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Ilhan Özgen
Franz Simons
Jiaheng Zhao
Reinhard Hinkelmann

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MODELING SHALLOW WATER FLOW AND TRANSPORT PROCESSES WITH SMALL WATER DEPTHS USING THE HYDROINFORMATICS MODELLING SYSTEM

ILHAN ÖZGEN, FRANZ SIMONS, JIAHENG ZHAO, REINHARD HINKELMANN
Chair of Water Resources Management and Modeling of Hydrosystems, Technische Universität Berlin, Gustav-Meyer Allee 25, 13353 Berlin, Germany

In hydro- and environmental systems modelling, there are several application cases where very small water depths occur, for example rainfall and runoff in natural or urban catchments, possibly associated with tracer transport. In these cases, the water depth may be in the range of millimeters to a few centimeters. The numerical simulation of the associated processes is complex, therefore robust numerical schemes are required. Two test cases using high resolution topography data are investigated with the Hydroinformatics Modelling System (HMS). In the first case, the influence of microtopography and local depressions were analyzed in an idealized urban catchment; both had a strong impact on the hydrograph. In the second one, rainfall runoff experiments, which were carried out by Mügler et al. [10] were simulated. Through parameter optimization an overall good agreement between computed and measured breakthrough curves was achieved.

INTRODUCTION

The Hydroinformatics Modelling System (HMS) is a Java-based, object-oriented software framework that has been developed at the Chair of Water Resources Management and Modeling of Hydrosystems, Technische Universität Berlin (Busse et al. [2], Simons et al. [11,12]). HMS follows the idea of a component-based system that allows the integrative coupling of different processes. The generalised design enables the implementation of single and multiple processes in different spatial and temporal resolutions, as well as the interactions with geospatial information data bases. Currently, the implemented processes in HMS are shallow water flow including rainfall and infiltration, tracer and sediment transport. The overall goal is to provide a computational framework for developing and applying different conceptual approaches for various shallow water flows and transport processes ranging from classical river hydraulics, over flood protection to new approaches in catchment hydrology, urban water management and to environmental fluid mechanics.

Small water depths (in the range of millimeters to a few centimeters) are associated with a number of difficulties such as the occurrence of wetting and drying of cells, flow transitions and high gradients which may cause instabilities in the simulation. Furthermore, these processes often occur above complex topography with much local detail. Therefore, sophisticated
numerical methods have to be applied to ensure the robustness of the model. In this context, Godunov type schemes have been found to behave very robust (Hou et al. [4,5], Liang [8]).

In this contribution, the Hydroinformatics Modelling System (HMS) is used to simulate rainfall runoff in a small urban and natural catchment on high resolution grids to study the implemented numerical scheme for two application cases with small water depths.

GOVERNING EQUATIONS

The general form of two dimensional conservation laws can be expressed as:

$$\frac{\partial q}{\partial t} + \frac{\partial f}{\partial x} + \frac{\partial g}{\partial y} = s$$ (1)

where $q$ is the vector of conserved flow variables and $t$ is the time; $f$ denotes the flux vector and $\nabla = (\partial/\partial x, \partial/\partial y)^T$, where $x$ and $y$ are the Cartesian coordinates. The vector $s$ represents the source terms.

The general form of conservation laws can be used to describe a series of different physical processes by choosing suitable expressions for the vectors $q$, $f$, $g$ and $s$. For conservation laws, the resulting system of partial differential equations is usually hyperbolic and therefore, the same numerical methods can be applied to model different processes as long as they can be expressed by conservation laws. HMS exploits this fact to obtain a flexible framework that provides the possibility to easily implement new processes and to test different numerical methods.

Shallow water flow

Using the general conservation law (Eq. 1) the depth-averaged incompressible shallow water equations are written with the vectors:

$$q = \begin{bmatrix} h \\
uh \\
vh \end{bmatrix}, \quad f = \begin{bmatrix} uh + \frac{1}{2} gh^2 - v\nabla(uh) \\
uh \\
vh \end{bmatrix},$$

$$g = \begin{bmatrix} vh \\
vuh \\
vvh + \frac{1}{2} gh^2 - v\nabla(vh) \end{bmatrix}, \quad s = \begin{bmatrix} r_w \\
-\tau_B/\rho - gh\partial z_B/\partial x \\
-\tau_B/\rho - gh\partial z_B/\partial y \end{bmatrix}$$ (2)

Herein, $h$ is the water depth, $z_B$ the bottom elevation above datum and $u$ and $v$ are the velocity vector components in $x$ and $y$ direction, respectively. $r_w$ is a mass source/sink term, the gravity constant is denoted with $g$, the kinematic viscosity with $\nu$ and the density of water with $\rho$.

The bed friction term can be written as

$$\tau_B/\rho = g/C^2|v|v$$ (3)

where $C$ is the Chézy coefficient which can be expressed as $C = h^{\frac{1}{6}} n$ in Manning’s law. Here, $n$ is the Manning coefficient.
Shallow water transport

Using the general conservation law Eq. (1) the conservation of a conservative tracer can be expressed as:

\[
\begin{align*}
q &= \begin{bmatrix} ch \end{bmatrix}, \\
f &= \begin{bmatrix} uch - D \nabla (ch) \end{bmatrix}, \\
g &= \begin{bmatrix} vch - D \nabla (ch) \end{bmatrix}, \\
s &= \begin{bmatrix} r \end{bmatrix}.
\end{align*}
\]

Herein, \( c \) is the dimensionless tracer concentration, \( D \) is the diffusion coefficient and \( r \), a tracer concentration source / sink term.

NUMERICAL METHODS

In HMS the general form of the conservation laws (Eq. 1) is discretized with a cell-centered Finite–Volume Method in space. If an explicit forward Euler method is chosen for time discretization, the conserved variables \( q \) in Eq. (1) can be written as follows:

\[
q^{n+1} = q^n - \Delta t \sum_k \frac{F^n_k \cdot n_k}{A} l_k + \Delta ts^n
\]

Herein, \( n+1 \) and \( n \) denote the new and the old time level, respectively. \( F_k \) stands for the flux vector over edge \( k \) and is calculated as follows:

\[
F_k n_k = fn_x + gn_y
\]

The time step is expressed as \( \Delta t \) and the area of the considered cell is \( A \); \( k \) is the index of a face of the considered cell; \( n_k \) is the normal vector pointing outward of a face with the components \( n_x \) and \( n_y \) in \( x \) and \( y \) direction, respectively and \( l_k \) is the length of a face. The time step is constrained by the Courant–Friedrichs–Lewy (CFL) criterion to ensure stability.

The implementation of the Finite–Volume solver in HMS is independent of the mesh type, so it is possible to use either structured or unstructured meshes.

For solving Eq. (5) a cell-centered second order MUSCL scheme of Hou et al. [4,5] is used. The flux in the shallow water equations is computed with the Harten, Lax, van Leer approximate Riemann solver with the contact wave restored (HLLC). Riemann solvers are able to handle discontinuities in the flow field like they appear in hydraulic jumps or at wet–dry fronts and they are therefore very suitable for simulating flow with small water depths. The state variables at cell interfaces are reconstructed with the hydrostatic reconstruction method which ensures non–negative water depths. A total variation diminishing scheme developed by Hou et al. [6,7] is applied to avoid spurious oscillations. In combination with the slope source treatment by Hou et al. [4], these numerical methods have been proven to preserve the C–property, which indicates that the model is well-balanced.

Wet–dry fronts are often a source for numerical instabilities (Hou et al. [5]). To ensure robustness at wet–dry fronts, numerical models often introduce a certain water depth threshold. If the threshold is undershot, the cell is defined dry and is taken out of the calculation. Additionally, at wet–dry fronts and high gradients, our scheme switches from second order to first order accuracy (Hou et al. [5], Murillo et al. [9]).
APPLICATIONS

In this section we present two numerical rainfall runoff simulations which were carried out with HMS. The first one considers shallow water flow only, while in the second one, shallow water flow and transport occur. In the following cases, turbulence has been neglected as it is expected to be of minor importance.

Simplified urban catchment

In the following test case the influence of microtopography and depressions on urban runoff is investigated. An inclined plane (6×3 m, slope 0.05) is considered and the microtopography is described with a sine wave. The catchment geometry is sketched in Figure 1. The amplitude \( A \) of the microtopography has been varied between 0.001 to 0.003 m and the wave length was set to \( \lambda = 0.2 \) m. The Manning coefficient for all simulations was chosen as \( n = 0.016 \), which corresponds to the friction coefficient of asphalt. A constant rainfall of \( 2.7 \times 10^{-5} \) mmh\(^{-1}\) with a duration of 61.25 s, which corresponds to the concentration time of the domain, was imposed. A reference solution is computed with a quadratic grid with an element size of 0.01 m. The CFL criterion was set to 0.1 and the drying criterion for cells was set to \( 10^{-6} \) m for all simulations.

Figure 2 shows results measured at the outlet of the domain for water depth \( h \) and discharge \( Q \) plotted over time for varying amplitudes of microtopography (cf. Figure 1). An analytical solution for the case without microtopography is given by Gottardi & Venutelli [3] and is plotted in Figure 2 too. The numerical solution for the domain without microtopography approximates the analytical solution with a very good agreement. A small amplitude \( (A=0.001 \) m) only has negligible influence on the runoff. The sensitivity of the system to increasing the amplitude of the microtopography can be seen in the large differences between the curves. As the amplitude of the microtopography increases to \( A = 0.002, 0.003 \) m, the peaks for both discharge and water depth strongly decrease. Further, the microtopography can act as retention space and cause diffusion in the hydrograph (cf. Figure 2) as water is captured in local depressions. For smaller rain intensities it was observed that no runoff occurred and the water was completely stored in these depressions. Therefore, local depressions are investigated further.

\[
f(y, A, \lambda) = A \sin \left( \frac{2\pi y}{\lambda} \right)
\]
In the following, the influence of the spatial distribution of local depressions on the hydrograph is analyzed. The same microtopography (sine wave with $\lambda=0.2$ m, $A=0.01$ m) as in the previous case was chosen, however only in varying parts of the domain (cf. Figure 3, left); outside of subdomains 1, 2, and 3, no microtopography was generated. The location of the square is varied in the domain and results are compared in Figure 3. It can be seen that the location of the microtopography and hence the local depression storage effects the hydrograph at the outlet of the domain. Especially, varying the position in slope direction was found to have a big impact on the hydrograph. The reason is that when rainfall is imposed, the amount of water flowing through the microtopography solely depends on the position in slope direction. Therefore, a microtopography closer to the outlet is able to store more water. Variation of the position perpendicular to the slope direction was found to be less significant. The changes in the hydrograph here result mainly from disturbances of the flow field due to microtopography which lead to a more two dimensional flow field. This means, that the length of the flow paths change and therefore water particles need more time to travel inside the domain. We suspect also a strong correlation between inundation ratio (ratio of the mean water depth to the amplitude of the depression) and microtopography on the hydrograph. This will be investigated in the future.

**Natural catchment**

The following test case shows a rainfall simulation with conservative tracer transport in a natural catchment in Thies, Senegal. The topography of the domain is shown in Figure 4. The domain has a length of 10 m and a width of 4 m with an average slope of 1 %. In the year 2004, the catchment was subject to a series of experiments regarding surface runoff and tracer...
transport. A detailed discussion of the experiments is found in Mügler et al. [10], where also measurement data and numerical results are provided. Numerical studies of the runoff on the site have also been carried out by Simons et al. [12] who modeled stationary and instationary runoff experiments. Later, Adamczak [1] conducted a sensitivity analysis and optimization for tracer transport and flow parameters. The domain consists of 4141 quadratic grid cells with an element length of 0.1 m.

A constant rainfall of 48.5 mm/h is applied over the whole simulation time of \( t=600 \) s. An alternative friction formulation, called variable Manning law (cf. Eq. 3), was chosen here as it was also chosen by Mügler et al. [10] and Simons et al. [12]. The variable Manning law is as follows:

\[
C = h^{1/6} n_0 \left( \frac{h}{h_0} \right)^c
\]

Here, \( n_0 \) is the minimum bottom roughness coefficient, \( h_0 \) is the minimum water depth and \( c \) is the vegetation drag coefficient.

After 300 s a tracer is injected with a constant rate of 1 g/s at different injection points in the domain for 30 s (cf. Figure 4). Bottom friction stress is described with a variable Manning approach. The CFL criterion was set to 0.4 and the drying threshold for cells was set to \( 10^{-7} \) m for all simulations. The parameters used for the optimization are the parameters of the variable Manning law and the diffusion coefficient \( D \).

Results of the optimization using a least square method are shown in Figure 5. The optimized parameters are determined as \( n_0=0.01685 \) s/m\(^{1/3} \), \( h_0=1.643 \) mm, \( c=0.3423 \) and \( D=2.982\times10^{-9} \) m\(^2\)/s. Figure 5, top, shows the tracer concentration fields for different injection cases. Note that the images are rotated. In Figure 5, bottom, measured and computed breakthrough curves are plotted. The overall agreement of the curves is good. Especially for point E and point I very good agreement between measurements and simulation results has been achieved. Point H shows the worst agreement. The optimized results show a better agreement with the measured breakthrough curves than those of Mügler et al. [10].

![Figure 4. Domain topography and position of tracer injection points](image)
CONCLUSIONS

Small water depths occur in various water and environmental related fields. They require robust numerical schemes which were implemented in the HMS framework and applied to two test cases with high resolution topography data. In the first one, the influence of microtopography and local depressions was analyzed in an idealized urban catchment; both had a strong impact on the hydrograph. In the second one, rainfall runoff experiments, which were carried out by Mügler et al. [10] were simulated. Through parameter optimization an overall good agreement between computed and measured breakthrough curves was achieved.

A comprehensive parameter study including, for example, different inundation ratios and scaling approaches will be carried out for the idealized urban catchment.

Figure 5. Tracer concentration fields for different injection cases (top), comparison of computed (blue) and measured (red) breakthrough curves after Adamczak [1] (bottom)
REFERENCES


