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DEVELOPMENT AND APPLICATION OF A SHALLOW WATER FLOW MODEL HDM-2D

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A shallow water flow model, HDM-2D (HydroDynamic Model-2D) was developed, and this study focuses on the description of the model and practical applications in terms of general aspect, apart from the specific functions to reproduce the particular physical phenomena. The HDM-2D uses the Petrov-Galerkin stabilizing test function of which the shape is deformed by the current direction to introduce balancing diffusion only in the flow direction. The resulting linear set of equations was solved by frontal method with a fully implicit time-discretized method. Three applications were considered to test the applicability of HDM-2D. In the first problem, the accuracy was checked by comparison with exact solution. In the second and third applications, the subcritical or super-critical flow regimes, evolved in non-uniform channels with expanding or converging side wall, were reproduced.

1. INTRODUCTION

Most open-channel flows can be regarded as shallow water problem, because the effect of vertical motions is usually insignificant and depth-averaged equations are generally accepted for analyzing the open channel flows with reasonable accuracy and efficiency. Accordingly, simulation of shallow water systems can serve numerous purposes: (1) understanding hydraulic, biological and other processes; (2) predicting impacts of development works and natural events to minimize the damage; and (3) environmental management. In addition, this shallow water hydrodynamic model can be coupled to a transport model in considering flow and transport phenomenon thus making it possible to study remediation options for polluted streams and estuaries to predict the impact of commercial projects on the environment and ecosystem and to study allocation of allowable discharges by municipalities and by industries in meeting water quality controls [1-2].

In this study, a shallow water flow model, HDM-2D (HydroDynamic Model-2D) was developed. The HDM-2D features (1) the incorporation of secondary current effect by dispersion stresses [3]; (2) the adjustment of the internal wall velocity by Navier-slip condition
(4); (3) the imposition of skewed inflow velocity profiles by beta function [5]; (4) the reproduction of convection-dominated or supercritical flow by SU/PG scheme [6]; (5) the provision of eddy viscosity by constant, parabolic, and Smagorinsky turbulence models; (6) the inclusion of bottom shear stress by either quadratic Manning’s law or bed friction factor; and (7) the representation of wetting and drying by flux blocking method, which can be extended into inundation analysis model. In particular, the first, second, and third features in the above statements are unique and the originality of the proposed model lies in them since these properties are unprecedented in commercial hydrodynamic models. This study focuses on the description of the model and practical applications in terms of general aspect.

2. DESCRIPTION OF HDM-2D

2.1 Governing equations

The present mathematical model of a free surface flow consists of the 2D shallow water equations, which are obtained by averaging momentum and mass balance equations along the vertical direction under barotropic and hydrostatic assumptions. The set of variables \( g(x,y,t) = (u_1(x,y,t), u_2(x,y,t)) \) and \( h(x,y,t) \) is a solution of the following continuity and momentum equations:

\[
\frac{\partial h}{\partial t} + h \nabla \cdot u + u \cdot (\nabla h) = 0 \tag{1}
\]

\[
\frac{\partial u}{\partial t} + (u \cdot \nabla) u = -g \nabla (H + h) + \frac{1}{h} \nabla \cdot (h |\nabla u|) - \frac{g n^2}{h^{4/3}} \|u\| \tag{2}
\]

where \( t = \) time; \( u = (u_1, u_2) \) is vertically averaged velocity vector in \( x \), \( y \) -directions, respectively; \( g = \) acceleration of gravity; \( H = \) bottom elevation; \( h = \) flow depth; \( n = \) kinematic viscosity; \( n = \) Manning’s roughness coefficient; and \( \|u\| = \) Euclid norm of velocity.

2.2 Discretization and numerical schemes

To construct a numerical model, and the continuity equation (3a) was discretized by the Galerkin method while the momentum equation (3b) was by the Petrov-Galerkin stabilizing test function of which the shape is deformed by the current direction to introduce balancing diffusion only in the flow direction [7-8]. After multiplying shallow water equations by the weighting function \( w \) and integrating over the flow domain \( \Omega \), we applied Green’s theorem to the viscous stress terms in the momentum equation to arrive at weak formulation

\[
\int_\Omega w \left[ \frac{\partial h}{\partial t} + h \nabla \cdot u + u \cdot (\nabla h) \right] d\Omega = 0 \tag{3a}
\]
where \( p_i = \frac{\bar{h}_e}{\alpha} (u \cdot \nabla w_i) \) is the perturbation function with \( \bar{h} \) = element characteristic length; 
\( \alpha = \coth\left( \gamma / 2 \right) - 2 / \gamma \) is the quadrature points; and \( \gamma = \|u\| / \nu \) is the element Reynolds number. The nonlinearity of the discretized momentum equations was linearized by Newton-Raphson method. Triangular or rectangular element with \( C_0 \) interpolation function can be mixed together in the construction of geometry. The linear set of equations was solved by frontal method with a fully implicit time-discretized method.

### 2.3 GUI interfaces

To increase the applicability and user convenience, the HDM-2D was coupled to a 2D advection-dispersion model, CTM-2D to consider the flow and the transport phenomenon. The flow and transport models were combined into a software suite RAMS (River Analysis and Modeling System) to interface the solvers with the pre- and post-processor. Therefore, RAMS supports the mesh generation, the model control, the result view, and the exportation to SMS geometry or Tecplot data files. All the numerical results reported herein were produced by Tecplot 360.

### 3. MODEL APPLICATIONS

The HDM-2D model has been applied in several flow problems, which were documented in the previously published papers [3-6]. Those include the incorporation of secondary current effect by dispersion stresses [3], the adjustment of the internal wall velocity by Navier-slip condition [4], the imposition of skewed inflow velocity profiles by beta function [5], and the reproduction of convection-dominated or supercritical flow by SU/PG scheme [6]. Accordingly, present study mainly focused on another general aspect of model accuracy and performance. Following three applications were considered to test the applicability of HDM-2D. In the first problem, the accuracy was checked by comparison with exact solution. In the second and third applications, the subcritical or supercritical flow regimes, evolved in non-uniform channels with expanding or converging side wall, were reproduced.

#### 3.1 Accuracy check

The accuracy of HDM-2D was examined by comparing the numerical results with analytical solutions in the steady flow problem over a Gaussian bump. Since the steady solver included in HDM-2D was used, only the spatial accuracy was checked excluding the temporal accuracy. The channel length is 30 m long, and bottom elevation varied by Gaussian function so that it has maximum value \( H=0.2 \) at \( x=15 \) m as shown in Figure 1.
\[ H(x) = \frac{1}{2\sqrt{2\pi}} e^{-\left(\frac{x^2}{2}\right)} \]  

(4)

Figure 1. A Gaussian bump channel for the analysis of spatial accuracy

The upstream boundary condition is assigned by discharge per unit width, \( q = 4.42 \, \text{m}^2/\text{s} \), and the downstream one is fixed as constant depth, \( h = 2 \, \text{m} \). Three different mesh layouts were generated with the numbers of longitudinal element being varied by 32, 128, and 512. The exact solution can be obtained by applying the Bernoulli equation incorporating with constant discharge to provide continuity. The \( L_1 \) and \( L_2 \) norms of the depth error were computed by the following formula to find the spatial accuracy of HDM-2D.

\[
L_1 = \sum_i |h_i^{\text{exact}} - h_i^{\text{numerical}}| \Delta x
\]

(5a)

\[
L_2 = \sqrt{\sum_i (h_i^{\text{exact}} - h_i^{\text{numerical}})^2} \Delta x
\]

(5b)

Table 1 shows the accuracy result. The second-order accuracy was achieved in the average sense. Although HDM-2D does not have higher accuracy such as fourth or fifth order, it has another purpose of applicability for the reproduction of physical phenomena [3-4] and for general uses [5-6].

Table 1. Accuracy of HDM-2D

<table>
<thead>
<tr>
<th>Elements</th>
<th>( \text{ln(Elements)} )</th>
<th>( \text{L1} )</th>
<th>( \text{ln(L1)} )</th>
<th>( \text{Order} )</th>
<th>( \text{L2} )</th>
<th>( \text{ln(L2)} )</th>
<th>( \text{Order} )</th>
</tr>
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<td>32</td>
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<td>-4.6520</td>
<td></td>
<td></td>
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<tr>
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<td>-9.3711</td>
<td>1.7020</td>
</tr>
</tbody>
</table>

3.2 Recirculating flow with side-wall expansion

The flow problem passing through a flume with side wall expansion was considered to assess the performance of the developed model under subcritical flow environment with non-uniform
geometry. The upstream boundary condition was assigned as total discharge rate of 0.01815 m³/s to impose a mean flow velocity of 0.30 m/s, and the downstream one was constant water depth of \( h = 0.101 \) m. The dimension of the channel was 6.5 m in length and 1.2 m in width with mild bed slope, \( S = 0.0001 \) as shown in Figure 2. The element layout used in the numerical simulation was displayed in Figure 2. A total of 3,900 rectangular elements with 4,066 nodes were produced to capture the recirculating behavior after the wall expansion.

![Finite element layout for the simulation of recirculating flow with a side-wall expansion](image)

Figure 2. Finite element layout for the simulation of recirculating flow with a side-wall expansion

The flow patterns including streamlines and velocity contours are shown in Figure 3. Xie’s experimental measurements \([9]\) reported a recirculation zone length of 4.6 m, and present model showed the recirculation length equal to 4.6 m according to the Figure 3. Consequently, they are in a good agreement and HDM-2D gave satisfactory result for the prediction of recirculating flow with a side-wall expansion.

![Velocity field and recirculation zone](image)

Figure 3. Velocity field and recirculation zone

### 3.3 Super-critical flow in a converging channel

Supercritical flow presents a formidable problem to the design engineer because of the presence of standing waves which assume particular importance in high-velocity flow. High velocity channels arise in man-made flood control structures that are required to convey the flow at supercritical velocities along a specified reach. Predicting the possible locations of oblique
standing waves and hydraulic jumps and determining the elevations of the water surface is necessary to design the required wall heights to avoid overtopping [10].

HDM-2D employing the Petrov-Galerkin stabilizing scheme was applied to a flow problem in a contraction channel for a supercritical condition as shown in Figure 4. This problem involves multiple shock reflection. The geometry includes two identical, but oppositely-positioned circular arcs having a radius of 1.905 m. To impose the super-critical inflow boundary condition, the flow velocity of 2.17 m/s and the water depth of 0.03 m were assigned at the upstream boundary, which results in the approach Froude number being 4.0. The outflow boundary does not require any conditions due to the super-critical exit property.

![Figure 4. Geometry and dimension of the converging channel](image)

Figure 4. Geometry and dimension of the converging channel

Figure 5 shows the 3D depth formation with cross waves in the channel. Two oblique standing shocks were formed at the two sidewalls in the contraction part of the channel. The curved oblique shocks crossed in the middle of the channel, and then impinged on the opposite sidewalls, propagating with a series of reflections downstream of the contraction exit. Each reflection led to a rise in water depth, although the largest single increase in water depth occurs within the first reflection.

![Figure 5. Depth formation of supercritical flow passing converging channel](image)

Figure 5. Depth formation of supercritical flow passing converging channel

4. CONCLUSION
In this study, a shallow water flow model, HDM-2D (HydroDynamic Model-2D) was developed. The description of the model and practical applications in terms of general aspect were highlighted in this article. The model used the non-conservative shallow water equations as the governing equations, and Petrov-Galerkin stabilizing scheme was employed to discretize the equations. The resulting linear set of equations was solved by frontal method with a fully implicit time-discretized method.

Three applications were considered to test the applicability of HDM-2D. In the first problem, the accuracy was checked by comparison with exact solution. In the second and third applications, the subcritical or super-critical flow regimes, evolved in non-uniform channels with expanding or converging side wall, were closely reproduced. Applications of HDM-2D have shown that the model gives accurate prediction of the flow characteristics in a variety of free surface problems and expected to be served as a useful tool for practical design. To increase the applicability and user convenience, the HDM-2D is being coupled to a 2D advection-dispersion model, CTM-2D to consider the flow and the transport phenomenon.

In the near future, the flow and transport models will be combined into a commercial software suite RAMS (River Analysis and Modeling System) to interface the solvers with the pre- and post-processor. The RAMS is expected to provide the linkage of water resources management with computational modeling method in the study field of hydroinformatics.

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