAL STORAGE REQUIRED=':112,'=', 112,' BYTES'), (1)
        Pl=3.14159
    ERROR SETTING ROUTIME-- UNDERFLOW ERROR
    CALL ERRSET (208,0,5,1)
    INITIALIZE -- IDUMP=1 MEANS GENERATE DETAILED NUMERICAL SUMMARY
    ONLY ON FREQ'S WITH MULTIPLES OF 10.
        IDUMP = 1
        INITIALIZE FLAG TO INDICATE NUMERICAL DIFFICULTIES
    1BOMB = 0
C
*******************************
* PROGRAM MAY be TERMINATED gY THE COMMAND '#CP STORE 464 1'
* INITIALIZE NUSERFWD IN NUCON (LOC X'464 J TO 0.
        IZERO= 1TYBIT (-1)
**
    SYSTEM TIME READING ROUTINE -- FORTRAN UTILITIES TXTLIB
    VARIABLES USEO:
    ETCPU = ELAPSED TOTAL CPU
    ETIME = ELAPSED CONNECT TIME
    STIME = STARTING TIME THAT PROBLEM WAS RUN (CPU)
    TIMNOW = CURRENT TIME (CPU)
    TIMLAS = LAST VALUE OF CURRENT TIMF (CPU)
    RUNTIM = TIMNOW - TIMLAS
    CONNOW = CURRENT CONNECT TIME
    CONLAS = LAST CONNECT TIME
    CONTIM = CONNOW - CONLAST
    all times are stored in minutes.
C
```

PORO1520
PORO1530
PRO1540
PORO 1550
PORO1560
PORO1570
Poro 1580
oro
ORO1600
PORO 1610
PORO1620
PORO1630
ORO 1640
PORO1650
PORO1660
ORO1670
ORO1680
PORO1690
ORO1700
PORO 1710
ORO1720
ORO1730
PRO1740
PORO1750
PORO1760
PORO1770
PORO1780
PORO1790
PORO 1800
PORO1810
PRO1820
PORO1830
PORO1840
PORD1850
PORO1860
PORO1870
PORO1880
PORO1890
PORO1900
PORO1910
PORO1920
POHO1930
PORO 1940
PORO1950
PORO1960
PORO1970
PORO1980
PORO1990
PORO2000
PORO2010

```
5 CALL DATTIM (EVCPU, ETIME, ETCPU)
WE'RE ONLY INTERESTED IN TOTAL CPU TIME
        INITIALIZE TIME COUNTERS AND CONVERT TO MINUTES
        STIME = ETCPU/60.0
        CONLAS = ETIME/60.0
        TIMLAS = STIME
C
```



```
        NRPT=0
C FIRST SET UP LATTICE PARAMETERS
    1 0
        READ (5, 10, END=900) IM, JM, KM, POR, POREND, PORINC
        FORMAT (312,3F3.2)
        READ (5,19) MODEL
    19 FORMAT (13)
        IF (MODEL.LT.O) THEN
            I SOLVE = 0
            MODEL = |ABS(MODEL)
        ELSE
        ISOLVE = 1
        ENDIF
        F (IM.GT.MAXSIZ) THEN
            WRITE (7,11) IM, MAXSIZ
            GOTO 910
            ENDIF
    11 FORMAJ ('FERROR-- IM= ', I3,: IS GT MAXSIZ= ',13)
        IF (PORINC.EQ.O) PORINC'=, ?
            EXTRA PARAMETER DEFINES HOW HIGH LP IN THE
            EXTRA PARAMETER DEFINES HOW HIGH LP IN THE
C--
        MEAD (5,20) KMSTOP
        FORMAT (12)
        IF (KMSTOP.GT.KM) KMSTOP = KM
C
        IS = IM*JM*KM
        ISM = IS - 1
    NOW READ IN ELECTRICAL PARAMETERS
        READ (5,50) RELIN, RMETIN,RSIN,RIIN,CPIN
    50. FORMAT (5E9. 1)
C PORSIZE OF 10 MICRON= 1E-3 CM; WE USE UNIT IN CM
```

PORO2020 PORO2030 PORO2040 PORO2040 PORO2050 PORO2060 Роя02070 PORO2080 POR02090 PORO2100 PORO2110 PORO2120
POR02130
POROZ 140
PORO2 150
PORO2 160
PORO2 170
PURO2180
POR()2190
PORO2200
POR02210
PORO2220
PORO2230
PORO22ll
PORO2250
PORO2260
POROR270
POR02280
POR02290
PORO2300
PORO2310
PORO2320
PORO2330 PORO2310 PORO2350 POKO2360 PORO2370 PORO2380 PORO2390 PORO2400 PORU24 10 PORO2420 PORO2430 PORO2440 PORT02450 PORO2460 PORO2470 PORO2480 POR02490 PORO2500


```
C
    NOW SET UP AND DISPLAY PHYSICAL SYSTEM
    WRITE(6,124) IM.JM, KM, POR
    WRITE(7, 124) IM,JM,KM,POR, 314,' POROSITY = ',F3.21
    READ (5,125) DSEED
    125 FORMAT(DI3.0
c INITIALIZE 'NEW LATtICE' flag
C
        IWENT = 0
C
c
        STORE INITIAL DSEED VALUE fOR THIS LATtICE
        DHOLD=DSEED
        ------------ WRITE SUMMARY to DATABASE FILE
```

$\qquad$

PORO3010
PORO3020
PORO3020
PORO3030
PORO3040

PORO3050 PORO3060 PORO3070 PORO3080 PORO3090 PORO3 100 \begin{tabular}{l}
PORO3 3110 <br>
PORO <br>
\hline

 PORO3120 PORO 3130 PORO3140 

POROB 3150 <br>
PORO <br>
\hline
\end{tabular} PORO3160 PORO3170 PORO3180

PORO 190 PORO3190 PORO3200 PORO3210 PORO3220
PORO 3230 PORO3230 PORO3240 PORO3250 PORO3260
PORO3270 PORO3270 PORO3280 PORU3290 PORO3300 PORO3310 PORO3320 PORO3330 PORO3340
PORO PORO335U PORO3360 PORO3370 PORO3380 PORO3390
PORO 3400 PORO3400 PORO3410 PORO3420 PORO3430 PORO3440 PORO3450 PORG3460 PORO3470 PORO3480
PORO 3490 PORO 3490
PORO 3500 PORO3500 PORO3510

```
130 IF (IDUMP.EQ.1) THEN
*)
IF (MOD(INT(FRCNT),IPPD),EQ.0) WRITE (9.131) FREQ
1 3 1
ENDIF
WRITE (6,135), 'H################################################
135 FORMAT (6, ', ///,'HH
C NOW SET UP IMPEDANCES OF DIFFERENT CIRCUIT ELEMENTS AT THIS FREQ
    OMEGA = 2*PI* FREO
    OMEGA = 2 * OMEGA * CP * AP)**2
*HERE WE HAVE THE OPTION OF AN IDEALLY POLARIZED ELECTRODE
#HERE WE HAVE THE OPTION OF AN IDEALLY POLARIZED ELECTRODE
    IPOLAR = .TRUE.
        ZREAL = RS + RP/DENOM
        ZREAL = RS + -OMEGA # CP * RP * RP /DENOM
*.....i\ddot{F iiPOMAARj THEON}
            ZREAL = RS
            ZIMAG = - 1.0 / (OMEGA*CP)
        ENOIF
***
C
Z(1,N) IS METAL RESISTANCE; Z (2,N) IS ELECTROLYTE RESIST
c
Z(1,1)=RMET
Z(1,2)=0.0
Z (2,1)=REL
Z(2,2)=0,0
Z(3,1)=ZREAL+RMET
Z ( 3,2)=Z\MAG
Z (4,1)=ZREAL+REL
Z(4,2)=ZIMAG
FLAG:#6 DENOTES METAL ON BOTTOM WHICH IS NOT TO HAVE SC INTFC
C FLAGV IT-G IT'S PURPOSE IS JUST TO DISTRIBUTE THE CURRENT
ABOVE THE PLANE ABOVE IT
Z (6,1) = RMET
z(6,2) = 0.0
WRITE(6,132)REL,RMET, RS, RP,CP
PORO352D
* ONLY LIST EVERY MULTIPLE OF 10
ORO3530
PORO3540
PORO3550
PORO3570
ORO3570
135 FORMA ( 3 ,
C NOW SET UP IMPEDANCES OF DIFFERENT CIRCUIT ELEMENTS AT THIS fREO DMEGA \(=2\) * PI * FREC
PORO3600
    ORO3620
    0RO3630
ORO3630
ORO3640
ORO3650
ORO3660
PORO3670
```



```
PORO3690
PRO3690
ORO3700
ORO3710
ORO3720
ORO373
PORO3740
PORO3750
ORO3760
ORO3770
poro3780
PORO3780
ORO3790
ORO3800
ORO3810
por03820
PORO3830
PORO3840
PORO3850
PORO3860
PORO3870
PORO3880
POR03890
PORO3900
ORO3910
132 FORMAT (' REL, RMET,RS,RP,CP= ',1P,5E13.3)
PORO3920
PORO3930
WRITE(6,138) CURE1),CUR(2)
WRITE(6 138) CUR(1) CUR(2) , T,SEI3.3)
PORO3930
WR1TE(6;138) INPUT CURRENT= ',1P,E12.2,' +(J*) ',E12.2)
PORO3940
138
WRITE (6,150)
PORO3950
PORO3950
50 FRITE (6,1 IMPEDANCES, REAL & IMAC, (')
```



```
PORO3970
WRITE (6, 200) ((Z(1,J)), , =1,4), J=1,2)
PORO3970
00 FORMAT (: Z= (ZE13.4)
PORO3980
If (IDUMP.EQ. I) THEN
PORO3990
PORO4000
```



```
C
    SET fLAG TO IndICATE THAT THIS LATTICE HAS MOOO AND MSSO SOLVED
    IWENT = 1
        ISN= IB(1)
    380
        IERIMD MNSO '3
        CALL MNNO (N,ISW,IA,JA,G,IAT,JAT,IAP, IPM,IU,JU,U,DI;IER,IUP,
        ISL=0
C IF FLAG=OIDIST' IS SET, THEN CUREENT IS TO BE UNIFORMLY DISTRIBUTED
C
    IF (IDIST.NE.1) COTO 410
C
        WAREA = IM * JM
    WHITE (6,395) HAREA 
        FORMAT(' CURRENT DISTRIBUTED EVENLY THROUGH BOTTOM*,F4.O,' NODES')PORO4G70
        CURNOR(2) = CUR(2)/WAREA POROL690
        IEND = 2** IM# JM WNKL_ PORO4700
        DO 400 ||=1,IEND, 2
        B(11) = CURNOR(1)
        B(11+1)= - CURNOR(2)
    400 CONTINUE
    410 CALL MBSO(N,ISL, IPM, IU,JU,U,DI, B,G,X)
        DIV1=CUR(1)**2 + CUR(2)**2
        ZREAL = (G(1)*CUR(1) +G(2)*CUR(2))/DIV1
        ZIMAG =GG(2)*CUR(1) -G(1)*CUR(2))/DIV1
        WRITE (6,503) DHOLD
        WRITE(6,172) ZREAL,ZIMAG
    72 FORMAT (' COMPL IMPED.= ',1P,E12.4,' (+J*)',E12.4)
    IF (IDUMP.EQ. 1) THEN
        DUMP V OF ALL NODES
            IF (MOD(INT(FRCNT),IPPD).EQ.0) WRITE (9,182) (G(11),1I=1,N)
    182 FORMAT (1P,6E13.4)
        ENDIF
        CALL CURCAL (A,G,MODEL)
        WRITE (6,420)REALI, RMAGI
    420 FORMAT (' CALCULATED CURRENT = ',1P,E12.2,' +(J*)',E12.2)
C
    IORMALIZE IMPEDANCE TO GIVE 1 CM**\dot{2}}\mathrm{ READING
        ZRLNOR = ZREAL / SCALE
            ZIMNOR = ZIMAG / SCALE
            WR1TE (6,190) SCALE
    190 FORMAT !' NORMALIZEO 1 SQ CM IMPEDANCE BY ',F8.0,' NORM FACTOR:'
    WRITE (6,172) ZRLNOR,ZIMNOR
C
    DOCUMENTATION PAGE
    PORO4710
    PORO4710
    PORO4730
    PORO4}74
    PORO4750
    PORO4750
    PORO4760
    PORO4780
    PORO4}79
    POR04790
    PORO4810
    PORO4810
    PORO4820
    PORO4830
    PORO4840
    PORO4850
    PORO4850
    PORO4850
    POROI!870
    POROII870
    POR(04880
    POR!4880
    PORO4900
    PORO4900
    POROL4910
    POROt/920
    PORO4930
    PORO4930
    PORO4950
    PORO4960
    PORO4970
    PORO4970
    POR04990
    POROS000
    PORO5010
    |
    PORO4920
```

```
            ||=1
            WR+TE(7,500)
```

POR05020 PORO5030 POR05040 POR05050 POROS060 POROS060 PORO5070
PORO5080 PORO5080
PORO5090 POR05090 POROS 100 POR05110 PORO5120 PORO5130 PORO5140 PORO5150 PORO5160 PORO5170 POROS 180
POROS 190 POROS 190
POROS200 PORO5200 POROS210
PORO5220 PORO5220 PORO5230 PORO5210

PORO5250 PORO5250 | PORO5260 |
| :--- |
| PORO52 | POROS2 10 PORO5290 PORO5290 PORO5300 POROS310

POROS PORO5320
POROS 330 POROS340 POROS340 POROS350 PORUS360 POROS3 380 PORO5380 PORO5390 PORO5400 PORO5410 PORO5420 POROS430 PORO5440 PORO5450 PORO5460 PORO5470 PORO5480 PORO5490
POR05500



```
    15 CALL LATSET{A, POR,MODEL)
        GOTO 505
C =========================
    17 KMIN = 2
        KMAX = KM - 1
        CALL PLANE (A,1,5)
        CALL PLANE
        CALL RNDMM(A,MB, DSEED, KMIN, KMAX, 1)
        gOTO 505
C ==========2
C MODELS 1 8c 2 -- OLD FASHIONED WAY
C SET BOTTOM PLANE = ELECTROLYTE ; TOP PLANE = METAL
            CALL PLANE(A,1,2)
C
    **世###############
    NOW ChECK FOR CONTINUATION OF METAL --NO ISLANDS ALLOWED
C THE (KM-1) PLANE IS ALL CONNEGtED to the top metal
        DO 50 :=1,1M
        IF (A(I,J,KM-1).EQ.5) A(I,J,KM-1)=1
    50
    C
    60 KFND=0
C THIS LOOP LOOPS DOWN FROM K=(KM-2) TO K=2
        DO 400 KT=4, KM
        K=KM-KT+2
c
    FIRST CONNECI FROM ABOVE
        D0 75 I=1, IM
        IF (A(I,J,K).NE.5) GотO 75
        IF (A(I,J,K+i).EQ,1) A(1,J,K)=1
    75 CONTINUE
C MOW GO CONNECT METAL TOGETHER ON THE REST OF KTH PLANE
    100 CALL CONECT (A,K,5,1, I FND, KFND)
C FINISHED PLANE YET? ?
IF IF (IFND.NE.O) GOTO 100
c if finished, go to next plane
400 CONTINUE
C
    FINISHED GOING THRU ALL PLANES .CO AAIM?
        FIF (KFND.NE.O) GOTO 60
c
C FINISHED CONNEGTIN ALL METAL; NOW DEAL W/ ISOLATED PTS
    C FINISHED CONNECTIN ALL METAL; NON DEAL 
                -14-
```

POR06500
POR06510
POR06520
PORO6530
POR06540
POR06550
POROG560
POR06570
PORO6580
POR06590
PORU6600
PORO6610
PORU6620
POR06630
PORO6640
POR06650
PORO6660
POR06670
PORO6680
POR06690
PORT06700
PON16710
PORU6 720
POROG730
PRO6740
PORO6750
ORO6760
ORO6770
PRO6780
ORO6790
ORO6800
PORO6810
PRO6820
POR06830
PRO6840
ORO6850
PORO6860
ORO6870
PORC16880
POR06890
ORO6900
PORU6910
ORO6920
ORO6930
PORU6940
PORO6950
PORO6960
POR06970
POR16980
ORUG990
POR07000

```
    ICTR=0
    KK=KM=2 K=2,KK
    MO 500 K=2,KK
    00 500 l=1, 1M
    IF (A(I,J,K).NE.5) GOTO 500
    IF (A(I,J,K),NE.5) GOTO 500
    A( }1,J,K)=
    450 CAL, R,K)#0 (ANPT (A, DSEED,KMIN,KMAX, IT, JT, KT)
        | FND=0
        CALL NEARN(A, IT, JT, KT, O, 1, I FND, KFND)
        CALL NEARN(A, IT,JT,KT,0,
        ICTR=ICTR+1
        WRITE (6,460) I,J,K,IT,JT,KT
    460 FORMAT (: PT #, 3|3,' REPLACED BY ',3I3)
    5 0 0 ~ C O N T I N U E ~
    501 FORMAT (; 501) ICTR % PF PTS REPLACED= ',16)
C
C ======================================
    NOW FIND ELECTROLYTE CONNECTIONS
    505 CONTINUE
C WE HAVE TWO METHODS--- 1) FILL IN ONLY CONNECTING ELEC PATHS
GOTO (540,515,540,515
C WE HAVE CHOSEN MODEL H2 OR H4-- FILL IN ALL NON-METAL
C
    515 SPACE WITH ELECTROLYTE
    515 DO 520 K = KMIN, KM
    520 CALL PLANE (A,K,2)
        CALL FILABL(A,LBLA)
        RETURN
C===========================================
M MODEL W1 WATCH FOR ELECTROLYTE FLOW
    540 CALL PLANE (A,2,2)
C NOW GO THROUGH ALI PLANES BOTTOM TO TOP
    550 KFND=0
            KT=KMF-1
            KT=KM-1 K=3,KT
        FIRST MAKE CONNECTIONS FROM BELOW
            DO 600 I= 1, IM
            00 600 J= 1,JM
            IF (A(I,J,K).NE.O) GOTO 600
            IF (A(I,J,K).NE.0) GOTO 600
        A(I,J,K)
C
    6 0 0
NOW DEAL HITH NN ON PLANE
```

POR07010
PORO7010 POR07020 POR07040 PORO7040 POR07050
PORO7060 PORO7060 PORO7070 PORO 7080 PORO7090 PORO7 100 PORO7110 PORO7120 POR07130 POR07140 PORO7150 PORO7160 PORO7170 PORO7180 PORO7190 PORO7200
PORO 7210 PORO7210
PORO 7220 PORO7220
PORO7230 PORO7230 PORO 7240 PORO7250 PORO 1260 POHO 1260 PORO7270 PORO7280 PORO7290 PORO7300 PORO7310 PORO7320 PORO7330 PORO7340 PORO7350 PORO7360 PORO7370 PORO7380 PORO7390 PORO7400 PORO7410 PORO7420 PORO7430 PORO7440 PORO7450 PORO7460 PORO7470 PORO7480
PORO7490 PORO7490

```
610 CALL CONECT(A,K,0,2,1FND,KFND)
C NOW GO TO NEXT PIANE
```

PORO7500
PORO7510 PORO7520 PORO7520 PORO7530
PORO7540 PORO7540
PORO 7550 PORO7550
PORO 7560 PORO7560
PORO 7570 PORN7570 PORO7580 POR07590 POR07600 PORO76 10 PORO7620 PORO7630 PORO7640 PORO7650 PORO7660 PORO7670
PORO7680 PORO 7680
PORO 7690
C IF A PREVIOUS CALL TO NEARN FOUND NO NEW VALUES THEN WE'RE FINISHEDPORO7690
IFND 15 THE FLAG TO INDICATE WHETHER A NEH VALUE HAS BEEN FOUND PORO7700
IFND=0
FND=0
$00325 \quad t=1,1 \mathrm{M}$
PORO 7710
PORO7720
$\begin{array}{ll}\text { DO } 325 \mathrm{~J}=1, \mathrm{JM} & \text { PORO7730 } \\ \text { CALL NEARN(A, I, J, K, ISTAY, ILOOK, I FND, KFND) } & \text { PORO7740 }\end{array}$
325 CONT INUE
IF (IFND.NE. 1) GOTO 400
FND $=0$
DO $335 \mathrm{~J}=1, \mathrm{JM}$
DO $335 \quad \mathrm{I}=1$, IM
$I T=1 M-1+1$
CALL NEARN(A, IT, JT, K, ISTAY, I LOOK, I FND, KFND)
335 CONTINUE
CONTINUE
IF (IFND.NE.1) GOTO 400
1 F 345
$00345 \quad 1=1,1 \mathrm{M}$
DO $345 \mathrm{~J}=1$, JM
$\mathrm{JT}=\mathrm{JM}-\mathrm{J}+$
(A, I, JT, K, ISTAY, I LOOK, I FND, KFND)
45 CALL NEARN (A, $1, J, K$,
CONT INUE
IF (IFND.NE.I) GOTO 400
1FND=0
DO $355 \mathrm{~J}=1$, JM
DO $355 \mathrm{l}=1, \mathrm{Im}$
$1 \mathrm{~T}=1 \mathrm{M}-\mathrm{I}+1$
CALL NEARN(A, IT, J, K, ISTAY, I LOOK, I FND, KFND)
355 CONTINUE
400 CONTINUE

PORO7700 PORO7720 PORO7740 PORO7750
PORO7760 PORO7760 PORO7770
PORO7780 PORO7780

PORO7790 PORO7800 PORO7800 POR07810 PORO7820 POR07830 | PORO784 |
| :--- |
| PORO |
| 850 | PORO7850 PRRO7860 PORO7870 PORO7880 PORO7890 PORO7900 PORO7910 PORO7920 POR07930 PORO7940 PORO7950 PORO7960 400 CONTINUE

PORO7970 POR07980


```
            IF (A(I+1,J,K).EQ.1LOOK) GOTO 300
            (J-1.LT,i) GOTO 40
            A(1,J-1,K).EQ.ILOOK) GOTO 300
            (J+1.GT.JM) GOTO 50
            (A(I,J+1,K).EQ. ILOOK) GOTO 300
            (K-1.LT. 1) GOTO }6
            (A(1,J,K-1).EQ.ILOOK) GOTO 300
    0 IF (K+1.GT.KM) GOTO 100
    NO (A(I,J,K+1).EQ. ILOOK) GOTO 300
    C NO NN VALUE OF ILOOK FOUND:
    100 GOTO 400
    C ONE OF ADJACENT NODES = ILOOK
    300 A(I,J,K) = ILOOK
        |FND=1
    O0 RETURN
    400 RETUR
    *---m-
    C SUBROUTINE RANPT (A, DSEED, KMIN, KMAX, I,J,K)
C SUBROUTINE TO GENERATE RANDOM LATTICE PT (I,N,K)
    C SUBROUTINE TO GENERATE RANDOM LATTICE PT (I,N,K)
        NUMBER 8ETHEEN, 
        INTEGER*4 A(IM,JM, KM)
        DIMENSION R(3)
        COMMON/LATICE/IM,JM, KM, KMSTOP
    C
        CALL GGUBS(DSEED, 3,R)
        l=INT(R(1)*IM + 1.0)
        K=INT(R(2)*JM+_1.0)
        K=INTG
        REND
    C-
C SUBROUTINE CHOP (A)
C SUBROUTINE TO EFFECTIVELY CHOP OFF THE UPPER PLANES
            AND ELIMINATE THEM FROM OUR PROBLEM
        INTEGER#4 A(IM,JM, KM)
    C
        COMMON/LATICE/IM,JM, KM, KMSTOP
        COMMON/ELEC/Z(6,2),CUR(2),NCUR,NGND, REALI, RMAGI
C COMMON/ELEC/Z(6,2)
    CHANGE KM TO
C KM = KMSTOP TO NEH LOCATION

POR08480
PORO8490 PORO8500 PORO8510 POROB520 POROB520
PORO8530 PORO8530
PORO8540 PORO8540 POR08550 POR08560 PORO8570
PORO8580 PORO8580
PORO8590 POR08590 POR08600 POR08610 POR08620
POR08630 PORO8630
POROB640 POR08640 POR08650 PORO8660 POR08670 POR08680 PORO8690 PORO8690
PORO8700 PORO8 700
PORO8 710 PORO8 710
POR08 720 PORO8720 PORO8730 PORO8740 PORO8750
PORO8760
PORO8770
PORO8780
PORO8790
PORO8800
PORO88 10
PORO8820
POR08830
PORU8840
PORO8850
PORO8860
PORO8870 PORO8880
PORO8890 POROB890
PORO8900 PORO8900 PORO8910 PORO8920 PORO8930 PORO8940 PORO8950 POR08960 PORO8970 PORO8970
POR08980
```

NGND = IM\#JM*KM
C
CHECK tO SEE THAT CORNER NODE is CONNECTED TO METAL
IF (A(IM, JM, KM).EQ. 1) GOTO 100
20 FORMAT (I ELECTROLYTE AT CORMER TO
FORMAT ( ELECTROLYTE AT CORNER T!OP- PLANE DISPLACED FOR GND*)
DO 50 I=1, IM
OO \$0 IM=1+1
II=IMM-I+1
A(1I,JJ,KM) =1
IFND = O
CALL NEARN(A, 11,JJ,KM, 1,1, I FND, KFND)
IFND IS SET (=1) IF THIS PT IS CONNECTED TO
ANOTHER METAL.O) GOTO 100
IF (IFND
c
100 HRITE (6,175) KMSTOP
175 FORMAT (; ',/%,' LATtice truncated above PLANE \#',13)
C COMPUTE NEW STATISTICS; NO NEED TO RE-DISPLAY LATtICE
C
ISHOW = 0
C
CALL DISPLA (A, ISHOW, MODEL)
RETURN
END
C
SUBROUTINE DISPLA (A, ISHOH,MODEL)
INTEGER*4 A( IM,JM, KM)
DIMENSION LBL(7),LAB1(20)
COMMON/LATICE/IM, NM, KM, KMSTOP
COMMON/LATICE/IM,JM, KM, KMSTOP'X','B'/
C
C
C ISHOW = O IHE LATTIGE FLAG THAT JUST CAUSES PRINT OUT OF STATISTICS
WRITE (6,4)
WRIIE (6,4)
C INITIALIZE COUNTÉRS FOR DISTRIBUTION TOTALS
NMET =0
NELEC = 0
NAlR=0
C
NOW go through each plaNE :

```
PORO9000
PORO9010
POR09020
PORO9030
PORO9040
Роно9050
PORO9060
PORO9070
PORO9080
PORO9090
PORO9100
PORO9110
PORO9120
port09130
PORO9140
PORO9150
PORO9160
PORO9170
PORO9180
POR09190
PORO9200
PORH9210
POR09220
PORO9230
POR09240
PORO9250
POR09260
PORO9270
PORO9280
PORO9290
porogion
PORO9310
PORO9320
PORO9330
PORO9340
PORO9350
PORO9 360
PORO9370
PORO9380
POH09390
PORO9400
PORO94 10
PORO9420
PORO9430
PORO9440
PORO9450
POR09450
PORO9460
PORO9470
POHO9480
PORO9480
PORO9490
```

                DO 600 K=KM, 1,=1
    C \#***************䋨**********
NOW COMPUTE THE DISTRIBUTIONS ON THIS PLANE :
C
ICONT = 0 _ (MODEL.GT.50) ICONT = 1
KMET=0
KELEC=0
SC=0
KSC=0
NO 250 I=1,IM
AT }=\textrm{A}(1,J,K
IF (IAT.EQ.O) GOTO 250
IF (IAT.EQ. I) KMET=KMET+
IF (IAT.EQ. I) KHET=KMET+1
C NOW CHECK IN THE 3 FORWARD DIRECTIONS FOR SC INTFC
IF (I+1.GT.IM) THEN
I+1.GT.IM) THEN
NEXT = A(1, K)
IF (IAT+NEXT.EQ. 3) KSC = KSC + 1
ELSE ENDIF
IF (IAT+NEXT.EQ.3) KSC = KSC + 1
ENDIF
IF(ICONT.EQ. 1) THEN
NEXT = A(I,1,K)
ENDIF
LSE
NEXT = A(I,J+1,K)
ENDIF
140 IF (K+1.GT.KM) GOTO 250
IF (IAT+A(I,J,K+1).EQ.3) KSC=KSC+1
C
250 CONTINUE
250 CONTINUE IM*JM) - KMET - KELEC'
C NOW ACCUMULATE TALLY FOR WHOLE LATTICE
NMET = NMET + KMET
NMET = NMET + KMET
NELEC = NELEC + KELEC
NSC = NSC + KSC
C END PLANE DISTRIBUTION CALC
C END PLANE DISTRIBUTION CALC
C
IF ISHOW IS SET THEN DISPLAY LATTICE---

```

POR09500
POR09510 POR09520 PORO9530 POR09540 PORO9550 PORO9560 POR09570 POR09580 POR09590 PORO9600 POR09610 PORO9620 POR09630 PORO9640 PORO9650 PORO9660 POR09670 PORO9680 PORO9690 PORO9700 POR09710 PORO9720 POR09730 PORU9 740 POR09750 POR09760 PORO9770 POR09780 PORO9780
PORO 790 PORO 9790
PORO 900 PORG9810 PORO9820 PORO9820
POR09830 POR09830
POR09840 POR09840
POR09850 PORO9850
POR09860 POR09860 PORO9870
POR09880 POR09880
POR09890 PORO9890
PORO9900 PORO9900
PORO9910 PORO9910
PORO9920 PORO9920 POR09930
POR09940 PORO9950 POR09950 POR09960 POR09970 POR09990
－Lて－

```

        GOTO 600
        22 WR1TE(6,52) {LAB1(J), J=1,JM)
    52 FORMAT (T28,20A2)
    GOTO 600
    23 WRITE(6,53) (LAB1(J), J=1,JM)
    53 format (r27,zOA2)
    GOTO 600
    24 WR1TE(6,54) (LABT(J), J=1,JM)
    fORMAT (T26,20A2)
    coto 600
    25 HRITEE(6.55) (LAB1(J), J=1,JM)
    55 FORMAT (T25,20AZ)
    GOTO 600
    26 MRTTE (6,56) (LAB1(J),J=1,JM)
    56 FORMAT (T24,20A2)
    coro 600
    27 HRITE (6,57) (LAB1(J),J=1,JM)
    57 FORMAT (T23,20A2)
    coto 600
    28 WRITE (6,58) (LAB1(J),J=1,JM)
    FORMAT (T22,20A2)
    GOT0 600
    29 GR1TE (6,59) (LAB1(J), J=1,JM)
    59 forMAT (IT21,20A2)
    600 CONTINUE
    C SUMMARY
C
IS=IM\#JM*KM
C COMPUTE TOTAL \# INTFC SITES POSSIBLE
1NTOT = (KM-2)*"(IM* (JM-1) + JM*(IM-1)) + IM*JNM*(KM-1)
INTOT = (KM-2
PAREA = IM*JM
WRITE (6,700) NELEC, NAIR, NMET, NSC, ROUGH
WRITE (7,700) NELEC, NAIR MMET'NSC, ROUGH
700 FORMAT ('OTOTALS: (E/A/M)=1,14,1/1,14,1/', 14,10K,'\# SC INTFC=',
| FORMAT ROUGH. = , F5,2)
II6, ROUGH. = 'F5.2)
WRITE(6.710) IS, INIOT
coil 600 (5) (LABI(J), J=1,JM)
H)
POR10500
OR10520
POR10520
OR10540
POR10540
POR10550
POR10560
POR10570
OR10590
POR10600
POR10600
POR10610
OOR10620
POR10630
OR 10650
POR10650
POR10670
POR10670
POR10680
POR10690
POR10700
POR10710
POR1U720
POR10730
POR10740
POR10750
POR10750
POR10760
POR10780
POR10780
POR10790
POR10800
POR10810
POR10820
POR10830
POR10840
POR10850
POR10860
POR10860
POR10870
POR 10880
POR10890
10 FORMAT(f TOTAL LATTICE PTS - *, 16,5X, " INTERFACE SITES- *, 16)
RETURN
END
RETURN (HOTAL LATTICEPGS

```

```

POR10900
C
POR10910
POR10910
SUBROUTIME FILABL(A, LBLA)
SUBROUT INE FILABL(A, LBLA)
POR10920
POR10930
SNM,
COMMON/LATICE/IM,NM, Km, KNSTOP
POR 10950
POR10960
POR10970
POR10980

```
```

* icNtr counts the number of nonzero nodes in the lattice
ICNTR = O
ICNTR = O
DO 100 K=1, KM
DO 100 l=3,MM
* -- IF A(1,J,K) = 0 THEN LBLA IS ALSO 0
LBLA(i,J,k) =A(I,J,k)
IF (A(i,j,K).NE.O) THEN
CNTR = ICNTR + 1
ENDIF
100 continue
RETURN
END


# 

        SUBROUTINE CONDUC(A, G,JA, IA, B,N,NT, MODEL)
        S.f. TO CONSTRUCT G MATRIX REV 5/2b/84
        INTEGER*4 A(IM,JM, KM)
        COMMON/DIMS/IDIMI,IDIM2, IDIM3, IDIM4
        COMMON/LATICE/IM, JM, KM, KMSTOP
        CDMMON/ELEC/Z (6,2), CUR(2), NCUR, NGND, REALI, RMAGI
        COMMON/GTEMP/ NG1,NG2,NG3,NG4,NG5,ICOL(6),GREAL(6),GIMAG(6)
        DIMENSION G(IDIM3),JA(IDIM3)
    C See subroutine nodes for the significance of model
C ---> HERE-IF MODEL > }50\mathrm{ THEN PERIODIC BOUNDARY CONDITIONS
c ---> HAVE BEEN REQUESTED
ICONT = O
IF (MODEL.GT.50) ICONT =1
c
G IS COEFFICIENT MATRIX STORED IN SPARSE-SYMMETRIC MODE
JA IS COLUMN ADDRESSES OF EACH ELEMENT OF PACKED G MATRIX
A IS THE ROW POINTER TO THE BEGINNING OF EACH ROW--
IA(N) CONTAINS THE INDEX OF THE ELEMENT IN G THAT
STARTS THE N'TH ROW OF MATRIXG
B is the rhS OF the simultaneous equ.
D0 50 1=1, 10/m3
G(1)=0.0

```

POR 10990
POR11000
POR1 1000
POR 11020
POR11020
POR 11030
POR11040
POR 11050
POR11060
POR 11080
POR11080
POR11100
POR11100
POR11120
POR11130
POR11140
POR 11140
POR 11150
POR 11170
POR 11170
POR11190
POR 11200
POR 11200
POR11220
ORI 1220
POR11230
OR11250
POR11250
ORI 1260
POR11270
POR 11280
POR 11290
POR11300
OR11310
POR11320
POR11330
OR11310
POR 11350
POR11360
POR 11370
POR11380
POR11390
POR 11400
POR11410
POR11420
POR 11430
POR 11440
POR 11450
POR 11460
POR11470
```

JA(1)=0
CONTINUE
DO 60 I=1,IDIM2
B(1)=0.0
CAN'I=0
60
c
NNA COUNTS WHICH ABS NODE NO. WE'RE UP TO (INCL. O'S)
NN COUNTS WHICH NODE \& WE'RE UP TO (EXCLUD. O'S)
N COUNTS WHICH LINE OF MATRIX WE'RE UP TO
NT COUNTS THE OF ELEMENTS IN G
NNA =0
NN =0
NN=0
C
initialize address of gnd adjacent nodes
NG1 =0
D0 500 K=1,KM
DO 500 I=1,1M
DO 500 J=1,JM
NNA = NNA + I I
IAT = A(I,J,K)
NNA IS ALSO DEFINED AS:
NNHA =(K-1)*JM*IM + (1-1)*JM + J J COTO 5,00
NN = NN + 1
POR11480 POR11490 POR11500 POR11510 POR11520 POR11530 POR11540 POR11550 POR 11560 POR11570 POR11580 POR11590 PORI 1600 POR11610 POR11620 POR11630 POR11640 POR11650 POR11660 POR11670 POR11680 POR11690 POR11700 POR11710 POR11720 POR11730 POR11740 POR11750 POR11760 POR 11770 POR11780 POR11790 POR11800 POR11810 POR11820 POR11830
FIRST FIND OUT THE CONDUCTANCE OF BRANCHES IN SURROUNDING NODES POR11840

```
``` POR11850 POR11860
```



``` POR11880
POR11890 \begin{tabular}{c} 
NODE \\
DEF \\
\hline
\end{tabular}
```


.$\quad \begin{gathered}\mathbf{C} \\ \mathbf{c}\end{gathered}$
-24-

-92-



```
C ICOL(II) WILL CONTAIN COLUMN INFORMATION ABOUT THE ADJ. HODES
    IF ICOL(II)=0 THEN EITHER: 1-THERE IS NO NODE THERE (AIR)
        OR: 2-IT BELONGS TO LOWER HALF TRIANGLE
        OF THE MATRIX--LEAVE OUT
        OR: 3-IT IS THE GROUND NODE
        00 160 11=1,6
        160
        160 CONT INUE
            IZERO =0
            1ZERO=0
C
    HOW MANY ZERO ELEMENTS ARE IN THE REST OF THIS ROW?
            IF (J.EQ.JM) GOTO 162
            DO 175 JJ=J,JMM, (A (1,JJ,K).EQ. O) ITEMP=1TEMP+1
    c
    175 CONTINUE
C
    162 IF (J+1.GT.JM) GоTO 170
        if (A(1,J+i,K).EQ.O) COTO 170
        IF (NNA+1.EQ.NGND) NG2 = N
    ICOL(2)=N+2
    170
    If (1+1.GT. IM) GOTO 190
    IF NNA+JM.EQ.NGNDI NG1 = N
    IF (NNA+JM.EQ.NGND) GOTO 190
    If (A(1+1,j,K).EQ. O) COTO 190
c HOW MANY ELEMENTS BTHN HERE && NODE IN FRONT OF THIS (NON-ZERO)
    IZERO=1TEMP
    11=1+1
    IF (A(II,JJ,K).EQ.0) IZERO=IZERO+1
    CONTINUE
    ICOL(3)}=N+2*(JM-IZERO
    190 IF (K+1.GT.KM) COTO 220
        IF ((NNA+TM*JM).EQ.NGND) NG3 = N
        IF ( NNA+IM*JM).EQ.NGND) GOTO 220
        IF (A(1,J,K+1),EQ.O) GOTO 220
    HOW MANY ZEROS BTWN. HERE & NODE UN TOP?
        RST COUNT ZEROS ON REST OF CURRENT PLANE
            IZERO=ITEMP
    FIRST COUNT ZEROS ON REST OF CURRENT PLANE 
```



```
        (START=i+1.M)GOTO 202
        ISTART=1+1
        DO 200 II=ISTART,IM
        DO 200 JJ=1,JM
        IF (A( II,JJ,KM. EQ.0) IZERO=IZERO+1
    200 CONTINUE
C (F (IZERO.GT.O) WRITE (6,998) N, IZERO
M988 FORMAT (''LINE #',13,(6,9%8T#2-N--IZERO=', 13)
                -27-
```

POR 12970 POR12980 POR 12990 POR13000 POR13010 POR13020 POR13030 POR13040 POR13050 POR13060 POR13070 POR13080

POR13090
POR 13100
POR13110
POR13120
POR13130
POR13140
POR13150
POR13160
POR13170
POR13180
POR13190
PORI 3200
POR13210
POR13220
POR13230
POR13240
POR13250
POR13260
POR13270
POR13280
POR 13290
POR13300
POR13310
POR13320
POR13330
POR13340
POR13350
POR13360
POR13370
POR13380
POR13390
POR 13400
POR13410
POR13420
POR13430
POR13140
POR13450
POR13460
POR13470


F (il.EQ.IM.AND.JJ.GE. J) GOTO 225 IF (A(IJ,JJ,K).EQ.O) IZERO $=$ IZERO CONTINUE $\mid \operatorname{COL}(1)=N+2 *(J M *(1 M-1 \mid-1$ ZFRO $)$ END IF

END IF
END IF
C
because of periodic b.c. COLUMM indices may be dut of order * SORT SO THAT THEY ARE IN ASCENDING ORDER
$00240 \mathrm{LL}=1,5$
$00240 \mathrm{iN}=\mathrm{LL}+1,6$
IF (ICOL(LLJ.GT. ICOL(IN)) THEN SWAP
$1 \mathrm{HCOL}=1 \mathrm{COL}(\mathrm{L} . \mathrm{L})$
TEMPGR = GREAL(LL)
TEMPGI = GIMAG(LL)
ICOL(LL) $=$ ICOL(IN)
GREAL(LL)
GIMAG(LL) GREAL(IN)
GIMAG(LL) $=$ GIMAC
ICOL $(I N)=I H C O L$
GREAL (IN) = TEMPGR
GIMAG(IN) $=$ TEMPGI

## END

240 CONTIMUE
C NOW START THE 1ST ROW OF REAL-IMAG PAIR
250 NT=NT+1
C POINTER TO FIRST ELEMENT IN ROW
IA(N) $=$ ENT
C COLUMN INDEX
JA(NT) $=\mathrm{N}$
C NOW MATRIX ITSELF
G(NT) $=$ GREALO
IF (GIMAGO. EQ. O. O) GOTO 380
C IMCREMENT POSITION IN G MATRIX
$\mathrm{NT}=\mathrm{NT}+1$
$\mathrm{JA}(N T)=\mathrm{N}+1$
$\mathrm{G}(\mathrm{NT})=-\mathrm{GIMAGO}$.
C
380 D0 $390 \mathrm{LL}=1,6$
If (ICOL(LL).EQ.0) GOTO 390
$\mathrm{NT}=\mathrm{NT}+1$
JA(NT) $=1 \operatorname{COL}(L L)$
$G(N T)=G R E A L(L L)$
$G(N T)=G R E A L(L L)$
IF (GIMAG(LL).EQ.0.0) GOTO 390
$\mathrm{NT}=N T+1$
$\mathrm{JA}(\mathrm{NT})=1 \mathrm{COL}(L L)+1$

```
            G(NT) = -GIMAG(LL)
    390 CONTINUE
C
DO NEXT LINE (IMAG LINE)
    N}=N+
    NT = NT + 1
    IA(N)=NT
    JA(NT)=N N
C
    DO 400 LL=1,6
    IF (ICOL(LL').EQ.0) GOTO 400
    IF (GMMAG(LLi.EQ.O.0) GOTO 395
    NT =NT + 1
    JA(NT) = ICOL(LL)
    395 NT = NT + 
    JA(NT) = ICOL(LL) + 1
    G(NT)=-GREAL(LL)
    400 CONTINUE
c 500 CONTINUE ELEMENT
    MARK LAST ELEMENT
        IA(N+1) =NT + + - MCD NG3 NC4 NC5
        FORMAT (1 ADJACENT GND NODE MATRIX ADDRESSES: ',316)
```



```
        WRITE (6,505) NNA,NN,N,NT
    505 FORMAT (' TOTAL NODES=',15,' NONZERO NODES= ', 15,
        1' f OF EQU-S= ,i8,' PACKÉD ELEMENTS=,18) ;
*********** EQUEGIN CONDITIONAL SECTION
C. NOW LIST OFF MATRIX
    L=1
    L=1
*510 FORMAT (; ***************',//,' ROW ',14)
    DO 580 }t=1,N
    WRITE (6,560) JA(1);G(1),
    560 FORMAT (I ELEMENT H',14,' = ',1PE13.4)
        IF (I+1.LT.IA(L+1)) GOTÓ 580
    L =|LE + '1
* 580 CONTINUE'
******* END CONDITIONAL SECTION ****************
c
    SET UP RHS = INPUT CURRENTS
        B(NCUR) = CUR(1)
        B(NCUR+1) = -CUR(2)
        RETURM
    END
*---------------------
```

POR 14460
POR 14470 POR14480 POR14480 POR14490
POR 14500 POR 14500 POR14510
POR14520 POR14520 POR 14530
POR 14540 POR14540 POR 14550 POR 14560
POR 14570 POR 14570
POR 14580 POR 14580
POR 14590 POR 14590
POR 14600 POR 14600
POR 14610 POR 14610 POR14620 POR 14630 POR 14640 POR14650 POR 14660 POR 14670 POR14680 POR 14690
POR 14700 POR14700 POR14710 POR 14720 POR 14730 POR14740 PORI4750 POR 14760 POR 14770
POR 14780 POR14780 POR 14790 POR 14800 POR 14810
POR 14820 POR 14820 POR 14830 POR 14840 POR 14850
POR 14860 POR14860 POR 14870 POR14880 POR 14890 POR 14900 POR 14910
POR 14920 POR 14930 POR14940 POR14950

```
SUBROUTINE NODI46(A,IPOS,IAT,NEIGH,IGND,Z1,Z2)
    SR to evaluate conductance elements at nodes 1,4,6
    INTEGER"4 A(IM,JM, KM)
    COMMON/LAT ICE/IM, JM, KM, KMSTOP
    COMMON/ELEC/Z (6,2),CUR(2),NCUR, NGND, REALI, RMAGI
    COMMON/GTEMP/ NG1,NG2,NG3',NG4,NG5,1COL(6),GREAL(6),GIMAG(6)
C
    IAGND=A(IM, JM, KH)
        G1 = Z(1AT,1)
    G2 = Z (|AT',',
C NOTHING THERE AT POS IPOS
    IF (NEIGG.EQ.O) COTO 100
C GND INTERFACE?
SEMIC (IGND.EQ.NGND) COTO 10
    SEMIC INTFC?
    IF (IAT+NEIGH.EQ.3) GOTO 20
    NORMAL INTFC
    GOTO 30.
    10
            GND NODE
    Z1 = G1+
    Z2=G2+ZZ(IAAGND,2)
    IF (IAT+NEIGH.NE.3) GOTO }10
    Z1 = G1 + 2(NEIGH+2,1)
    Z2}=\mathbf{G2 + Z(NEICH+2,2)
    GOTO 100
C}2
    AZ = Z(NEIGH+2,1)+G1
        A2 = Z(NEIGH+2,2) +G2
        GOTO 50
    NORMAL INTFC
    30
    A1 =Z(NEIGH,1) + G1
    AZ = Z(NEIGH;2) + G2
    50 CREAL(IPOS) = -A1/(A1**2+A2**2)
    GIMAG(iPOS) = A2/(A1**2+A2**2)
    100 RETURN
    END
#
    SUBROUTINE NOD235(A, IPOS, IAT, NEIGH, IGND,Z1,Z2)
C
C SR tO EVALUATE G MATRIX AT NODES 2,3,5
    INTEGER*4 A(IM,JM, KM)
    COMMON/LATICE/IM,JM, KM, KMSTOP
    COMMON/ELEC/Z(6,2),CUR(2),NCUR,NGND,REALI,RMAGI
    COMMON/GLEC/Z/ ''2),NG2,NG3,NG4,NG5,ICOL(6),GREAL(6),GIMAG(6)
C
```

POR 14960
POR14970
POR 14970
POR 14980
POR 14990
POR 15000
POR15010
POR 15020
POR 15030
OR15040
POR15050
POR 15060
POR15070
POR 15080
POR15090
POR15100
POR 15110
POR15120
POR15130
POR15140
POR15150
POR15160
POR 15170
POR15180
POR15190
POR15200
POR15210
POR15220
POR15230
POR15240
POR15250
POR15260
POR15270
POR15290
POR 15290
POR15300
POR15310
POR15320
POR15330
POR15340
POR15350
POR15360
POR 15360
POR 15370
POR15380
POR15390
POR15100
POR15410
POR15420 POR15430 POR15440 POR15450


[^2]```
53
    CONTINUOUS BOUND. COND. ?
    ICONT \(=0\)
IF (MODEL.GT.50) ICONT \(=1\)
IAGND \(=\) A(1M, JM, KM)
WRITE ( 7,10 ) NG1, NG2, NG3, NG4, NG5
    WRITE ( 7 (10) NG1, NG2, NG3, NG4, NGS
FORMAT ( MATRIX ADDRESS OF ADJACENT GND NODES \(=1.517\) )
    \(\{A 1=A(I M-1, J M, K M)\)
    IA1 \(=A(I M-T, J M, K M)\)
\(I A Z=A(I M, J M=1, K M)\)
    IAZ \(=A(I M, J M=1, K H)\)
\(\mid A 3=A(I M, J M, K M-1)\)
    A3 \(=A\left(I M_{,} J M_{1} K M-1\right)\)
IF (ICONT.EQ.1) THEN
            \(\{A 4=A\{I M, 1, K M\}\)
    ELSE
            IA5 \(=A(1, J M, K M)\)
            IA4 \(=0\)
1A5 \(=0\)
    END IF
    REALI \(=0.0\)
RMAGI \(=0.0\)
C
    ASSUME GND NODE \(=\) METAL
    IF (IA1.EQ.0) GOTO 50
    \(\mathbf{V 1 R}=\mathbf{G}(N G 1)\)
    IF (IAT+IAGND.EQ. 3\() \quad|A|=|A|+2\)
    IF (IAI+IAGND.EQ. 3 ) \(\mid A 1=1\)
\(Z R=Z(I A 1,1)+Z(I A G N D, 1)\)
    \(Z R=Z(\mid A 1,1)+Z(\mid A\)
\(Z I=Z(\mid A 1,2)\)
    DIV = ZR*ZR \(+2 I^{*}\) ZI
    REALI \(=\left(V 1 R^{*} Z R+V 1 I * Z I\right) / D I V\)
    REALI \(=\left(V 1 R^{*} Z R+V 1 I^{*} Z I\right) / D I V\)
RMAGI \(=\left(V 1 I{ }^{*} Z R-V 1 R^{*} Z I\right) / D I V\)
    50 IF (IAC.EQ.O) GOTO 100
    V2R \(=G(\) NG2 \()\)
    \(\mathrm{V} 21=\mathrm{G}(\mathrm{NG} 2+1)\)
    IF (IAZ+|AGND.EQ. 3) IAZ = \(1 A 2+1\)
    \(Z R=Z(\mid A Q, 1)+Z(I A G N D, 1)\)
    \(Z 1=Z(\mid A 2,2)\)
    DIV \(=\mathbf{Z R} R^{*} \mathbf{Z R}+\mathbf{Z I * Z}\)
    REALI \(=\) REALI \(+\left(V 2 R^{* Z R R}+V 2 I^{* Z} \mid\right) / D I V\)
    RMAGI = RMAGI + (VZI\#ZR = V2R*ZI)/DIV
    100
    If (iA3.EQ.O) GOTO 150
    V3R \(=\mathrm{G}(\mathrm{NG} 3)\)
    \(\vee 3 t=G(N G 3+1)\)
    \(V 3 t=G(N G 3+1)\)
\(I F(I A 3+I A G N D . E Q .3) \quad I A 3=1 A 3+2\)
    \(Z R=Z(I A B, 1)+Z(I A G N D, 1)\)
\(Z I=Z(I A 3,2)\)
    \(Z R=Z(\mid A 3,1)\)
\(Z \mid=Z(\mid A 3,2)\)
```

POR15960
POR15970
POR15970
POR 15980
POR 15980
POR15990
POR15990
POR 16000
POR 16010
POR 16010
POR16020
POR 16030
POR 16040
POR 16040
POR 16050
POR 16050
POR 16060
POR 16060
POR 16070
POR 16070
POR 16080
POR 16090
POR 16090 POR16100 POR16110 POR16120 POR 16130 POR16140
POR16150 POR16150 POR16160
POR 16170 POR16170 POR16180 POR16190 POR162U0 POR16210 POR16220
POR16230 POR16230 POR 16240
POR 16250 POR 16250 POR 16260 POR 16270 POR16280
POR16290 POR16290 POR 16300 POR 16310 POR16320 POR16330 POR 16340 PQR 16350 POR 16360 POR16370 POR16380 POR16390 POR16400 POR16410 POR 16420 POR 16430
POR 16440

```
            DIV = ZR*ZR + ZI#Z
            REALI = REALI + (V3R*ZR + V3I*Z')/DIV
            RMAGI = RMAGI + (V3I*ZR - V3R*ZI)/DIV
    150 IF (IA4.EQ.0) GOTO 200
    V4R=G(NG4)
    IF (|A4+|AGND.EQ.3) IA4 = IA4+2
    ZR = Z(IA4,1) + Z(IAGND,1)
    ZI=Z(|A4,2)
    DIV = ZR*ZR'R + ZI*ZI
    REALI = REALI + (V4R*ZR + V4I*ZI)/OIV
    RMMAGI = RMAGI + (V4I*ZR - V4R*ZI)/DIV
200 IF (IAS.EQ.O) GOTO 250
    V5R = G(NG5)
    IF (IAS+IAGND.EQ.3) 1A5 = IA5+2
    ZR=Z(|A5,1)+2(IAGND,1)
    Z| = Z(|A5,2)
    Z|= Z(|A5,2)
    REALI = REAL! + (V5R*ZR + V5!*ZI)/DIV
    RMAGI = RMMAGI + (V5I*ZR - V5R*ZI)/DIV
```



```
c
WRITE (7, 110) V1R,V1I,V2R,V21,V3R,V3I,V4R,V4I,V5R,V5I
C
C 110 FORMAT,I'VON NODES = ','/l'.2./))
    RETU
*----NND
```



```
* STORED ON FILE: CURPRF FORTRAN 12/5/84
POR16450
POR16460
POR16470
POR16480
POR16490
POR16490
POR16500
POR16510
POR16520
POR16530
POR16540
POR16550
POR16570
POR16570
PDR16590
POR16590
POR16610
POR16620
POR16620
POR16630
POR16650
POR16650
POR16660
POR16680
POR16680
POR16690
POR16690
POR16710
POR16710
POR16720
POR16730
POR16740
POR16750
POR16760
POR16770
POR16780
POR16790
POR16800
POR16810
POR16820
POR16830
POR16840
POR16850
POR16860
POR16860
POR16870
POR16880
POR16890
POR16900
POR16910
POR16920
POR16930
```





```
    IATT1 = IAT1
    MNDX1 = 2*NNDE1 - 1
    NDX2 = 2*NODE2 -
    V1R=G(INDXI)
    V11 =G( INDX1+1)
    V2R =G(|NDX2)
    CV1 = DCMPLX(V1R,V1I)
    CV2 = DCMPLX(V2R;,V2I)
    CV0 = CV2 - CV1
    IF (IATT1+|AT2.EQ.3) IATT| = |ATT\+2
    ZR = Z(|ATT1,1) + Z(IAT2,1)
    ZI = Z(|ATT1,'Z) + Z(|ATZ,Z)
    CZ = DCMPLX(ZR,ZI)
I = V/Z IN COMPLEX ARITHMETIC
C! = CVO//CZ CZ,',',14,' CUR (%)=',1P,E13.3,3X,E13.3,E13.3)
RETURN
END
SUBROUTINE TO CALCULATE LATTICE CORRELATIONS
SUBROUTINE TO CALCULATE LATOPE AND QUANTIFY THE PROPERTIES OF
THE GEOMETRICAL CHARACTERISTICS OF EACH POROUS ELECTRODE
-- DENSITY-DENSITY CORRELATION CALCULATION ...
-- OF THE SURFACE SHELL
WRITIEN ON 10/30/85 IN VS FORTRAN77
* Subroutine shlcor (A, LbLa, LPLANE)
PARAMETER (MAXR=100)
PARAMETER (MAXR=100)
INTEGER A(IM, JM, KM), LBLA(IM, JM, KM)
INTEGER A(IM, JM, KM), LBLA(IM,
COMMON /LATICE/ IM, JM, KM, KM
DIMENSION COR(MAXR), R(MAXR)
*
SET UP PARAMETERS fOR NEARN SEARCH
ISTAY \(=1\)
ILOAK = 2
NPART \(=\)
\(\mathrm{N}=\)
DO \(100 \mathrm{~K}=1\), KM
DO \(100 \mathrm{~J}=1\), JM
Do \(100 \mathrm{I}=1\), IM
CALL \(\operatorname{NEARN}(A, 1, J, K\), ISTAY, I LOOK, I FND, KFND ) IF (IFND.EQ. 1 . AND. K.LT.KM) THEN WE WANT TO LEAVE A() THE WAY THAT IT WAS \(A(1, J, K)=1\)
```

POR18420
POR18420 POR18430 POR18440
POR18450 POR18450 POR 18460 POR 18470 POR 18480 POR 18490
POR 18500 POR 18500
POR 18510 POR18510
POR18520 POR 18520
POR 18530 POR 18530 POR18540 POR18550 POR 18560 POR 18570 POR18580
POR 18590 POR18590 POR 18600 POR18610 POR18620 POR18630 POR18640 POR 18650
POR18660 POR18660 POR18670 POR 18680
POR 18690 POR 18690 POR18700 POR18710 POR18720 POR18730 POR18740
PORR18750 POR18750 POR18760
POR18770 POR 18770 POR18780 FOR18790 PORTB800 POR 18810 POR18820 POR18830 POR 18840 POR 18850 POR18860 POR18870 POR18880 POR18890
POR18900


ELSE
$\operatorname{LBLA}(1, J, k)=0$
ENDIF
POR 18910
POR18920
POR 18930
POR 18940
POR 18950
POR 18960
POR 18970
POR 18980
POR 18990
POR 19000
POR19010
POR19020
POR19030
POR 19040
POR 19050
POR 19060
POR19070
POR 19080
POR 19090
POR 19100
POR19110
POR19120
POR19130
POR 19140
POR19150
POR 19160
POR19170
POR19180
POR19190
POR 19200
POR 19210
POR19220
POR 19230
POR 19240
POR 19250
POR19260
POR19270
POR19280
POR 19290
POR 19300
POR19310
POR 19320
POR19330
POR 19340
POR 19350
POR 19360
POR 19370
POR 19380
POR 19390
POR 19400





```
    WRITE (*,*) '1- INITIALIZE A NEW PROBLEM"
    WRITE (*,*),2* RESUME AN OLD PROBLEM'
    WRITE
    WRITE
    WRITE
    WRITE (*,*)
    WRITE (**) 6- LIST THE CURRENTLY DEFINED PARAMETERS'
    WRITE (***)'7- GRAPHICAL DISPLAY OF THE CURRENT SIMULATION'
    IF (EMPTYMM(IOPT,DUM,CDUM,O,O,*20)
    F (EMPTY.AND. (IOPT.GT.3)) THEN
            WRITE (*,*) "NO}\mathrm{ SIMULATION CURRENTLY DEFINED'
    END:F
    ENTO (1000,2000, 3000,4000,5000,6000,7000), IOPT
    GOTO (100
*================= INITIALIZE A NEW PROBLEM ====================
1000 WRITE (*,*), ENTER ID NAME FOR THIS RUN (DATAID-8 CHAR ,
    READ (*,*) IDNAM
    READ (***) IDNAM (*)*) 'ENTER DESIRED CLUSTER ATTACHMENT:',
    WRITE (*,*) ENTER DESIRED CLUSTER ATTACHMEN
    READ (***) ICLUST
    WRITE (*,*) "ENTER DESIRED PARTICLE CONCENTRATION (<.5):'
    READ (*,*) CONC
    WRITE (***) "ENTER DESIRED NUCLEI CONCENTRATION :'
    READ (***) CONUC (ENTER DESIRED # OF LAYERS :'
    READ (*,*) NLAY
    IF (NLAY.GT.2*NYSPAC/3) THEN
                    WRITE (***) TOO MANY LAYERS OF BULK SPECIFIED'
                    WRITE
    ENDIF
    WRITE (***) 'ENTER DESIRED STICKING PROBABILITY .'
    WRITE (***) EEN
    READ (*,*) PROB
    WRITE (***) 'ENTER DESIRED UN-STICKING PROBABILITY:'
    READ (**) PROB2
    HRITE (*,*) "ENTER DESIRED MOVING PROBABILITY:'
    READ (*,*) PROB3
        DSEEDO = 2345.0DO
        DSEED = DSEEDO
        NNUC=CONUG*HXSPAC
        IZ=NMUC+(NLAY-1)*NXSPAC
        NFREE= CONC*NXSPAC* (NYSPAC-NLAY-CONUC)
        NPART= NFREE+IZ
    * NTOP IS THE NUMBER OF PARTICLES ON THE TOP (NON-NUCLEATING) PLANE
* THIS MAY BE KEPT CONSTANT IN SUCCEEDING UENERATIONS BY ADDING LOST
C PARTICLES, COUNTEO BY'CNTTOP'
    NTOP = CONC*NXSPAC+.5
    NPARTO = NPART
    DO 1050 J=1, NYSPAC
```

POR21380 POR2 1380
POR21390 POR2 1390
POR2 1400 POR2 1400
POR2 1410 POR2 1410 POR2 1420 POR21430 POR2 1440
POA2 1450 POR21450
POR2 1460 POR2 1460
POR21470 POR21470 POR21480
POR2 1490 POR2 1490
POR2 1500 POR2 1500
POR2 1510 POR2 1510 POR21520 POR21530
POR21540 POR21540 POR21550 POR21560 POR2 1570
POR2 1580 POR21580
POR21590 POR2 1590 POR2 1600 POR2 1610
POR21620 POR21620 POR21630
POR2 16140 POR2 1640 POR2 1650
POR 21660 POR2 1660
POR2 1670 POR2 1670 POR2 1680 POR2 1690 POR21700
POR2 1710 POR2 1710
POR2 1720 POR2 1720
POR2 1730 POR21730 POR2 1740 POR21750 POR21760 POR21770 POR2 1780 POR21790 POR2 1800 POR21820 POR2 1820 PORZ 1830 POR21840 POR21850 POR21860

* MXIY KEEPS TRACK OF THE MAXIMUM LEVEL THAT DENDRITE HAS GROWN TO MXIY $=$ NLAY
* IAMBLK KEEPS TRACK OF WHERE THE BULK APROXIMATELY BEGINS POR2190 POR2 1920 POR2 1930 PORえ1940 POR2 1950 |AMBLK $=$ NLAY
KGENO $=1$
PR21960 POR21970 POR21980 P0R21990 PORT2000 POR2?uIO POR22020 POR22030 POR22030 POR22050 POR220G0
* COTO 20 RE= $==================$ READ IN
2000 READ $(7,2005)$ IDNAM DUMMY READ (7, 2010) REV EZMAN
EAD 17,2020 ) NXSPAC, NYSPAC, MXNPRT
 WRITE $\left({ }^{*}, *\right)$ ERROR: STORED SIMULATION SPACE, DIMENSION INCORRECT POR22070
WRITE $\left({ }^{*}, *\right)$ EXPECTING DIMENSIONS: I MXSPAC, $x$, MYSPAC
 WRITE ${ }^{(*,}{ }^{*}$ ) BURPRT STORED SIMULATION IS: , NXSPAC, $X$,NYSPAC PRT POR22100 MYSPAC = MYSPAC PORZ 110
NYSPAC $=$ MYSPAC $\quad$ POR22120
NXSPAC $=$ MXSPAC
GOTO 20
ENDIF
READ (7,2030) CONC, CONUC, NPARTO, NPART, HRAC
READ $(7,2040)$ DSEEDO, DSEED, PROB3
READ (7, 2050 ) ICLUST PROB PROB2 DUMMY
READ (7,2060) KGEN, MXGEN, IZ, MXIY,MNIY, NSTOR, HLAY
READ $(7,2060)$
READ $(7,2070)$
(JENCNT( 11 ), NPSTUK( 11 ), NPFRE(11),NSURPT(II)

, MAXHT(II), RUF(II), MINIY(II), MXFREE $||\mid, I I=1$,NSTOR)
READ $(7,2075)$ MSG1 2078 DCONC(1)
I PTO = DCONC (i)
READ $(7,2079)$ (DCONC( 11$), 11=2,1 \operatorname{PTO})$
READ (7,2078) SCONC(i)
lPT = SCONC(1)
$\begin{array}{ll}\text { READ }(7,2079) & \text { (SCONC }(11), I I=2,1 P T) \\ \text { POR } & \text { POR222260 }\end{array}$
READ ( 7,2075 ) MSG2 $\quad$ POR22290
C IORD
D IS THE STARTING POINT FOR FUTURE FILLING OF THE CORREL. MATRICES
I ORD =NSTOR-1
IORD =NSTOR-1
ICOR = IORD*MAXR
POR22310
POR22320
READ $(7,2075)$ MSG3 $\quad$ POR22330
$\begin{array}{ll}\text { READ }(7,2075) \\ \text { READ }(7,2100) & \text { (COR1 }(I), I=1, I C O R)\end{array} \quad$ POR22340 $\quad$ POR22350

```
    READ (7,2075) MSG4
    READ (7,2100) (COR2(I), I=1, ICOR)
    2100 FORMAT (1X,9FB.5)
    2005 FORMAT (1X,AB,F5.2)
    2010 FORMAT (1X,F6.2,F10.2
    2020 FORMAT (1x,215,18
    X,2F5.2,218,F5.21
    2050 FORMAT 1X,2D15.0.F5.
    2060 FORMAT (1x, 21,30,18,215,17,15)
    2070 FORMAT (1\times,517,F7,3,217,
    075 f0RMAT (1x,517,F7.3,217)
    2078 FORMAT (1x,F8,0
    2079 FOPMAT 1x, %.O
    2080 FORMAT {1X,215,19,215,19,215,19,215,19)
    DO 2200 J=1,NYSPAC
    DO 2200 l=1;NXSPAC
    2200 A(1,J) = l2ZERO
* now decode the information from the stored b matrix.
    DO 2300 I = 1,NPART
        IX = B(1,1)
        A(IX B(Y)=2)-120NE
* PARTICLE STUCK??
    IF (BGROW(I).GT.0) A(IX,IY) = I2ONE
2300 CONTINUE
2300 CONTINUE APROXIMATION; WILL GET SET EXACTLY NEXT GENERATION
    IRST APROXIMATIO
    IAMBLK = MNIY 
    NTOP = CONC * NXSPAC +..5
    KGENO = KGEN + 1
    EMPTY = .FALSE.
    COTO 20
    POR22360
    POR22370
    POR22370
    POR22390
    POR22400
    POR22400
    POR22410
    POR22420
    POR22430
    POR22440
    POR22450
    POR22460
    POR22470
    POR22480
    POR22490
    POR22500
    POR22510
    POR22520
    POR22530
    POR22540
    POR22550
    POR22560
    POR22570
    POR22580
    POR22590
    POR22600
    POR22610
    POR22620
    POR22630
    POR22640
    P0R22650
    POR226G0
    POR22670
    POR22G80
    POR22690
    POR22700
POR22710
POR22720
POR22730
POR22740
    POR22750
    POR22760
    POR22770
    POR22780
    POR22790
    POR22800
    POR22810
    PORZ2810
    POR22820
POR22830
POR22840
```

        READ (*,*) MXGEN MAXIMUM ALLOWED LOST FRACTION: : POR22910
        WRITE (**) ENTER MAXIMUH ALLOWED LOST FRAGTION: PRAC POR22920
        READ (*,*) FRAC
    * INITIALIZE INTERRUPT FLAG TO ZERO:
INIT $=$ ITYBIT $(-1)$
C INIT = ITYBIT $(-1)$ IS
MITIALIZE DISSOLUTION FLAG
IDIS=120NE
POR22930
POR22940
POR22940
$*$
$*$
POR22950
POR22960
POR22970
- inITIALIZE TIMER VARIABLES
POR22990
OGEN = KGEN
* INITIAL TIME FOR TOTAL SIMULATION CALCULATION.
ZMANO WILL ALWAYS BE SUBTRACTED FROM CURRENT TIME.
ZMANO = GETT IM(COTMIN)
WRITE (***) 'ENTER DEVICE H:
$1(0=\text { NONE, } 1=P C, 2=V M / T E K, 3=V M / P R T, 4=V M / S C R E E N)^{\prime}$
READ (*, *) DEVICE
POR23000
IBOTOM IS' THE BOTTOM BELOW WHICH WE DONT ALLOW THIS TO PROGRESS
IBOTOM = MLAY/ 10
IBOTOM $=$ NLAY 10
C DISPLAY IN LOGARITIMIC SCALE, E.G. 1 FREQ $=3==>$ GENERATIONS DISPLAYEO
C HILL BE $3,6,9,30,60,90,300,600, ~ E T C$.
C WILL BE 3, 6, 9, 30, 60, 90, 300,600 , ETC.
* SET UP TO AUTOMATICALLY QUIT AT THE END OF SIMULATION:
* INITIALIZE DECADE COUNTER
POR231110
POR23020
POR23030
POR23040
POR23050
POR23060
POR23070
POR23080
POR23090
POR2 3100
POR23110
POR23 320
POR23 130
POR231110
POR23150
- INITIALIZE DECADE COUNTER :
* IDECAD = $10^{* * *}$ INT(ALOG10(FLOAT(KGENO) I)
POR23160
$\begin{array}{ll}* & \text { *OECAD }=10 * * \\ * & \text { PNT(ALOG10(FLOAT(KGENO) i) POR23170 } \\ \text { POR23180 }\end{array}$
DO $40501=$ NPART +1 , MXNPRT $\quad$ POR23 180
$\begin{array}{ll}\text { BGROW( } 1)=0 & \text { POR23200 } \\ \text { POR23210 }\end{array}$
4050 CONTINUE
POR2 3210
POR23220
DO 4500 KGEN=KGENO, MXGEN
C IF ONE WANTS TO KEEP THE TOP ROW AT CONSTANT CONC THEN:
POR23220
DO 4500 KGEN=KGENO,MXGEN
$C$ IF ONE WANTS TO KEEP THE TOP ROW AT CONSTANT CONC THEN: POR23230
$\begin{array}{llll}C & \text { IF ONE WANTS TO KEEP THE TOP ROW AT CONSTANT CONC THEN: } & \text { POR23240 } \\ \text { C IF CALL CNTTOP (A, B, BGROW, NTOP, DSEED) } & \text { POR23250 } \\ \text { C IF HANTS TO KEEP THE BOTTOM ROW AT CONSTANT CONC THEN: }\end{array}$
$\begin{array}{lcll}\text { C CALL CNTTOP (A,B, BGROW, NTOP, DSEED) } & & \text { POR23250 } \\ \text { C IF ONE WANTS TO KEEP THE BOTTOM ROW AT CONSTANT CONC THEN: } & \text { POR23260 } \\ \text { C CALL CNTBOT (A, B, BGROW, NTOP, DSEED) } & \text { POR23270 }\end{array}$
C CALL CNTBOT (A,B, BGROW, NTOP, DSEED)
POR23270
POR23270
IF (NPART.LE.O) GOTO 20
CALL UPDATE
POR23280
$\begin{array}{lll}\text { CALL UPDATE } & & \text { POR23290 } \\ \text { NOW UPDATE BGROW, IZ TO BE PROPERLY RECORDED } & \text { POR23300 } \\ \text { DO } 4200 ~ I=1, ~ M P A R T ~ & \text { POR23310 }\end{array}$
$\begin{array}{ll}\text { DO } 42001=1, \text { NPART } & \text { POR23310 } \\ 1 \times=\mathrm{B}(1,1) & \text { POR23320 }\end{array}$
IX $=B(\overline{1}, 1)$
$1 Y=B(1,2)$
POR23330

```
    IF (A(IX, IY),GT. IZZERO) THEN
ELSE IF (A IX, IY).LT. I2ZERO) THEN POR23390 PARTICLE IS FREE - MAKE SURE BGROW IS 0 AND MXFRE IS CORRECT POR23400 F (BGROW(1).GT. O) THEN POR23410 BGROW (I) \(=0 \quad\) POR23420
                                    POR23430
                                    POR23440
POR23450
            ENDIF
IF (IY.GT.MXFRE) MXFRE = IY
POR23460
4200
CONTINUE
POR23470
*200 CONTINUE
POR23480
* NOW FIND IAMBLK VALUE POR POM490
    NOW FIND IAMBLK VALUE
    POR23490
    IAMBLK = MKIY
4210 ICNTR(1) = 0 
            ICNTR(-1)= I2ZERO
    POR23500
POR23510
ICNTR(O)=12ZERO
* COUNT TMENR(O) = I2ZERO PARTICLES PER LIME - IN ICNTR(1)
```



```
4250 DO 4250 I{=1,NXSPAC
POR23550
    POR23560
    IAMBLK = IAMBLK =-1
    POR23570
    IF (ICNTR(1).LT.IDBULK) GOTO 4210
* IF (ICNTR(1).LT. IDBULK) GOTO 4210
```



```
    POR23580
    POR23590
```



```
POR23590
C IF IANBLK IS TOO LOW, EXIT
    POR23600
IF (IAMBLK. LE. IBOTOM) GOTO 4700
        if (KGEN.GT. IDECAD*10) 1DECAD = IDECAD* 10
    POR23610
    POR23620
    POR23620
            IF (MOD(KGEN, IFREQ*IDECAD), EQ.0) THEN
* RESET COUNTERS EVERY NOW AND THEN SO THAT THEY DONT OVERILOW
* RESET COUNTERS EVERY NOW AND THEN SO THAT THEY DONT OVERILOW
* RESET COUNTERS EVERY NOW AND THEN SO THAT THEY DONT OVERILOW
            CLA ALSO CALCULATES 
            CALL CPROF(IPTO)
            NSTOR = NSTOR +
            JENCNT(NSTOR) = KGEN
            NPSTUK(NSTOR) = IZ
            MINIY(NSTOR) = MNIY
            NPFRE (NSTOR) = NPART - IZ
            MAXHT(NSTOR) = MXIY
            RUF(NSTOR) = FLOAT(NSURF) / FLOAT(NXSPAC)
            NSURPT (NSTOR)=NSURFI
            MXFREE(NSTOR)=HXFRE
            NDIF
* LOOP EXIT CONDITION:
    POR23640
    POR23650
    POR23660
* DISPLA ALSO CALCULATES NSURF, MNIY, MXIY, NSURF1, COR1, COR2
    POR23670
    POR23680
    POR23690
    POR23690
    POR23710
    POR23720
    POR23730
    POR23740
    POR23750
    POR23760
    POR23770
    POR23780
    POR23790
    POR23800
    POR23810
(IF (NPART.LT.NPARTO*(1-FRAC).OR.MXIY+1.GE.NYSPAC) GOTO 4700 POR23820
- CHECK INTERRUPT FLAG - HAS THE USER REQUESTED A HALT? POR23830 ITHIS MAY BE DONE BY THE CP COMMAND 'ST \(4641^{\prime \prime}\) AT A CP ATTN INTRPPOR23840

RQUTI \(=\). FALSE.
WRITE (***) 'USER REQUESTED HALT IN PROGRAM EXECUTION' NOW SET UP LOOP PARAMETER FOR POSSIBLE LATER CONTINUE KGENO \(=\) KGEN +1 GOTO 4700
ENDIF
4500 CONT INUE
4700 10RD=10RD+1
CALL DISPLA (NSURF)
I CPROF ( I PTO)
F (JENCNT (NSTOR). NE. KGEN) THEN
NSTOR = NSTOR +1
NPSTUK(NSTOR) \(=1 \mathrm{Z}\)
MINIY(NSTOR) \(=\) MNIY
NPFRE (NSTOR) \(=\) NPART \(-I Z\)
MAXHT (NSTOR) = MXIY
RUX (NSTOR) \()=\) FLOAT(NSURF) / FLOAT (NXSPAC)
RUF (NSTOR) \(=\) FLOAT(NSURF) / FLOAT (NXSPAC)
MSFREE(NSTOR)=MXFRE
MXFR
ENDIF
* NOH GO AND STORE AND QUIT/MENU
 5000 WRITE (8,5005) IDNAM, DUMMY

EZMAN \(=\) ZMAM \(/ 60\). DO
WRITE \((8,5010)\) REV, EZMAN
WRITE (8,5020) MXSPAC,MYSPAC, MXMPRT
WRITE (8,5030) CONC, CONUC, NPARTO, NPART, FRAC
WRITE (8,5040) DSEEDO, DSEED, PROB 3
WRITE \((8,5050)\) ICLUST, PROB, PROB2, DUMMY
WRITE \((8,5060)\) KGEN, MXGEN, IZ, MXIY, MNIY, NSTOR, NLAY
WRITE \((8,5060)\)
WRITE \((8,5070)\)
WRITE (8,5070)
1 (JENCNT(II),NPSTUK(II), NPFRE(: 1), NSURPT(11)
1, MAXHT (II), RUF (II), MINIY(II), MXFREE(II), II=1, NSTOR)
WRITE \((8,5075)\) MSG
WRITE \((8,5078)\) DCONC(1)
JPT \(=\) DCONC( 1)
WRITE(8,5079) (DCONC (1!), \(11=2,1\) PT)
WRITE(8,5078) SCONC(1)
IPT = SCONC(1)
WRITE(8.5079) (SCONC(11), \(11=2,1\) PT)
WRITE (8,5075) MSG2
WRITE \((8,5080)((B(11, J J), J J=1,2), \operatorname{BGROW}(I I), 11=1\), NPART)
ICOR \(=(\) IORD+1 \()\) MAX
WRITE(8,5075) MSG3
WRITE (8,5100) (COR1 ( 1 ), \(1=1,1 \operatorname{COR}\) )
WRITE (8,5075) MSG4
WRITE 8,5100\()\) (COR2(1), \(1=1,1\) COR)

PORR3850
POR23860 POR23870 POR23870 POR23890 POR23890 POR23900 POR23910 POR23920 POR23930 POR23940 POR23950 POR23960 POR23970 POR23980 POR23990 POR24000 POR2 4010 POR24020 POR24040 POR24040 POR24050 POR24660 POR24070 POR24090 POR24100 POR24100 POR24110 POR24120 POR24130 POR24140 POR24150 POR24170 POR24170 POR24180 POR2 4190 POR24290 POR24210 POR24220 POR24230 POR24240 POR24250 POR24260 POR24270 POR24280 POR24290 POR24300 POR24310 POR24320 POR24330
```

5100 FORMAT {: ,9F8,5)
NOTSAY = .FALSE.
*============== LIST THE CURRENTLY DEFINED PARAMETERS ==============
*============= LIST THE CURRENTLY DEFINED PARAM
GOTO 20
*============== GRAPHICAL DISPLAY OF SIMULATION ======================
700Q WRITE (*,*) 'ENTER DEVICE F:(1=PC,2=VM/TEK, 3=VM/PRT,4=VM/SCREEN)'

- INITIALIZE'TIMER VARIABLES
OGEN = KGEN
TIM = GETIIM (GOTMIN)
CALL DISPLA (NSURF)
GOTO 20
END
SUBROUTINE GENER (A,B,BGROW,DSEED)
SUBROUTINE GENER (A,B,BGROW
C POPULATE A USING NPART PARTICLES AND INITIALIZE
C POPULATE A USING N
COMMON /SIZPRM/ NXSPAC,NYSPAC,MXNPRT, NPART, NNUC, NLAY
COHMON/SIZPRM/ NXSPAC,NYSPAC,MXNPRT,NPARI,NNUC,NLAY
1 MN IY,MXFRE, IAMBLK
1 MNTY,MXFRE,IAMBLK
INTEGER*2 A(NXSPAC,NYSPAC), B(MXNPRT,2)
SET UP THE FIXED PARTICLES
C SET UP THE
NSTUK=0
DO 50 JJ=1, NLAY-1

```
```

    NSTUK=NSTUK+1
        B(NSTUK,1)=11
        (NSTUK, 2)=\J
        BGROW(NSTUK)=1
    A(IINJJ
    IF (NNUC.NE.MXSPAC) THEN
        DD 500 I=NSTUK+1,12
        lx=1N
        If (A(IX,IY).EQ. I2ZERO) THEN
                    A(IX,IY)=120NE
                        B(1,1)=1X
                BGROW(1) = 1
                ELSE
                GOTO 400
            ENDIF
            CONTINUE
            GOTO 600
    ELSE
        DO 550 I=1,NXSPAC
            NSTUK=NSTUK+
            B(NSTUK,1)=1
            B(NSTUK,2)=NLAY
            BGROW(NSTUK)=1
            A(1,NLAY)= (2ONE
    5 5 0
CONTINUE
ENDIf

* NOW JO SET UP THE fREE PARTICLES:
IF (NPART, EQ. IZ) RETURN
DO 1000 i={Z+1,NPARI
800 }\mathcal{X}=(\mathrm{ NXSPAC\#GGUBFS(DSEED)+1.)
IY=(NYSPAC*GGUBFS(DSEED)+1.)
IF (A(IX,IY), EQ.IZZERO) THEN
A(IX,IY)=-120NE
B(1,1)=ix
B(1,2)=1Y
ElSE
BGROW(1) = 0
GOTO 800
ENDIF
CONTINUE
RETURN
END
SUBROUTINE ISHUFL (DSEED,B,BGROW)

```

POR24830
POR24840
POR24850
POR24860
POR24870
POR24880
POR24890
POR24900
POR24910
POR24920
POR24930
POR24940
POR24950
POR24960
POR24970
POR24980
POR24990
POR25000
POR25010
POR25020
POR25030
POR25040
POR25050
POR25060
POR25070
POR25080 POR25090
POR25 100 POR25 100 POR25110 POR25 120 POR25130 POR25140 POR25150 POR25160 POR2S170 POR25180 POR25190 POR25200 POR25210 POR25220 POR25230 POR252H0 POR25250 POR25260 POR25270 POR25280 POR25290 POR25300 POR25310

INTEGER"4 NPART NXSPAC, NYSPAC, MXNPRT, NPART, NNUC, NLAY
COMMON /SIZPRM/ NPAK
COMMON /PARAM/ NPARTO, PROB, PROBR, PROB3, CONC, IZ, MXIY, ICLUST, KGEN,
COMMON / PARAM/ NPAR
MNIY, MXFRE, I
INTEGER*2 B(MXNPRT, 2), I TEMP
INTEGER*2 B(MXNPRT, 2), IT
INTEGER*4 BGROW(MXNPRT)
INTEGEP
RETURN
ENTRY SHUFLE
N=NPART
M=NPART
\(\mathrm{KM1}=\) NPART-1
C
DO \(10 \quad I=1\) NOW RANDOMLY PERMUTE
\(101=1, K M 1\)
\(J=1+G G U B F S(D S E E D) * M\)
1 TEMP \(=B(M, 1)\)
\(B(M, 1)=B(J, T)\)
\(B(J, 1)=1 T E M P)\)
ITEMP \(=B(M, 2)\)
\(\mathrm{B}(\mathrm{M}, 2)=\mathrm{B}(J, 2)\)
B(J, 2) = ITEMP
1JEMP \(=\) BGROW \((M)\)
\(\operatorname{ITEMP}=\operatorname{BGROH}(\mathrm{M})\)
BGROW \((\mathrm{M})=\operatorname{BGROW}(\mathrm{J})\)
BGRON(J) = ITEMP
\(M=M-\)
0 CONTINUE
RETURN
END

SUBROUTINE IUPDAT (A, B, BGROH, DSEED, D, KCOUNT, JCOUNT)
INTEGER*2 I2ZERO, T2ONE, IDIS, KCOUNT, I2FOUR, I2TWO, I2THRE, JCOUNT PARAMETER (I2ZERO=D, IŹONE=1, 12FOUR=4, 12TWO=2, I2THRE=3)
C UPDATES POSITION OF THE PARTICLES; IZ IS A STUCK PARTICLE COUNTER
G MXIY RECORDS THE MAXIMUM HEIGHT OF DENDRITE, MNIY - THE FLOOR LEVEL
COMMON/SIZPRM/ NXSPAC, NYSPAC, MXNPRT, NPART, HNUC, NLAY
COMMON /PARAM/ NPARTO, PROB, PROB2, PROB3, CONC, IZ, MXIY, ICLUST, KGEN,
1 MNIY, MXFRE, IAMBLK
NTEGER"4 NXSPAC, NYSPAC, MXNPRT, NPART, BGROW(MXNPRT)
( NTEGER*2 A(NXSPAC, NYSPAC), B(MXNPRT, 2), D(MXNPRT, 2)
INTEGER*2 ILIST(4), JLIST(4)
REAL"8 DSEED
CALL ISHUFL (DSEED,B,BGROW)
CALL IMOVE (A, B, RGROW, I, IX, IY, DSEED, I IRY)
CALL INCLUS ( \(A, D\) )
CALL INEIGH (A)
RETURN
ENTRY UPDATE

POR25320
POR25320
POR25330 POR25330
POR25340 POR2 5340
POR25350 POR25350 POR25360
POR2 5370 POR25370 POR25380 POR25390 POR25400
POR25410 POR25410 POR25420 POR25430 POR25440
POR25450 POR254460 POR25460 POR25470 POR251480 POR25490 POR25500 POR25510 POR25520 PORR5530 POR25540 POR25550 POR25560 POR25570 POR25580 PORZ5590 POR25600 POR25610 POR25620 POR25630 POR25640 POR25650 POR25670 POR25670 POR25690 POR25690
POR25700 POR25700 POR25710 POR25720 POR25730 POR25740 POR25750 POR25760 POR25770 POR25780 POR2S 790

\title{
CALL SHUFLE \\ * the value of npart changes as particles leave and enter the box DO \(1000 \quad i=1\), MXNPRT
}
* EVEN THOUGH NPART MAY have decreased WE still want to get all the particles checked in this generation
5 If ( \(1 ; \mathrm{GT}\). NPART) RETURN
\(\mathrm{X}=\mathrm{B}(1,1)\)
\(\mathrm{Y}=\mathrm{B}(1,2)\)
C IS THE PARTICLE STUCK??
*\%\% \(5 \% 55558 \% \% 6 \%\) IF SO, BEGIN ATTEMPT TO DISLODGE STUCK PARTICLE
UHLESS IT IS ON THE BOTTOM PLANE
IF (IY.EQ. 1) GOTO 1000
F (A 1 IX, IY).GT. I2ZERO) THEN
CAI SURROUNDED IT STAYS STUCK - AND GOTO
CALL COUNT ( \(1 X, 1 Y\), KCOUNT, JCOUNT, 1)
IF (KCOUNT. EQ. I2FOUR) COTO 100
IF IKCOUTAR DISSOLVE
IF (KCOUNT.EQ. I2ZERO) THEN
TRY TO MOVE, iY \(=-120 \mathrm{NE}\)
* try to move it

GOTO 15
ENDIF
* particle is stuck and on the surface

DOES IT DISSOLVE?
R=GGUBFS (DSEED)
* probz is the dissolution probability
- IF IT DOESNT dISSOLVE, THEN FORGET THIS PARTICLE.
* If (R.GE. PROB2) GOTO 1000 THE PARTICLE DISSOLVES
- NEXT A(IX,IY) \(=-120 \mathrm{NE}\)
* NEXT QUESTION - DID DISSOLVED PARTICLE DISCONNECT A CLUSTER?

C IDIS IS A DISSOLUTION (1)/PRECIPITATION (-1) FLAG
C IF IT IS A TIP OF DENDRITE, JUST TRY TO MOVE IT IF (KCOUNT, EQ. I2ONE) CÓTO 15

POR25810
POR25810
POR25820
OR25830
POR25840
POR25850
POR25860
POR25870
POR25890
POR25900
POR25910
POR25910
POR25920
POR25930
POR25940
POR25950
POR25960
POR25970
POR25980
POR25990
POR26010
POR26020
POR26020
POR26030
POR26040
POR26050
POR26060
POR26070
POR26080
POR26090
POR26100
POR26110
POR26120
POR26130
POR26140
POR26150
POR26160
POR26170
POR26180
POR26190 POR26200 POR26210 POR26220 POR26230 POR26240 POR26250 POR26260 POR26270 PDR26280 POR26290


POR26800
POR26800
POR268 10
POR26820
POR26830
POR26840
POR26850
POR26860
POR26870
POR26880
POR26890
POR26900
POR26910
POR26920
POR26930
POR26940
POR26950
POR26960
POR26970
POR26980
POR26990
POR27000
POR27010
POR27020
POR27030 POR27010 POR27050 POR27060 POR27080 POR27080 POR27090 POR27100 POR27110 POR27120 POR27130 POR27140 POR27150 POR27160 POR27170 POR27180 POR27190 POR27200 POR27210 POR27220 POR27230 POR27240
\(101 R=(4 * G G U B F S(D S E E D)+1\).
GOTO \((1000,4000,2000,3000)\) IDIR POR27250 1000 ERIODIC BOUNDARY CONDITIONS ON X AXIS \(1000 \quad 1 Q=1 X+1\)

IF (IQ.GT.NXSPAC) \(10=1\) POR2726 POR27270 POR27280
```

C EMPTY THE PREVIOUS POSIGN

```

POR27290 POR27300 POR27310 POR27320 POR27330 POR27340 POR27350 POR27360 POR27370 POR27380 POR27390 POR27400 POR27410 POR27420 POR27440 POR27440 POR27450 POR27460 POR27470 POR2 7480 POR27490 POR27500 POR27510 POR27520 POR27530 POR27540 POR27550 POR27560 POR27570 POR27580 POR27590 POR27600 POR27610 POR27620 POR27630 POR27640 POR27650 POR27660 POR27670 POR27680 POR27690 POR27700 POR27710 POR27720 POR27730 POR27740 POR27750 POR27760 POR27770
\begin{tabular}{|c|c|c|c|}
\hline \multicolumn{4}{|l|}{\[
4000 \text { IF (IY-1.GE.1) THEN }
\]} \\
\hline & A(ix, \(\mid \gamma-1\) ) & -I20NE & \\
\hline \multicolumn{4}{|l|}{C Empty the previous position} \\
\hline & \(A(1 X, \mid Y)=1\) & 2ZERO & \\
\hline \multicolumn{4}{|l|}{C CHANGE COORD OF THE I PARTICLE} \\
\hline \multicolumn{4}{|l|}{\multirow[t]{2}{*}{}} \\
\hline & & & \\
\hline \multicolumn{4}{|c|}{ENDIF} \\
\hline \multicolumn{4}{|c|}{\multirow[t]{2}{*}{ENDIF}} \\
\hline & & & \\
\hline \multicolumn{4}{|c|}{END} \\
\hline
\end{tabular}

POR27780 POR27790 POR27790
POR27800 POR27800
POR27810 POR27810
POR27820 POR27830 POR27840 POR27840 POR27850
POR27860 POR27860
POR27870 POR27870
POR27880 POR27880
POR27890 POR2 7890
POR2 7900 POR27900
POR2 7910 POR2 7910
POR27920 POR27920
POR27930 POR27930
POR27940 POR27940 POR27950 POR27960 POR27970 POR27980 POR27990 POR28000 POR28010 POR28020 SPOR28030 POR28040 POR28050 POR28060 POR28010 POR28080 POR28090 POR28100 POR28 110 POR28120 POR28 130 POR28 140 POR28150 POR26160 POR28170 POR28180 POR28190 POR28200 POR28210 POR28220 POR28230 POR28240 POR28250
POR28260
```

                A(IQ,J)=12FIVE
                    JCLUST=JCLUST+1
                    D(JCLUST, 1)=19
            D(JCCLUST,2)=J ( MAX(1Q, IXMAX)
            IXMAAX = MAX(IG, IXM
                :XMAX=NXSPAC
                | XMAX=NX
            ENDIF
            ENDIF
            HAND SIDE
            1Q=1-1
    IF (1Q.EQ.0) IQ=NXSPAC
        F (A(1Q,J).EQ. 1DIS) THEN
            A(IQ,J)=12FIVE
                JCLUST=JCLUST+1
                DCLUST=JCLUST+
            D(JCLUST, 1)=10
            |XMIN = MIN(1O, IXMIN
            IXMIN = MIN(IQ, IXMIN)
                IXMAX=NXSPAC
                    | XHAX=NX
            IXHI
    ENDIF
    c
Y AXIS (J.EQ.NYSPAC) GOTO 75
IF (J.EQ.NYSPAC) GOTO 75
IF (A(I.\&Q).EQ.IDIS) THEN
A(I, IQ )=12FIVE
JCLUST=JCLUST+
D{JCL.UST, 1)=1
D(JCLUST,2)=1Q
ENDIF.ea.1) coto 90

```

```

    IF (A(I, IQ).EQ.IDIS) THEN
                A(1,1Q).EQ.IDIS
                JCLUST=JCLUST+1
                D(JCLUST, 1)=1
                D(JCLUST,'2)=1Q
    ENDIF
    C NOW EMPTY THE CHECKED SPOT, WITHOUT EMPTYING A

```

POR28270
POR28280 POR28280 POR28290
POR28 300 POR28300
POR28310 POR283 10
POR28320 POR28320 POR28330 POR28340
POR28350 POR28350 POR28360 POR28370 POR28380 POR28390 POR28400 POR28410 POR28420 POR28430 POR28440 POR28450 POR28460 POR28470 POR284180 POR28490 POR28500 POR28510 POR28520 POR28530 POR28540 POR28550 POR2B560 POR28570 POR28580 POR28590 POR28600 POR28610 POR26620 PUR28630 POR28640 POR28650 POR28660 POR28670
POR28680 POR28680
POR28690 POR28690
POR28700 POR28710 POR28710 POR28720
POR28730 POR28730 POR28740
POR28750 POR28750
POR2 760 POR28760
```

                    IF (IYMIN LT IAMBLK AND IDIS EQ I2ONE) GOTO 240
                    HFCK WHETHER NEW PARTICLES STUCK TO THE CLUSTER
    IF (IFOUND.NE.0) GOTO }5
    * 240 WE ARE TRYING TO DISSOLVE - IS CLUSTER CONNECTED TO BULK?
WE AR (IYMIN IT IAMBLK) THEN
* YUP - PART OF BULLK - LEAVE THEM STUCK
DO 500 |l=1 JCLUST
A(D(11,1),D(11,2))=120NE
ELSE
C if Not CONNEctED tO THE buLK, dissOLVE IT
DO 550 | |=1,JCLUST
DO 550 1I=1,JCLUST
5 5 0
ENDIF
ELSE
* WE ARE TRYING TO STICK CLUSTER - IS IT BIG ENOUGH?
IF (JCLUST.LT, ICLUST) THEN
C UN-STICK THE ALREADY STUCK PARTICLES INCLUDING THE INITIAL ONE
C STORED
THE ALREADY STUCK
DO 250 1=1,JGLUST
CONTINUE
ELSE
DO 255 I=1,JCLUST

```

```

    255
        ENDIF
        ENDIF
    RETURN
    END
        SUBROUTINE IDISP (A,AS,B,BGROW, DEVICE, REV,OLDGEN,ZMANO, CONUC
        1,COR1,COR2, IORO,NSURF1, IDNAM,ZMAN,DCONC, SCONC, IPTO,MAXR)
        INTEGER*2 I2ZERO, I2ONE, I2THRE, I2FOUR, IDIS, I2EIGT
    NTEGER*2 I2ZERO, 12ONE, I2THRE,12FOUR, IDIS,I2EIGT 
    PARAME /ER (SIZPRM/ NXSPAC,NYSPAC,MXNPRT,NPART,NNUC,NLAY
    COMMON /PARAM/ NPARTO, PROB, PROB2, PROB3,CONC, IZ,MXIY, ICLUST, KGEN,
    1 MNIY,MXFRE, (AMBLK
    REALW4 DCONG(10000), SCONC(10000), COR1 (100000), COR2(100000)
    REAL*8 ZMAN,GETTIM,GOTMIN,ZMANO,ZMANOW,DELTAI
    INTEGERW4 NXSPAC,NYSPAC, BGROW(MXNPRT)
    INTEGER*2 A(NXSPAC,NYSPAC),B(MXNPRT,2), (CNTR!-1:1), KCOUNT, JCOUNT
    INTEGER*Z AS(NXSPAC,NYSPAC)
    NTEGER*2 MODE, IPALET, ICOLOR(3), ICLR, II,JJ, IZERO, IFIVH
    INTEGER*4 NPART,MXGEN, KGEN
    POR28770 C NOW CHECK WHETHER NEW PARTICLES STUCK TO THE CLUSTER POR28790 POR28790 POR28800 POR28810 POR28820 POR28830 POR28840 POR28850 POR28860 POR28870 POR28880 POR28890 POR28900 POR28910 POR28920 POR28930 POR28940 POR28950 POR28960 POR28970 POR28980 POR28990 POR29000 POR29010 POR29020 POR29040 POR29040 POR29060 POR29060 POR29070 POR29080 POR29090 POR29100 POR29110 POR29120 POR29130 POR29140 POR29150 POR29160 PORR29170 POR29 180 POR2919 POR29200 POR29210 POR29220 POR29230 POR29240 POR29250

```
```

    INTEGER*2 ITWO, DEVICE, IONE,MAXR
    CHARACTER*1 GRAP(-1:5)
    * IDNAM
    CHARACTER*130 CPLOT, CBLANK
    DATA CBLANK
    1
    ```


```

    DATA MODE, I PALET, ICOLOR /1,0, 1,0,2/' 
    CALL ICORE (AS,MAKR,NSURF1,CORI,COR2, IORD
    RETURN
        ENTRY DISPLA (NSURF)
            DO 10 I=1, NXSPAC
            DO 10 J=1, NYSPAC
    10
    FIMD AVERAGE EXECUTION TIME FOR SINGLE GENERATION
    LONG SIMULATION RUN FROM BEGINNING TO NOW?
            ZMANOW = GETTIM(GOTMIN)
                IF (ZMANOW.LT.ZMANO) THEM
                    WRITE (*,*) 'ACCOUNTING BOUNDARY CROSSED1'
                    ZMANO = ZMANON
            ENDIF
            DELTAT = ZMANOW - ZMANO
        ZMAN = ZMAN + DELTAT
    IN MINUTES
EZMAN = ZMAN / 60.DO
EZMAN = ZMAN / 60.DO
SPEFO = 0.0
If (DELTAG.NE.O.) SPEED = DELTAT / DELTAG
OLDGEN = KGEN

* UPDATE THE MAXIMUM HEIGHT COUNTER
DO 400 J=NYSPAC, 1,-1
DO 400 I=1,NXSPAC (I,N).EQ. I2ONE) GOTO 410
400 CONIINUE
410 MONIINUE
MXXIY = UPDATE MNIY COUNTER
DO 500 J= 1, NYSPAC
ICNT = 0
DO 490 I = 1,NXSPAC
ICNT = ICNT + A(I,J)
490 CONTICNT
CONTINUE (IF.NXSPAC) GOTO 510
500 CONTENUE
500 CONT INUE
510 MNIY =
C NSURF IS THE OF INTERFACES EXPOSED BY THE BULK

POR29260
POR29270
PR29280
OR29290
POR29300
OR29310
POR29320
OR29330
POR29340
POR29350
POR29360
POR29370
OR29380
POR29390
POR29400
POR29410
POR291120
POR29430
POR29440
POR29450
POR29460
POR29470
POR29480
POR29490
POR29500
POR29510
POR29520
POR29530
POR29540
POR29550
POR29560
POR29570
POR29580
POR29590
POR29600
POR29610
POR29620
POR29630
POR29640
POR29650
POR29660
POR29670
POR29680
POR29680
POR29690
POR29700
POR29710
POR29720
POR29730 POR29740 POR29750
AS(IX,IY) = 12FOUR-KCOUNT+12ONE POR29920
ENDIF
300 CONT INUE
C CALCULATE THE CORRELATION FUNCTION POR29960

* NOW OISPLAY CORE POR29970
* NOW DISPLAY -
* COTO (1000,2000,3000,4000) DEVICE POR29990
* DEVICE O - JUST RETURN'4000) DEVICE
*_----- PETURN VERSION:
.1000 CONTINUE
* CALL CLS,*),' ', KGEN,NPARIO,NPART
* CALLTCLS,*),' ', KGEN,NPARIO,NPART


# DO 1115 KK=1,NPART (BIKK,1),B(KK,2))+2)

* DO 1115 KK=1,NPART
** DO 1115 KK=1,NPART ( ICOLOR(A(BIKK,1),B(KK,2))+2)
CALL PSET
RETURN
*---N---- MAINfrAME VERSION - TEK SCREEN
2000 1ONE =1
ICLR=1
1 THO = =2
1ZERO = 0
IFIVH = 500
CALL INIT (IZERO,TWO)
11=1
jJ= = * (NYSPAC + 5}
CALL PLOTT (11,JJ,IZERO)
11 = 500
CALL PLOT (II,JJ,ITWO)
POR29760

```
```

```
NSURF = O
```

```
NSURF = O
C NSURF1 IS THE OF SURFACE POINTS
C NSURF1 IS THE OF SURFACE POINTS
NSURF1 = O = N00 NPART
NSURF1 = O = N00 NPART
        IX = B(1,1
        IX = B(1,1
        IY = B(I,2) ,GT. I2ZERO.AND. IY.GE.MNIY-2) THEN
        IY = B(I,2) ,GT. I2ZERO.AND. IY.GE.MNIY-2) THEN
        CALL, COUNT (iX, IY, KCOUNT, JCOUNT, 2)
        CALL, COUNT (iX, IY, KCOUNT, JCOUNT, 2)
c the true 'roughaESS' IS THE fof INTERfacES Stuck/rest or the world
c the true 'roughaESS' IS THE fof INTERfacES Stuck/rest or the world
C (FREE PART OR EMPTY SPACE)
C (FREE PART OR EMPTY SPACE)
    KJCNT = NSURF=NSURF+(4-KCOUNT
    KJCNT = NSURF=NSURF+(4-KCOUNT
    KJCNT = KCOUNT + JCOUNT
    KJCNT = KCOUNT + JCOUNT
    IF (KJCNT.LT.I2EIGT. AHD. KJCNT.GT. I2ZERO) THEN
    IF (KJCNT.LT.I2EIGT. AHD. KJCNT.GT. I2ZERO) THEN
                NSURF1=NSURF 1+1
                NSURF1=NSURF 1+1
    C FILLING THE 'SURFACE MAP" AS(I,J) FOR 4-COORDINATED SURTACE PARTICLES POR29900
    C FILLING THE 'SURFACE MAP" AS(I,J) FOR 4-COORDINATED SURTACE PARTICLES POR29900
```

    POR29770
    OR29780
    DO 300 I = 1,NPART
    DO 300 I = 1,NPART
    POR29790
        IX = B(1,1)
        IX = B(1,1)
    POR29800
    OR29810
    POR29820
    POR29830
    POR29840
    POR29850
    POR29860
    POR29870
    OR29880
    OR29890
    POR29900
    MENGS(IX,IY) = I2FOUR-KCOUNT+I2ONE
MENGS(IX,IY) = I2FOUR-KCOUNT+I2ONE
MENGS(IX,IY) = I2FOUR-KCOUNT+I2ONE
POR29950
M POR301110
POR29950
POR29960
POR29970
OR29990
POR30!110
POR30020
POR30030
POR30040
POR30050
POR30060
POR30070
POR30080
POR30090
POR30100
*/115
POR30110
POR30120
POR30130
POR30140
I RAK vERSION G TEK SCREEN
POR30140
POR30150
POR30160
POR30170
POR30170
POR30180
POR30190
POR30200
POR30210
POR30220
POR30230
POR30240
POR30250

```

```

        ASSIGN 4007 TO IFORM
    ENDIF
    4009 I = IPTO+1, IPT
    4009 I = IPTO+1, IPT
    C IF IPOS IS TOO BIG, IPOS+2 IS LARGER THAN IEND AND NEEDS ADJUSTMENT
OS IS TOO BIG,i IPOS+2 IS LARGER
CPLOT = CBLANK(1: |POS) // "*"// CBLANK( (POS+2:IEND)
IPOS = SCONC(1)*(iEND-1) +
IF (IPOS.EQ. I END) IPOS=IEND-2 +1, (I)
CPLOT = CPLOT(1:IPOS) // '+'/// CPLOT(IPOS+2: IEND)
WRITE ( IO, IFORM) CPLOT
4007 FORMAT (1X,A80)
4008 FORMAT
4010 FORMAT (' :;50A1)

```

```

4105 FORMAT (';',AB,', TMME:',F8.2,'MIN.',' SPEED=',F8.3,'SEC/GEN', POR309000
1'VERSION'',F5:3,'GENH',17)
4120 FORMAT ('','LAT. SZE', IF,' X', '15,' HLYRS', 15,' NPRTO',17, POR30930
"'CLST SZE',14,''PRB (PP,DIS,MOVE)',3F5.2 POR30940
4140 FORMAT ,FS.3, CONUC,FS.3)
4140 FORMAT (' AVG.HT=',F7.3,' MAXHT=', 15,' MNHT=', 15,' PORO=',F7.3, POR30960
1' ROUGH=',F9.3) POM, POM30970
4150 FORMAT ("TOTAL \#', 18,' STUCK=',18,'/F FREE=',18) POR30980
RETURN PO, POR30990
END
SUBROUTINE ICOUNT (A)
INTEGER*2 I2ZERO, IZONE, IDIS, KCOUNT, ICNTR(-1: 1 }, JCOUNT, I2TWO
PARAMETER (I2ZERO=0, IZONE=1, 12THO=2)
COMMON SITPRM/ NOROP
COMHON /SIZPRM/ NXSPAC, NYSPAC, MXNPRT,NPART,NNUC,NLAY NOLST KGEN POR31050
COMMON/PARAM/ NPARTO, PROB, PROB2, PROB3, CONC,IZ,MX!Y, ICLUST, KGEN,
1 MNIY,MXFRE,IAMBLK
INTEGER*4 NXSPAC, NYSPAC,MXNPRT, NPART POR31070
NNTEGER,2 NOST,
RETURN
C
ENTRY COUNT (IX, IY, KCOUNT, JCOUNT, JUHP)
KCOUNT= I2ZERO
JCOUNT=12ZERO
|CNTR(1)=12ZERO
\R(1)={2ZERO = I2ZERO
POR3 1000
POR31010
POR31020
POR31030
POR31040
POR31060
POR3107%
POR31080
POR31690
POR31100
POR31110
POR3}112
POR31130
POR31140
POR31150
POR31160
POR31170
c LOOK TO THE RIGHT
POR31180
lQ=1X+1
POR31190
POR31190
IF (IX.EQ.NXSPAC) IQ=1
POR31200
IF (IX.EQ.NXSPAC) IQ=1
POR31210
C
TO THE LEFT

IF (IQ.EQ.O) IQ=NXSPAC

```
    C LOOK UP
    C THE SPACE IS CONSTDERED VOID OF 'SOLUTE' OUTSIDE THE BOX
    IF (IY.EQ. NYSPAC) COTO 100
```


C LOOK 100 DOWN
$10=1 \mathrm{Y}-1$
C THE BOX IS INFINITELY FULL UNDER THE BOTTOM
IF (IG.EQ. O) JHEN 1 CNTR 1 ) (CNTR(1) + I20NE
KCOUNT = $=\operatorname{ICNTR}(1)$
KCOUNT
GOTO
150
ENDIF
$\operatorname{ENDIF}(A)|X| Q),)=\operatorname{ICNTR}(A(|X| Q))+,120 N E$
$\operatorname{ICNTR}(A)$
$\operatorname{KCOUNT}=\operatorname{ICNTR}(1)$
I
150 IF (JUMP.EQ.1) RETUR
$c$ LOOKING ON THE DIAGONAL
C LOOKING ON THE DIAGONT
C ICNTR SHOULD BE RESET
ICNTR(1)=12ZERO
ICNJR(-1) $=$ 12ZERO
ICNTR(0) $=$ I2ZERO
C MORTH-EAST
$10 x=1 x+1$

IF (IX. EQ. NXSPAC) $\operatorname{IQX=1}$
ICNTR (A(IQX, IQY))=ICNTR(A(IQX, IQY)) $+120 N E$
C NORTH-WEST
$1 Q x=1 X-1$
IF (1QX.EQ.0) IQX=NXSPAC
IF (IQX.EQ.O) IQX=NXSPAC
ICNTR (Al IQX, IQY) $)=I C N T R(A(I Q X, I Q Y))+120 N E$
C SOUTH-EAST
$200 \quad \mid Q X=1 X+1$
$10 x=1 x+1$
$10 Y=19-1$
IF (IQY.EQ.0) GOTO 300
IF (1X EO NXSPAC) $1 Q X=1$

C SOUTH-HEST
IGX=1 $x-1$
$1 \mathrm{QY}=1 \mathrm{Y}-1$
IF (IX.EQ.1) IQX=NXSPAC
ICNTR (AQ (IQX, IQY))=ICNTR(A(IQX, IQY)) + I2ONE
CCNTR (A1 IQX, IQY) ) $=1$ CNTR
GOTO 350
GOTO 350
$\operatorname{ICNTR}(1)=\operatorname{ICNTR}(1)+12$ TWO
$\begin{array}{ll}300 & \text { ICNTR } 1 \text { 1)=ICNTR } \\ 350 & \text { JCOUNT=ICNTR(1) } \\ 400 & \text { RETURN }\end{array}$
RETURN

POR31240
POR31240
POR31250
POR31250
POR31260
POR31270
POR31280
POR31280
POR31290
POR31300
POR31310
POR3 1320
POR31330
POR3 1340
POR31350
POR3 1360
POR31370
POR3 1380
POR3 1390
POR31400
POR31410
POR31420
POR31430
POR31440
POR31450
POR311160
POR31470
POR3 1180
POR3 1490
POR3 1490
POR31500
POR3 1510
POR3 1520
POR31520
POR31530
POR31540
POR31550
POR31560
POR3 1570
POR31580
POR3 1590
POR3 1610
POR31610
POR31630
POR31630
POR31650
POR3 1650
POR3 1660
POR31670
POR3 1680
POR31690
POR31700
POR31710
POR31720
SUBROUTINE INEIGH (A)

## INTEGER*2 I2ZERO, IZONE, IDIS, ILIST(4), JLIST(4)

POR3 1760
POR 31770
POR31770
POR31780
PARAMETER ( $122 E R O=0$, $120 N E=1$
POR3 1780
POR31790
POR31800
COMMON/PARAM/ NPARTO, PROB, PROB2, PROB3, CONC, IZ, MXIY, ICLUST, KGEN, 1 MNIY, MXFRE, IAMBLK

POR31810
POR3 1820
$\begin{array}{ll}\text { IMTEGER"2 A(NXSPAC, NYSPAC) } & \text { POR31830 } \\ \text { ETURN } & \\ \text { POR31840 }\end{array}$
C
ENTRY NEIGH (IX, IY, ILIST,JLIST)
POR31850
POR31860
RETURNS THE COORDINATES OF THE FOUR NEIGHBORS IF THEY EXIST
POR3 1870
POR31880
1- TOP 2-RIGHT 3-BOTTOM 4-LEFT
POR31890
C LOOK TO THE RIGHT
POR31900
I $Q=1 X+1$
IF ( $1 \times$. EQ. NXSPAC) $1 Q=1$
POR31910
POR31920
POR31930
POR3 1940
POR31950
POR3 1960
POR31970
POR3 1980
C LOOK TO THE LEFT
$10=1 x-1$
If (1Q.EQ.0) IQ=NXSPAC
POR3 1990
POR32000
POR32000
POR32010
POR32020
POR32030
POR32040
POR32050
POR32060
POR32070
POR32080
POR32090
POR32 100
POR32110
POR32 120
POR32130
POR3214
POR32 150
POR32160
POR32170
POR32180
POR32 190
POR32200
POR 32210
POR32220
POR32230



```
            IF (MFRET) = 1-FLOAT(NSTIK) / FLOAT(NXSPAC)
            IF (NFRE+NAIR.EQ. O) THEN POR32750
            ELSE DCONC(IPT) = 0.001
                ENDIF DCONG(IPT)= FLOAT(NFRE) / FLOAT(NFRE + NAIR)
            ENDIF
            CONTINUE 
            DCONC(IPTO) = IPT - IPTO
            SCONC(1) = IPT
            SCONC(IPTO) = IPT = IPTO
            RETURM
            END
            SUBROUTINE ICORE (AS,MAKR,NSURF1,COR1,COR2, IORD)
            COMMON /SIZPRM// NXSPAC,NYSPAC,MXNPRT',NPART,NNUC,NLAY
            COMHON /SIZPRM/ NPSPAC,NYSPAC,MXNPRT,NPARI,NNUC,NLAY (CLUST, KGEN,
    COMMON /PARAM/ NPAR
    REAL*4 COR1(100000), COR2(100000)
    INTEGER*2 AS(NXSPAC;NYSPAC),MAKR, I2ZERO
    PARAMETER (I2ZERO=0.
    INTEGER#4 MPART,MXGEN, KGEN, ICALC1, ICALC2
    RETURN
    EmTRY CORE
C CHOOSE A PARTICLE
    OO 200 |R=1, MAXR
        ICRR=|ARD+iR
            |CALC]= 0
            CALC1= 0
            DO 100 I=1,NXSPAC
            00 100 J=1'NYSPAC
            IF=(AS(I,J).EQ.I2ZERO) GOTO :OO
C X -NEIGHBORS
            10=1+1f
            IF (1Q.GT.NXSPAC) 1Q=1Q-NXSPAC+1
            F (AS(IQ,J).NE.I2ZERO) THEM
                    ICALC'= 'CCALCCl+1
                    |CALC2=1CALC2+AS(1Q,J )-1
                    EmDIF
C left
    IQ = M - IR (IG. LE.O) IQ=NXSPAC+IQ-1
    POR32760
POR32760
POR32770
POR32780
POR32790
    100
POR32800
POR32800
POR32810
POR32820
POR32830
POR32840
POR32850
POR32860
POR32860
POR32880
POR32890
POR32990
POR32910
POR32920
POR32930
POR32940
POR32950
POR32950
POR32960
POR32970
POR32980
POR32990
POR33000
POR33010
POR33020
POR33030
POR33030
POR33040
POR33060
POR33070
POR33080
POR33090
POR33090
POR33100
POR33110
POR33120
POR33130
POR33140
POR33i40
POR33150
POR33160
OR33170
POR33170
POR33180
POR33190
POR33200
POR33210
```

ICALCT=| CALC $1+1$
ICALC2= ICALC2+AS(1Q,J)-1

## ENDIF

C Y Y -NEIGHBORS
c UP
$19=J+1 R$
IF (IQ.GT. NYSPAC) $10=10$-NYSPAC+1
F (AS(I, iQ).NE. I2ZERO) THEN
ICALĆI=1CALC $1+1$
ICALC2 $=\operatorname{ICALC} 2+A S(1,19)-1$
ENDIF

if (ASil,iq).NE. IZZERO) THEN
ICALĆI=1CALC1+1
$\left.\begin{array}{l}\text { ICALC1 }=1 \text { CALC1+1 } \\ \text { |CALC2 } \\ \text { CALC2+AS }\end{array} 1,19\right)-1$ ENDIF
C ${ }^{100}$ AVERAGE CORTINUE
C AVERAGE COR1(R) OVER 4 NEIGHBOR OIRECTIONS
C AVERAGE COR2(R) DVER B HEIGHBOR DIRECTIONS
200 continue CONTINU
RETUR
END

POR33220
POR33230 POR33240 POR33250 POR33260 POR33270 POR33280 POR33290 POR33300 POR33310 POR33320 POR33330 POR33340 POR33350 POR33360 POR33370
POR33380
POR33390
POR33400
POR33410
POR33420
POR3 3430
POR33440
POR33450
POR33460
POR33470

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